Using proportional odds models for semiparametric ROC surface estimation

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Abstract

We propose a semiparametric ROC surface estimation method derived from proportional odds models. Some simulation results show that our proposed method is a sound choice whether the usual normality assumption holds or not. Analysis of a real data set is also provided.

Keywords: ROC curve; ROC surface; VUS; Proportional odds model

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1 Introduction

In clinical practice, clinicians commonly face situations that require a decision among three or more diagnostic alternatives. The multiple test outcomes would render the conventional ROC curve analysis unable to assess the accuracy of the test. Recently, ROC curve analysis was extended to three- and multiple-class medical diagnostic problems; see for example Mossman (1999), Dreiseitl et al. (2000), Heckerling (2001), Nakas and Yiannoutsos (2004), Xiong et al. (2006), Li and Zhou (2009), Wan (2012), and Wan and Zhang (2013). Here, an usual assumption is that subjects from class 3 tend to have higher measurements than subjects from class 2 and the latter tend to have higher measurements than subjects from class 1. In this paper, we try to incorporate order information of the three classes and establish a semiparametric ROC surface analysis method by modeling test results using proportional

odds models. Although our study is focused on three-class diagnostic problems, the proposed method can be naturally generalized to the multiple-class case.

The paper is structured as follows. In Section 2, we propose our methodology for semi-parametric ROC surface estimation. In Section 3, we compare our proposed method to the nonparametric, semiparametric and parametric counterparts via a simulation study. In Section 4, analysis of a real data set is provided.

2 Methodology

Let $X_{k1}, \dots, X_{kn_k}$ denote independent and identically distributed test results from the k th class for $k = 1, 2, 3$. Assume that $\{(X_{k1}, \dots, X_{kn_k}) : k = 1, 2, 3\}$ are jointly independent and that subjects from class 3 tend to have higher test results than those in class 2; the latter tend to have higher test results than subjects in class 1. If F_1 , F_2 , and F_3 represent, respectively, the distribution functions of X_{11} , X_{21} , and X_{31} , Nakas and Yiannoutsos (2004) proposed the following closed-form expression for the ROC surface:

$$R(s_1, s_2) = F_2(F_3^{-1}(1 - s_2)) - F_2(F_1^{-1}(s_1)), \quad 0 \leq s_1, s_2 \leq 1.$$

The volume under the ROC surface denoted by VUS is a summary index of the probability that three test results, one from each class, will be in the correct order. Let $\hat{F}_k(x) = \frac{1}{n_k} \sum_{i=1}^{n_k} I(X_{ki} \leq x)$ be the nonparametric maximum likelihood estimator of $F_k(x)$ for $k = 1, 2, 3$. Then the nonparametric maximum likelihood estimators of $R(s_1, s_2)$ is given by $\hat{R}(s_1, s_2) = \hat{F}_2(\hat{F}_3^{-1}(1 - s_2)) - \hat{F}_2(\hat{F}_1^{-1}(s_1))$. Under normal assumptions, a parametric ROC surface estimator $\bar{R}(s_1, s_2)$ can be naturally constructed as $\bar{R}(s_1, s_2) = \bar{F}_2(\bar{F}_3^{-1}(1 - s_2)) - \bar{F}_2(\bar{F}_1^{-1}(s_1))$ using the parametric estimators for the distribution functions such as $\bar{F}_1(x)$, $\bar{F}_2(x)$, and $\bar{F}_3(x)$.

Proportional odds models are commonly used in modeling multicategory ordinal responses. Let $D = k$ denote the k th class for $k = 1, 2, 3$. For a given test result $X = x$, the proportional odds model has the form

$$\log\left(\frac{P(D \leq m | X = x)}{P(D \geq m + 1 | X = x)}\right) = \alpha_m + \beta^T r(x), \quad m = 1, 2, \quad (1)$$

where α_1 and α_2 are scale parameters, β is a $p \times 1$ vector parameter, and $r(x)$ is a $p \times 1$ smooth function in x . In most applications, $r(x) = x$ or $r(x) = (x, x^2)^T$. Then $F_k(x) = P(X_{k1} \leq x | D = k)$, $k = 1, 2, 3$. Let $f_k(x)$ denote the density functions corresponding to $F_k(x)$ for $k = 1, 2, 3$. Then it follows easily by Bayes' rule that model (1) is equivalent to the following three-sample semiparametric density ratio model in which the unknown density functions $f_k(x)$ and $f_3(x)$ are linked by an "exponential tilt" $\exp\{\theta_k + s_k(x; \eta)\}$:

$$X_{31}, \dots, X_{3n_3} \text{ are i.i.d. with } f_3(x),$$

$$X_{k1}, \dots, X_{kn_k} \text{ are i.i.d. with } f_k(x) = \exp\{\theta_k + s_k(x; \eta)\} f_3(x), \quad k = 1, 2, \quad (2)$$

where $\theta_k = \log[P(D = 3)/P(D = k)]$, $\eta = (\alpha_1, \alpha_2, \beta^T)^T$, and

$$\gamma(x; \alpha_k, \beta) = \frac{\exp(\alpha_k + \beta^T r(x))}{1 + \exp(\alpha_k + \beta^T r(x))}, \quad s_k(x; \eta) = \log \left[\frac{\gamma(x; \alpha_k, \beta) - \gamma(x; \alpha_{k-1}, \beta)}{1 - \gamma(x; \alpha_2, \beta)} \right], \quad k = 1, 2,$$

with $\gamma(x; \alpha_0, \beta) = 0$.

Let $\{T_1, \dots, T_n\}$ denote the pooled sample $\{X_{11}, \dots, X_{1n_1}; X_{21}, \dots, X_{2n_2}; X_{31}, \dots, X_{3n_3}\}$ with $n = n_1 + n_2 + n_3$. Moreover, let $\rho_k = n_k/n_3$ for $k = 1, 2$. Based on the observed data in (2), we can write the semiparametric likelihood function as

$$\begin{aligned} \mathbf{L}(\theta, \eta, F_3) &= \prod_{i=1}^{n_3} dF_3(X_{3i}) \prod_{j=1}^{n_1} \exp\{\theta_1 + s_1(X_{1j}; \eta)\} dF_3(X_{1j}) \prod_{k=1}^{n_2} \exp\{\theta_2 + s_2(X_{2k}; \eta)\} dF_3(X_{2k}) \\ &= \left\{ \prod_{i=1}^n p_i \right\} \left\{ \prod_{j=1}^2 \prod_{k=1}^{n_j} \exp\{\theta_j + s_j(X_{jk}; \eta)\} \right\}, \end{aligned}$$

where $p_i = dF_3(T_i)$ for $i = 1, \dots, n$ are jumps with total mass being unity. Similar to Qin and Zhang (1997), it can be shown by using the method of Lagrange multipliers that the maximum value of $\mathbf{L}$, subject to constraints

$$\sum_{i=1}^n p_i = 1, \quad p_i \geq 0, \quad \sum_{i=1}^n p_i [\exp\{\theta_k + s_k(T_i, \eta)\} - 1] = 0, \quad k = 1, 2,$$

is attained at

$$\tilde{p}_i = \frac{1}{n_3} \frac{1}{1 + \sum_{k=1}^2 \rho_k \exp\{\tilde{\theta}_k + s_k(T_i; \tilde{\eta})\}},$$

where $(\tilde{\theta}_k, \tilde{\eta})$ is the maximum semiparametric likelihood estimator of (θ_k, η) obtained as the solution to the following system of score equations:

$$\frac{\partial l(\theta, \eta)}{\partial \theta_u} = n_u - \sum_{k=1}^n \frac{\rho_u \exp\{\theta_u + s_u(T_k; \eta)\}}{1 + \sum_{j=1}^2 \rho_j \exp\{\theta_j + s_j(T_k; \eta)\}} = 0, \quad (u = 1, 2)$$

$$\frac{\partial l(\theta, \eta)}{\partial \eta} = \sum_{j=1}^2 \sum_{k=1}^{n_j} d_j(X_{jk}; \eta) + \sum_{i=1}^n \frac{\sum_{k=1}^2 \rho_k \exp\{\theta_k + s_k(T_i; \eta)\} d_k(T_i; \eta)}{1 + \sum_{k=1}^2 \rho_k \exp\{\theta_k + s_k(T_i; \eta)\}} = 0, \quad (3)$$

where $d_i(t; \eta) = \partial s_i(t; \eta) / \partial \eta$ for $i = 1, 2$, and $l(\theta, \eta)$ is the profile log-likelihood function given by

$$l(\theta, \eta) \propto \sum_{j=1}^2 \sum_{k=1}^{n_j} [\theta_j + s_j(X_{jk}; \eta)] - \sum_{i=1}^n \log \left\{ 1 + \sum_{k=1}^2 \rho_k \exp\{\theta_k + s_k(T_i; \eta)\} \right\}.$$

Note that for $k = 1, 2$, the constraint $\sum_{i=1}^n p_i [\exp\{\theta_k + s_k(T_i, \eta)\} - 1] = 0$ is equivalent to the constraint $\sum_{i=1}^n p_i \exp\{\theta_k + s_k(T_i, \eta)\} = 1$ and reflects the fact that $\exp\{\theta_k + s_k(T_i, \eta)\} dF_3(t)$ is a distribution function.

As a result, the maximum semiparametric likelihood estimator of $F_3(t)$ is

$$\tilde{F}_3(t) = \sum_{i=1}^n \tilde{p}_i I(T_i \leq t) = \frac{1}{n_3} \sum_{i=1}^n \frac{I(T_i \leq t)}{1 + \sum_{k=1}^2 \rho_k \exp\{\tilde{\theta}_k + s_k(T_i; \tilde{\eta})\}}.$$

Similarly, the maximum semiparametric likelihood estimator of $F_j(t)$, $j = 1, 2$, is

$$\tilde{F}_j(t) = \frac{1}{n_3} \sum_{i=1}^n \frac{\exp\{\tilde{\theta}_j + s_j(T_i; \tilde{\eta})\}}{1 + \sum_{k=1}^2 \rho_k \exp\{\tilde{\theta}_k + s_k(T_i; \tilde{\eta})\}} I(T_i \leq t).$$

On the basis of $(\tilde{F}_1, \tilde{F}_2, \tilde{F}_3)$, we propose under the three-sample density ratio model (2) to estimate the ROC surface $R(s_1, s_2)$ by

$$\tilde{R}(s_1, s_2) = \tilde{F}_2(\tilde{F}_3^{-1}(1 - s_2)) - \tilde{F}_2(\tilde{F}_1^{-1}(s_1)), \quad s_1, s_2 \in [0, 1]$$

and to estimate the volume under $R(s_1, s_2)$ by

$$\widetilde{\text{VUS}} = \int_0^1 \int_0^1 \tilde{R}(s_1, s_2) ds_1 ds_2.$$

According to the invariance property of maximum likelihood estimation, $\tilde{R}(s_1, s_2)$ and $\widetilde{\text{VUS}}$ are, respectively, the maximum semiparametric likelihood estimators of the ROC surface $R(s_1, s_2)$ and its volume VUS under model (2). In practice, the computation of $\widetilde{\text{VUS}}$ does not require numerical integration due to the fact that the semiparametric distribution function estimators $\tilde{F}_k$, $k = 1, 2, 3$ are all step functions. After a simple manipulation, we have the following expression for calculating $\widetilde{\text{VUS}}$ under model (2):

$$\widetilde{\text{VUS}} = \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \tilde{p}_i \tilde{p}_j \tilde{p}_k \exp\{\tilde{\theta}_1 + s_1(T_i; \tilde{\eta})\} \exp\{\tilde{\theta}_2 + s_2(T_i; \tilde{\eta})\} I(T_i \leq T_j \leq T_k).$$

Next we study the issue of statistical inferences on our proposed semiparametric ROC surface estimator $\tilde{R}(s_1, s_2)$. Unfortunately, an analytic expression for $\text{Var}(\tilde{R}(s_1, s_2))$ appears very difficult to obtain. Here we present a bootstrap method for constructing confidence intervals for the ROC surface $\tilde{R}(s_1, s_2)$. Let $X_{11}^*, \dots, X_{1n_1}^*$ be a random sample from $\tilde{F}_1$; in a similar way and independently, we can generate $X_{21}^*, \dots, X_{2n_2}^*$ and $X_{31}^*, \dots, X_{3n_3}^*$ from $\tilde{F}_2$ and $\tilde{F}_3$, respectively. Furthermore, let $T_1^*, \dots, T_n^*$ be the combined bootstrap sample and let $(\tilde{\theta}_k^*, \tilde{\eta}^*; k = 1, 2)$ be the solution of the score equations in (3) with the T_i replaced by the T_i^* . Moreover, let

$$\tilde{F}_3^*(t) = \frac{1}{n_3} \sum_{i=1}^n \frac{I(T_i^* \leq t)}{1 + \sum_{k=1}^2 \rho_k \exp\{\tilde{\theta}_k^* + s_k(T_i^*; \tilde{\eta}^*)\}},$$

and

$$\tilde{F}_j^*(t) = \frac{1}{n_3} \sum_{i=1}^n \frac{\exp\{\tilde{\theta}_j^* + s_j(T_i^*; \tilde{\eta}^*)\}}{1 + \sum_{k=1}^2 \rho_k \exp\{\tilde{\theta}_k^* + s_k(T_i^*; \tilde{\eta}^*)\}} I(T_i^* \leq t), \quad j = 1, 2.$$

Then the corresponding bootstrap version of $\tilde{R}(s_1, s_2)$ is

$$\tilde{R}^*(s_1, s_2) = \tilde{F}_2^*(\tilde{F}_3^{*-1}(1 - s_2)) - \tilde{F}_2^*(\tilde{F}_1^{*-1}(s_1)).$$

We propose to approximate the critical values of $\tilde{R}(s_1, s_2)$ based on the values of $\tilde{R}^*(s_1, s_2)$.

3 A simulation study

In this section, we report a simulation study of the finite sample performance of the proposed semiparametric ROC surface estimator $\tilde{R}(s_1, s_2)$. We compare the nonparametric estimator $\hat{R}(s_1, s_2)$, the semiparametric estimator $\tilde{R}_{Li}(s_1, s_2)$ of Li and Zhou (2009), our proposed semiparametric estimator $\tilde{R}(s_1, s_2)$ and the parametric estimator $\bar{R}(s_1, s_2)$. The parameters of $\tilde{R}_{Li}(s_1, s_2)$ are estimated using method 1 of Li and Zhou (2009).

For our simulation, one case is to assume that $f_i(x)$ is the density function of a $N(\mu_i, \sigma^2)$ distribution for $i = 1, 2, 3$. We consider sample sizes of $(n_1, n_2, n_3) = (30, 30, 30)$ and $(n_1, n_2, n_3) = (60, 60, 60)$. Moreover, we let $\mu_1 = 0$, $\mu_2 = 1$, $\mu_3 = 2$ and $\sigma = 1$. For each pair (n_1, n_2, n_3) , we generate 1000 independent sets of combined random samples from the $N(\mu_i, \sigma^2)$ ($i = 1, 2, 3$) distributions. For the ROC surface estimators of $R(s_1, s_2)$, we consider $(s_1, s_2) \in \{(0.8, 0.5), (0.8, 0.2), (0.5, 0.8), (0.5, 0.2), (0.2, 0.8), (0.2, 0.5)\}$, and the corresponding simulation results are summarized in Table 1. In Table 1, $\text{BS}(\hat{R}(s_1, s_2))$ and

$SE(\hat{R}(s_1, s_2))$ stand for, respectively, the average of 1000 biases of $\hat{R}(s_1, s_2)$ and the sample standard error of 1000 estimates $\hat{R}(s_1, s_2)$; $e(\tilde{R}, \hat{R})$ stands for the relative efficiency of $\tilde{R}(s_1, s_2)$ with respect to $\hat{R}(s_1, s_2)$; other notations are defined in a similar way. From Table 1, all estimators produce biases that are close to 0, and $\tilde{R}(s_1, s_2)$ generally produces a smaller standard error than both $\hat{R}(s_1, s_2)$, $\tilde{R}_{Li}(s_1, s_2)$ and $\bar{R}(s_1, s_2)$ for each (s_1, s_2) though the standard errors of $\tilde{R}(s_1, s_2)$ and $\bar{R}(s_1, s_2)$ are close to each other. As a result, the relative efficiency $e(\tilde{R}, \hat{R})$ is always greater than 1 in Table 1, indicating that our semiparametric approach is consistently better than the nonparametric approach for estimating $R(s_1, s_2)$ when data are normal. In addition, the relative efficiency $e(\tilde{R}, \tilde{R}_{Li})$ is always larger than 1, suggesting that our semiparametric method is slightly superior to $\tilde{R}_{Li}$ in terms of efficiency when the normal assumption is correctly made. Finally, $e(\tilde{R}, \bar{R})$ is sometimes larger than 1 and sometimes smaller than 1, suggesting that our semiparametric estimator is comparable to the parametric one when the normal assumption is correctly made.

We also consider a case against normal assumption, in which we assume that $f_i(x)$ is the density function of an $E(\theta_i)$ distribution for $i = 1, 2, 3$. For our simulation, we consider sample sizes of $(n_1, n_2, n_3) = (30, 30, 30)$ and $(n_1, n_2, n_3) = (60, 60, 60)$. Furthermore, we let $\theta_1 = 1$, $\theta_2 = 2$, $\theta_3 = 3$. For each pair (n_1, n_2, n_3) , we generate 1000 independent sets of combined random samples from the $E(\theta_i)$ ($i = 1, 2, 3$) distributions. The simulation results are summarized in Table 2. From Table 2, large values of $BS(\bar{R}(s_1, s_2))$ indicate that the parametric estimator is biased when the parametric assumption is wrong. In terms of relative efficiency, our proposed semiparametric estimator has the most satisfactory performance since all the values of $e(\tilde{R}, \bar{R})$ are larger than 1 and most values of $e(\tilde{R}, \hat{R})$ and $e(\tilde{R}, \tilde{R}_{Li})$ are larger than 1. It is interesting to notice that our proposed semiparametric estimator and the semiparametric one of Li and Zhou (2009) have similar performance since lots of values of $e(\tilde{R}, \tilde{R}_{Li})$ are around 1. In all, usage of a semiparametric method is a sound choice when the normality assumption is wrong.

4 A real example

We apply our proposed method to analyze a real data set, which was also analyzed by Reaven and Miller (1979) in a study on diabetes. In this data, 145 non-obese adults were clinically classified into three populations, with 76 being normal, 36 diagnosed as chemical diabetic and 33 overt diabetes. We take fasting plasma glucose (PLG) as an example for ROC surface analysis. A log transformation on the data is taken since the original data varies large in magnitude. The semiparametric ROC surface estimator $\tilde{R}_{Li}$ of Li and Zhou (2009) is found to be $\Phi(1.9645 + 1.8188\Phi^{-1}(1 - s_2)) - \Phi(-2.0061 + 1.3676\Phi^{-1}(s_1))$. However, both estimator $\tilde{R}_{Li}$ and parametric estimator $\bar{R}$ are not recommended in this example since the normality assumption does not hold even for the transformed data with a p-value of less than 0.005 for normality test. For model (2), we apply a goodness-of-fit test of Zhang (2004) and find that model (2) holds with a p-value of 0.45. We then fit model (2) and find that the semiparametric estimates are $\tilde{\theta}_1 = 4.0344$, $\tilde{\theta}_2 = -2.2114$, $\tilde{\alpha}_1 = 13.0308$, $\tilde{\alpha}_2 = 10.5246$, $\tilde{\beta} = -3.8570$. For ROC surface estimation, we take $R(0.2, 0.4)$ as an example and find that $\tilde{R}(0.2, 0.4) = 0.9151$, and its 95% confidence interval is (0.8646, 0.9753). The plot of estimated ROC surface is given in Figure 1. As to VUS, we find the semiparametric estimate is $\widetilde{VUS} = 0.6662$, close to the nonparametric estimate 0.7161, and its 95% confidence interval is (0.5788, 0.7730). The fact of VUS larger than 1/6 indicates that PLG is a very useful diagnostic test.

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Table 1 Biases, standard errors and the relative efficiencies of $\hat{R}(s_1, s_2)$, $\tilde{R}_{Li}(s_1, s_2)$, $\tilde{R}(s_1, s_2)$ and $\bar{R}(s_1, s_2)$ when samples are generated from normal distributions.

(s_1, s_2)	(0.8, 0.5)	(0.8, 0.2)	(0.5, 0.8)	(0.5, 0.2)	(0.2, 0.8)	(0.2, 0.5)
$(n_1, n_2, n_3) = (30, 30, 30)$						
$BS(\hat{R}(s_1, s_2))$	0.02372	0.01518	0.00907	-0.00458	-0.00482	-0.00991
$BS(\tilde{R}_{Li}(s_1, s_2))$	-0.00005	-0.01074	0.03007	-0.01503	0.02344	-0.01086
$BS(\tilde{R}(s_1, s_2))$	-0.00273	-0.02502	0.04047	-0.00653	0.04731	0.02261
$BS(\bar{R}(s_1, s_2))$	0.00076	-0.00799	0.01138	-0.02296	0.01429	-0.01129
$SE(\hat{R}(s_1, s_2))$	0.18224	0.14965	0.13437	0.08183	0.12483	0.10992
$SE(\tilde{R}_{Li}(s_1, s_2))$	0.15795	0.13962	0.13302	0.09439	0.11708	0.08947
$SE(\tilde{R}(s_1, s_2))$	0.11945	0.10225	0.12057	0.06545	0.09834	0.05954
$SE(\bar{R}(s_1, s_2))$	0.12180	0.11049	0.11547	0.08245	0.10394	0.07071
$e(\tilde{R}, \hat{R})$	2.366	2.042	1.121	1.552	1.310	3.003
$e(\tilde{R}, \bar{R})$	1.039	1.107	0.832	1.693	0.924	1.264
$e(\tilde{R}, \tilde{R}_{Li})$	1.746	1.770	1.150	2.111	1.197	2.003
$(n_1, n_2, n_3) = (60, 60, 60)$						
$BS(\hat{R}(s_1, s_2))$	-0.00560	-0.00682	-0.02693	-0.02058	-0.00949	-0.00191
$BS(\tilde{R}_{Li}(s_1, s_2))$	-0.01982	-0.02918	0.00023	-0.01964	0.00744	-0.00306
$BS(\tilde{R}(s_1, s_2))$	-0.02178	-0.03681	0.02255	-0.01189	0.03675	0.01735
$BS(\bar{R}(s_1, s_2))$	-0.01688	-0.02309	0.00705	-0.01388	0.01289	-0.00182
$SE(\hat{R}(s_1, s_2))$	0.11745	0.09694	0.09929	0.07047	0.10794	0.08151
$SE(\tilde{R}_{Li}(s_1, s_2))$	0.08941	0.09168	0.08280	0.06087	0.09380	0.06738
$SE(\tilde{R}(s_1, s_2))$	0.07480	0.07898	0.06615	0.04302	0.06659	0.03942
$SE(\bar{R}(s_1, s_2))$	0.06636	0.06700	0.07488	0.04917	0.07490	0.05508
$e(\tilde{R}, \hat{R})$	2.278	1.244	2.167	2.706	2.029	3.584
$e(\tilde{R}, \bar{R})$	0.773	0.661	1.158	1.310	1.118	1.637
$e(\tilde{R}, \tilde{R}_{Li})$	1.382	1.219	1.404	2.054	1.530	2.453

Table 2 Biases, standard errors and the relative efficiencies of $\hat{R}(s_1, s_2)$, $\tilde{R}_{Li}(s_1, s_2)$, $\tilde{R}(s_1, s_2)$ and $\bar{R}(s_1, s_2)$ when samples are generated from exponential distributions.

(s_1, s_2)	(0.8, 0.5)	(0.8, 0.2)	(0.5, 0.8)	(0.5, 0.2)	(0.2, 0.8)	(0.2, 0.5)
$(n_1, n_2, n_3) = (30, 30, 30)$						
$BS(\hat{R}(s_1, s_2))$	0.00901	0.02623	0.00310	-0.00300	-0.02155	-0.04487
$BS(\tilde{R}_{Li}(s_1, s_2))$	-0.01406	0.00845	-0.01917	0.00038	-0.04073	-0.04369
$BS(\tilde{R}(s_1, s_2))$	-0.03781	-0.08434	-0.02370	-0.05525	0.02428	0.03925
$BS(\bar{R}(s_1, s_2))$	0.12637	0.12206	-0.07419	0.02598	-0.13181	-0.02734
$SE(\hat{R}(s_1, s_2))$	0.14967	0.17589	0.12934	0.15338	0.12639	0.14153
$SE(\tilde{R}_{Li}(s_1, s_2))$	0.14678	0.14744	0.13660	0.13393	0.12312	0.11482
$SE(\tilde{R}(s_1, s_2))$	0.10777	0.06468	0.09129	0.05673	0.07446	0.08333
$SE(\bar{R}(s_1, s_2))$	0.11518	0.11806	0.09439	0.09578	0.08369	0.12474
$e(\tilde{R}, \hat{R})$	1.723	2.800	1.882	3.753	2.680	2.598
$e(\tilde{R}, \bar{R})$	2.241	2.553	1.620	1.570	3.974	1.922
$e(\tilde{R}, \tilde{R}_{Li})$	1.667	1.931	2.139	2.860	2.741	1.779
$(n_1, n_2, n_3) = (60, 60, 60)$						
$BS(\hat{R}(s_1, s_2))$	0.00234	-0.01644	-0.01490	-0.02900	-0.02222	-0.01754
$BS(\tilde{R}_{Li}(s_1, s_2))$	0.00007	0.00169	0.00082	-0.00528	-0.01932	-0.02704
$BS(\tilde{R}(s_1, s_2))$	-0.03058	-0.07716	-0.02698	-0.05096	0.02165	0.04425
$BS(\bar{R}(s_1, s_2))$	0.13443	0.12881	-0.07160	0.02681	-0.13407	-0.03003
$SE(\hat{R}(s_1, s_2))$	0.09602	0.07548	0.08485	0.09962	0.08471	0.09884
$SE(\tilde{R}_{Li}(s_1, s_2))$	0.07517	0.06079	0.06254	0.06803	0.04939	0.07782
$SE(\tilde{R}(s_1, s_2))$	0.06604	0.03753	0.05439	0.03499	0.04554	0.05454
$SE(\bar{R}(s_1, s_2))$	0.08452	0.06747	0.05331	0.06417	0.03524	0.11695
$e(\tilde{R}, \hat{R})$	1.742	0.811	2.013	2.817	3.016	2.043
$e(\tilde{R}, \bar{R})$	4.760	2.872	2.162	1.266	7.556	2.956
$e(\tilde{R}, \tilde{R}_{Li})$	1.067	0.502	1.061	1.218	1.106	1.376

Figure 1 Semiparametric ROC surface estimator based on the PLG data.