Correlated Statistical Uncertainties in Coded-aperture Imaging

Matthew C. Fleenor^{a,*}, Matthew A. Blackston^b, Klaus P. Ziock^b

^a*Roanoke College, Physics Group, 221 College Lane, Salem, Virginia 24153*

^b*Oak Ridge National Laboratory, P.O. Box 2008, Oak Ridge, Tennessee 37831*

Abstract

In nuclear security applications, coded-aperture imagers can provide a wealth of information regarding the attributes of both the radioactive and nonradioactive components of the objects being imaged. However, for optimum benefit to the community, spatial attributes need to be determined in a quantitative and statistically meaningful manner. To address a deficiency of quantifiable errors in coded-aperture imaging, we present uncertainty matrices containing covariance terms between image pixels for MURA mask patterns. We calculated these correlated uncertainties as functions of variation in mask rank, mask pattern over-sampling, and whether or not anti-mask data are included. Utilizing simulated point source data, we found that correlations arose when two or more image pixels were summed. Furthermore, we found that the presence of correlations was heightened by the process of over-sampling, while correlations were suppressed by the inclusion of anti-mask data and with increased mask rank. As an application of this result, we

*Principal corresponding author

Email addresses: `fleenor@roanoke.edu` (Matthew C. Fleenor),
`blackstonma@ornl.gov` (Matthew A. Blackston), `ziockk@ornl.gov` (Klaus P. Ziock)

explored how statistics-based alarming is impacted in a radiological search scenario.

Keywords: coded-aperture; uncertainties; detection significance; correlation

1. Scope, Motivation, & Focus

Originally designed as a method to observe high-energy photons in astronomical applications [1, 2], coded-aperture imaging is a mature, indirect technique for obtaining spatial information in a variety of imaging applications [3, 4]. As an elaborate extrapolation from a single pinhole aperture, a mask with structured pattern is placed between a radiation source and a position-sensitive detector, resulting in a modulated hit pattern on the detector [1, 2, 3, 4]. The resulting hit pattern is computationally decoded to produce an image of radiation emanating from the field-of-view.

Coded apertures are currently utilized across a spectrum of imaging applications, including astrophysics [5, 6] and medicine [7, 8, 9]. The nuclear security community has successfully applied coded-aperture techniques to gamma-ray imaging of radioisotopes for arms control and nuclear nonproliferation applications [10]. In certain cases, the coded-aperture technique provides simultaneously collected spectral information in addition to the spatial location of the sources in question [4, 11]. Both of these capabilities are valuable to the nuclear community since they provide access to the shape, size, isotopic composition, and activity of the radioisotopes of interest.

In order to be of optimum benefit to the community, quantitative results that rely on statistically meaningful analyses of imaged data are needed.

21 However, coded-aperture imaging was designed for point source imaging,
22 and for cases where sources occupy a single pixel, uncertainties are well-
23 known. Coded-aperture imaging can also be used for extended sources. For
24 point sources that contain strength in adjacent image pixels or for extended
25 sources that cover multiple image pixels, a measure of the total strength of
26 a source requires summing multiple image pixels together. If correlations
27 between image pixels exist, then covariance terms must be properly included
28 in the uncertainty calculation.

29 The focus of the current study is on the calculation of the uncertainties
30 for the sum of multiple image pixels and these uncertainties as a function
31 of a few important imaging variables. Unlike previous studies, we do not
32 presuppose the absence of correlations when summing multiple image pixels
33 during specific calculations of total source uncertainty. The article maintains
34 the following outline: §2 introduces the imaging variables manipulated in the
35 current study. In §3, mathematical formalism of the covariance problem is
36 developed, while §4 examines image pixel values for degrees of correlation. §5
37 presents the degree of correlation as a function of the three image variables,
38 and §6 discusses applications related to the detection significance within the
39 nuclear security sector.

40 **2. Coded-Aperture Imaging Variables of Interest**

41 The imaging variables adjusted in the current work are mask rank, whether
42 or not anti-mask data are included, and mask-pattern oversampling. Each
43 of these variables are important for coded-aperture imaging performed with
44 MURA mask, and may not be applicable to other mask types. A modi-

45 fied uniformly redundant array (MURA) is a common mask pattern yielding
 46 high signal-to-noise images [12]. They consist of an equal number of open
 47 and closed mask elements, giving an open fraction of about 50%.

48 The rank of the mask, R , refers to the prime number on which the MURA
 49 pattern is based. Even though other mask pattern arrangements have been
 50 found to equal and/or surpass MURA signal-to-noise ratios [13, 8, 14, 15],
 51 MURA patterns exhibit ideal imaging properties because the number of ar-
 52 tifacts are minimized due to auto-correlative properties [16]. In addition,
 53 due to the anti-symmetry of the MURA pattern under rotation, background
 54 subtraction is uncomplicated and efficient [17].

55 The accumulation of anti-mask detector counts afforded by MURA pat-
 56 terns provides a significant advantage for image quality [10]. In rotating an
 57 anti-symmetric mask by 90° and exposing the detector plane, an anti-mask
 58 exposure is obtained because the spatial locations of the open and closed
 59 mask positions are exchanged [17, 18]. Subtracting the anti-mask counts
 60 from the mask counts suppresses image artifacts and provides an *in situ*
 61 background subtraction [11].

62 The concept of over-sampling relates to the number of detector pixels
 63 that sample each mask opening. In a one-to-one mapping between the detec-
 64 tor plane and mask, each opening or closure in the mask is resolved by one
 65 detector pixel. This relationship between mask and detector is referred to as
 66 single-sampling. When an image is double-sampled, the number of detector
 67 pixels covering each mask opening is increased by 2^2 (two times increase in
 68 each dimension). The process of over-sampling serves two purposes related
 69 to image quality. First, over-sampling aids in the aesthetic problem of pixela-

tion. For detector planes and instrumentation with an inherently large pixel size, the fourfold increase in the number of image pixels increases the spatial sampling, allowing the shape of objects in the field-of-view to be more easily observed. Second, the number of effective lines-of-sight forming the image increases, which adds to the information present when reconstructing. By remembering that the mask is an effective conglomeration of individual pinholes, each pinhole forms a unique response on the detector (i.e., a basis vector). When the detector plane is over-sampled, there is an increase in the number of basis vectors available to reconstruct the image. However, these basis vectors are not completely independent; rather, they are linear combinations of the original, single-sampled, orthogonal basis vectors.

3. Correlation & Covariance within Coded-aperture Imaging

Since coded-aperture image reconstruction involves a cross-correlation of the detector hit pattern with the MURA mask pattern [3, 12], the value of each reconstructed image pixel (I_n) can be expressed as

$$I_n = \sum_i M_{n,i} D_i, \quad (1)$$

where D_i represents the measured detector counts in pixel i , and $M_{n,i}$ is the transfer (or mask) function which is based on the MURA mask pattern with a value of +1 for open mask elements and -1 for closed mask elements (with the exception of the central mask element, which is closed but always set to +1 in order to achieve a delta function response for a point source [19]). Note that in our notation, a single index is used for an image pixel; it should be understood that two-dimensional image pixel indices are collapsed into one

index for clarity in the expressions that follow. If the *detector* pixels (D_i) are assumed to have independent errors, i.e., no covariance ($\sigma_{D_i D_j}^2 = 0$, for $i \neq j$), the uncertainty on any single image pixel follows as

$$\sigma_{I_n}^2 = \sum_i \left(\frac{\partial I_n}{\partial D_i} \right)^2 \sigma_{D_i}^2, \quad (2)$$

where the partial derivatives are given by the decoding array sampled for a particular image pixel:

$$\left(\frac{\partial I_n}{\partial D_i} \right) = M_{n,i} = \pm 1.$$

Also since the associated errors for the detector pixels are determined by Poisson statistics (i.e., $\sigma_{D_i} \approx \sqrt{D_i}$),

$$\sum_i \sigma_{D_i}^2 = N, \quad (3)$$

where N is simply the total number of counts in the detector from both source and background. Therefore, the uncertainty on any single-image pixel simplifies to

$$\sigma_{I_n}^2 = N, \quad (4)$$

i.e., the variance. When including anti-mask data (see §4.1), the total error for single-image pixels will take the form

$$\sigma_{I_n}^2 = \sum_i \sigma_{D_i}^2 = \sum_i [(\sigma_{D_i}^M)^2 + (\sigma_{D_i}^A)^2] = N^M + N^A, \quad (5)$$

where the superscripts “M” and “A” represent the detector counts in a given mask or anti-mask exposure, respectively.

At this point, we may be tempted to naively assume that the total error for the sum of two image pixels is

$$\sigma_{I_m + I_n}^2 = \sigma_{I_m}^2 + \sigma_{I_n}^2 = 2N. \quad (6)$$

109 However Eq. (6) assumes an independent nature for the image pixel uncer-
 110 tainties and the absence of any correlation between them. Until this as-
 111 sumption can be confirmed for multiple image pixels, we must calculate the
 112 covariances between image pixels, namely, $\sigma_{I_m I_n}^2$ [20]. Note that in Eq. (1)
 113 the vector containing detector counts is the same for each image pixel calcu-
 114 lation. The reapplication of the entire dataset to calculate each image pixel
 115 value ensures that the image pixel uncertainties will be correlated, which
 116 points to the need for the use of the full covariance matrix. Therefore, if two
 117 image pixels are summed, the uncertainty for the sum is

$$\sigma_{I_m+I_n}^2 = \sigma_{I_m}^2 + \sigma_{I_n}^2 + 2\sigma_{I_m I_n}^2. \quad (7)$$

118 Explicitly, the covariance term in Eq. (7) takes the following form:

$$\sigma_{I_m I_n}^2 = \sum_i \left[\left(\frac{\partial I_m}{\partial D_i} \right) \left(\frac{\partial I_n}{\partial D_i} \right) \sigma_{D_i}^2 \right] + \sum_i \sum_{j \neq i} \left[\left(\frac{\partial I_m}{\partial D_i} \right) \left(\frac{\partial I_n}{\partial D_j} \right) \sigma_{D_i D_j}^2 \right]. \quad (8)$$

119 Because we are assuming the detector pixel uncertainties are uncorrelated
 120 ($\sigma_{D_i D_j}^2 = 0$), the second term in Eq. (8) goes to zero and the uncertainty for
 121 the sum of two image pixels becomes

$$\sigma_{I_m+I_n}^2 = \sum_i \left[\left(\frac{\partial I_m}{\partial D_i} \right)^2 \sigma_{D_i}^2 + \left(\frac{\partial I_n}{\partial D_i} \right)^2 \sigma_{D_i}^2 + 2 \left(\frac{\partial I_m}{\partial D_i} \right) \left(\frac{\partial I_n}{\partial D_i} \right) \sigma_{D_i}^2 \right], \quad (9)$$

122 where the first two terms are instances of Eq. (2), and the third term is the
 123 remainder of Eq. (8). Having established the values of the partial derivatives
 124 as decoder elements of ± 1 and knowing the detector pixel counts, the un-
 125 certainties for image pixel sums are calculable. Further, it is clear from Eq.
 126 (9) that the first two partial derivative coefficients are squared, giving always
 127 +1, while the partials in front of the third term could be ± 1 . To verify the

128 mathematical result, an examination of simulated coded-aperture images is
129 now presented.

130 4. Evidence of Image Pixel Correlation

131 4.1. Pixel Value Scatter Plots

132 One convenient way to visualize correlations between two values is to plot
133 one value versus the other for a large number of independent measurements.
134 In Figure 1, image pixel pairs are plotted for a rank-3 coded aperture with 10^4
135 distinct simulated measurements. The Upper Left panel of Figure 1 shows a
136 sample simulated image from which the pairs were drawn. The simulations
137 included partial mask transmission and independent Poisson statistics within
138 each detector pixel. Figure 1 (Upper Right) shows an example of nearly
139 uncorrelated pairs of image pixels (e.g., the lower right and lower left image
140 pixels). When one pixel deviates from its mean value in a given direction,
141 there is no directional preference (positive or negative) for the deviation of
142 the other pixel from its mean. The expected uncorrelated distribution is
143 nearly symmetric in both horizontal and vertical directions.

144 The Bottom panels of Figure 1 show image pixel pairs with observable
145 correlation between them. For image pixel pairs that are positively corre-
146 lated (Fig. 1, Bottom Left), an increase (decrease) of one pixel from its mean
147 value is typically accompanied by an increase (decrease) of the other pixel.
148 For negatively correlated image pixel pairs (Fig. 1, Bottom Right), an in-
149 crease (decrease) in pixel value 1 is accompanied by a decrease (increase)
150 in pixel value 2. It should be noted that the correlations exist regardless of
151 whether the image pixels in question contain the source. We now turn to

an examination of how a few important variables for coded-aperture imaging (the inclusion of the anti-mask dataset, mask rank, and over-sampling) influence the degree of correlation.

5. Normalized Covariance

For all the simulated results shown here, the full covariance was calculated as given in Eq. (9). The diagonal terms (the variances) were equal to the total number of counts in the detector, as given in Eq. (4) for mask-only counts or Eq. (5) for mask+anti-mask counts. In what follows, the covariance matrix was normalized by dividing the maximum value (i.e., the variance, equal for all diagonal elements) into all other terms. This will be referred to as the normalized covariance. In this way, the strength of the covariance was displayed as a fraction of the variance, being either negative or positive.

Depending on the context, Figure 2 displays two convenient ways to show the normalized covariance within a given image. For covariance matrices of moderate ranks, it is straightforward to plot the normalized covariance as a two-dimensional array (Fig. 2, Left). These arrays are constructed by plotting the normalized covariance of each image pixel (indexed on the y-axis) with every other image pixel (indexed on the x-axis). The diagonal terms (i.e., the variance) maintain a value of 1.0, while the off-diagonal terms display fractional values. For covariance matrices of larger ranks, it is helpful to display the normalized covariance of a given image pixel with respect to all other image pixels in image space (Fig. 2, Right). This given image pixel always maintains a value of 1.0 because the covariance of a pixel with itself is simply its own variance. Thus, the image on the right of Fig. 2 is represented

Figure 1: Example simulated rank-3 image, with single-sampling, and image pixel pair scatter plots for 10^4 distinct measurements: **(Top Left)** example simulated image with centered source (black), one of 10^4 , from which the image pixel pairs were taken; **(Top Right)** an example of a nearly uncorrelated pair of image pixels, taken from the bottom right and bottom left pixels in the images, marked by symmetry about both x and y axes; **(Bottom Left)** an example of a positively correlated pair of image pixels, taken from the bottom right and upper left pixels in the images, marked by positive slope; **(Bottom Right)** an example of a negatively correlated pair of image pixels, taken from the bottom right and middle left pixels in the images, marked by negative slope.

by one row (or column) in the array display (Fig. 2, Left).

Figure 2: Displays of normalized covariance for a coded-aperture image of rank 7, with single-sampling, and the inclusion of mask counts only: **(Left)** normalized covariance array displaying the covariance between each pixel pair in the image; **(Right)** normalized covariance image displaying the covariance between the primary image pixel (equal to 1.0) and every other image pixel. This image corresponds to either Row or Column 19 of the covariance array representation on the left.

5.1. Covariance as a Function of Anti-Mask Data Inclusion

Remembering that the anti-mask data are generated by exposing the detector under a 90° rotation of the MURA mask pattern, the covariance for summed image pixels was examined as a function of including the anti-mask exposure. Figure 3 shows a normalized covariance array for a rank-7 simulated measurement, with single-sampling, and the inclusion of mask and anti-mask counts (other mask ranks show similar behavior). When compared with Figure 2 (Left), it is clear that reconstructing the image by subtracting

185 the anti-mask dataset has advantages for reducing correlations. In what
 186 follows, all normalized covariances include mask and anti-mask datasets.

Figure 3: Normalized covariance array for simulated coded-aperture data of rank 7, with single-sampling, and the inclusion of mask and anti-mask datasets. Note that the inclusion of the anti-mask data significantly reduces the variation in the off-diagonal covariance values, bringing them all to nearly zero, when compared with the mask-only array in Fig. 2 (Left). For an explanation of the values in the array, please consult §4.

187 5.2. Covariance as a Function of Mask rank

188 The crux of the coded-aperture technique depends on the insertion of a
 189 shadow mask with a structured pattern of open and closed elements between
 190 the object and detector. The rank of the mask, R , refers to the prime num-
 191 ber upon which the MURA pattern is based; as the mask rank increases, the
 192 number of directions (i.e., the number of image pixels) being imaged also in-
 193 creases. Figure 4 displays the normalized covariance for three different mask
 194 ranks, increasing from left to right. All three plots represent single-sampled

195 data. This figure shows that increasing the mask rank reduces the covariance,
 196 which is not unexpected since the number of detector (and image) pixels has
 197 R^2 dependence, meaning that fluctuations of individual detector pixel values
 198 produce less correlation in image pixel values because the overall contribution
 199 of any one detector pixel is diminished. During the cross-correlation given
 200 by Eq. (1), this increased number of mask elements reduces the effect of any
 201 one contribution by R^{-2} .

Figure 4: Normalized covariance images with single-sampling, and the inclusion of mask and anti-mask datasets. These images of normalized covariance show the effect of increasing mask rank, while the image pixel of interest (black) remains in the same general location: **(Left)** rank-3 mask; **(Center)** rank-7 mask; **(Right)** rank-11 mask.

202 5.3. Covariance as a Function of Over-Sampling

203 The intrinsic angular resolution of a coded-aperture imager is determined
 204 by the size of a single mask element divided by the focal length (the distance
 205 between the detector and the mask). When there are ample numbers of
 206 detector counts, the ability to locate a source can be improved if the amount
 207 of radiation through each mask element is sampled more finely than the
 208 size of the mask element itself. When more than one detector pixel samples

209 a mask opening, the dataset is said to be over-sampled. If the dataset is
210 double-sampled, a 2×2 array of detector pixels views each mask element.

211 Figure 5 displays a normalized covariance image of rank 11 with double-
212 sampling, and the inclusion of mask and anti-mask datasets. In comparison
213 to the single-sampled counterpart in Figure 4 (Right), the effects of double-
214 sampling are made clear. The process of double-sampling creates a region of
215 increased (positive) covariance with the nearest neighbors of the selected pixel
216 of interest. In general for masks of higher rank, the perpendicular nearest
217 neighbors contain a normalized covariance of $+0.5$, while the diagonal near-
218 est neighbors are $+0.25$. Note that the magnitude of the nearest-neighbor
219 covariance does not depend on the presence of a source in either pixel.

220 While improved ability to localize sources with double-sampled images is
221 desirable, the source strength is spread over multiple pixels and the process
222 of summing multiple pixels introduces covariance. Within the context of
223 statistics-based alarming, one might question whether or not double-sampling
224 is a hindrance due to the introduction of covariance terms. Considering
225 the few cases studied so far, detection significance calculations are similar
226 when comparing images with double-sampling to those with single-sampling.
227 Whether or not alarming sensitivities are reduced or increased by double-
228 sampling more generally, likely depends on several factors (such as where the
229 source is located within an image pixel) that need to be studied.

230 6. Impact on Source Detection Significance

231 When calculating the statistical significance of a source detection, the
232 detection significance is defined as a ratio of the sum of the image counts in

Figure 5: Normalized covariance image of rank 11 with double-sampling, and the inclusion of mask and anti-mask datasets. When compared with the rank-11, single-sampled covariance image in Fig. 4 (Right), the effect of over-sampling is observed. Note that the image with double-sampling shows positive covariance with the nearest eight neighbors to the pixel of interest.

the source region divided by the total uncertainty on that sum, where the resulting quantity is given in units of sigma (e.g., a five-sigma detection). For a single-image pixel, the uncertainty is straightforward and given as the square root of Eq. (3). However, for sources that have strength in multiple pixels, it was demonstrated mathematically and through simulated data that the covariance is not negligible when multiple image pixels are summed. In the following examples, the detection significance is examined in light of the non-zero covariance between summed image pixels.

6.1. Total Uncertainty of Summed Image Pixels

In §5.3 the effect of image over-sampling revealed that the covariance is positive for the eight nearest neighbors of an image pixel. Figure 6 (Left) displays an example image of a double-sampled point source, where the source

245 strength is distributed among nine image pixels. When considering the to-
 246 tal source strength and its associated uncertainty (i.e., the source detection
 247 significance), all nine pixels must be included in the sum. Figure 6 (Right)
 248 is a histogram of the sum for those nine pixels for 10^4 distinct simulated im-
 249 ages. When only variance terms are used, the calculated expected spread is
 250 too narrow and does not correctly predict the observed spread (lighter curve
 251 labeled “Variance-only”). However, the calculated expected spread models
 252 the observed behavior correctly when covariance terms are included (darker
 253 curve labeled “Full Covariance”). This figure demonstrates that the observed
 254 spread is accurately predicted only if correlated uncertainties are included.
 255 Without accurately calculating these uncertainties, the source detection sig-
 256 nificance is miscalculated by about 60%.

Figure 6: **(Left)** Example simulated image of a point source for a rank-19 mask, with double-sampling. Note that the values within each image pixel are source counts. **(Right)** Histogram showing the sum of nine central image pixels for 10^4 simulated images.

Since the detection significance is given as a ratio of signal over uncertainty, the effect of uncertainty miscalculation has a deleterious effect on the alarm rate. Figure 6 (Right) shows that the total uncertainty is underestimated when only the variance terms are considered. When the alarm threshold is given by the detection significance, an underestimated uncertainty leads to an inflated detection significance and an overconfidence when sounding an alarm. The final section intends to show how this miscalculated uncertainty can lead to false alarms in real-world situations.

6.2. Detection Alarms & Coded-aperture Imaging

Understanding that the correct calculation of the total source uncertainty is intimately connected to alarming, we demonstrate how false alarms could be generated from incorrect calculations of uncertainties in the source. Figure 7 displays measured data from a fast-neutron, coded-aperture imager of mask-rank 11 [21], with double-sampling, and the inclusion of mask and anti-mask counts. The image of the ^{252}Cf source was taken at a mask-source distance of 27 m and a mask-detector distance of 25 cm. The source extends over four image pixels and is framed by the broken white box (at $\Delta x \approx 60$ cm, $\Delta y \approx 1000$ cm). Each individual pixel has a significance near 3σ and should be summed to obtain the total strength of the source. When only the variance is considered for the uncertainty on the sum, a value of 190 ± 30 counts is calculated. However, when considering the covariance terms for the four image pixels, a value of 190 ± 40 counts is calculated. As an example, consider a detection system that maintains an alarm threshold value of 5σ . The variance-only detection significance equals $190/30 \approx 6.3\sigma$, which would sound an alarm if the threshold were 5σ . However, the detection significance

282 for an uncertainty that incorporates the covariance yields a value of $\approx 4.75\sigma$
 283 ($^{190}_{40}$), which would *not* signal a 5σ alarm. This demonstrates how an im-
 284 proper accounting of the total uncertainty for summed image pixels could
 285 lead to an excess of false alarms within nuclear security applications.

Figure 7: Measured image data for a rank-11, double-sampled coded-aperture image with mask and anti-mask counts. The source extends over four total pixels and is framed by the broken white box (at $\Delta x \approx 60$ cm, $\Delta y \approx 1000$ cm), each with a single pixel significance of $\sim 3\sigma$. Without the covariance included, the detection significance for the entire extended source would falsely sound an alarm set at 5σ , see text in §6.2.

286 7. Conclusions

287 The current study has demonstrated that correlated uncertainties are
 288 reduced in coded-aperture imaging with MURA masks by including the

289 anti-mask data and increasing mask rank. However, even when utilizing
290 higher mask ranks and including anti-mask data, correlations between non-
291 neighboring pixels are suppressed but are not entirely negated. Double-
292 sampled data create the largest correlation effects, but almost exclusively
293 in the nearest neighboring pixels (see Fig. 5). This is not unexpected since
294 double-sampling the data breaks the orthogonality of each image pixel with
295 respect to its neighbors. Nearest neighbor correlation exists independently
296 of whether or not source pixels are being examined.

297 We have calculated the effects of ignoring the covariance when calcu-
298 lating the detection significance, finding that a variance-only uncertainty is
299 narrower than the observed width of simulated data. For these cases, the de-
300 tection significance is overestimated when the covariance is not incorporated
301 into the uncertainty measurement. The example presented in §5.2 illustrates
302 how false alarms can arise in radiological search applications if the total un-
303 certainty for summed image pixels is not calculated correctly. This points
304 to the need for the inclusion of covariance terms in situations where over-
305 sampling is employed and source strength is summed across multiple image
306 pixels.

307 8. Acknowledgments

308 We thank the Department of Energy (DOE) for its support of the Vis-
309 iting Faculty Program (VFP) and Oak Ridge Associated Universities for its
310 administration of the program. We thank Paul Hausladen for helpful dis-
311 cussions. MCF also thanks the Roanoke College Sabbatical Program for its
312 support and the Nuclear Security and Isotope Technology Division for its

313 hospitality. This manuscript has been authored by the Oak Ridge National
 314 Laboratory, managed by UT-Battelle LLC under Contract No. DE-AC05-
 315 00OR22725 with the US DOE. The US government retains and the pub-
 316 lisher, by accepting the article for publication, acknowledges that the US
 317 government retains a nonexclusive, paid-up, irrevocable, worldwide license
 318 to publish or reproduce the published form of this manuscript, or allow oth-
 319 ers to do so, for US government purposes. This work was supported in part
 320 by the US Office of Science and the Office of Workforce Development for
 321 Teachers and Scientists under the VFP.

322 REFERENCES

- 323 [1] R. H. Dicke, “Scatter-hole Cameras for X-rays and Gamma Rays”, *As-*
 324 *trophysical Journal* 153 (1968) L101.
- 325 [2] J. G. Ables, “Fourier Transform Photography: A New Method for X-ray
 326 Astronomy”, *Proc. Astronomical Soc. Australia* 1, 4 (1968) 172.
- 327 [3] E. E. Fenimore & T. M. Cannon, “Coded Aperture Imaging with Uni-
 328 formly Redundant Arrays”, *Applied Optics* 17, 3 (1978) 337.
- 329 [4] J. Gunson & G. Polychronopolous, “Optimum Design of a Coded Mask
 330 X-ray Telescope for Rocket Applications”, *Monthly Notices Royal Astro-*
 331 *nomical Soc.* 177 (1976) 485.
- 332 [5] C. B. Wunderer et al., “Imaging with the Coded Aperture Gamma-ray
 333 Spectrometer SPI Aboard INTEGRAL”, *Proc. SPIE 4851* (2003) 1269.

- 334 [6] J.E. Grindlay & J. Hong, “Enhanced Wide-field Coded aperture Imaging:
335 Scanning and Radial Holes”, *Proc. SPIE 5168* (2003) 402.
- 336 [7] Zhang, L., Lanza, R. C., Horn, B. K. P., Robert E. Zimmerman, R.
337 E., “Three-dimensional coded-aperture techniques in diagnostic nuclear
338 medicine imaging”, *Proc. SPIE 3336, Medical Imaging* (1998) 1532.
- 339 [8] R. Accorsi, F. Gasparini, & R. C. Lanza, “Optimal Coded Aperture Pat-
340 terns for Improved SNR in Nuclear Medicine Imaging”, *Nuclear Instru-*
341 *ments & Methods in Physics Research, Section A (NIM A)* 474 (2001)
342 273.
- 343 [9] Starfield, D. M., Rubin, D. M., Marwala, T., “Design of an Ultra-Near-
344 Field System for Planar Coded Aperture Nuclear Medicine Imaging”, *Proc.*
345 *IFMBE 20* (2008) 590.
- 346 [10] K. P. Ziock, C. J. Hailey, T. B. Gosnell, J. H. Lupton, & E A. Harrison
347 “A Gamma-Ray Imager for Arms Control”, *IEEE Trans. Nuc. Sci.* 39, 4
348 (1992)
- 349 [11] K. P. Ziock, M. T. Burks, W. Craig, L. Fabris, E. L. Hull, & N. W.
350 Madden, “Real-time Generation of Images with Pixel-by-pixel Spectra for
351 a Coded Aperture Imager with High Spectral Resolution”, *NIM A* 505
352 (2003) 420.
- 353 [12] E. E. Fenimore, “Coded Aperture Imaging: Predicted Performance of
354 Uniformly Redundant Arrays”, *Applied Optics* 17, 22 (1978) 3562.
- 355 [13] J. J. in’t Zand, J. Heise, & R. Jager, “The Optimum Open Fraction of

356 Coded Apertures. With an Application to the Wide Field X-ray Cameras
357 of SAX”, *Astronomy & Astrophysics* 288 (1994) 665.

358 [14] G. K. Skinner, “Sensitivity of Coded Mask Telescopes”, *Applied Optics*
359 47, 15 (2008) 2740.

360 [15] P. M. E. Shutler, A. Talebitaher, & S. V. Springham, “Signal-to-noise
361 Ratio in Coded Aperture Imaging”, *NIM A* 669 (2012) 22.

362 [16] S. R. Gottesman & E. E. Fenimore, “New Family of Binary Arrays for
363 Coded Aperture Imaging”, *Applied Optics* 28, 20 (1989) 4344.

364 [17] J. Braga, T. Villela, U. B. Jayanthi, F. D’Amico, & J. A. Neri, “A
365 New Mask-antimask Coded-aperture Telescope for Hard X-ray Astron-
366 omy”, *Experimental Astronomy* 2, 2 (1991) 101.

367 [18] R. Accorsi & R. C. Lanza, “Near-field Artifact Reduction in Planar
368 Coded Aperture Imaging”, *Applied Optics* 40, 28 (2001) 4697.

369 [19] E. E. Fenimore, “Coded Aperture Imaging: The Modulation Transfer
370 Function for Uniformly Redundant Arrays”, *Applied Optics* 19, (1980)
371 2465.

372 [20] Bevington, P. R. & Robinson, D. K. “Data Reduction and Error Analysis
373 for the Physical Sciences, 3rd ed.”, McGraw-Hill:New York (2003) 41.

374 [21] Hausladen, P. A., Newby, J., Liang F., Blackston, M. A., “The Deploy-
375 able Fast-Neutron Coded-Aperture Imager: Demonstration of Locating
376 One or More Sources in Three Dimensions”, ORNL/TM-2013/446 (2013).