Local Linear Neighbor Reconstruction for Multi-view Data

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ABSTRACT

Graph based multi-view data analysis has become a hot topic in the past decade, and multi-view similarity matrix is fundamental for such tasks. Existing multi-view similarity matrix construction methods can not learn local geometrical information in the original data space from multiple views simultaneously. Considering the fact that an appropriate similarity matrix is block-wise with intra-class similarity, it is more reasonable to learn a similarity matrix by using local geometrical information in multiple original data space. In this paper, we propose to construct a unified similarity matrix by using local linear neighbors in multiple views. In each view, the similarity matrix can be reconstructed with the weights of the neighbors of each data point in the original space. In multiple views, we seek for a unified similarity matrix which consists of the similarity matrix in each view. The unified similarity matrix can be used for spectral clustering, label propagation and other graph based learning algorithms. Experimental results show that spectral clustering and label propagation algorithms using the unified similarity matrix outperform those using other multi-view similarity matrices, they also outperform typical multi-view spectral clustering algorithms and typical multi-view label propagation algorithms.

Keyword: Multi-view similarity, Similarity construction, Local linear neighbor

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1. Introduction

Real-world datasets are often collected from multiple views which are compatible and complementary. For example, journalists may report the same news in different ways. A webpage can be described by its content and the hyperlinks linking to it. An object can be photographed by several monitors. Multi-view data analysis has become a hot topic in the past decade and many algorithms have been proposed [4, 5, 27, 12, 26, 25, 13, 38, 29].

Multi-view similarity matrix is fundamental for graph based multi-view data analysis such as spectral clustering [32] aiming to partition a dataset into groups according to the pairwise similarities between data points, and label propagation [47] predicting the labels of the unlabeled points according to the labeled

points and similarity matrix. Recently, several algorithms have been proposed to construct similarity matrix by integrating information from multiple views. Kumar and Daum [19] constructed the combined similarity matrix in one view by using the eigenvectors in another view. Bisson and Grimal [6] simultaneously handled all the information contained in multi-view datasets by using several instances of an existing co-similarity algorithm which transforms the global data matrix into a similarity matrix. Xia et al. [35] conducted Markov chain based spectral clustering and learned the consensus transition probability matrix via low-rank and sparse decomposition. Multiple kernel learning algorithms [30, 15] learned a combined kernel as a function from the input kernels. Tzortzis and Likas [30] learned the coefficient of the linear combination by minimizing the objective function values of multiple views. Guo et al. [15] learned a linear combination of multiple kernels and determined the coefficient by minimizing the kernel alignment between the learned kernel and the kernel of each view. De Sa et al. [14] generated a multipartite graph based on the Minimizing-Disagreement algorithm [25, 13]. Li et al. [22] approximated the similarity graphs using bipartite graphs. How-

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ever, these multi-view similarity matrix construction methods can not learn local geometrical information in the original data space from multiple views simultaneously. Considering the fact that an appropriate similarity matrix is block-wise with intra-class similarity, it is more reasonable to learn the similarity matrix by using local geometrical information. As shown in [24], the local geometry can be characterized by the local linear neighbors. Recently, many methods have been proposed to construct similarity matrix by using local linear neighbors [24, 33, 44, 8, 43]. However, those methods are designed for single-view data. In addition, learning from the multiple original data spaces will reduce the information loss in the process of constructing multi-view similarity matrix.

In this paper, we propose to construct a unified similarity matrix by using local linear neighbors in multiple original spaces. In each view, the similarity matrix can be reconstructed with the weight of neighbors of each data point in the original data space [24, 44]. Intuitively, for the same set of data points with multiple views, the similarity matrix in each view should consist with a ground truth similarity matrix. Thus we seek for an approximated ground truth similarity matrix that is reconstructed with neighbors in multiple original data spaces and consistent with the similarity matrix in each view. The unified similarity matrix and the single-view similarity matrix are learned iteratively. The unified similarity matrix can be used for spectral clustering, label propagation and other graph based algorithms. Experimental results show that spectral clustering and label propagation algorithms using the learned similarity matrix outperform those using other multi-view similarity matrices, they also outperform typical multi-view spectral clustering algorithms and typical multi-view label propagation algorithms.

2. Local Linear Single-view Neighbor Reconstruction

Local linear single-view neighbor reconstruction is based on the local linear neighbor assumption, which insists that each data point can be reconstructed by combining its selected neighbors in the original data space. This assumption is widely used for similarity matrix learning [24, 33, 44, 8, 43]. The work in [24, 33] fixed the number of neighbors and used l_2 -norm in the objective function. Generally, the objective function is shown as Eq.(1).

$$\begin{aligned} \varepsilon(W) &= \sum_i \|x_i - \sum_{j: x_j \in N(x_i)} W_{ji} x_j\|^2 \\ s.t. & \sum_{j: x_j \in N(x_i)} W_{ji} = 1, W_{ji} \geq 0 \end{aligned} \quad (1)$$

In Eq.(1), $W_{ji} = 0$ if $x_j \notin N(x_i)$. $N(x_i)$ is the set of k nearest neighbors of x_i . To avoid using fixed number of neighbors, the work in [44, 8, 43] minimized the l_1 -norm and was expected to learn a sparse adjacency matrix without fixed neighborhoods for each point. The work in [8, 43] tried to optimize Eq.(2), but the weights can not be ensured to be positive. Here $W_{.i}$ is the i -th column of W .

$$\min_W \|W_{.i}\|_{l_1} \quad s.t. x_i = \sum_{j: j \neq i} W_{ji} x_j \quad (2)$$

Fig. 1. The interaction between two variables. $X^{(v)}$ is the dataset of v -th view. $W^{(v)}$ is the similarity matrix of v -th view. W^* is the approximated ground truth similarity matrix. μ_v is the weight of v -th view.

The algorithm in [44] relaxed the restriction of [8, 43] and added the non-negative constraint. Its objective function is Eq.(3).

$$\min_W \|x_i - \sum_{j: j \neq i} W_{ji} x_j\|_2^2 + \lambda \|W_{.i}\|_{l_1} \quad s.t. W_{.i} \geq 0 \quad (3)$$

The similarity matrices learned by Eq.(1) and Eq.(3) are asymmetric. To ensure symmetry, W_{ij} and W_{ji} can be set as $(W_{ij} + W_{ji})/2$.

3. Local Linear Multi-view Neighbor Reconstruction

3.1. Local Linear Multi-view Neighbor Reconstruction

Data points with multiple views have multiple similarity matrices. As the information in multiple views is compatible and complementary, it is reasonable to learn a unified similarity matrix containing the information of each view.

For the same set of data points, the ground truth similarity between each pair of points should be the same in each view. Intuitively, we seek for an approximated ground truth similarity matrix that consists of the similarity matrix in each view. Given a dataset X with n_v views. Let $X^{(v)} = [x_1^{(v)}, x_2^{(v)}, \dots, x_n^{(v)}]$ be the set of data points in v -th view, where n is the number of data points, $W^{(v)}$ be the similarity matrix in v -th view, W^* be the approximated ground truth similarity matrix, and μ_v be the weight of v -th view. The interactions among $X^{(v)}$, $W^{(v)}$, W^* and μ_v are shown in Fig.1. W^* proportionally learns information from $W^{(v)}$ in each view. $W^{(v)}$ is constructed from $X^{(v)}$ and part of W^* .

In each view, the appropriate similarity matrix is block-wise with intra-class similarity. Considering the fact that neighborhood contains the local geometrical information [24], we use the local linear neighbor reconstruction method to construct similarity matrix in each view. Let $A_{-i}^{(v)}$ be the matrix obtained from $X^{(v)}$ by removing its i -th column. We learn the unified similarity between $x_i^{(v)}$ and other points by minimizing Eq.(4).

$$\begin{aligned} O &= \sum_v (\|x_i^{(v)} - A_{-i}^{(v)} W_{.i}^{(v)}\|^2 + 2\lambda \|W_{.i}^{(v)}\|_{l_1} \\ &+ \mu_v \|W_{.i}^{(v)} - W_{.i}^*\|^2) + \|\mu\|^2 \\ s.t. & W_{.i}^* \geq 0, W_{.i}^{(v)} \geq 0, \\ & \|\mu\|_1 = 1, \mu = (\mu_1, \dots, \mu_{n_v}) \end{aligned} \quad (4)$$

In Eq.(4), $\|\mu\|^2$ is used to avoid overfitting on a single view. The coefficient of $\|W_{.i}^{(v)}\|_1$ is set as 2λ to simplify the following optimization.

Eq.(4) extends Eq.(3) to multiple views. The approximated ground truth similarity matrix W^* consists of the similarity matrix $W^{(v)}$ in each view. $W^{(v)}$ is sparsely reconstructed with the local linear neighbors of each data point in the original data space. Then, W^* implicitly learns the local geometrical information in original data space from multiple views simultaneously.

3.2. Multiplicative Update Rule

Eq.(4) is not jointly convex but convex on each variable individually, thus it can be solved by the multiplicative update rule. We present an iterative method, which optimizes one variable while fixing others, for solving it.

3.2.1. Fixing $W_{.i}^*$, μ , compute $W_{.i}^{(v)}$

For brevity, we only care about terms that are relevant to $W_{.i}^{(v)}$. Then the objective function reduce to Eq.(5).

$$O_{W_{.i}^{(v)}} = -2X_{.i}^{(v)T} A_{-i}^{(v)} W_{.i}^{(v)} + W_{.i}^{(v)T} A_{-i}^{(v)T} A_{-i}^{(v)} W_{.i}^{(v)} + 2\lambda \|W_{.i}^{(v)}\|_1 + \mu_v \|W_{.i}^{(v)} - W_{.i}^*\|^2 \quad (5)$$

Let Θ be the Lagrange multiplier for $W_{.i}^{(v)}$, then the Lagrangian function of Eq.(5) is Eq.(6).

$$L = O_{W_{.i}^{(v)}} + \Theta^T W_{.i}^{(v)} \quad (6)$$

The first-order derivative of Eq.(6) is shown in Eq.(7).

$$L' = -2A_{-i}^{(v)T} X_{.i}^{(v)} + 2A_{-i}^{(v)T} A_{-i}^{(v)} W_{.i}^{(v)} + 2\lambda + 2\mu_v (W_{.i}^{(v)} - W_{.i}^*) + \Theta \quad (7)$$

Then, the update rule of $W_{.i}^{(v)}$ can be expressed as follows.

$$W_{.i}^{(v)} \leftarrow W_{.i}^{(v)} \odot \frac{A_{-i}^{(v)T} X_{.i}^{(v)} + \mu_v W_{.i}^*}{A_{-i}^{(v)T} A_{-i}^{(v)} W_{.i}^{(v)} + \lambda + \mu_v W_{.i}^{(v)}} \quad (8)$$

where $\odot$ is the dot product between two matrices.

3.2.2. Fixing $W_{.i}^{(v)}$, μ , compute $W_{.i}^*$

Similarly, we get the following objective function Eq.(9). Eq.(9) is a quadratic function of $W_{.i}^*$, then we derive Eq.(10).

$$O_{W_{.i}^*} = \sum_v \mu_v \|W_{.i}^{(v)} - W_{.i}^*\|^2 \quad (9)$$

$$W_{.i}^* = \frac{\sum_v \mu_v W_{.i}^{(v)}}{\sum_v \mu_v} \quad (10)$$

3.2.3. Fixing $W_{.i}^*$, $W_{.i}^{(v)}$, compute μ

The objective function with respect to μ is Eq.(11).

$$O_\mu = \sum_v (\mu_v \|W_{.i}^{(v)} - W_{.i}^*\|^2) + \|\mu\|^2 \quad (11)$$

s.t. $\|\mu\|_1 = 1, \mu = (\mu_1, \dots, \mu_{n_v})$

μ can be solved by the entropic mirror descent algorithm (EMDA) [3]. EMDA requires the Lipschitz constant as input. For a fixed norm(for example 1-norm), the Lipschitz constant of μ is as follows.

$$L_f(\mu) = \sup_{\mu} \|f'(\mu)\|_1 = \sum_v \|W_{.i}^{(v)} - W_{.i}^*\|^2 + 2 \quad (12)$$

3.3. The Algorithm

We summarize the flow of local linear multi-view neighbor reconstruction (LMNR) algorithm in Algorithm 1. We initially set the element of $W^{(v)}$ and W^* as $1/(n-1)$ and element of μ as $1/n_v$.

Algorithm 1: LMNR algorithm

Input:

$X^{(a)} = \{x_1^{(a)}, x_2^{(a)}, \dots, x_{n_a}^{(a)}\}$: the multi-view data sets

Output:

$W^{(v)}$: the similarity matrix in view a

W^* : the approximated ground truth similarity matrix

1: For each view a , normalize $X^{(a)}$;

2: Initialize $W^{(a)}$, W^* , μ ;

3: **for** $i = 1$ to n **do**

4: **repeat**

5: Update W^* using Eq.(10);

6: **for** $v = 1$ to n_v **do**

7: Update $W^{(v)}$ using Eq.(8);

8: **end for**

9: Update μ using EMDA [3];

10: **until** Eq.(4) is convergent.

11: **end for**

12: $W^* = \frac{W^{*T} + W^*}{2}$;

In Fig.2, we demonstrate the benefit of LMNR on a simple synthetic dataset involving 100 samples represented in two views. Fig.2(a) and Fig.2(b) separately show the distribution of the two views. Each view has two clusters and the first 50 samples belong to one cluster (represented by red points) while the others belong to the other cluster (represented by blue points). Specifically, we mark several points to facilitate the discussion. The presentations of the same point in two views are marked by the same shape. In the example, the two clusters seem to be well separated in each view. However, the boundary points (e.g., the point in view 1 marked by square and the point in view 2 marked by circle) are close to some points belonging to both of two clusters. Then, as shown in Fig.2(c) and Fig.2(d)¹, the intra-cluster similarities may be less than or nearly the same with the inter-cluster similarities. It is hard to separate the boundary points by using the information in a single view. That is, the similarity matrix in each view is not so good to separate the clusters. Fortunately, the two similarity matrices are complementary and capable of identifying individual points, i.e. the points that have high similarities in view 1 have low similarities in view 2 and vice versa (e.g. the point marked by circle and square). By using LMNR, the unified similarities merge together the similarities in two views and subsequently become better to partition clusters. As shown in Fig.2(e), the unified similarity matrix is a proximate block diagonal matrix, so the clusters are well separated. Results on the toy dataset indicate that LMNR leverages the information in multiple views to learn a better unified similarity matrix.

¹The representation of the similarity matrix is bad since the scale of the intra-cluster similarities calculated by Eq.(3) varies greatly. When graphed, the high similarities weaken the representations of low similarities. Actually, most of the intra-cluster similarities are still higher than inter-cluster similarities.

Fig. 2. Synthetic dataset and the similarity matrix. (a) and (b) are the distribution of the dataset. (c) and (d) are the sparse similarity matrices calculated by Eq.(3). (e) is the unified similarity matrix calculated by LMNR.

4. Application to Clustering and Label Propagation

4.1. Spectral Clustering

Graph-based clustering, such as spectral clustering [32], aims to partition a dataset into groups according to the pairwise similarities between pairs of data points. We apply the learned similarity matrix on spectral clustering to show its effectiveness. The flow of spectral clustering is shown as follows.

- (1) It calculates the Laplacian matrix $L = D^{-1/2}W^*D^{-1/2}$, where D is a diagonal matrix and $D_{ii} = \sum_j W_{ij}^*$.
- (2) It calculates the first k eigenvectors of L and forms Y by the eigenvectors. k is the number of clusters.
- (3) It clusters Y with k -means into k clusters.

4.2. Label Propagation

Label propagation [47] propagates the labels of labeled data to unlabeled data using the similarity matrix. The flow of label propagation is shown as follows.

- (1) It calculates the Laplacian matrix $L = D^{-1/2}W^*D^{-1/2}$, where D is a diagonal matrix and $D_{ii} = \sum_j W_{ij}^*$.
- (2) It iterates $F(t+1) = \alpha LF(t) + (1-\alpha)Y$ until convergence, where $\alpha \in (0, 1)$ is a parameter previously defined, $Y \in R^{n \times k}$ is the constrained matrix with $Y_{ij} = 1$ if x_i belongs to j -th cluster and $Y_{ij} = 0$, otherwise $F(0) = Y$.
- (3) Let F^* be the limit of $F(t)$. Label x_i as a label $y_i = \arg \max_{1 \leq j \leq k} F_{ij}^*$.

5. Experiments

5.1. Datasets

We use four datasets to evaluate the effectiveness of LMNR with application to spectral clustering and label propagation.

UCI Handwritten digits data² consists of 2000 examples, with view-1 being the 76 Fourier coefficients and view-2 being the 47 Zernike moments. Animal³ consists of 30475 images of 50 animals classes with six pre-extracted feature representations for each image. We choose images of 5 animals classes with Local Self-Similarity features and PyramidHOG features. Webkb⁴ is composed of web pages collected from computer science department of Washington university. A web page is made of two views: content and cites. The documents are described by 1703 words in the content view and 938 links between them in the cites views. M2⁴ contains subsets of the NG20 dataset with 3 different preprocessing, supervised mutual information preprocessing, partitioning around medoids preprocessing and unsupervised mutual information preprocessing.

5.2. Baselines

LMNR in spectral clustering and label propagation is compared with single-view similarity matrix reconstruction methods: Cos and Sparse, multi-view similarity matrix reconstruction methods: Cos-concat, Sparse-concat, MVSIM, Cospectral and MVSpec. Cos calculates cosine similarity on each view; Cos-concat first concatenates all the views, and then calculates cosine similarity on the concatenated data; Sparse calculates the similarity matrix on each view data using method in [44]; Sparse-concat first concatenates all the views, and then calculates the similarity matrix on each view data using method in

²<https://archive.ics.uci.edu/ml/datasets/Multiple+Features>

³<http://attributes.kyb.tuebingen.mpg.de/>

⁴<http://lig-membres.imag.fr/grimal/data.html>

[44]; MVSim [6] simultaneously deals with all the information contained in multi-view datasets by using several instances of an existing co-similarity algorithm; Cospectral [19] constructs the similarity matrix in one view by using the eigenvectors in another view; and MVSpec [30] learns a weighted combination of the given kernels in parallel to clustering.

When applied to spectral clustering, LMNR is also compared with the following multi-view spectral clustering algorithms. Coregspectral [20] co-regularizes the clustering hypotheses; MvSpecCE [37] extends clustering ensembles to multi-view clustering. KCCA [7] is based on kernel canonical correlation analysis. sM-D [14] generates a multipartite graph. CGC [9] minimizes cluster assignment inconsistency with the given cross domain relationships. RMSC [35] learns the consensus transition probability matrix via low-rank and sparse decomposition. To show the influence of μ , we also report the result of LMNR with equally treated views (LMNR- μ), i.e. we set the element of μ as $1/n_v$ for each data point.

When applied to label propagation, LMNR is also compared with one typical multi-view label propagation algorithm: SSM-DML [45]. SSM-DML learns multi-view distance metrics from multiple feature sets in the process of label propagation.

Cos, Sparse, MVSim, Cospectral, CGC and sM-D do not have a consensus result, and we report their best single view results. For the baselines, we use the parameters suggested in the original papers. Specifically, each view is equally treated in the baselines except for MVSpec and MvSpecCE. In MVSpec and MvSpecCE, the importance of each view is leveraged by an exponent with power $p = 1.5$. In MvSpecCE, we take a coarse grid search for scale parameter in the region $[2^0, 2^6]$ with exponent growth 0.5. In MVSim, the power is set as 0.8, the percentage of the smallest similarity values is set as 0.4, the damping parameter is set as 0.5, the number of iterations is set as 6. In CGC, the regularization parameter is set as 1. In KCCA, the Gram Schmidt precision parameter is fixed at 0.5. In RMSC, we set the trade-off parameter as 0.005. In LMNR, λ controls the importance of reconstruction and sparsity. To balance the two parts, we set λ as 0.5 for all the datasets. For each data point, we set the number of iterations as 10 since Eq.(4) generally converges in limited steps (≤ 10). In label propagation, we set α as 0.2. To avoid randomness, we conduct the algorithms 20 runs with different initializations and report the average results.

5.3. Evaluation Metrics

We use accuracy (ACC) and normalized mutual information (NMI) [39] to measure the performance.

5.3.1. Accuracy

The accuracy (ACC) measure calculates the largest rate of correct assignment by corresponding each data to the right cluster. Denote c_i as the label of an algorithm and l_i as the true label of x_i . ACC is defined as:

$$ACC = \frac{\sum_{i=1}^n \delta(l_i, \text{map}(c_i))}{n} \quad (13)$$

where $\delta(x, y)$ is the indicator function, and n is the total number of objects, $\text{map}(c_i)$ is the mapping function that permutes the clustering labels to match the ground truth data labels.

5.3.2. Normalized Mutual Information

The normalized mutual information (NMI) indicates the shared statistics between the label of an algorithm and the true label. Given the clustering result, NMI is computed as follows.

$$NMI = \frac{\sum_{c_i \in C, c'_j \in C'} p(c_i, c'_j) \cdot \log \frac{p(c_i, c'_j)}{p(c_i) \cdot p(c'_j)}}{\sqrt{\sum_{c_i \in C} p(c_i) \log p(c_i) \sum_{c'_j \in C'} p(c'_j) \log p(c'_j)}} \quad (14)$$

where $p(c_i)$ denotes the probabilities that an object belongs to the clusters c_i , and $p(c_i, c'_j)$ is the joint probability that the object belongs to the clusters c_i as well as c'_j at the same time.

5.4. Spectral Clustering Results

Table 1 shows the results of spectral clustering. The sixth column and the last column separately show the average ACC and NMI of every algorithm over the four datasets. Generally, Cos-concat and Sparse-concat have no advantage over their single view counterparts, which shows that simply concatenating all the features together may lose information of single view. The multi-view baselines perform better than Cos and Sparse in most cases, which indicates that combining information in multiple views improves the clustering performance. LMNR always performs better than the other single-view and multi-view similarity matrix reconstruction methods, and it outperforms the multi-view spectral clustering algorithms in most cases. As exceptional cases, for Handwritten, LMNR performs not as well as Cospectral in terms of ACC; for Webkb, LMNR performs worse than CGC and sM-D in terms of ACC. However, LMNR performs the best on all the datasets in terms of NMI. In addition, LMNR performs slightly better than LMNR- μ , which shows that the strategy of adjusting μ is effective. Overall, the average performance of LMNR is the best in terms of both ACC and NMI. We can conclude that LMNR performs the best among all the tested methods, which is due to integrating original local linear neighbor information from multiple views.

5.5. Label Propagation Results

Fig.3 shows the results of label propagation in terms of ACC and NMI. For each dataset, we randomly label 6% ~ 20% data points. From the results it can be seen that label propagation based on LMNR performs better than label propagation based on the other similarity matrix reconstruction methods, and the multi-view label propagation method, SSM-DML. The performance of LMNR based label propagation increases stably when increasing the percent of labeled data. Increasing the number of labeled data also generally improves the performance of other algorithms, but they increase slower than LMNR. This is because that label propagation mainly depends on the local geometrical structure, which could be represented by local linear neighbors. LMNR not only contains local linear neighbor information but also integrates information from multiple views, while other methods contain only one kind of information.

Table 1. Accuracy (ACC) and normalized mutual information (NMI) on all datasets. The best result of each column is highlighted in boldface.

Algorithm	ACC(%)					NMI(%)				
	Handwritten	Animal	Webkb	M2	Avg.	Handwritten	Animal	Webkb	M2	Avg.
Cos	53.96±0.20	35.59±0.00	51.57±0.50	93.60±0.00	58.68	49.75±0.16	10.44±0.00	31.33±0.36	65.95±0.00	39.37
Cos-concat	65.89±0.08	39.35±0.05	51.54±0.50	54.00±0.00	52.69	58.16±0.18	13.93±0.02	32.57±0.50	1.59±0.00	26.56
Sparse	26.16±2.28	33.88±0.27	54.35±1.93	95.00±0.00	52.35	23.33±1.61	10.63±0.02	34.10±1.68	72.03±0.00	35.02
Sparse-concat	13.89±0.14	39.38±0.00	54.24±1.81	98.20±0.00	51.43	7.85±0.27	13.76±0.00	33.81±0.70	87.14±0.00	35.64
MVSim	28.35±1.02	41.59±0.00	47.85±0.10	96.40±0.00	53.55	27.73±0.60	13.91±0.20	10.87±0.01	78.14±0.00	32.66
Coregspectral	59.02±3.35	38.46±0.63	44.39±3.20	96.00±0.13	59.47	53.96±1.46	13.36±0.10	23.34±2.50	75.89±0.58	41.64
Cospectral	68.99±0.14	41.25±0.02	51.16±6.97	95.20±0.00	64.15	59.95±0.06	12.36±0.00	29.82±6.15	79.57±0.00	45.42
MVSpec	54.35±0.00	36.43±0.00	56.96±0.00	52.00±0.00	49.93	50.45±0.00	11.13±0.00	35.49±0.00	0.45±0.00	24.38
CGC	36.55±2.52	33.74±2.38	62.09±5.36	75.81±5.17	52.05	33.57±2.25	9.73±0.95	35.63±2.51	30.48±9.98	27.35
MvSpecCE	54.65±0.00	36.43±0.00	42.61±0.00	73.60±0.00	51.82	50.10±0.00	10.83±0.00	7.70±0.00	17.77±0.00	21.60
KCCA	26.88±1.08	36.63±0.00	42.61±0.00	62.80±0.00	42.23	25.46±0.14	11.06±0.00	9.78±0.00	14.03±0.00	15.08
sM-D	66.15±3.87	35.95±0.19	62.52±0.79	95.50±0.00	65.03	58.02±1.36	12.31±0.18	38.59±1.77	73.71±0.00	45.66
RMSC	62.28±3.69	40.88±2.08	40.93±4.05	96.80±0.00	60.22	55.74±2.12	14.19±1.42	18.39±3.66	80.97±0.00	42.32
LMNR- μ	64.87±2.41	40.41±0.00	56.11±5.10	99.40±0.00	65.20	62.37±0.84	14.23±0.00	43.37±0.13	94.76±0.00	53.68
LMNR	65.01±1.17	42.41±0.00	56.61±5.03	99.40±0.00	65.86	62.46±0.19	14.23±0.00	43.38±0.13	94.76±0.00	53.71

Fig. 3. Accuracy (ACC) and normalized mutual information (NMI) of label propagation on all datasets. The horizontal axis is the percent of labeled data points and the vertical axis shows the corresponding performances.

6. Related Work

6.1. Multi-view Metric Learning

Multi-view metric learning constructs embedding projections from multi-view data into a shared feature space.

Following the classical principle of pulling samples together if they are related and pushing them apart if they are not, Quadrianto and Lampert [23] constructed Euclidean distance that provided a meaningful within-view as well as between-view similarity. Based on the theory of graph-based semi-supervised learning, Yu et al. [45] learned the multi-view distance metrics from multiple feature sets and the labeled characters. Zhai et al. [46] learned the multi-view metric with part of labeled sets and considered global consistency and local s-smoothness. Cui et al. [11] proposed a pairwise-constrained multiple metric learning method to effectively integrate the face region descriptors of all blocks. Hu et al. [16] maximized the correlations of different feature representations of each sample.

Wu and Shah [34] proposed a class-dependent distance between two multi-view data points by weighting the data in each view upon the classification impact of the specific viewpoint to the corresponding action class. Xie and Xing [36] embedded data of arbitrary modalities into a single latent space, and learned the distance metric by minimizing the distance between similar pairs whereas maximizing the distance between dissimilar.

Spectral clustering is a graph based unsupervised method, so we do not compare it with these Euclidean distance based methods and semi-supervised methods. For label propagation, we choose to compare with a typical multi-view label propagation method [45].

6.2. Multiple Kernel Learning

Multiple kernel learning (MKL) was first proposed in [21], where it considers conic combinations of kernel matrices. Later methods find either a linear combination or a nonlinear combination of base kernels.

Sonnenburg et al. [28] cast the method in [21] into a Semi-Definite Programming problem and used l_1 norm to regularize multiple kernels. Xu et al. [41] introduced smoothness to the optimization procedure. Kowalski et al. [18] involved a misclassification loss term and a regularization term that was expressed by means of a mixed norm. Xu et al. [40] formulated a closed-form solution for optimizing the kernel weights on basis of the equivalence between group-lasso and MKL. Yan et al. [42] generalized multiple kernel Fisher discriminant analysis such that the kernel weights can be regularised with l_p -norm for any $p \geq 1$. Tzortzis and Likas [30] learned a weighted combination of the given kernels in parallel to clustering. Guo et al. [15] learned a linear combination of multiple kernels and determined the coefficient by minimizing the kernel alignment between the learned kernel and the kernel of each view.

The methods in [31, 10] tried to find a polynomial combination of the base kernels. Jagarlapudi et al. [17] employed l_∞ regularization for promoting combinations at the component level and l_1 regularization for promoting sparsity among kernels in each component. Aflalo et al. [1] employed l_1 -norm regularization for promoting sparsity within RKHS norms of each group and l_p -norm, $p \geq 2$, regularization for promoting non-sparse combinations across groups. Bach [2] used l_1 -norm inside the feature space.

7. Conclusion

Similarity matrix is one of the frequently used technique in data analysis. For data with multiple views, it is better to learn a similarity matrix containing information in all views. In this paper, we propose to construct a similarity matrix by using local linear neighbors in multiple views. In each view, the similarity matrix can be reconstructed with the weight of neighbors. In multiple views, we seek for an approximated ground truth similarity matrix which is proportionally consistent with the similarity matrix in each view. The learned similarity matrix can be used for spectral clustering, label propagation and other graph based learning algorithms. Results on several datasets have shown that the learned similarity matrix enhances the performance of both spectral clustering and label propagation.

Acknowledgments

This work was supported by National Science Foundation of China (No. 61272374, 61300190, 61428202) and National High Technology Research and Development Program (863 Program) of China (No. 2015AA015403).

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