Reconstructing $t\bar{t}$ events with one lost jet

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Abstract

We present a technique for reconstructing the kinematics of pair-produced top quarks that decay to a charged lepton, a neutrino and four final state quarks in the subset of events where only three jets are reconstructed. We present a figure of merit that allows for a fair comparison of reconstruction algorithms without requiring their calibration. The new reconstruction of events with only three jets is fully competitive with the full reconstruction typically used for four-jet events.

Keywords: top, reconstruction, partial

1. Introduction

Several problems in top quark physics require a full reconstruction of the kinematics of the top quark–antiquark pair. For example, to measure the forward-backward (or charge) asymmetry in $t\bar{t}$ production, it is essential to know the direction of both the top quark and the antiquark.

We consider $t\bar{t}$ events where each top quark decays into a b quark and a W boson, and where one W boson decays hadronically ($W \rightarrow q'\bar{q}$) and one W

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8 boson decays leptonically ($W \rightarrow l\nu$). We classify top quarks as “leptonic”
 9 or “hadronic”, based on the mode of the W -boson decay. The final state
 10 contains a lepton, a neutrino and four quarks that subsequently shower and
 11 hadronize into jets. This channel is commonly referred to as “ l +jets”.

12 The four final state quarks do not always yield four reconstructed jets,
 13 which is the case, for example, when one of the quarks is too soft or when the
 14 angular separation between two of them is small. Though the signal purity
 15 is lower in the sample of events with exactly three jets than in the sample
 16 with at least four jets, it is still useful for measuring top properties [1, 2],
 17 effectively increasing the sample size by $\approx 55\%$. Furthermore, extending the
 18 event selection of a top property measurement in the l +jets channel to include
 19 three-jet events can reduce the acceptance bias [1] and reduces systematic
 20 uncertainties related to jet reconstruction, as events with one unreconstructed
 21 jet are still used. The three-jet sample can also be interesting in its own right.
 22 For example, at the Large Hadron Collider (LHC) $t\bar{t}$ production of events
 23 with at least four jets is dominated by initial states that contain gluons, while
 24 the three-jet sample is enriched in $t\bar{t}$ pairs produced from $q\bar{q}$ initial states.

25 When one of the jets from top decay is lost, it is not possible to fully
 26 reconstruct the $t\bar{t}$ decay chain, which has so far limited the use of lepton
 27 plus three jets (l +3jets) events to measurements of observables such as the
 28 production cross section [3] and the rapidity¹ of the lepton [1].

¹ The rapidity y is defined as $y(\theta, \beta) = \frac{1}{2} \ln [(1 + \beta \cos \theta) / (1 - \beta \cos \theta)]$, where θ is the polar angle and β is the ratio of a particle’s momentum to its energy. The pseudorapidity η is defined as $-\ln \tan \frac{\theta}{2}$. In this paper, pseudorapidities are defined relative to the center of the detector, while rapidities and all other angles are defined with respect to the primary

29 The kinematics of the $t\bar{t}$ events with a lepton and at least four jets
30 ($l+\geq 4\text{jets}$) is fully reconstructed by matching four of the jets to the four
31 final state quarks from $t\bar{t}$ decay (for example, see [4]). As for events with at
32 least four jets, the main challenge in fully reconstructing three-jet events is
33 to disentangle the two top-quark decay chains. That is, the main challenge is
34 to match the observed jets with the quarks from $t\bar{t}$ decay, though with only
35 three jets available a perfect 1-to-1 correspondence is impossible and partial
36 matchings are used instead.

37 In this paper we present a method to infer the direction and kinematics
38 of the top quark and antiquark in $l+\text{jets}$ events where only three jets are
39 reconstructed, and demonstrate the application of the method to simulated
40 $t\bar{t}$ events. We focus on $p\bar{p} \rightarrow t\bar{t}$ production at a center of mass energy of
41 1.96 TeV, as in the Tevatron. About half of the $t\bar{t} l+\text{jets}$ events produced at
42 the Tevatron contain only three jets.

43 The main steps of the method have been described in Ref. [2], where
44 it is used to reconstruct the directions of the top quark and antiquark and
45 the invariant mass of the $t\bar{t}$ system. This paper provides the details of the
46 method and quantifies its performance. We discuss the selection of the events
47 in Section 2. In Section 3 we detail the method to partially reconstruct the
48 $t\bar{t}$ pair using the invariant mass of various combinations of jets and jet life-
49 time observables [5]. We compare the performance of different reconstruction
50 algorithms in Section 4, for which we introduce a new figure of merit (FOM).

51 The reconstruction of $t\bar{t}$ events at the LHC poses different problems. The

collision point.

52 typical jet-selection threshold for the LHC is transverse momentum $p_T >$
 53 30 GeV , and only in $40(2)\%$ ² of the $t\bar{t}$ events do all four jets associated with
 54 $t\bar{t}$ -decay quarks pass this threshold. Yet, due to the initial state radiation
 55 (ISR) only a small fraction of $t\bar{t}$ events produced at the LHC end up in
 56 the three-jet sample. For the LHC $l+3\text{jets}$ events, a method similar to
 57 that presented here may suffice. Roughly 40% of the LHC $l+\geq 4\text{jets}$ events
 58 contain only three jets associated with $t\bar{t}$ decay quarks, with the other jets
 59 due to ISR. An extension of the algorithm discussed in this paper could be
 60 used to partially reconstruct these events and thus increase the number of
 61 reconstructed events by approximately a factor of two.

62 2. Samples and selection

63 The results shown in this paper are based on simulated $p\bar{p} \rightarrow t\bar{t}$ events
 64 with a collision center-of-mass energy of $\sqrt{s} = 1.96 \text{ TeV}$. The events were
 65 simulated with the MC@NLO event generator [6] and processed through a
 66 detector simulation and object reconstruction that largely correspond to but
 67 are not identical to that of the D0 experiment. In particular, some of the
 68 quality selection criteria are not applied since they are not relevant for the
 69 development of the method.

70 Simulated energy deposits in the calorimetry are clustered into jets using
 71 the “Run II Midpoint cone algorithm” [7] with a cone radius of 0.5 in the
 72 y - ϕ plane, where ϕ is the azimuthal angle and y is the rapidity. We select

²Here and later the presented fractions are representative, but are given only as example
 as they depend on the detector, the event selection, the jet algorithm, etc. The uncertainty
 on the last significant figure is given in parenthesis.

73 jets with $p_T > 20$ GeV and with pseudorapidity $|\eta| < 2.5$.

74 We select leptons from electron and muon candidates with $p_T > 20$ GeV
75 and with $|\eta_e| < 1.1$ or $|\eta_\mu| < 2.0$. We then select events with exactly one
76 lepton and exactly three jets. We require that the transverse momentum
77 imbalance measured by the calorimetry, $\cancel{E}_T$, is greater than 20 GeV. We
78 reject events where the $\cancel{E}_T$ is closely aligned with the lepton and events with
79 $\cancel{E}_T > 500$ GeV. These two cuts suppress multijet background and events with
80 misreconstructed $\cancel{E}_T$, respectively.

81 Generally, the signal purity is lower in the sample of events with exactly
82 three jets than in the sample with at least four jets. However additional selec-
83 tion criteria, e.g. identification of jets associated with b -quarks (b -tagging),
84 can improve the situation, making the $l+3$ jets sample useful for measuring
85 top properties. In particular, in [1, 2] it was shown that purity of $l+3$ jets
86 sample with two b -tags is similar to that of $l+\geq 4$ jets with one b -tag.

87 We further categorize the selected events by how well the reconstructed
88 jets match the quarks from $t\bar{t}$ decay, as that affects the quality of recon-
89 struction. We consider a jet to be matched to a quark when their angular
90 separation $\Delta R = \sqrt{(\Delta y)^2 + (\Delta \phi)^2}$ is less than 0.5. We classify an event as
91 “matchable” if all $t\bar{t}$ decay products assumed to be present by the recon-
92 struction algorithm were matched to reconstructed objects.

93 For the reconstruction of $l+\geq 4$ jets events at the Tevatron [4], a matchable
94 event is the one in which the four jets of highest p_T match the four final
95 state quarks from $t\bar{t}$ decay. Only 55(1)% of the $l+\geq 4$ jets events at the
96 Tevatron are matchable. In the context of this paper a $l+3$ jets event is
97 considered matchable if one jet matches the b quark from of the leptonic top

98 quark decay and the two other jets match two of the three quarks from the
 99 decay of the hadronically decaying top quark. 20(1)% of the $l+3$ jets events
 100 are classified as unmatchable because the b jet from the leptonic top decay,
 101 which is essential to the described algorithm, is lost. In 4.0(2)% of the events
 102 two jets were lost, while an extra one was gained from initial or final state
 103 radiation. Thus, 76(1)% of the $l+3$ jets events are considered matchable.

104 **3. Reconstructing $t\bar{t}$ in $l+3$ jets events**

105 For almost half of the simulated $p\bar{p} \rightarrow t\bar{t}$ pairs that decay in the l +jets
 106 channel, only three jets are reconstructed. In our study scenario, two quarks
 107 yield a single jet due to an accidental overlap in $\approx 18\%$ of these $l+3$ jets
 108 events. One of the quarks is too forward (high $|\eta|$) to yield a selected jet in
 109 $\approx 8\%$ of the events. In the remaining $\approx 74\%$ of the events, either one of the
 110 quarks was too soft (low p_T) to yield a selected jet or a jet was lost due to
 111 reconstruction and identification inefficiencies.

112 In Fig. 1 we show a schematic of a possible $t\bar{t}$ decay process. Instead
 113 of trying to infer the kinematics of the missing or merged jet in a $l+3$ jets
 114 event, we partially reconstruct the $t\bar{t}$ system by neglecting this jet altogether.
 115 Though there is some experimental sensitivity to the presence of two quarks
 116 in a single jet, e.g., through the jet width and mass, we found it too weak to
 117 be useful. Thus we do not attempt to “unmerge” any of the jets and assign
 118 two quarks to it. Events in the $l+\geq 4$ jets channel are often reconstructed
 119 using a “kinematic fit” algorithm, which modifies the measured momenta to
 120 satisfy the known resonance masses (e.g. Ref. [4]). Given that we neglect the
 121 missing jet, such refinements are of little use for $l+3$ jets events. Thus we

Figure 1: Cartoon depicting an example of $t\bar{t}$ decay: the leptonic top quark (t_l) decays to b -jet j_3 and to a W boson which decays to a lepton and a neutrino; the hadronic top quark (t_h) decays to b -jet j_1 and to a W boson which decays hadronically to jets j_2 and j_4 . In the depicted $l+3$ jets event, j_4 is lost.

122 employ a simpler approach to partially reconstruct the $t\bar{t}$ system in $l+3$ jets
 123 events.

124 3.1. Reconstructing the leptonic W boson

125 We start by reconstructing the leptonically decaying W boson using the
 126 lepton momentum and the $\cancel{E}_T$. The neutrino momentum in the plane trans-
 127 verse to the beam direction, $\vec{q}_T$, is initially set equal to the $\cancel{E}_T$. The longitu-
 128 dinal component of neutrino momentum, q_z , is calculated using a constraint
 129 on the W -boson mass, M_W . The resultant quadratic equation can have two
 130 solutions, which creates a two-fold ambiguity. Both solutions are considered.

131 Following Ref. [8], when the discriminant of the quadratic equation for
 132 q_z is negative, we scale $\vec{q}_T$ to satisfy the M_W constraint with a discriminant
 133 equal to zero. This results in another quadratic equation which yields two
 134 solutions for the scale, at least one of which is positive. When both solutions

135 are positive, we use the one that is closer to unity.

136 3.2. Reconstructing the top-quark candidates

137 The next step is to form leptonic and hadronic top quark candidates. To
 138 do so, we assume that the lost jet is from the decay of the hadronic top quark.
 139 One of the jets is combined with the leptonic W boson to form a leptonic top
 140 candidate. The two remaining jets are combined to form a “proxy” for the
 141 hadronic top quark, which serves instead of a fully reconstructed candidate.
 142 The assignment is completely defined by the choice of leptonic b jet. If the
 143 previous step yielded two q_z solutions, for each assignment we choose the
 144 solution where the combination of the leptonic b jet, the lepton and the
 145 neutrino yields an invariant mass closer to the nominal top quark mass [9].

146 3.2.1. χ^2 method

147 Invariant mass distributions on both the leptonic and hadronic sides have
 148 characteristic shapes as shown in Fig. 2. Both can be used to find the best
 149 jet assignment. The distributions were made using an adaptive kernel esti-
 150 mator [10].

A simple way to choose an assignment is to use a χ^2 test statistic for the
 masses reconstructed for the leptonic top candidate (m_t) and for the proxy
 (m_p):

$$\chi^2 = \left(\frac{m_t - m_t^0}{\sigma_t} \right)^2 + \left(\frac{m_p - m_p^0}{\sigma_p} \right)^2, \quad (1)$$

151 where m_t^0 (m_p^0) and σ_t (σ_p) are the mean and width of the Gaussian fits for
 152 leptonic (proxy) masses shown in Fig. 2. This approach picks the correct
 153 assignment in 66.0(1)% of the cases where such an assignment exists. Below
 154 we discuss more detailed treatments that improve upon this basic technique.

Figure 2: Invariant mass distribution of (a) lepton, neutrino and b jet from leptonically decaying top quark, and (b) invariant mass of two remaining jets from the hadronic top decay. In both cases, fits to Gaussian distributions are shown by the dashed curves.

155 3.2.2. Complete likelihood method

156 We improve the choice of the assignment by replacing the χ^2 with a
 157 likelihood function. The likelihood formalism allows us to take into account
 158 additional information. The use of the invariant masses of the incorrect
 159 assignments, which too have distinct shapes, is detailed below. The use of
 160 “ b -tagging” observables that attempt to identify jets likely to arise from a b
 161 quark is detailed further on.

162 Figure 3 shows the distributions in top candidate mass on the leptonic
 163 side for three situations: when the leptonic W boson is (correctly) combined
 164 with the b jet from leptonic top decay ($P_{t:l}$), when it is (wrongly) combined
 165 with the hadronic b jet ($P_{t:h}$), and when it is (wrongly) combined with a
 166 jet from hadronic W -boson decay ($P_{t:q}$). Using the distinct shape of a pre-
 167 sumably “incorrect” assignment means we need to keep track of two types
 168 of assignments which may disagree. We will introduce notation for the as-
 169 signment used to combine the jets into the mass observables and for the

assignment hypothesized to be correct.

Figure 3: Distributions of the mass of the leptonic top candidate, which comprises the lepton, neutrino and leptonic b -jet candidate. The distribution is shown for events where the jet assigned to the leptonic b quark is the correct one ($P_{t:l}$, solid curve), the hadronic b jet ($P_{t:h}$, dot-dashed curve), or a jet from hadronic W -boson decay ($P_{t;q}$, dashed curve).

170

171 Depending on which jet is lost and which jet is picked to form the lep-
 172 tonic top candidate there are four possible two-jet combinations for the proxy
 173 side. The probability distributions for the invariant mass on the proxy side
 174 are shown in Fig. 4 for hadronic and leptonic b jets ($P_{p:hl}$), leptonic b jet
 175 and a jet from W -boson decay ($P_{p:lq}$), hadronic b jet and a jet from W -boson
 176 decay ($P_{p:hq}$), and both jets from W -boson decay ($P_{p:qq}$). The first two com-
 177 binations are incorrect, as they include the leptonic b jet. The last two
 178 combinations are correct, and under the assumption that the leptonic b jet
 179 was reconstructed, they cannot both be available in the same event.

These shapes can be used to maximize the probability P of selecting the correct assignment a given the data d , which according to Bayes' theorem is:

$$P(a | d) = \frac{P(d | a) P(a)}{\sum_b P(d | b) P(b)} = \frac{P(d | a)}{\sum_b P(d | b)}, \quad (2)$$

Figure 4: Distributions of the mass of the proxy for the hadronic top quark, which comprises two jets. The distribution is shown for events where the jets assigned to the proxy are the hadronic and leptonic b jets ($P_{p:hl}$, solid curve), the leptonic b jet and a jet from W -boson decay ($P_{p:lq}$, dot-dashed curve), the hadronic b jet and a jet from W -boson decay ($P_{p:hq}$, long dashes), or both jets from the W -boson decay ($P_{p:qq}$, short dashes). In the last case, the W resonance is clearly seen.

180 where b is any assignment and the second equality uses the fact that a priori
181 all assignments are equally probable.

There are three possible jet assignments per event ($i = 1, 2, 3$), corresponding to the choice of the candidate for the leptonic b jet. Each event is characterized by three possible masses on the leptonic side (t_1, t_2, t_3) and three possible masses on the proxy side (p_1, p_2, p_3). In addition to this kinematic information, b -tagging algorithms [5] can also help to identify the origins of the jets. The results of the b -tagging algorithms can usually be expressed as a single continuous variable per jet, which discriminates between light and b -flavored jets. We label the b -tagging discriminant for the i -th jet

as b_i . Thus, data are presented by nine variables:

$$d = (t_1, t_2, t_3; p_1, p_2, p_3; b_1, b_2, b_3) \quad (3)$$

In matchable events the lost jet is either the hadronic b jet or a jet from hadronic W -boson decay. We label the former as $Q = b_l q q$ and the latter $H = b_l b_h q$. For a matchable event, the probability for assignment a is a weighted sum of the probabilities of H and Q types:

$$P(d | a) = (1 - f_Q)P(d | a, H) + f_Q P(d | a, Q), \quad (4)$$

182 where f_Q is the fraction of matchable events that are type Q , which in our
183 study scenario is 20.5(2)%.

Each jet assignment hypothesis specifies the type of each jet: either a b jet, or a jet from hadronic W -boson decay. The latter category includes jets that arise from c quarks, and are somewhat similar to b jets [5]. The correlations between the b -tagging discriminants (b_j) are small. Furthermore, these correlations are mostly independent of the true jet flavors, hence they are irrelevant for our purposes. Thus, the b -tagging probabilities can be factorized:

$$P(d | a, C) = P(t_1, t_2, t_3; p_1, p_2, p_3 | a, C) P(b_1, b_2, b_3 | a, C) \quad (5)$$

$$= P(t_1, t_2, t_3; p_1, p_2, p_3 | a, C) \prod_{j=1}^3 P(b_j | a, C) \quad (6)$$

where $C = H$ or Q is the hypothesized class of the event. By neglecting the correlations between the remaining variables we can factorize the first two terms into six of the one-dimensional distributions shown in Figs. 3 and 4

($P_{t:y}$ and $P_{p:y}$):

$$P(d | a, C) = \prod_{j=1}^3 P_{t:f(j,a,C)} \prod_{j=1}^3 P_{p:g(j,a,C)} \prod_{j=1}^3 P(b_j | a, C) \quad (7)$$

where $f(j, a, C) \in \{l, h, q\}$ gives the type of the j -th jet (i.e., the jet assumed to be the leptonic b jet when building the t_j observable) according to assignment a and event class C , and $g(j, a, C) \in \{hq, lq, hl, qq\}$ gives the types of the non- j -th jets (i.e., the jets combined to form the proxy for the p_j observable) according to a and C . Though we neglected some of the correlations between the observables in Eq. 7, the structure of the likelihood preserves the dominant correlations, such as having at most one W -boson resonance, and the correlation between the presence of a W -boson resonance and the b -tagging variables. Using the described algorithm, the correct jet assignment is chosen for 69.1(2)% of the matchable events, which is to be compared to 66.0(1)% of correct assignments using a simple χ^2 method discussed in Section 3.2.1.

Returning to the example of Fig. 1, the following terms help identify the correct event class (H) and assignment ($a = 3$, i.e. j_3 is the leptonic b jet):

- the invariant mass formed by combining the leptonic W candidate (W_l) and the jet j_1 , $t_1 = m(W_l + j_1)$, should be consistent with the $P_{t:h}$ distribution from Fig. 3;
- $t_2 = m(W_l + j_2)$ should be consistent with $P_{t:q}$ (same figure);
- $t_3 = m(W_l + j_3)$ should be consistent with $P_{t:l}$ (same figure);
- the invariant mass formed by the jets j_2 and j_3 , $p_1 = m(j_2 + j_3)$, should be consistent with the $P_{p:lq}$ distribution from Fig. 4;

- 205 • $p_2 = m(j_1 + j_3)$, invariant mass of leptonic b jet and a light jet should
206 be consistent with $P_{p:hl}$ (same figure);
- 207 • $p_3 = m(j_1 + j_2)$, invariant mass of leptonic and hadronic b jets should
208 be consistent with $P_{p:hq}$ (same figure);
- 209 • b_1 , the b -tagging discriminant of j_1 , should be consistent with the dis-
210 tribution for a b jet;
- 211 • b_2 should be consistent with the distribution for a jet from hadronic
212 W -boson decay;
- 213 • b_3 should be consistent with the distribution for a b jet.

The inclusion of the rarer Q events in the likelihood can distort the reconstruction of the more common case, the H events. But this risk is mitigated when the likelihood contains enough information to distinguish between the two cases on an event-by-event basis. To demonstrate that, we calculate the a posteriori probability that a matchable event is of type Q as:

$$P_Q = \frac{f_Q P(d | a, Q)}{(1 - f_Q) P(d | a, H) + f_Q P(d | a, Q)} \quad (8)$$

214 As Fig. 5 demonstrates the separation between the two cases is quite good.
215 This separation is mostly due to the b -tagging discriminants. It is also useful
216 to check the modeling of P_Q against collider data, as all the terms in $P(d | a)$
217 also appear in P_Q .

218 3.2.3. *Scaling the proxy*

219 Given a specific jet to quark assignment we have a candidate for the
220 leptonic top t with the energy E_t , momentum $\vec{P}_t$ and invariant mass $m_t =$

Figure 5: Distribution in the a posteriori probability for a Q -type event, shown for H - and Q -type events.

221 $\sqrt{E_t^2 - \vec{P}_t^2}$ and a proxy p for the hadronic top with the energy E_p , momen-
 222 tum $\vec{P}_p$ and invariant mass $m_p = \sqrt{E_p^2 - \vec{P}_p^2}$. Since the proxy tends to
 223 underestimate the 4-vector of the hadronic top quark, the invariant mass of
 224 these two objects, $m(t+p)$, is likely to underestimate the generated invariant
 225 mass of the $t\bar{t}$ system, $m_{t\bar{t}}^{\text{gen}}$, as shown in Fig. 6. Additional scaling can be
 226 applied to the proxy 4-vector to partially correct for this underestimation.
 227 Furthermore, since the reconstructed proxy mass, m_p , indicates the size of
 228 the underestimation in each event, this scaling can be parametrized as a
 229 function of m_p .

230 For each simulated event, we define the ideal scaling of the proxy 4-vector,
 231 α , as the scale that will bring the reconstructed $m(t+p)$ to the peak position³
 232 of the reconstructed mass, $m_{t\bar{t}}^{\text{peak}}$ (see Fig. 6). Since $m_{t\bar{t}}^{\text{peak}}$ is a function of $m_{t\bar{t}}^{\text{gen}}$,
 233 this scale is unavailable in collider data. Instead, we reconstruct events using

³It is tempting to define the ideal as $m(t+p) = m_{t\bar{t}}^{\text{gen}}$, which will also calibrate the reconstructed $m_{t\bar{t}}$. But it is more important to reduce the scatter in the reconstructed $m_{t\bar{t}}$, and needlessly introducing the calibration lowers the effectiveness of the derived scaling.

234 a scale $\hat{\alpha}$ which is an estimate of α based on the observable m_p .

To derive this estimate, we solve for α in simulated events, which results in a quadratic equation:

$$\alpha^2 m_p^2 + 2\alpha \left(E_t E_p - \vec{P}_t \vec{P}_p \right) + \left(m_t^2 - m_{t\bar{t}}^{\text{peak}^2} \right) = 0. \quad (9)$$

235 We then plot, in Fig. 7, the two-dimensional distribution of the proxy mass
 236 scaled by $\alpha(m_{t\bar{t}}^{\text{gen}})$ and the unscaled m_p . From this distribution we parametrize
 237 the most probable value of α as a function of m_p to find our estimated $\hat{\alpha}$. The
 238 parametrization of $\hat{\alpha}(m_p)$ was chosen from polynomial functions that were
 239 constrained so that the scaled mass, $\hat{\alpha}m_p$, is non-decreasing ⁴. Finally, we
 240 construct the invariant mass of the $t\bar{t}$ system from the sum of the 4-vector of
 241 the proxy, scaled by $\hat{\alpha}(m_p)$, and the 4-vector of the leptonic top candidate.

242 3.2.4. Averaging the assignments

The most significant improvement is from considering more than one jet assignment. The algorithms described so far considered only the most likely assignment, the one that minimizes the χ^2 in Eq. 1 or that maximizes $P(a | d)$ in Eq. 2. But we can also use all the possible assignments weighted by their a posteriori probabilities. For example:

$$m_{t\bar{t}} = \sum_a m_{t\bar{t}}^a P(a | d). \quad (10)$$

243 These averaged reconstructions tend to have the advantage of a spread lower
 244 than that of the single-assignment reconstructions, and the disadvantage of a

⁴This is enforced only at the edge of the distributions. Though the middle of the function was allowed to decrease, the best-fit function does not do so.

Figure 6: The distribution of the generated and reconstructed $t\bar{t}$ invariant masses, without any scaling of the proxy, for all selected events. The light-gray line shows a fit to the peak position of the reconstructed mass, $m_{t\bar{t}}^{\text{peak}}$.

Figure 7: The distribution of the proxy mass before and after scaling by α , for all selected events. The dashed black and white curve shows a fit to the peak position of α , $\hat{\alpha}(m_p)$.

245 lower response. Here we define the “response” for an observable as the deriva-
 246 tive of the average reconstructed value as a function of the true, generated
 247 value and the “spread” as the RMS of the distribution of the reconstructed
 248 value for a fixed true, generated value.

249 4. Performance

250 4.1. Definition of the figure of merit

251 To compare the performance of different reconstruction algorithms, we
 252 require an appropriate figure of merit. Algorithm performance is usually
 253 quantified by summarizing the distribution of the difference (or the ratio)
 254 between the reconstructed and generated observable into its RMS, or into
 255 the width of a Gaussian fit to the core of the distribution. However, this
 256 quantification presumes that the reconstruction is unbiased and centered

257 around the true value. For the reconstruction algorithms discussed here⁵
 258 the difference distributions are intrinsically bimodal, since the performance
 259 differs for matchable and unmatchable events.

260 For matchable events, the reconstruction typically has a response that is
 261 close to one and a narrow spread, while for the unmatchable events it typically
 262 has a low response and a wide spread. Hence the average reconstruction is
 263 biased, while the peak position is almost unbiased, and the reconstruction
 264 can not be calibrated so it is both unbiased and peaks at the generated value.

265 To quantify the quality of the reconstruction without relying on the prop-
 266 erties of its calibration, we contrast the reconstructed observable for two cat-
 267 egories of events, defined by the quantiles of the generated observable. This
 268 is demonstrated in Fig. 8. Each category contains 10% of the events, and
 269 they are defined according to an offset, s , so that one category is generated
 270 between the s and $s + 0.1$ quantiles and the other between the $0.9 - s$ and
 271 $1 - s$ quantiles (see Fig. 8a where the 2nd and 9th deciles are used). The
 272 FOM quantifies how well the reconstruction separates these two categories.

We denote the distributions of the reconstructed observable for these
 categories f_L and f_H . An example is shown in Fig. 8b. Were these distribu-
 tions Gaussian and identical, it would be natural to quantify the separation
 in terms of N_σ , the number of standard deviations between their peaks. To
 generalize this concept to arbitrary distributions and to focus on the possible
 misclassification of events between the two categories, we define $T(x)$ as the
 overlap between these distributions at observable value x and the minimal

⁵And also for other $t\bar{t}$ reconstruction algorithms for $l+\geq 4$ jets events.

overlap M :

$$M = \min_x T(x), \quad T(x) = \max \left(\int_x^{+\infty} f_L(x') dx', \int_{-\infty}^x f_H(x') dx' \right). \quad (11)$$

These too are shown in Fig. 8b. Smaller M values indicate less misclassification and hence better performance of the reconstruction algorithm.

Figure 8: An example of the minimal overlap as a figure of merit for a reconstruction. The events of f_L are shaded in (a) and shown with the solid curve in (b); the events of f_H are hatched in (a) and shown with the dashed curve in (b). Overflows are shown in the edge bins. The offset is $s = 0.1$, the point of minimal overlap is at $m_{t\bar{t}} = 427$ GeV, as shown in the lower panel of (b), and $N_\sigma = 2.65$.

We can translate M to the more familiar “number of σ s” by considering M for two Gaussian distributions of width one, whose means are separated by N_σ :

$$M(N_\sigma) = \int_{\frac{1}{2}N_\sigma}^{\infty} G(x) dx = \frac{1}{2} \left(1 - \operatorname{erf} \left(\frac{N_\sigma}{2\sqrt{2}} \right) \right), \quad (12)$$

where G is the normal distribution (see Fig. 9a). By inverting this relationship (see Fig. 9b), we can present the minimal overlap in terms of N_σ .

Figure 9: The minimal overlap and the number of σ s.

277 This FOM has another, incidental advantage. Unlike RMS values, it
 278 can be interpreted without referring to the width and shape of the expected
 279 generated distribution.

280 4.2. Comparison of the algorithms

281 Figures 10 and 11 compare the reconstruction of different classes of events
 282 with the new algorithm. For ease of display, a rough linear calibration of $m_{t\bar{t}}$
 283 is used when displaying the resolutions of the partial reconstruction algo-
 284 rithm. Both classes of matchable events (case H and case Q) are recon-
 285 structed well, and the reconstruction of unmatchable events is not much
 286 worse. As 76(1)% of the events are matchable, the reconstruction for all
 287 events is almost as good as for matchable events. The reconstruction of the
 288 hadronic-top rapidity is especially weak for events of type Q , indicating that
 289 a missing “hadronic” b jet is more problematic than a missing jet from W -
 290 boson decay. The reconstruction of the leptonic-top rapidity is especially
 291 weak for unmatchable events, since for most of these events the “leptonic” b
 292 jet is lost.

293 No partial reconstruction algorithm was previously applied to $t\bar{t} \rightarrow l + \text{jets}$

Figure 10: Resolution in $m_{t\bar{t}}$. The y axis in the left-hand plots is on a logarithmic scale, while the right-hand plots show the peak region on a linear scale. Events where one of the jets from W -boson decay is lost (case H) are shown by the long-dashed curves, events where the hadronic b jet is lost (case Q) are shown by the dashed-dotted curves, unmatchable events are shown by the dashed curves, and the solid curves show all events.

294 events, so we choose to compare the performance of the algorithm described
 295 in this paper to that of a kinematic fit algorithm that was used to fully
 296 reconstruct $l+\geq 4$ jets events [4] in many top measurements (e.g. in Refs. [11]
 297 and [12]). As with the new algorithm, we can either use the most likely
 298 assignment from the kinematic fit algorithm or use a weighted average of all
 299 assignments. The relative weight of each assignment is $\exp(-\chi^2/2)$, as in
 300 Ref. [11].

301 We compare the performance of the two algorithms for the ability to
 302 reconstruct the following observable: the invariant mass of the $t\bar{t}$ system
 303 ($m_{t\bar{t}}$), the rapidity of the leptonically decaying top quark (y_l), the rapid-
 304 ity of the hadronically decaying top quark (y_h) and the rapidity difference
 305 ($\Delta y = y_l - y_h$). The distributions of the differences and ratio between re-
 306 constructed and generated observables for these two algorithms, shown in
 307 Fig. 12, illustrate that the partial reconstruction provides a performance

Figure 11: Resolution in top-quark rapidity on the leptonic side (a,b) and the proxy side (c,d), and in Δy (e,f). The y axis in the left-hand plots is on a logarithmic scale, while the right-hand plots show the peak region on a linear scale. Events where one of the jets from W -boson decay is lost (case H) are shown by the long-dashed curves, events where the hadronic b jet is lost (case Q) are shown by the dashed-dotted curves, unmatchable events are shown by the dashed curves, and the solid curves show all events. As we expect symmetric resolution functions, we construct all curves to be symmetric.

Figure 12: Resolution in (a) y_l , (b) y_h , (c) Δy , and in (d) $m_{t\bar{t}}$ for $l+3$ jets events (solid curve) and for $l+\geq 4$ jets events reconstructed with a kinematic fit algorithm [4] (dot-dashed curve). In both cases, the weighted average of all assignments is used.

308 similar in quality to that of the full reconstruction.

309 Table 1 uses the FOM introduced in Section 4.1 to quantitatively compare
310 the performance of the two algorithms. As the generated distributions differ
311 between the $l+3$ jets and the $l+\geq 4$ jets samples, there is some arbitrariness
312 in such a comparison. To quantify this arbitrariness, for the $l+\geq 4$ jets sam-
313 ples each FOM was evaluated twice, once using the quantiles found in the
314 $l+\geq 4$ jets sample and once using the quantiles found in the $l+3$ jets sample.

315 Table 1 also lists the performance of simpler versions of the new algorithm,
316 corresponding to Sections 3.2.1, 3.2.2, 3.2.3, and 3.2.4. A constant offset,
317 s , was chosen for each observable ($m_{t\bar{t}}$, y_l , y_h and Δy). The offsets were
318 chosen so the resulting N_σ values are ≈ 2 , a level of separation where further
319 improvements are still useful (see Fig. 8b). Though the tail behavior of
320 the reconstructions varies, the variations are limited to a fraction of events
321 much smaller than the 10% we consider in each category. Thus the choice of
322 offsets has little effect on the comparison of reconstruction techniques. We
323 find that the partial reconstruction of $m_{t\bar{t}}$ and Δy in $l+3$ jets sample is fully
324 competitive with that of the full reconstruction in the $l+\geq 4$ jets events.

325 The $l+3$ jets channel has the obvious disadvantage of missing a jet. On
326 the other hand, it has the advantage of fewer jets from initial state radiation,
327 and for the algorithm outlined here, of fewer unmatchable events. These
328 advantages compensate quite well for the missing jet. It may be that the
329 reconstruction of $l+\geq 4$ jets can be improved by considering additional recon-
330 struction hypotheses, in particular, events where one jet is lost and a jet from
331 initial state radiation was selected.

Table 1: Performance of the various reconstruction algorithms for all selected events. The ranges listed for the $l+\geq 4$ jets samples are between the two FOMs derived to ensure a fair comparison, as detailed in the text.

		$m_{t\bar{t}}$	Δy	y_l	y_h
offset		0.1	0	0.2	0.2
Separation power in N_σ					
$l+3$ jets	χ^2 based	2.52	2.10	2.00	2.70
	complete likelihood	2.53	2.21	2.05	2.69
	scaled proxy	2.60	2.21	2.05	2.69
	averaged	2.65	2.61	2.26	2.92
$l+\geq 4$ jets	best assignment	2.43–2.45	1.66–1.68	1.57	2.26
	averaged	2.53–2.56	2.46–2.51	1.85–1.86	2.70–2.71

332 5. Summary

333 We present an algorithm that partially reconstructs $t\bar{t}$ events in the l +jets
334 channel in the case when one of the jets is lost, resulting in a l +3 jets topology.
335 Probabilities for correct and incorrect jet assignment are formed based on b -
336 tagging discriminants and on all possible mass combinations on the leptonic
337 and hadronic sides. The algorithm can be applied to measure the forward-
338 backward asymmetry in $t\bar{t}$ production, the invariant mass spectrum of the $t\bar{t}$
339 system and for a number of other analyses that require a full reconstruction.
340 The performance of the partial reconstruction algorithm is competitive with
341 that commonly achieved for fully reconstructed l + ≥ 4 jets events. The inclu-
342 sion of l +3 jets events can improve the statistical strength and reduce the
343 systematic uncertainties of a top properties measurement. Gains equivalent
344 to having 50% more data were achieved at the Tevatron [2].

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