1 An ultimate storage ring lattice with vertical emittance 2 generated by damping wigglers

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5 Abstract

We discuss the approach of generating round beams for ultimate storage rings using vertical damping wigglers (with horizontal magnetic field). The vertical damping wigglers provide damping and excite vertical emittance. This eliminates the need to generate large linear coupling that is impractical with traditional off-axis injection. We use a PEP-X compatible lattice to demonstrate the approach. This lattice uses separate quadrupole and sextupole magnets with realistic gradient strengths. Intrabeam scattering effects are calculated. The horizontal and vertical emittances are 22.3 pm and 10.3 pm, respectively, for a 200 mA, 4.5 GeV beam, with a vertical damping wiggler of a total length of 90 meters, peak field of 1.5 T and wiggler period of 100 mm.

6 *Keywords:* ultimate storage ring, vertical emittance, damping wiggler

7 1. Introduction

8 In present day third generation light sources, the vertical emittance is usually
9 small compared to the horizontal emittance. It is typically a few percent of the
10 latter or below without coupling correction and can reach pico-meter level with
11 coupling correction. For ultimate storage rings (USR), it is not advisable to
12 maintain the same level vertical-to-horizontal emittance ratio. This is because
13 the horizontal emittance will already be diffraction-limited and hence there is
14 no need to make the vertical emittance any smaller. In addition, a smaller
15 vertical emittance would cause significant emittance growth due to intrabeam
16 scattering (IBS) and also short Touschek lifetime. Many USR designs to-date

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17 (such as PEP-X [3]) assume the vertical emittance to be equal to the horizontal
 18 emittance, resulting in a round beam.

19 Vertical emittance in a storage ring can be generated with linear coupling or
 20 vertical dispersion. A round beam can be achieved with 100% linear coupling,
 21 in which case the horizontal and vertical emittances are 50% of the natural
 22 emittance. The reduction of horizontal emittance by a factor of 2 is a significant
 23 benefit of this approach. However, large coupling between the two transverse
 24 directions will cause injection difficulties for off-axis injection. The injected
 25 beam, initially at a large horizontal offset, will take large vertical oscillation
 26 and likely get lost to small vertical apertures such as the small-gap insertion
 27 devices. Effectively, large coupling with small vertical apertures causes the
 28 dynamic aperture to decrease. This is experimentally demonstrated on the
 29 SPEAR3 storage ring as is shown in Figure 1, which shows that the injection
 30 efficiency drops to zero at or before the coupling ratio is increased to 26%.
 31 Large coupling may also reduce the local momentum aperture and in turn the
 32 Touschek lifetime for a beam with a given vertical emittance since the horizontal
 33 oscillation of the Touschek particles will be coupled to the vertical plane which
 usually has smaller apertures.

Figure 1: Injection efficiency vs. coupling ratio at SPEAR3.

34
 35 The second approach to generate vertical emittance is to create vertical
 36 dispersion inside dipole magnets. This will not cause injection and lifetime

37 difficulties, but will lose the benefit of horizontal emittance reduction.

38 We have studied a third approach which can mitigate the negative effects of
39 both of the above approaches. In this approach we use vertical damping wigglers
40 (with horizontal magnetic field) to achieve both the reduction of horizontal
41 emittance and the generation of vertical emittance. Damping wigglers may be
42 desirable for USRs because the dipole bending radius of USRs could be large
43 and hence the radiation energy loss from dipole magnets may be too small for
44 fast enough damping, which is required for controlling collective effects such as
45 intrabeam scattering and beam instabilities. An additional benefit of damping
46 wigglers is that they can reduce the sensitivity of beam parameters (such as
47 emittances) to status of user insertion devices as they provide a large anchor
48 term to the radiation damping. Usually damping wigglers have vertical magnet
49 field that causes wiggling beam motion on the horizontal plane. The intrinsic
50 horizontal dispersion generated by the damping wiggler itself contributes to an
51 increase of the horizontal emittance (even though overall the emittance may
52 be still decreasing due to extra damping). The relative emittance contribution
53 from the damping wiggler can be significant when the natural emittance is
54 small. Choosing to use small period damping wigglers alleviates the emittance
55 growth problem to some extent. But it puts a challenge to the damping wiggler
56 design and increases the cost. A vertical damping wiggler does not increase
57 the horizontal emittance and in the same time generates the desirable vertical
58 emittance. Therefore it is reasonable to use vertical damping wigglers for USRs.
59 The idea of using vertical damping wigglers to generate vertical emittance has
60 been independently proposed in Refs. [1, 2].

61 In this study we demonstrate this approach with a lattice that is compatible
62 with the PEP tunnel at SLAC National Accelerator Laboratory. In section 2 we
63 use a simple model to calculate and compare the emittances with the horizontal
64 and vertical damping wiggler approaches. In section 3 the PEP compatible
65 lattice with vertical damping wigglers is presented. Emittance parameters with
66 intrabeam scattering effects are given in section 4. The conclusions are given
67 in section 5.

68 2. Theoretic calculation

69 The effects of vertical damping wigglers can be analytically estimated. Suppose the wiggler peak field is B_w , its length is L_w , ignoring effects of the wiggler
70 end termination, the horizontal field inside the damping wiggler may be given
71 by
72

$$B_x = B_w \cosh kx \cos ks, \quad B_y = 0, \quad (1)$$

73 where $k = 2\pi/\lambda_w$ and λ_w is the wiggler period. The corresponding vertical
74 closed orbit inside the damping wiggler (DW) is [4]

$$y_{co} = \frac{1}{\rho_w k^2} (1 - \cos ks), \quad y'_{co} = \frac{1}{\rho_w k} \sin ks, \quad (2)$$

75 where $\rho_w = B\rho/B_w$ is the minimum bending radius. The vertical dispersion
76 generated by the DW itself is

$$D_y = -\frac{1}{\rho_w k^2} (1 - \cos ks), \quad D'_y = -\frac{1}{\rho_w k} \sin ks, \quad (3)$$

77 Consequently the radiation integral [5] contributions are

$$\begin{aligned} I_{2w} &= \frac{L_w}{2\rho_w^2}, & I_{3w} &= \frac{4L_w}{3\pi\rho_w^3}, \\ I_{4wy} &= \frac{3L_w}{8\pi\rho_w^4 k^2}, & I_{5wy} &= \frac{4\langle\beta_y\rangle L_w}{15\pi\rho_w^5 k^2}, \end{aligned} \quad (4)$$

78 where $\langle\beta_y\rangle$ is the average vertical beta function across the DW. Assuming
79 no lattice errors, the emittances and momentum spread are given by

$$\sigma_\delta^2 = \gamma^2 C_q \frac{I_3 + I_{3w}}{I_2 + I_{2w}} \frac{1}{2 + \mathcal{D}_x + \mathcal{D}_y}, \quad (5)$$

$$\epsilon_x = \gamma^2 C_q \frac{I_5}{I_2 + I_{2w}} \frac{1}{1 - \mathcal{D}_x}, \quad (6)$$

$$\epsilon_y = \gamma^2 C_q \frac{I_{5wy}}{I_2 + I_{2w}} \frac{1}{1 - \mathcal{D}_y}, \quad (7)$$

80 where I_{2-5} are radiation integrals for the bare lattice and

$$\mathcal{D}_x = \frac{I_4}{I_2 + I_{2w}}, \quad \mathcal{D}_y = \frac{I_{4wy}}{I_2 + I_{2w}}. \quad (8)$$

81 We now consider a PEP-X compatible lattice at 4.5 GeV (see section 3).
82 The relevant radiation integrals without DWs are

$$\begin{aligned} I_2 &= 0.1026 \text{ m}^{-1}, & I_3 &= 1.674 \times 10^{-3} \text{ m}^{-2}, \\ I_4 &= -0.1215 \text{ m}^{-1}, & I_5 &= 3.092 \times 10^{-7} \text{ m}^{-1}. \end{aligned} \quad (9)$$

83 Assuming the average beta function over the DW is 10 m, the emittances as a
84 function of wiggler length is calculated and compared to the case with a regular
85 horizontal damping wiggler for various sets of peak magnetic field and wiggler
86 period values. The results are shown in Figure 2. Clearly the vertical DW
87 provides damping of the horizontal emittance and in the meantime generates
88 vertical emittance. The total emittance is only slightly larger than the case with
89 a regular horizontal DW. The difference is smaller for smaller wiggler periods.

90 3. Application to a PEP-X compatible lattice

91 We have applied the vertical DW approach in a design study for a PEP-X
92 compatible lattice with the design beam energy at 4.5 GeV. This lattice is similar
93 to the PEP-X USR design as it adopts the same MBA and fourth order achromat
94 approach [3]. The 2.2-km long PEP tunnel has a hexagonal geometry. There are
95 six 120-m long straight sections which can be used to host long damping wigglers.
96 The lattice has 6 arcs, each consists of 8 MBA (with $M = 7$) cells. An MBA cell
97 is composed of 5 identical TME cells in the middle and two matching cells at the
98 ends. The MBA cell and the TME cell are shown in Figure 3. The TME and
99 7BA cell lengths are 3.12 m and 30.4 m, respectively. The TME dipole magnet
100 is 1.12 m in length and its bending angle is 1.0475° . This dipole is a combined-
101 function magnet with a defocusing quadrupole component and the normalized
102 gradient is -0.7989 m^{-2} . The focusing quadrupole (QF) is split into two halves
103 to put the SF sextupole in between. The length of each half is 0.18 m. The
104 length of SF is 0.30 m. One SD sextupole magnet is put at each end of the dipole.
105 Its length is 0.21 m. The matching dipole has no quadrupole gradient. Its length
106 is 8% longer than the TME dipole. At each end of the MBA cell, outside of the

Figure 2: Comparison of the emittances of a PEP-X ring with vertical or horizontal damping wigglers. Top row with $B_w = 1.2$ T and $\lambda_w = 200$ mm; middle row with $B_w = 1.2$ T and $\lambda_w = 100$ mm; bottom row with $B_w = 1.5$ T and $\lambda_w = 100$ mm. A 100% coupling (i.e., $\epsilon_x = \epsilon_y = \epsilon_{x0}/2$, where ϵ_{x0} is uncoupled horizontal emittance) is assumed for the regular horizontal DW case.

107 matching dipole, there is a quadrupole triplet. Three harmonic sextupoles are
 108 put between these magnets. The minimum edge-to-edge distance for magnets
 109 is 8 cm to accommodate coils and BPMs [6]. The quadrupole strength is below
 110 51 T/m and the sextupole strength is below 7500 T/m². With a bore radius
 111 of 12.5 mm, the pole tip magnetic field would be below 0.64 T for quadrupoles
 and below 0.59 T for sextupoles.

Figure 3: The TME cell (top) and 7BA cell (bottom) for the PEP-X compatible lattice.

112

113 The insertion device straight sections between the MBA cells are 5 meter
 114 long. The horizontal and vertical beta functions at the centers of these straight
 115 sections are 0.8 m and 2.0 m, respectively. The horizontal beta function is made

116 very small (at the optimal value of $L/2\pi = 0.8$ m) to provide better matching
 117 of the electron and photon optics. But we keep the vertical beta at a level close
 118 to half of the ID straight length to allow small gap insertion devices [7].

119 The 120-m long straight sections are filled with FODO cells. One of the
 120 long straight sections houses the damping wigglers. The wiggler sections are
 121 4.06 meter long and are put between the quadrupoles of the FODO cells. The
 122 optics functions are shown in Figure 4 for a FODO cell for the case with wiggler
 123 period at 200 mm and peak field at 1.2 T. Optics function for one half of the
 long straight section is as shown in Figure 5.

Figure 4: One FODO lattice period with damping wigglers.

124

125 The ring lattice parameters for three wiggler settings are compared in Table
 126 1. The parameters were calculated with MAD8 [8]. The results agree with
 127 the analytic prediction given in Figure 2. For the vertical DW sets of (1.2 T,
 128 200 mm) and (1.5 T, 100 mm), the horizontal and vertical emittances are nearly
 129 equal, with values down to 17/17 pm and 13/10 pm, respectively.

130 4. Intrabeam scattering calculation

131 Intrabeam scattering (IBS) can significantly increase the emittance and en-
 132 ergy spread for very low emittance beams at high current. To examine how the

Figure 5: Half of the DW straight section.

Table 1: Ring parameters with or without vertical damping wigglers

parameters	no DW	VDW-1	VDW-2	VDW-3
Energy (GeV)	4.5			
Circumference (m)	2199.3			
$\nu_{x,y}$	130.15/73.30			
α_c	0.38×10^{-4}			
VDW length (m)	0	90.8	89.4	89.4
VDW B_w (T)		1.2	1.2	1.5
VDW λ_w (m)		0.2	0.1	0.1
U_0 (MeV)	0.59	2.30	2.26	3.19
ϵ_x (pm)	36.5	16.5	16.7	12.7
ϵ_y (pm)	0	17.1	4.3	10.2
σ_δ ($\times 0.001$)	0.77	1.05	1.05	1.10
damping time τ_x (ms)	47	21	22	17
damping time τ_y (ms)	112	29	29	21
damping time τ_s (ms)	176	17	18	12

133 IBS effects may differ for the two approaches of generating vertical emittance,
 134 i.e., with full coupling or with vertical damping wiggler, we did IBS calculation
 135 for both cases for the PEP-X compatible lattice with the high energy approx-
 136 imation model [3, 9]. Similar to the nominal PEP-X, we assume the Coulomb
 137 log function is $(\log) = 11$. For the full coupling case, a horizontal damping wig-
 138 gler is put into the model to reduce emittance. The wiggler parameters are the
 139 same as the vertical damping wiggler. For the case corresponding to VDW-3 in
 140 Table 1 (i.e., with peak field 1.5 T, wiggler period 100 mm and wiggler length
 141 89.4 m), the emittances are 10.5 pm for both planes. For the vertical wiggler
 142 case, a small linear coupling ratio of $\epsilon_y/\epsilon_x = 0.001$ is assumed. The vertical
 143 emittance is almost entirely generated with the vertical damping wiggler. In
 144 existing third generation light sources, linear coupling and vertical dispersion
 145 correction can routinely keep the emittance ratio at the 0.1% level or below.

146 The emittances vs. beam current for the two cases with IBS effects are
 147 shown in Figure 6. The bunch length is assumed to be $\sigma_z = 2.7$ mm at the zero
 148 current limit, corresponding to an RF gap voltage of 6 MV with the 476.0 MHz
 149 rf system. The ratio of bunch length over momentum spread is kept constant
 150 in the IBS calculation. The total number of bunches is assumed to be 3300.
 151 For the vertical DW case, the horizontal emittance has a significant increase.
 152 But the vertical emittance growth is very small. This is because the vertical
 153 dispersion is confined to inside the damping wiggler, which constitutes only a
 154 small fraction of the ring, while the horizontal dispersion is present at all arc
 155 areas. In addition, the vertical dispersion is much smaller than the horizontal
 156 dispersion while the horizontal and vertical emittances at zero current are nearly
 157 the same. This is because the average bending field in the damping wiggler is
 158 much stronger than in the bending magnets. Overall the vertical IBS growth
 159 rate is much smaller than the horizontal plane because the IBS growth rate is
 160 proportional to the dispersion invariant averaged over the ring circumference.

161 The distribution of the IBS growth rate for the vertical DW case is shown
 162 in Figure 7 for the case with a 200 mA total current. For this case, the average
 163 IBS growth rates for x , y , p directions are 26.1 s^{-1} , 0.20 s^{-1} and 8.4 s^{-1} ,

Figure 6: Emittance growth vs total beam current, assuming a uniform current distribution in 3300 bunches and a zero-current bunch length of 2.7 mm.

164 respectively. The corresponding emittances are $\epsilon_x = 22.3$ pm and $\epsilon_y = 10.3$ pm
165 and the momentum spread is $\sigma_\delta = 1.16 \times 10^{-3}$. If harmonic cavities are used to
166 lengthen the bunch to $\sigma_z = 5$ mm, the x , y , p IBS growth rates become 18.8 s^{-1} ,
167 0.13 s^{-1} and 5.5 s^{-1} , respectively, and the emittances and the momentum spread
168 become $\epsilon_x = 18.4$ pm, $\epsilon_y = 10.25$ pm and $\sigma_\delta = 1.13 \times 10^{-3}$.

169 5. Conclusion

170 For ultimate storage rings that plan for off-axis injection, we propose the use
171 of vertical damping wiggler (with horizontal magnetic field) to generate vertical
172 emittance in order to obtain round beams. This approach has an advantage over
173 the approach of generating round beams with 100% coupling because it does
174 not couple the large amplitude horizontal oscillation of the injected beam to the
175 vertical plane and therefore the small vertical apertures in the ring does not pose
176 severe limitation to the dynamic aperture. It is shown that for damping wigglers
177 with reasonably small wiggler period (e.g., $\lambda_w = 100$ mm), the total emittances
178 of the two approaches are nearly equal. The vertical damping wigglers causes an
179 increase to the momentum spread. The amount of momentum spread increase
180 would be the same if horizontal damping wigglers of identical parameters are
181 used.

Figure 7: The distribution of local IBS growth rate for the three dimensions for a beam current of 200 mA in 3300 bunches.

182 A PEP-X compatible lattice is designed to demonstrate this approach. The
 183 bare lattice horizontal emittance is 36.5 pm at 4.5 GeV. When a 90-m long
 184 vertical damping wiggler with peak field at 1.5 T and wiggler period at 100
 185 mm is put into one of the long straight sections, the horizontal and vertical
 186 emittances are 13 pm and 10 pm, respectively. The rms momentum spread is
 187 1.1×10^{-3} . Intrabeam scattering is calculated for this case. For 200 mA beam
 188 current in 3300 bunches and a zero-current bunch length of $\sigma_z = 2.7$ mm, the
 189 horizontal and vertical emittance become 22.3 pm and 10.3 pm, respectively.
 190 The rms momentum spread is 1.16×10^{-3} .

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