Anti-disturbance control for nonlinear systems subject to input saturation via disturbance observer [★]

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Abstract

This paper is concerned with the problem of the disturbance observer based control for a class of continuous-time uncertain systems subject to input saturation and nonlinearity. The input of the system includes two parts, the control input and the disturbance input. The nonlinearity of the system, which satisfies a global Lipschitz condition, is considered as two cases of known nonlinearity and unknown nonlinearity. By virtue of the technique of the disturbance observer based controller, the anti-disturbance controllers are designed respectively with both the polytopic and dead-zone representations of the saturation function, which ensure that the resulting closed-loop systems are asymptotically stable with an estimation of the domain of attraction described by the level set of the Lyapunov function. Further, an iterative optimization method is used to obtain the maximum estimation of the domain of the set of initial states. An example of application design for a flight control system illustrates the effectiveness of the proposed results.

Key words: Disturbance uncertainties, disturbance observer based control, disturbance attenuation, input saturation.

1 Introduction

As one kind of uncertainties, external disturbances are often encountered in industrial control systems and can lead to degradation of the system performance, or even instability. Therefore, the study of control for systems with external disturbance uncertainties is of great importance from both theoretical and engineering points of view [1–12]. With the development of high-precision control, some effective disturbance rejection methods have been proposed, such as feedforward compensation control [1, 3, 6, 7, 11–13]. However, a restriction on implementation of those disturbance rejection methods is that most of uncertainties in practical engineering systems cannot be measured. Consequently, the disturbance estimation technique is required in control processes [11].

During the past decades, the problem of disturbance observer based control for nonlinear systems with disturbances has been extensively investigated (see, e.g., [1, 3, 6, 7] and the references therein). In [7], the problem of disturbance observer based control for a class of multi-input multi-output (MIMO) nonlinear systems was studied, where the unknown external disturbances were supposed to be generated by an exogenous system. As further research, a method of composite disturbance observer based control for nonlinear systems with two types of disturbances was proposed in [8], while

the problem of H_∞ control was examined in [10]. Recently, in [13] a composite anti-disturbance control methodology was introduced for Markovian jump systems with nonlinearity and multiple disturbances. It is worth noting that the above-mentioned results are obtained without consideration of constraint on the system input.

Input saturation is a ubiquitous problem in control systems due to the physical impossibility of applying unlimited wide input channel. Meanwhile, input saturation has a great negative impact on the performance and stability of a control system. Recently, some results of stability analysis and control synthesis of the systems subject to input saturation have appeared in the literature [14–28]. In [14], a semi-global low-and-high gain design technique for a class of linear systems with input saturation and disturbances was developed, where the external disturbances and the control input used the same channel with saturating constraint. In [29], by expressing the saturated linear feedback on a convex hull of a group of linear feedback laws, the conditions were proposed for determining whether a given ellipsoid was contractively invariant for a linear system under a saturated linear feedback, followed by further development of analysis and design methods for the closed-loop system with disturbances. The problem of robust exponential convergence for uncertain discrete-time time-delayed systems with input saturation and disturbances was tackled in [23] by designing a state feedback controller, while a similar problem was investigated in the continuous-time domain in [21]. To the best of the authors' knowledge, most of the existing results about the controller design for the systems subject to input saturation and disturbance uncertainties are based on the feedback control methods, which are not applicable to the control problem of the systems subject to saturated input affected by disturbances. How to solve the control problem for the systems with saturated input including two parts (i.e., control

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input and disturbance input) is the motivation for this paper.

In this paper, we consider the control problem of a class of nonlinear systems subject to input saturation and external disturbance uncertainties from a new perspective, namely, via disturbance observer based control (DOBC). Two cases of known and unknown nonlinearities satisfying a global Lipschitz condition are investigated, respectively, and both the polytopic and dead-zone representation are used to obtain stabilization criteria. It is assumed that external disturbances satisfy the matching condition, that is, the disturbances enter into the system via the same channel as the control input [2, 14]. With disturbance observers being first designed, attention is then focused on constructing the disturbance observer based controllers for the nonlinear uncertain systems subject to input saturation such that there exists an initial condition domain ensuring that for every initial condition from this domain, the composite closed-loop system combined with estimation error of disturbances is asymptotically stable. Moreover, the disturbance observer based controllers can be designed by using an iterative optimization algorithm so as to make an initial domain of attraction as large as possible. Finally, a simulation example for a flight control system demonstrates the efficacy of the proposed method.

The remainder of the paper is organized as follows. In Section 2, we give some preliminaries and a formulation of the problem under consideration. Section 3 and Section 4 present the main results of the paper based on polytopic bounding and sector bounding, respectively. In Section 5, a simulation example is provided to illustrate the usefulness of the proposed technique. Finally, concluding remarks are given in Section 6.

2 Preliminaries

2.1 System description

Consider a class of uncertain systems subject to saturating input and nonlinearity as follows

$$\dot{x}(t) = Ax(t) + Ff(x(t)) + B\text{sat}(u(t) + d(t)), \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^m$ is the control input, $d(t) \in \mathbb{R}^m$ is the unknown disturbance input, and A , B and F are known real constant matrices. $\text{sat}(\cdot)$ is the standard saturation function defined by

$$\text{sat}(\cdot) = [\text{sat}(\cdot)_1 \cdots \text{sat}(\cdot)_m]^T,$$

where $\text{sat}(\cdot)_i = \text{sign}(\cdot) \min(|\cdot|, 1)$ with $\text{sign}(\cdot)$ being the signum function. By slight abuse of notation, we use $\text{sat}(\cdot)$ to denote both the scalar valued and the vector valued saturation functions, and assume the unity saturation level.

The function $f(x(t)) : \mathbb{R}^n \rightarrow \mathbb{R}^l$ stands for the nonlinearity of the system and satisfies the following assumption.

Assumption 1 $f(0) = 0$, and $f(x(t))$ satisfies the global Lipschitz condition with respect to x :

$$\|f(x_1) - f(x_2)\| \leq \|U(x_1 - x_2)\|, \quad \forall x_1, x_2 \in \mathbb{R}^n,$$

where U is a known matrix with appropriate dimension.

Remark 1 In [14], the linear system comprising a chain of integrators subject to input saturation was considered. In this paper, we study a class of nonlinear systems described by (1) which can include the system in [14] as a special case, so the system (1) is very general. On the other hand, due to

consideration of the input saturation constraint, the system under investigation herein has more practical significance when compared with the system in [7].

Remark 2 In [9], the problem of saturating composite disturbance-observer-based control for discrete-time system was investigated. However, the saturation constraint is added onto the feedback input which is a part of the system input. Different from [9], here we consider that the system is subjected to system input saturation.

In this paper, the disturbance input is assumed to be generated by the following dynamic system

$$\begin{aligned} \dot{\omega}(t) &= W\omega(t), \\ d(t) &= V\omega(t), \end{aligned} \quad (2)$$

where $\omega(t) \in \mathbb{R}^r$ is the disturbance system state, and W and V are known real constant matrices with appropriate dimension.

Remark 3 The disturbances generating by system (2) have been extensively investigated and may contain many types of disturbances in practical engineering (see [7, 31] and the references therein). When W is chosen as

$$W = \begin{bmatrix} 0 & c \\ -c & 0 \end{bmatrix}, \quad c > 0,$$

$d(t)$ is a harmonic disturbance with the frequency c .

In the subsequent study, we consider the system (1) and the disturbance dynamic system (2) under the following assumption.

Assumption 2 (A, B) is controllable and (W, V) is observable.

Lemma 1 ([30]) Let D , S and F be real matrices of appropriate dimensions with $F^T F \leq I$. Then, for any scalar $\lambda > 0$, we have

$$DFS + (DFS)^T \leq \lambda^{-1}DD^T + \lambda S^T S.$$

2.2 Approximations of the saturation function

In this subsection, we will present two classical approximations of the saturation.

2.2.1 Polytopic bounding

Let E be the set of $m \times m$ diagonal matrices with elements being either 1 or 0. There are 2^m elements in E . Suppose that each element of E is labeled as E_i , $i \in Q = \{1, \dots, 2^m\}$, and denote $E_i^- = I - E_i$. Obviously, E_i^- is also an element of E . Furthermore, for a family of scalars η_i satisfying $0 \leq \eta_i \leq 1$ and $\sum_{i=1}^{2^m} \eta_i = 1$, it is easy to get that

$$\sum_{i=1}^{2^m} \eta_i (E_i + E_i^-) = I, \quad \forall E_i \in E, \quad i \in 1, 2, \dots, 2^m. \quad (3)$$

For further discussion, we denote an ellipsoid for a positive definite matrix $P \in \mathbb{R}^{n \times n}$ by

$$\Omega(P) = \{x \in \mathbb{R}^n : x^T P x \leq 1\} \quad (4)$$

and a symmetric polyhedron by

$$L(H) = \{x \in \mathbb{R}^n : |H^l x| \leq 1, l \in Q_m\}, \quad (5)$$

where $Q_m = \{1, \dots, m\}$ and H^l is the l th row of the matrix $H \in \mathbb{R}^{m \times n}$.

Now let us recall the convex hull representation of a saturated linear feedback law.

Lemma 2 ([15]) Let $K, H \in \mathbb{R}^{m \times n}$ be given. For an $x \in \mathbb{R}^n$, if $x \in L(H)$, then

$$\text{sat}(Kx) \in \text{co}\{E_i Kx + E_i^- Hx, i \in Q\},$$

where $\text{co}(\cdot)$ denotes the convex hull of a set.

2.2.2 Sector bounding

Denote a dead-zone nonlinear vector function as $\psi(v) = v - \text{sat}(v)$. For given matrices K and H , define a polyhedral set as

$$L(K, H) = \{x \in \mathbb{R}^n : |(K^l - H^l)x| \leq 1, l \in \{1, 2, \dots, m\}\}, \quad (6)$$

where K^l and H^l are the l th row of the matrices K and H , respectively.

Lemma 3 ([16]) Consider the function $\psi(v)$ defined above. If $x \in L(K, H)$, then the relation

$$\psi^T(Kx)T[\psi(Kx) - Hx] \leq 0,$$

holds for any diagonal and positive definite matrix $T \in \mathbb{R}^{m \times m}$.

3 Results Based on Polytopic Bounding Approach

In this section, we will design the disturbance observer based controllers for the system (1) with known nonlinearity and unknown nonlinearity, respectively, by the polytopic bounding approach, such that the resulting closed-loop systems are asymptotically stable at the equilibrium point. Meanwhile, the corresponding domain of attraction of the closed-loop systems in the above-mentioned two cases of nonlinearities will also be estimated.

3.1 The case of known nonlinearity

First, we consider the case that $f(x(t))$ is the known nonlinear vector valued function satisfying Assumption 1. In this case, the disturbance observer is designed as

$$\begin{aligned} \hat{d}(t) &= V\hat{\omega}(t), \\ \hat{\omega}(t) &= \nu(t) - Lx(t), \\ \dot{\nu}(t) &= (W + LBV)(\nu(t) - Lx(t)) \\ &\quad - L[-Ax(t) - Bu(t) - Ff(x(t))]. \end{aligned} \quad (7)$$

Based on (7), a disturbance observer based controller (DOBC) is designed in the following form as

$$u(t) = -\hat{d}(t) + Kx(t). \quad (8)$$

Let us denote the disturbance estimation error by $e_\omega(t) = \omega(t) - \hat{\omega}(t)$ and introduce an extended matrix as

$$\xi(t) = \begin{bmatrix} x(t) \\ e_\omega(t) \end{bmatrix}.$$

By using Lemma 2 and combining (2), (3) with (5), for $\forall \xi(t) \in L(H)$ with $H = [H_1, V] \in \mathbb{R}^{m \times (n+r)}$, the satura-

tion $\text{sat}(u(t) + d(t))$ can be expressed as

$$\begin{aligned} \text{sat}(u(t) + d(t)) &= \text{sat}((K, V)\xi(t)) \\ &= \sum_{i=1}^{2^m} \eta_i [(E_i(K, V)\xi(t) + E_i^- H\xi(t))] \\ &= \sum_{i=1}^{2^m} \eta_i [(E_i(Kx(t) + Ve_\omega(t)) \\ &\quad + E_i^-(H_1x(t) + Ve_\omega(t)))] \\ &= \sum_{i=1}^{2^m} \eta_i (E_i K + E_i^- H_1)x(t) + Ve_\omega(t), \end{aligned} \quad (9)$$

where $\sum_{i=1}^{2^m} \eta_i = 1, 0 \leq \eta_i \leq 1$.

Based on the above discussion and the fact of (3), for any $\xi(t) \in L(H)$, the dynamic of the disturbance estimation error is given by

$$\begin{aligned} \dot{e}_\omega(t) &= (W + LBV)e_\omega(t) \\ &\quad + \sum_{i=1}^{2^m} \eta_i LBE_i^-(H_1 - K)x(t), \end{aligned} \quad (10)$$

and further, the closed-loop system can be described by

$$\dot{\xi}(t) = \tilde{A}\xi(t) + \tilde{F}f(\xi(t)), \quad \forall \xi(t) \in L(H), \quad (11)$$

where

$$\begin{aligned} \tilde{A} &= \begin{bmatrix} A + \sum_{i=1}^{2^m} \eta_i B(E_i K + E_i^- H_1) & BV \\ \sum_{i=1}^{2^m} \eta_i LBE_i^-(H_1 - K) & W + LBV \end{bmatrix}, \\ \tilde{F} &= \begin{bmatrix} F \\ 0 \end{bmatrix}, \quad f(\xi(t)) = f(x(t)). \end{aligned}$$

Remark 4 The construction of the closed-loop system (11) is based on the expression of the saturating input in the local region. That is to say, like in [15, 17, 23, 29], the results of this paper are not global either. Because of this, we need to make an estimation of the domain of the initial states such that the domain is included in the set $L(H)$ and the trajectories of the closed-loop system (11) from the domain of the initial states always stay in the set $L(H)$.

Now we are in a position to present the first main result of the paper in the following theorem.

Theorem 1 If there exist positive definite matrices $X > 0, P_e > 0$, matrices $R_i, i = 1, 2, 3$ and scalar $\lambda > 0$ satisfying the following conditions

$$\begin{bmatrix} -\lambda I & UX & 0 \\ * & \Lambda_{1i} + \lambda FF^T & \Lambda_{3i} \\ * & * & \Lambda_2 \end{bmatrix} < 0, \quad (12)$$

$$\begin{bmatrix} 1 & R_2^l & V^l \\ * & X & 0 \\ * & * & P_e \end{bmatrix} \geq 0, \quad (13)$$

where R_2^l and V^l are the l th row of the matrix R_2 and V , respectively, and

$$\begin{aligned}\Lambda_{1i} &= He\{AX + BE_i R_1 + BE_i^- R_2\}, \\ \Lambda_2 &= He\{P_e W + R_3 B V\}, \\ \Lambda_{3i} &= B V + [R_3 B E_i^- (R_2 - R_1)]^T,\end{aligned}$$

with the notation $He\{\cdot\} = \cdot + \cdot^T$, then the closed-loop system (11) under the disturbance observer (7) and the DOBC (8) is asymptotically stable with $H_1 = R_2 X^{-1}$, gain $K = R_1 X^{-1}$ and gain $L = P_e^{-1} R_3$. Moreover, an estimate of the domain of attraction of the system (11) is given by $\Omega(\bar{P})$, where $\bar{P} = \text{diag}\{X^{-1}, P_e\}$.

Proof. First of all, we set $X = P^{-1} > 0, P_e > 0, R_1 = KX, R_2 = H_1 X, R_3 = P_e L$ that satisfy the linear matrix inequalities (LMIs) (12) and (13).

To proceed, a Lyapunov function candidate is chosen as

$$V(\xi(t)) = \xi(t)^T \bar{P} \xi(t). \quad (14)$$

For $\xi(t) \in L(H), H = [H_1, V]$, the time-derivative of $V(\xi(t))$ along the solution of (11) is given by

$$\begin{aligned}\dot{V}(\xi(t)) &= 2\xi^T(t) \bar{P} \dot{\xi}(t) \\ &= 2\xi^T(t) \bar{P} [\bar{A}\xi(t) + \bar{F}f(\xi(t))] \\ &= \xi^T(t) He\{\bar{P}\bar{A}\}\xi(t) + 2\xi^T(t) \bar{P} \bar{F} f(\xi(t)).\end{aligned} \quad (15)$$

By Assumption 1 and Lemma 1, there exists $\lambda > 0$ such that

$$\begin{aligned}\dot{V}(\xi(t)) &\leq \xi^T(t) He\{\bar{P}\bar{A}\}\xi(t) + \lambda \xi^T(t) \bar{P} \bar{F} \bar{F}^T \bar{P} \xi(t) \\ &\quad + \frac{1}{\lambda} f^T(\xi(t), t) f(\xi(t), t) \\ &\leq \xi^T(t) \begin{bmatrix} \varphi_{11} & \varphi_{12} \\ * & He\{P_e W + P_e L B V\} \end{bmatrix} \xi(t) \\ &\leq \max_{i \in Q} \{\eta_i \xi^T(t) \Omega_i \xi(t)\},\end{aligned} \quad (16)$$

where

$$\begin{aligned}\varphi_{11} &= \sum_{i=1}^{2^m} \eta_i He\{PA + PB(E_i K + E_i^- H_1)\} \\ &\quad + \lambda P F F^T P + \frac{1}{\lambda} U^T U, \\ \varphi_{12} &= \sum_{i=1}^{2^m} \eta_i \{P B V + [P_e L B E_i^- (H_1 - K)]^T\} \\ \Omega_i &= \begin{bmatrix} \Delta_i & P B V + [P_e L B E_i^- (H_1 - K)]^T \\ * & He\{P_e W + P_e L B V\} \end{bmatrix}\end{aligned}$$

with $\Delta_i = He\{PA + PB(E_i K + E_i^- H_1)\} + \lambda P F F^T P + \frac{1}{\lambda} U^T U$.

Pre-and post-multiplying (12) by $\text{diag}\{I, P, I\}$ and using the Schur complement, it is easy to get that

$$\Omega_i < 0. \quad (17)$$

Combining (17) with (16), we can deduce that

$$\dot{V}(\xi(t)) < 0, \quad \forall \xi(t) \in L(H) \text{ with } \xi(t) \neq 0. \quad (18)$$

At the next step, we prove $\Omega(P) \subset L(H)$ with $H = [H_1, V]$, if (13) is satisfied. Pre- and post-multiplying (13) by $\text{diag}\{I, P, I\}$, we have

$$\begin{bmatrix} 1 & H_1^l & V^l \\ * & P & 0 \\ * & * & P_e \end{bmatrix} \geq 0. \quad (19)$$

From [15], it follows that (19) is equivalent to $\Omega(P) \subset L(H)$. That is to say, for any $\xi(t) \in \Omega(P)$, we have $\xi(t) \in L(H)$.

Finally, the closed-loop system (11) is asymptotically stable with $\Omega(\bar{P})$ as an estimate of the domain of attraction. This completes the proof. $\blacksquare$

3.2 The case of unknown nonlinearity

For the case of the system (1) with unknown nonlinearity, the disturbance observer with an auxiliary dynamics is designed in the following form as

$$\begin{aligned}\hat{d}(t) &= V \hat{\omega}(t), \\ \hat{\omega}(t) &= \nu(t) - Lx(t), \\ \dot{\nu}(t) &= (W + L B V)(\nu(t) - Lx(t)) \\ &\quad - L[-Ax(t) - Bu(t)].\end{aligned} \quad (20)$$

Then, $\forall \xi(t) \in L(H)$, the corresponding dynamic of the disturbance estimation error may be described by

$$\begin{aligned}\dot{e}_\omega(t) &= (W + L B V)e_\omega(t) \\ &\quad + \sum_{i=1}^{2^m} \eta_i L B E_i^- (K - H_1)x(t) - L F f(\xi(t)).\end{aligned} \quad (21)$$

Combining (21) with the system (1) and the DOBC (8), the resulting closed-loop system is formulated by

$$\dot{\xi}(t) = \tilde{A}\xi(t) + \bar{F}f(\xi(t)), \quad \forall \xi(t) \in L(H), \quad (22)$$

where $\tilde{A}$ is the same as in (11) and

$$\bar{F} = \begin{bmatrix} F \\ -L F \end{bmatrix}. \quad (23)$$

Now, we provide the conditions in the following theorem for designing the controller gains under which the closed-loop system (22) is asymptotically stable with an estimation of the domain of initial states.

Theorem 2 *If there exist positive definite matrices $X > 0, P_e > 0$, matrices $R_i, i = 1, 2, 3$ and scalar $\lambda > 0$ satisfying the following conditions (13) and*

$$\begin{bmatrix} -\lambda I & 0 & \lambda F^T & -\lambda F^T R_3^T \\ * & -\lambda I & U X & 0 \\ * & * & \Lambda_{1i} & \Lambda_{3i} \\ * & * & * & \Lambda_2 \end{bmatrix} < 0, \quad (24)$$

where $\Lambda_j, j = 1, 2, 3$ are as defined in Theorem 1, then the closed-loop system (22) under the disturbance observer

(20) and the DOBC (8) is asymptotically stable with $H_1 = R_2 X^{-1}$, gain $K = R_1 X^{-1}$ and gain $L = P_e^{-1} R_3$. Moreover, an estimate of the domain of attraction of the system (22) is given by $\Omega(\bar{P})$, where $\bar{P} = \text{diag}\{X^{-1}, P_e\}$.

Proof. The proof follows a similar procedure to the proof of Theorem 1. Noticing the difference between $\tilde{F}$ and $\bar{F}$, we can deduce from (15) that, $\forall \xi(t) \in L(H)$,

$$\begin{aligned}\dot{V}(\xi(t)) &= 2\xi^T(t)\bar{P}\dot{\xi}(t) \\ &= 2\xi^T(t)\bar{P}[\tilde{A}\xi(t) + \bar{F}f(\xi(t))] \\ &= \xi^T(t)He\{\bar{P}\tilde{A}\}\xi(t) + 2\xi^T(t)\bar{P}\bar{F}f(\xi(t)) \\ &\leq \xi^T(t)He\{\bar{P}\tilde{A}\}\xi(t) + \lambda\xi^T(t)\bar{P}\bar{F}\bar{F}^T\bar{P}\xi(t) \\ &\quad + \frac{1}{\lambda}f(\xi(t), t)^T f(\xi(t), t) \\ &\leq \xi^T(t) \begin{bmatrix} \psi_{11} & \psi_{12} \\ * & \psi_{22} \end{bmatrix} \xi(t) \\ &\leq \max_{i \in Q} \{\eta_i \xi^T(t) \Omega_i \xi(t)\},\end{aligned}\quad (25)$$

where

$$\begin{aligned}\psi_{11} &= \sum_{i=1}^{2^m} \eta_i He\{PA + PB(E_i K + E_i^- H_1)\} \\ &\quad + \lambda P F F^T P + \frac{1}{\lambda} U^T U, \\ \psi_{12} &= PBV - \lambda P F F^T L^T P_e \\ &\quad + \sum_{i=1}^{2^m} \eta_i [P_e L B E_i^- (H_1 - K)]^T, \\ \psi_{22} &= He\{P_e W + P_e L B V\} + \lambda P_e L F F^T L^T P_e, \\ \Pi_i &= \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ * & He\{P_e W + P_e L B V\} + \lambda P_e L F F^T L^T P_e \end{bmatrix}\end{aligned}\quad (26)$$

with

$$\begin{aligned}\Theta_{11} &= He\{PA + PB(E_i K + E_i^- H_1)\} \\ &\quad + \lambda P F F^T P + \frac{1}{\lambda} U^T U, \\ \Theta_{12} &= PBV + [P_e L B E_i^- (H_1 - K)]^T - \lambda P F F^T L^T P_e.\end{aligned}$$

Set $X = P^{-1}$ and note that $K = R_1 X^{-1}$, $H_1 = R_2 X^{-1}$ and $L = P_e^{-1} R_3$. Then pre- and post-multiplying (24) by $\text{diag}\{\lambda^{-1} I, I, P, I\}$ yields

$$\begin{bmatrix} -\frac{1}{\lambda} I & 0 & F^T P & -F^T L^T P_e \\ * & -\lambda I & U & 0 \\ * & * & \tilde{\Lambda}_1 & PBV + [P_e L B E_i^- (H_1 - K)]^T \\ * & * & * & \tilde{\Lambda}_2 \end{bmatrix} < 0, \quad (27)$$

where

$$\begin{aligned}\tilde{\Lambda}_1 &= He\{PA + P B E_i K + P B E_i^- H_1\}, \\ \tilde{\Lambda}_2 &= He\{P_e W + P_e L B V\}.\end{aligned}\quad (28)$$

Comparing the notation Θ_{11} in (26) with $\tilde{\Lambda}_1$ in (28), we have

$$\theta_{11} = \tilde{\Lambda}_1 + \lambda P F F^T P + \frac{1}{\lambda} U^T U.$$

By using the Schur complement, (27) is equivalent to

$$\Pi_i < 0. \quad (29)$$

Therefore, from (25) and (29), the following inequality can be ensured:

$$\dot{V}(\xi(t)) < 0, \quad \forall \xi(t) \in L(H) \text{ with } \xi(t) \neq 0. \quad (30)$$

According to the proof of Theorem 1, we know that the condition (13) can ensure that $\Omega(\bar{P}) \subset L(H)$. Hence, the closed-loop system (22) is asymptotically stable with $\Omega(\bar{P})$ as an estimate of the domain of attraction under the conditions (24) and (13). This completes the proof. ■

Remark 5 In this subsection, the polytopic approach is applied to represent the saturation function. The conditions in Theorem 1 and Theorem 2 are not in the form of standard LMIs, and they are actually bilinear matrix inequalities (BLMIs). However, the BLMIs can be solved if we fix the matrix R_3 beforehand by the results in [7] or the matrices R_1 and R_2 by the method in [29]. Therefore, the results in Theorem 1 and Theorem 2 can be seen as the compensatory design for the system in [7] subject to input saturation or the systems in [29] subject to input disturbances.

When the conditions of Theorem 1 and Theorem 2 are satisfied, we can obtain the estimates of the domain of attraction of the closed-loop systems with known nonlinearity and unknown nonlinearity, respectively, under the DOBC. An implicit objective of this paper is to maximize the estimation of the domain of initial states associated with the closed-loop system. In order to achieve this goal, we present the following four-step iterative algorithm:

Algorithm 1:

- Step 1:** Initialization: Choose the appropriate matrix $R > 0$ and the appropriate nonzero initial matrices R_3 .
- Step 2:** Solve inequalities (12) and (13) for R_1 and R_2 .
- Step 3:** Use R_1 and R_2 obtained from Step 2 to solve the following problem for R_3 and γ :

$$\begin{aligned} & \inf_{X>0, P_e>0, R_3, \lambda>0, \gamma>0} \gamma \\ \text{s.t. a)} & \begin{bmatrix} \gamma R & I \\ I & X \end{bmatrix} \geq 0, \\ & \text{b) inequalities (12) and (13) hold.} \end{aligned}$$

Step 4: If there is no further improvement about the scalar γ (e.g., $|\gamma_{\text{new}} - \gamma_{\text{old}}| < \varepsilon$, where ε is a very small positive number), stop the iteration; otherwise, go to Step 5.

Step 5: Use R_3 obtained from Step 3 and go back to Step 2.

Remark 6 In implementation of Algorithm 1, the initial value of the matrix R_3 may be chosen based on the results in [7], as mentioned in Remark 5. The matrix R should be a positive definite matrix and it can be set as an identity matrix I generally. Note that R remains unchanged during the iteration process. On the other hand, for the case of the

system with unknown nonlinearity, we just need to change the conditions (12) into (24) in the above algorithm and include the scalar λ into the iteration process.

4 Results Based on Sector Bounding Approach

Parallel to the preceding section, we now consider the DOBC design by the sector bounding approach.

4.1 The case of known nonlinearity

For the case of known nonlinearity for system (1), the disturbance observer based controller is still designed as the form of (21) with the disturbance observer (20). Based on the vector function $\psi(\cdot)$ defined before, the closed-loop system (1) under the controller (21) is equivalent to

$$\dot{\xi}(t) = \bar{A}\xi(t) + \bar{F}f(\xi(t)) + \bar{E}\psi(\bar{K}\xi(t)), \quad (31)$$

where

$$\bar{A} = \begin{bmatrix} A + BK & BV \\ 0 & W + LBV \end{bmatrix}, \quad \bar{E} = \begin{bmatrix} -B \\ -LB \end{bmatrix},$$

$$\bar{K} = [K \ V].$$

Theorem 3 Given $\alpha > 0$, if there exist positive definite matrices $X > 0, P_e > 0$, matrices $H_2, R_i, i = 1, 2, 3$ and scalar $\lambda > 0$ satisfying the following conditions

$$\begin{bmatrix} -\lambda I & UX & 0 & 0 \\ * & \Psi_{22} + \lambda F F^T & BV & -\alpha B + R_2^T \\ * & * & \Psi_{33} - \alpha R_3 B + H_2^T \\ * & * & * & -2\alpha I \end{bmatrix} < 0, \quad (32)$$

$$\begin{bmatrix} X & 0 & R_1^l T - R_2^l T \\ * & P_e & V^l T - H_2^l T \\ * & * & 1 \end{bmatrix} \geq 0, \quad (33)$$

where R_1^l, R_2^l, H_2^l and V^l are the l th row of the matrix R_1, R_2, H_2 and V , respectively, and

$$\Psi_{22} = He\{AX + BR_1\},$$

$$\Psi_{33} = He\{P_e W + R_3 BV\},$$

then the closed-loop system (31) under the disturbance observer (7) and the DOBC (8) is asymptotically stable with $H_1 = R_2 X^{-1}$, gain $K = R_1 X^{-1}$ and gain $L = P_e^{-1} R_3$. Moreover, an estimate of the domain of attraction of the system (31) is given by $\Omega(\bar{P})$, where $\bar{P} = \text{diag}\{X^{-1}, P_e\}$.

Proof. Set $X = P^{-1}$, $K = R_1 X^{-1}$, $H_1 = R_2 X^{-1}$, $L = P_e^{-1} R_3$ and $H = [H_1 \ H_2]$ and consider the same Lyapunov functional (14). For $\xi(t) \in L(\bar{K}, H)$, the derivative of $V(\xi(t))$ along the states for the closed-loop system (31) can be deduced as

$$\begin{aligned} \dot{V}(\xi(t)) &= 2\xi^T(t) \bar{P} \dot{\xi}(t) \\ &\leq 2\xi^T(t) \bar{P} [\bar{A}\xi(t) + \bar{F}f(\xi(t)) + \bar{E}\psi(\bar{K}\xi(t))] \\ &\quad - 2\psi^T(\bar{K}\xi(t)) T (\psi(\bar{K}\xi(t)) - H\xi(t)) \\ &\leq 2\xi^T(t) \bar{P} [\bar{A}\xi(t) + \bar{F}f(\xi(t)) + \bar{E}\psi(\bar{K}\xi(t))] \\ &\quad - 2\psi^T(\bar{K}\xi(t)) T (\psi(\bar{K}\xi(t)) - H\xi(t)). \end{aligned} \quad (34)$$

Using Lemma (1) and Assumption (1), for scalar $\lambda > 0$, one has

$$\begin{aligned} \dot{V}(\xi(t)) &\leq 2\xi^T(t) \bar{P} [\bar{A}\xi(t) + \bar{E}\psi(\bar{K}\xi(t))] \\ &\quad - 2\psi^T(\bar{K}\xi(t)) T (\psi(\bar{K}\xi(t)) - H\xi(t)) \\ &\quad + \lambda \xi^T \bar{P} \bar{F} \bar{F}^T \bar{P} \xi + \frac{1}{\lambda} f^T(\xi(t)) f(\xi(t)) \\ &\leq 2\xi^T(t) \bar{P} [\bar{A}\xi(t) + \bar{E}\psi(\bar{K}\xi(t))] \\ &\quad - 2\psi^T(\bar{K}\xi(t)) T (\psi(\bar{K}\xi(t)) - H\xi(t)) \\ &\quad + \lambda \xi^T \bar{P} \bar{F} \bar{F}^T \bar{P} \xi + \frac{1}{\lambda} \xi^T(t) \bar{U}^T \bar{U} \xi(t), \end{aligned} \quad (35)$$

where $\bar{U} = [U \ 0]$.

Pre- and post-multiplying (32) by $\text{diag}\{I, P, I, T\}$ with $T = \frac{I}{\alpha}$ and using the Schur complement, we have

$$\Xi = \begin{bmatrix} \bar{\Psi}_1 & P B V & -P B + H_1^T T \\ * & \Psi_{33} & -R_3 B + H_2^T T \\ * & * & -2T \end{bmatrix} < 0, \quad (36)$$

where

$$\bar{\Psi}_1 = He\{PA + PBK\} + \lambda P F F^T P + \frac{1}{\lambda} U^T U.$$

Combining (34) with (36), for any $\xi \in L(\bar{K}, H)$, it is easy to get that

$$\dot{V}(t) \leq \zeta^T(t) \Xi \zeta(t) < 0, \quad \forall \zeta(t) \neq 0, \quad (37)$$

where $\zeta(t) = [x^T(t) \ e_\omega^T(t) \ \psi(\bar{K}\xi(t))]^T$.

On the other hand, following the process above and on the basis of the result in [16], (33) can guarantee that

$$\Omega(\bar{P}) \subset L(\bar{K}, H). \quad (38)$$

Noting that (38), we can get that $\Omega(\bar{P})$ is a compact invariant set, namely, the trajectories of the states of system (31), when starting from $\Omega(\bar{P})$, always stay in this domain and asymptotically converge to the equilibrium point of the system. This completes the proof. ■

4.2 The case of unknown nonlinearity

Similar to the process in Section 3, the design conditions of the DOBC for the case of unknown nonlinearity can be generalized. When the nonlinearity is unknown, we consider the controller as (8) with the disturbance observer (20). In this case, the closed-loop system can be described by

$$\dot{\xi}(t) = \bar{A}\xi(t) + \bar{F}f(\xi(t)) + \bar{E}\psi(\bar{K}\xi(t)), \quad (39)$$

where the matrices $\bar{A}$, $\bar{F}$ and $\bar{E}$ have been defined above.

Theorem 4 Given $\alpha > 0$, if there exist positive definite matrices $X > 0, P_e > 0$, matrices $H_2, R_i, i = 1, 2, 3$ and

scalar $\lambda > 0$ satisfying (33) and the following condition

$$\begin{bmatrix} -\lambda I & 0 & \lambda F^T & -\lambda F^T R_3^T & 0 \\ * & -\lambda I & UX & 0 & 0 \\ * & * & \Psi_{22} & BV & -\alpha B + R_2^T \\ * & * & * & \Psi_{33} & -\alpha R_3 B + H_2^T \\ * & * & * & * & -2\alpha I \end{bmatrix} < 0, \quad (40)$$

where Ψ_{22} and Ψ_{33} are defined in Theorem 3, then the closed-loop system (31) under the disturbance observer (7) and the DOBC (8) is asymptotically stable with $H_1 = R_2 X^{-1}$, gain $K = R_1 X^{-1}$ and gain $L = P_e^{-1} R_3$. Moreover, an estimate of the domain of attraction of the system (31) is given by $\Omega(\bar{P})$, where $\bar{P} = \text{diag}\{X^{-1}, P_e\}$.

Proof. The proof can follow the similar processes of Theorem 2 as well as Theorem 3, thus is omitted here. ■

In what follows, we will present an iterative algorithm to design a stabilizing DOBC for system (1) with known nonlinearity such that the estimation of the domain of initial states is as large as possible.

Algorithm 2:

- Step 1:** Initialization: Choose the appropriate matrix $R > 0$ and the appropriate initial scalar $\alpha > 0$.
Step 2: Solve inequalities (32) and (33) for R_3 .
Step 3: Use R_3 obtained from Step 2 to solve the following problem for α and γ :

$$\begin{aligned} & \inf_{X>0, R_i, i=1,2,3, \lambda>0, \gamma>0} \gamma \\ \text{s.t. a)} & \begin{bmatrix} \gamma R & I \\ I & X \end{bmatrix} \geq 0, \\ & \text{b) inequalities (32) and (33) hold.} \end{aligned}$$

- Step 4:** If $|\gamma_{\text{new}} - \gamma_{\text{old}}| < \varepsilon$, where ε is a very small positive number, stop the iteration; otherwise, go to Step 5.
Step 5: Use α obtained from Step 3 and go back to Step 2.

Remark 7 In Algorithm 2, the initial value of the positive scalar α is usually chosen as $\alpha = 1$, while the selection of the positive definite matrix R is the same as discussed in Remark 6. For the case of with unknown nonlinearity, we can give a similar iterative algorithm by just replacing (32) with (40) and adding the scalar λ in the iteration process.

Remark 8 For linear systems subject to input saturation, it is known that the treatment of the polytopic representation of saturation nonlinearity can generally lead to less conservative results than description of the saturation nonlinearity as a local sector bound nonlinearity with different multipliers [27]. However, for the system under study in this paper, there arises a different situation. The reason for this is that, just as mentioned in Remark 5, the key parameter matrices R_1 , R_2 and R_3 are coupled in block Λ_{3i} of the conditions (12) and (24), while these three matrices are independent of each other in the conditions of Theorem 3 and Theorem 4. Since the use of BLMI may increase conservativeness, the results in Theorem 3 and 4 are less conservative than the results in Theorem 1 and Theorem 2 to some extent. This may be viewed as an advantage of the sector bounding approach over the polytopic bounding approach for the system

considered in this paper.

5 Numerical Example

For testing and verifying the applicability of the proposed results, we choose a A4D aircraft model subject to actuator saturation at a flight condition of 15000 *ft* altitude and 0.9 *Mach*. This system can be modeled as follows [7,14,32,33]:

$$\dot{x}(t) = Ax(t) + Ff(x(t)) + B\text{sat}(u(t) + d(t)), \quad (41)$$

where $x_1(t)$ is the forward velocity (*ft s*⁻¹), $x_2(t)$ is the angle of attack (*rad*), $x_3(t)$ is the pitching velocity (*rad s*⁻¹), $x_4(t)$ is the pitching angle (*rad*), u is the elevator deflection (*deg*), and

$$A = \begin{bmatrix} -0.0605 & 32.37 & 0 & 32.2 \\ -0.00014 & -1.475 & 1 & 0 \\ -0.0111 & -34.72 & -2.793 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad (42)$$

$$B = [0 \quad -0.1064 \quad -33.8 \quad 0]^T.$$

Here, a harmonic disturbance signal $d(t)$ is supposed to satisfy the dynamic (2) with

$$W = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}, \quad V = \begin{bmatrix} 25 \\ 0 \end{bmatrix}^T. \quad (43)$$

In what follows, based on our results, we will design DOBC controllers respectively for systems (41) with a known nonlinearity given as

$$F_1 = [0 \ 0 \ 50 \ 0]^T, \quad f(x(t)) = \sin(2\pi 5t)x_2(t), \quad (44)$$

and an unknown nonlinearity given as

$$F_2 = [0 \ 0 \ 10 \ 0]^T, \quad f(x(t)) = \text{Rand}(t)x_2(t), \quad (45)$$

where $\text{Rand}(t) \in [0, 50]$ is the random input.

Remark 9 The system (41) is a nominal linearized longitudinal dynamics of the A4D aircraft. In this system, the (3,2)*th* entry of the system matrix represents the change in pitching moment with varying angle of attack, which is referred to as the “longitudinal static stability derivative” and is a very important parameter in aircraft longitudinal dynamics [32]. In this paper, we denote the parameter as a nonlinear uncertainty $f(t, x(t))$, and the simulations for both cases are made by using our derived results.

First, by utilizing the proposed iterative Algorithm 1, we can get $\gamma = 1.7136$ and

$$K = [0.0030 \ 0.1230 \ 0.1170 \ 0.4425],$$

$$L = \begin{bmatrix} 0 & 0.0004 & 0.0053 & 0 \\ 0 & 0.0006 & 0.0669 & 0 \end{bmatrix}. \quad (46)$$

It can be observed from Figure 1 that the DOBC (8) can stabilize the system (41) subject to the known nonlinearity (44), disturbances and input saturation. Figure 2 shows the efficiency and accuracy of the disturbance observer (7).

Next, we consider the system (41) with unknown nonlinearity (45). Applying the proposed iterative algorithm, we design the related gains as follows

$$K = [0.0007 \ 0.0667 \ 0.0930 \ 0.4951],$$

$$L = \begin{bmatrix} 0 & 0.0011 & 0.0293 & 0 \\ 0 & 0.0017 & 0.0639 & 0 \end{bmatrix}, \quad (47)$$

and the scalar $\gamma = 1.7359$. The trajectories of the states of the closed-loop system (22) with nonlinearity (45) are displayed in Figure 3. Figure 4 shows the trajectory of the disturbance estimation error.

Then, the DOBC controllers are designed by the results based on the sector bounding representation via Algorithm 2. The corresponding design parameters can be obtained, respectively, for the case of with known nonlinearity (44) as $\gamma = 1.9758$,

$$K = [0.0118 \ 0.7257 \ 0.2239 \ 0.8514],$$

$$L = \begin{bmatrix} 0 & 0.0000 & 0.0152 & 0 \\ 0 & 0.0000 & 0.0035 & 0 \end{bmatrix}, \quad (48)$$

and for the case of unknown nonlinearity (45) as $\gamma = 1.1158$,

$$K = [0.0084 \ -0.3243 \ 0.0429 \ 0.5841],$$

$$L = \begin{bmatrix} 0 & 0.1036 & 0.0717 & 0 \\ 0 & 0.1950 & 0.0108 & 0 \end{bmatrix}. \quad (49)$$

Figure 5 and Figure 6 display the trajectories of the states of the closed-loop system (31) with nonlinearity (44) and that of the estimation error of the input disturbance, respectively. These two trajectories for the system (39) with nonlinearity (45) are plotted in Figure 7 and Figure 8, respectively.

Finally, from the simulation results it can be observed that the disturbance observer based controller designed with Algorithm 2 is more efficient than the one designed with Algorithm 1 to some extent in terms of faster convergence and smaller overshoot to achieve stabilization.

6 Conclusion

In this paper, we have developed two kinds of efficient methods to design the disturbance observer based controllers for a class of nonlinear systems subject to disturbance uncertainties and input saturation with the polytopic approach and the sector bounding approach. Two cases of nonlinearities in the system have been considered respectively, corresponding to the known and unknown nonlinear functions satisfying a global Lipschitz condition. With some parameters fixed in advance, the sufficient conditions in the LMI framework have been obtained, under which the disturbance observer gain and the DOBC gain can be designed such that the resulting closed-loop system is asymptotically stable with an estimation of the domain of attraction. The corresponding iterative algorithms have also been proposed to obtain the maximal estimation of the domain of attraction. Finally, a simulation example has been presented to demonstrate the usefulness of the derived results.

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Fig. 1. States of the system with nonlinearity (44) under controller (8) based on disturbance observer (7) with gains (46).

Fig. 2. Estimation error of the input disturbances under the disturbances observer (7) with gains (46).

Fig. 3. States of the system with nonlinearity (45) under controller (8) based on disturbance observer (20) with gains (47).

Fig. 4. Estimation error of the input disturbances under the disturbances observer (20) with gains (47).

Fig. 5. States of the system with nonlinearity (44) under controller (8) based on disturbance observer (7) with gains (48).

Fig. 6. Estimation error of the input disturbances under the disturbances observer (7) with gains (48).

Fig. 7. States of the system with nonlinearity (45) under controller (8) based on disturbance observer (20) with gains (49).

Fig. 8. Estimation error of the input disturbances under the disturbances observer (20) with gains (49).

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