Effects of crack surface electrostatic tractions on the fracture behaviour of magnetoelectric composite materials

Aibing Zhang^{a, *}, Baolin Wang^b

^a Piezoelectric Device Laboratory, School of Mechanical Engineering & Mechanics, Ningbo
University, Ningbo, 315211, P.R. China

^b Institute for Infrastructure Engineering, Western Sydney University, Penrith, NSW 2751,
Australia

Abstract

The fracture mechanics problem of a Griffith crack embedded in a two-dimensional magnetoelectric composite material subjected to coupling mechanical, electric and magnetic loads at infinity is investigated. The crack surfaces electrostatic tractions are taken into account and the crack is assumed to be electrically and magnetically semi-permeable. Explicit and closed form solutions of electric displacement and magnetic induction inside the crack, stress, electric displacement and magnetic induction intensity factors are derived based on the extended Stroh formalism and continuous distribution of generalized dislocation approach. Numerical computations are also carried out to illustrate the influence of electrostatic tractions on the crack tip field when the crack interior is filled with different dielectric medium. It is found that the electrostatic tractions on the crack surfaces have the tendency to close the crack thus retard the crack propagation, and the traditional traction-free crack model always overestimates the effect of applied magnetoelectric loads on the crack tip field intensity factors. Crack surfaces electrostatic tractions cannot be neglected for large applied electric or magnetic load to mechanical load ratio, and small dielectric constant and magnetic permeability of the crack interior.

* Corresponding author. Tel.: +86 754 87600980
E-mail address: zhangaibing@nbu.edu.cn (Aibing Zhang).

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1. Introduction

Magnetolectric composite materials made of piezoelectric and piezomagnetic phase have been found extensively applications in modern industries such as sensors, actuators, transducers, hydrophones, electric packaging and acoustic/ultrasonic devices due to coupling mechanical, electric and magnetic effect (Alshits et al., 1992; Huang and Kuo, 1997; Nan, 1994). However, because of their brittleness and low strength, cracks and flaws in such composites inevitably occur either during manufacturing process or when subjected to mechanical, electric and magnetic loads. Thus, fracture mechanics problems in magnetolectric composite materials have been received tremendous interest. Green's functions for an infinite two-dimensional magnetoelctroelastic medium containing an elliptical cavity or a crack were derived based on the extended Stroh formalism and complex function method by Liu et al. (2001). Crack initiation behavior in magnetolectric composite materials under in-plane deformation was studied by Song and Sih (2003). The interaction problem of multiple arbitrarily oriented and distributed cracks in magnetoelctroelastic materials was considered by Tian and Gabbert (2004). Green's functions for the various defects embedded in an infinite magnetoelctroelastic material under thermal loading were obtained with the framework of in-plane magnetoelctroelastic interactions by Qin (2005). García-Sánchez et al. (2007) analysed that the behavior of cracked linear magnetoelctroelastic materials by using the dual boundary element method approach. Recently, to eliminate the singularity behavior on the crack tips, a pre-fracture zone on electrically impermeable and magnetically permeable interface crack model between two dissimilar magnetoelctroelastic materials was developed by Ma et al.

(2013). The solutions for electrically and magnetically impermeable crack branching out of the crack plane in a magnetoelectric composite materials under thermo-electro-magneto-mechanical loads were presented by Zhang and Wang (2014a). Hu et al. (2015) investigated interface crack problem by extending the concept of Dugdale crack model and Yoffe model to a moving crack between two dissimilar magnetoelastoelectric materials. For more contributions in the literature on the fracture mechanics of magnetoelectric composite materials, see Wang and Shen (2002), Gao et al. (2003), Sih and Song (2003), Qin (2004), Soh and Liu (2005), Ootao and Tanigawa (2005), Tupholme (2009), Rojas-Díaz et al. (2012), Hu et al. (2014), Sladek et al. (2015), Bhargava et al. (2015), Rogowski (2015) and Ma et al. (2015).

In above-mentioned works, four different boundary conditions on the crack surfaces, there are electrically impermeable and magnetically permeable, electrically permeable and magnetically impermeable, electrically and magnetically impermeable and electrically and magnetically permeable are assumed to investigate the fracture mechanics problem in magnetoelectric composites. Wang and Mai (2007) discussed the applicability of the crack-face electromagnetic boundary conditions, and pointed out that the electric and magnetic permeabilities of air or vacuum inside the crack cannot be ignored. Taking electromagnetic properties of crack interior into consideration and using magnetoelectric boundary conditions at the crack face related to crack opening displacement, the fracture mechanics problem of magnetoelastoelectric solid was solved by Zhong and Li (2007). To date, the crack surfaces stress boundary conditions of most magnetoelectric composites crack problems are still considered to be free, which is identical with the typical assumption for the fracture mechanics analysis of purely elastic materials. Generally, an opened

crack filled with air or vacuum represents an obstacle against an electric field perpendicular to the crack plane, since the dielectric constant of air or vacuum is at least three orders of magnitude smaller than that of magnetoelectric composites. Electric field that passes the crack gap causes a normal electric potential jump across the crack and induces electrostatic stresses that pull the crack surfaces together (McMeeking and Landis, 2005; Neumeister et al., 2013). The electrostatic tractions acting upon crack faces should be introduced in piezoelectric medium fracture evaluation together with the electrically semi-permeable crack model, especially when the mechanical loads is not too large and the electric displacement loads is not too small (Ricoeur and Kuna, 2009). The axisymmetric problem of a penny-shaped crack embedded in an infinite three-dimensional piezoelectric body is considered by Li et al. (2011). The effect of electrostatic tractions on the fracture behavior of a piezoelectric material under mechanical and/or electric loading is analyzed by Xie et al. (2014) and Zhang and Wang (2014b).

However, in contrast to the piezoelectric materials, very few papers can be found on the influence of crack surfaces electrostatic tractions on magnetoelectric composite materials fracture mechanics problem. On the other hand, due to the pronounced piezoelectric, piezomagnetic and magnetoelectric properties, the electric displacement on the crack surfaces does not vanish and has a non-negligible magnitude even if only a mechanical (or magnetic induction) loads is applied at infinity to a magnetoelectric composite material with a crack filled with air/vacuum. Therefore, the primary objective of this paper is to investigate effect of crack surfaces electrostatic tractions on the fracture behaviour of magnetoelectric composites. The plan of the paper is as follows. The extended Stroh formalism is outlined in Section 2. In Section 3 the explicit solutions of stress, electric displacement and magnetic induction intensity

factors are derived for magnetoelectric composite materials problem. Some numerical results are presented in Section 4, and concluding remarks are made in Section 5.

2. The Stroh formalism

Consider a linear magnetoelectroelastic material in which all fields are assumed to depend only on the in-plane coordinates x_1 and x_2 . The basic equations (constitutive, divergence and gradient) of the linear magnetoelectric composite solids without any body force, electric charge and electric current are (Liu et al., 2001):

$$\begin{aligned}\sigma_{ij} &= c_{ijkl}u_{k,l} - e_{ij}\phi_{,l} - h_{ij}\varphi_{,l} \\ D_i &= e_{ikl}u_{k,l} + \kappa_{il}\phi_{,l} + \alpha_{il}\varphi_{,l} \\ B_i &= h_{ikl}u_{k,l} + \alpha_{il}\phi_{,l} + \mu_{il}\varphi_{,l}\end{aligned}\quad (1)$$

$$\sigma_{ij,i} = 0, D_{i,i} = 0, B_{i,i} = 0 \quad (2)$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), E_i = -\phi_{,i}, H_i = -\varphi_{,i} \quad (3)$$

In Eqs. (1)-(3), a comma denotes partial differentiation and repeated indices mean summation. σ_{ij} , ε_{ij} , D_i , E_i , B_i and H_i are elastic stress, strain, electric displacement, electric field, magnetic induction and magnetic field, respectively. u_i , ϕ and φ are elastic displacement, electric potential and magnetic potential, respectively. c_{ijkl} , e_{ikl} , h_{ikl} , α_{ij} , κ_{ij} and μ_{ij} are the elastic moduli, piezoelectric coefficients, piezomagnetic coefficients, magnetoelectric coefficients, dielectric constants and magnetic permeability coefficients, respectively.

For two-dimensional deformations, the general solution to the Eq. (1) can be written as

$$\mathbf{u} = \mathbf{A}\mathbf{f}(z) + \overline{\mathbf{A}\mathbf{f}(z)} \quad (4)$$

where $\mathbf{u} = [u_1, u_2, u_3, \phi, \varphi]^T$, $\mathbf{f}(z) = \text{diag}[f(z_1), f(z_2), f(z_3), f(z_4), f(z_5)]$ is arbitrary

analytic function, $z_i = x_1 + p_i x_2$, the superscript “T” and the overbars stand for transposition of a matrix and complex conjugation, respectively. p_i and $\mathbf{A}$ are constants determined by

$$\left[\mathbf{Q} + p_i(\mathbf{R} + \mathbf{R}^T) + p_i^2 \mathbf{T} \right] \mathbf{A}_i = 0 \quad (5)$$

in which $\mathbf{Q}$, $\mathbf{R}$, and $\mathbf{T}$ are defined by

$$\mathbf{Q} = \begin{bmatrix} c_{1ik1} & e_{1j1} & h_{1j1} \\ e_{1k1}^T & -\kappa_{11} & -\alpha_{11} \\ h_{1k1}^T & -\alpha_{11} & -\mu_{11} \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} c_{1ik2} & e_{2j1} & h_{2j1} \\ e_{1k2}^T & -\kappa_{12} & -\alpha_{12} \\ h_{1k2}^T & -\alpha_{12} & -\mu_{12} \end{bmatrix}, \quad \mathbf{T} = \begin{bmatrix} c_{2ik1} & e_{2j2} & h_{2j2} \\ e_{2k2}^T & -\kappa_{22} & -\alpha_{22} \\ h_{2k2}^T & -\alpha_{22} & -\mu_{22} \end{bmatrix}$$

The stress, electric displacement and magnetic induction obtained by inserting Eqs. (3) and (4) into Eq. (1) can be expressed in terms of the generalized stress function vector Φ as

$$\begin{aligned} \mathbf{t}_1 &= [\sigma_{11}, \sigma_{12}, \sigma_{13}, D_1, B_1]^T = -\Phi_{,2} \\ \mathbf{t}_2 &= [\sigma_{21}, \sigma_{22}, \sigma_{23}, D_2, B_2]^T = \Phi_{,1} \end{aligned} \quad (6)$$

where

$$\Phi = \mathbf{B}\mathbf{f}(z) + \overline{\mathbf{B}\mathbf{f}(z)} \quad (7)$$

$$\begin{aligned} \mathbf{B} &= \mathbf{R}^T \mathbf{A} + \mathbf{T} \mathbf{A} \mathbf{P} \\ \mathbf{P} &= \text{diag}[p_1, p_2, p_3, p_4, p_5] \end{aligned} \quad (8)$$

3. Fracture mechanics solution of magnetoelectric composite materials by considering crack surface electrostatic tractions

Consider a generalized 2D magnetoelectric composite material problem of a single crack of length $2a$ and filled with air or vacuum, as shown in Fig. 1. The opening displacement of the crack is always very small, the electric displacement and magnetic induction through crack should not be zero. Therefore, it is more reasonable

to consider the electric and magnetic fields inside the crack and the electric and magnetic potential jumps across the crack surfaces simultaneously, that is magnetoelectric semi-permeable crack faces boundary condition. The remote loading system is composed of stresses σ_{21}^∞ , σ_{22}^∞ and σ_{23}^∞ , electric displacement D_2^∞ and magnetic induction B_2^∞ , i.e.,

$$\mathbf{t}_2^\infty = [\sigma_{21}^\infty, \sigma_{22}^\infty, \sigma_{23}^\infty, D_2^\infty, B_2^\infty]^T \quad (9)$$

The faces of a non-conducting crack in a dielectric material are interfaces between a solid and air, vacuum or any other insulation matter. If either an electric loading is imposed or the material exhibits physical properties coupling magnetical/mechanical and electrical fields, such as piezoelectrics and magnetoelectrics, electrostatic tractions and related displacements are induced at the crack faces. Even a comparatively small crack opening stress induces a crack tip loading which might not be small compared to the low fracture toughness of ceramic materials. Therefore, electrostatic tractions are very essential for assessment of strength for piezoelectric or magnetoelectric materials, and they acting on the crack surfaces have a following form (Ricoeru and Kuna, 2009)

$$\sigma_{22}^c = \frac{1}{2} \frac{(D_2^c)^2}{\kappa_c} \left(1 - \frac{\kappa_c}{\kappa_{22}} \right) \quad (10)$$

where κ_c is the permittivity of air, vacuum or others dielectric medium filling the crack, D_2^c is the electric displacement inside the crack to be determined, calculated from the capacitor analogy model (Hao and Shen, 1994)

$$D_2^c = -\kappa_c \frac{\varphi^+ - \varphi^-}{u_2^+ - u_2^-} \quad (11)$$

The superscript “+” and “-” refer to the upper and lower main crack surfaces, respectively.

It is noted that the crack may be breakdown electrically by the mechanism of corona discharges under the applied mechanical and electrical loadings if the dielectric constant of the magnetoelectric composite is much higher than that of the crack interior. In this case $|D_2^c| \geq \kappa_c E_d$ (where E_d is the critical electric field level for discharge), the electrostatic tractions acting on the crack surfaces and electric displacement inside the crack are given by Landis (2004) from the viewpoint of energetically equivalent as

$$\sigma_{22}^c = E_d |\omega_d| + \frac{1}{2} \kappa_c E_d^2 \quad (12)$$

$$D_2^c = \kappa_c \operatorname{sgn}(D_2^c) E_d + \omega_d \quad (13)$$

where ω_d represents the amount of charge per unit area transferred between the crack surfaces. The purpose of this paper is to exam the effect of crack surface electrostatic tractions on the fracture behaviour of magnetoelectric composite materials for the case of $|D_2^c| \leq \kappa_c E_d$. Electrical breakdown of the crack is out of the scope of this research. Therefore the phenomenon of electrical discharge will not be taken into consideration in this paper.

In order to consider the magnetic effect of the medium inside the crack, it is more realistic to adopt the opening crack model (Gao et al., 2004)

$$B_2^c = -\mu_c \frac{\varphi^+ - \varphi^-}{u_2^+ - u_2^-} \quad (14)$$

In Eq. (14), B_2^c denotes the magnetic induction normal to the crack faces, and μ_c stands for the magnetic permeability of the medium inside the crack.

By the superposition principle, the crack surfaces boundary conditions for this problem can be expressed as

$$\begin{aligned} \mathbf{t}_2(x_1, x_2) &= -[\mathbf{t}_2^\infty - \mathbf{t}_2^c], \quad |x_1| \leq a, x_2 = 0 \\ \mathbf{t}_1 &= \mathbf{t}_2 = 0, \quad \text{at infinity} \end{aligned} \quad (15)$$

where $\mathbf{t}_2^c = [0, \sigma_{22}^c, 0, D_2^c, B_2^c]^T$.

A magneto-electroelastic crack can be constructed by continuous distribution of generalized dislocation (i.e., a mechanical dislocation, an electric dipole and a magnetic dipole at the same point) technique with appropriate weight function. The solution of single generalized dislocation in magnetoelectric composites has the similar expression with that in anisotropic materials obtained by Suo (1990). Stress, electric displacement and magnetic induction fields on the plane $x_2=0$ of a generalized dislocation located at coordinate origin have the following form

$$\mathbf{t}_2(x_1, 0) = \frac{1}{\pi} \mathbf{d} \left(\frac{1}{x_1} \right) \quad (16)$$

where $\mathbf{Y} = \mathbf{L} + \bar{\mathbf{L}}$, $\mathbf{L} = i\mathbf{A}\mathbf{B}^{-1}$, $\mathbf{b} = \mathbf{Y}\mathbf{d}$ and $\mathbf{b} = [b_1, b_2, b_3, b_4, b_5]^T$ is the Burgers vector. Enforcing the satisfaction of the boundary conditions (15) on the crack surfaces, a singular integral equation is obtained by distributing generalized dislocation of the preceding form with weight $\mathbf{d}(\xi)$ in the interval $|x_1| \leq a$ as

$$\int_{-a}^a \frac{\mathbf{d}(\xi)}{\pi(x_1 - \xi)} d\xi + \mathbf{t}_2^\infty - \mathbf{t}_2^c = 0, \quad |x_1| < a \quad (17)$$

The single value condition of the displacement, electric potential and magnetic potential around a closed contour surrounding the whole crack requires that

$$\int_{-a}^a \mathbf{d}(\xi) d\xi = 0, \quad |x_1| < a \quad (18)$$

Eqs. (17) and (18) have well known solution

$$\mathbf{d}(\xi) = [\mathbf{t}_2^\infty - \mathbf{t}_2^c] \frac{\xi}{\sqrt{a^2 - \xi^2}}, \quad |\xi| \leq a \quad (19)$$

Then the crack opening displacement, the jump of electric potential and magnetic potential can be obtained

$$\Delta \mathbf{u} = \int_{x_1}^a \mathbf{b}(\xi) d\xi = \mathbf{Y} [\mathbf{t}_2^\infty - \mathbf{t}_2^c] \sqrt{a^2 - x_1^2} \quad (20)$$

From Eqs. (11) and (14), we have

$$D_2^c = -\kappa_c \frac{Y_{42}(\sigma_{22}^\infty - \sigma_{22}^c) + Y_{44}(D_2^\infty - D_2^c) + Y_{45}(B_2^\infty - B_2^c)}{Y_{22}(\sigma_{22}^\infty - \sigma_{22}^c) + Y_{24}(D_2^\infty - D_2^c) + Y_{25}(B_2^\infty - B_2^c)} \quad (21)$$

$$B_2^c = -\mu_c \frac{Y_{52}(\sigma_{22}^\infty - \sigma_{22}^c) + Y_{54}(D_2^\infty - D_2^c) + Y_{55}(B_2^\infty - B_2^c)}{Y_{22}(\sigma_{22}^\infty - \sigma_{22}^c) + Y_{24}(D_2^\infty - D_2^c) + Y_{25}(B_2^\infty - B_2^c)} \quad (22)$$

Eq. (21) can be reduced to

$$\alpha_3 (D_2^c)^3 + \alpha_2 (D_2^c)^2 + \alpha_1 D_2^c + \alpha_0 = 0 \quad (23)$$

with

$$\alpha_3 = \frac{Y_{22}}{2\kappa_c} \left(1 - \frac{\kappa_c}{\kappa_{22}} \right), \alpha_2 = Y_{24} + \frac{Y_{42}}{2} \left(1 - \frac{\kappa_c}{\kappa_{22}} \right), \alpha_1 = -[Y_{22}\sigma_{22}^\infty + Y_{24}D_2^\infty + Y_{25}(Y_{24}D_2^\infty - Y_{24}B_2^c) - \kappa_c Y_{44}],$$

$$\alpha_0 = -\kappa_c [Y_{42}\sigma_{22}^\infty + Y_{44}D_2^\infty + Y_{45}(B_2^\infty - B_2^c)].$$

Similarly, Eq. (22) can be reduced to

$$\beta_2 (B_2^c)^2 + \beta_1 B_2^c + \beta_0 = 0 \quad (24)$$

with

$$\beta_2 = Y_{25}, \beta_1 = -\left\{ Y_{22} \left[\sigma_{22}^\infty - \frac{(D_2^c)^2}{2\kappa_c} \left(1 - \frac{\kappa_c}{\kappa_{22}} \right) \right] + Y_{24}(D_2^\infty - D_2^c) + Y_{25}B_2^\infty \right\},$$

$$\beta_0 = -\mu_c \left\{ Y_{52} \left[\sigma_{22}^\infty - \frac{(D_2^c)^2}{2\kappa_c} \left(1 - \frac{\kappa_c}{\kappa_{22}} \right) \right] + Y_{54}(D_2^\infty - D_2^c) + Y_{55}B_2^\infty \right\}.$$

Eq. (23) is a cubic equation and its general solution can be found from mathematics

handbook. For reasonable material constants and loading conditions, Eq. (23) has

three roots, but only one root is physically acceptable. In this paper, the solution has

the following form:

$$D_2^c = -\frac{\alpha_2}{3\alpha_3} + \frac{-1 - \sqrt{3}i}{2} \sqrt[3]{P + \sqrt{P^2 + Q^3}} + \frac{-1 + \sqrt{3}i}{2} \sqrt[3]{P - \sqrt{P^2 + Q^3}} \quad (25)$$

where P and Q are two known real constants depending on the applied loads, the

material constants, dielectric constants and the magnetic permeability of the medium inside the crack as formulated below:

$$P = \frac{\alpha_2 \alpha_1}{6\alpha_3^2} - \frac{\alpha_2^3}{27\alpha_3^3} - \frac{\alpha_0}{2\alpha_3}, \quad Q = \frac{\alpha_1}{3\alpha_3} - \frac{\alpha_2^2}{9\alpha_3^2} \quad (26)$$

It follows from Eq. (24) that the magnetic induction at the crack surfaces is

$$B_2^c = -\frac{\beta_1 + \sqrt{\beta_1^2 - 4\beta_2\beta_0}}{2\beta_2} \quad (27)$$

The stress, electric displacement and magnetic induction field on the plane $x_2=0$ are obtained by Eqs. (17) and (19)

$$\mathbf{t}_2(x_1, 0) = [\mathbf{t}_2^\infty - \mathbf{t}_2^c] \left[\frac{x_1}{\sqrt{x_1^2 - a^2}} - 1 \right] \quad (28)$$

Defining the field intensity factor vector

$$\mathbf{K} = [K_{II}, K_I, K_{III}, K_D, K_B]^T = \lim_{x_1 \rightarrow a^+} \sqrt{2\pi(x_1 - a)} \mathbf{t}_2 \quad (29)$$

Inserting Eq. (29) into Eq. (28) gives

$$\mathbf{K} = [\mathbf{t}_2^\infty - \mathbf{t}_2^c] \sqrt{\pi a} \quad (30)$$

4. Numerical results and discussion

Magnetoelectric coupling effect has been observed in single-phase multiferroic material at low temperature where simultaneous electric and magnetic ordering coexists, and in two phase composites where the participating phases are piezoelectric and piezomagnetic. Although over ten different compound families have been widely studied as multiferroic magnetoelectric materials, a high inherent coupling (especially above room temperature) has not yet been found in the single-phase compound, which hinders their applications (Nan et al., 2008). However, magnetoelectric composites made by combining piezoelectric and magnetic substances together can produce a

large magnetoelectric response at the room temperature (Nan, 1994). The magnetoelectric effect in composite materials is result of the product of the mechanical/electrical effect in the piezoelectric phase and the magnetic/mechanical effect in the piezomagnetic one. This is a coupled electrical and magnetic phenomenon via elastic interaction. It is noted that the applied constants of magnetoelectric material are controversy in the literature, and a complete set of material properties of single-phase multiferroic materials, such as BiFeO₃, YMnO₃, Cr₂O₃, LuFe₂O₃, etc., cannot be found. In this paper, the magnetoelectric composite material is made of piezoelectric phase BaTiO₃ and magnetostrictive phase CoFe₂O₄, and the ratio of BaTiO₃ to CoFe₂O₄ is roughly 0.5:0.5. Hence, the fundamental purpose of this paper is to present an analytical method to investigate the influence of crack surfaces electrostatic tractions on crack tip fields in the magnetoelectric medium. The variation tendency of intensity factors under applied magnetoelectroelastic loading is correct although the material properties may not be accuracy enough. The material constants are taken as (Sih and Chen, 2003),

Elastic constants:

$$c_{11} = 226\text{GPa}, c_{13} = 124\text{GPa}, c_{33} = 216\text{GPa}, c_{44} = 44\text{GPa}$$

Piezoelectric constants:

$$e_{31} = -2.2\text{C/m}^2, e_{33} = 9.3\text{C/m}^2, e_{15} = 5.8\text{C/m}^2$$

Dielectric constants:

$$\kappa_{11} = 56.4 \times 10^{-10} \text{C}^2/\text{Nm}^2, \quad \kappa_{33} = 63.5 \times 10^{-10} \text{C}^2/\text{Nm}^2$$

Piezomagnetic constants:

$$h_{31} = 290.2\text{N/Am}, h_{33} = 350\text{N/Am}, h_{15} = 275\text{N/Am}$$

Magnetic permeability constants:

$$\mu_{11} = 297 \times 10^{-6} \text{ N s}^2 / \text{C}^2, \quad \mu_{33} = 83.5 \times 10^{-6} \text{ N s}^2 / \text{C}^2$$

and all the absent material constants equal to zero. The dielectric constant and magnetic permeability of air or vacuum are, respectively, $\kappa_0 = 0.0885 \times 10^{-10} \text{ C}^2 / \text{N m}^2$ and $\mu_0 = 1.256 \times 10^{-6} \text{ N s}^2 / \text{C}^2$.

4.1 Effect of applied electric loads on crack tip field

In Figs. 2 and 3 normalized stress intensity factor K_I/K_{I0} is plotted versus the applied electric loads D_2^∞ for different dielectric constant and magnetic permeability of the crack interior, where $K_{I0} = 10\sqrt{\pi a} \text{ MN/m}^{3/2}$. The applied mechanical loads and magnetic induction are chosen to be $\sigma_{22}^\infty = 10 \text{ MPa}$ and $B_{22}^\infty = 100 \text{ N/(Am)}$, respectively. It can be found that the value of K_I/K_{I0} obtained is smaller than that without considering the crack surfaces electrostatic tractions, and it increases firstly then decreases with the applied electric loads. For $D_2^\infty = 0$ the stress on the crack surfaces emanates from the coupling feature between mechanical and electric field. The related electric field is compensated increasing D_2^∞ , which leads to a condition where $\sigma_{22}^c = 0$. The electrostatic tractions σ_{22}^c do not change its sign at further enhancement of D_2^∞ due to the square relation in Eq. (10), and crack surfaces electrostatic tractions may be neglected for large applied mechanical and small electric displacement loads. K_I/K_{I0} is approximated very well using the magnetoelectric impermeable crack model when the values of D_2^∞ varies from 0 to 0.002 under $\sigma_{22}^\infty = 10 \text{ MPa}$ and $B_{22}^\infty = 100 \text{ N/(Am)}$. It can also be seen that crack tip stress intensity factor K_I/K_{I0} has been greatly influenced by the dielectric constant κ_c and the magnetic permeability μ_c of medium filled inside the crack. Generally, K_I/K_{I0} increases with increasing the value of κ_c and decrease with increasing the value of μ_c .

Figs. 4-7 show that the numerical results for normalized electric displacement intensity factor K_D/K_{D0} and normalized magnetic induction factor K_B/K_{B0} with applied electric loads D_2^∞ for different κ_c and μ_c , where $K_{D0} = 0.01\sqrt{\pi a} C/m^{3/2}$ and $K_{B0} = 0.01\sqrt{\pi a} N/(Am^{3/2})$. It is noted that K_D/K_{D0} increases firstly then decreases with D_2^∞ for the case of $\kappa_c = \kappa_0$, $B_2^\infty = 100 N/(Am)$ and $\sigma_{22}^\infty = 10 MPa$, the possible reason is that the applied electric displacement plays an important role in crack tip K_D/K_{D0} when D_2^∞ is small, and effect of crack surfaces electrostatic tractions σ_{22}^c on K_D/K_{D0} gradually becomes dominant as D_2^∞ increases. Furthermore, magnetic permeability of the medium inside the crack also makes a large impact on crack tip electric displacement and magnetic induction intensity factors, one can observe that both K_D/K_{D0} and K_B/K_{B0} increase with the decreasing value of μ_c from Figs. 5 and 7.

4.2 Effect of applied magnetic loads on crack tip field

Normalized stress intensity factor K_I/K_{I0} is plotted versus the applied magnetic induction B_2^∞ for different dielectric constant and magnetic permeability of the crack interior from Figs. 8-13. Normalized factors K_{I0} , K_{D0} and K_{B0} are set to be the same values in section 4.1. From Figs. 8 and 9 we can see that both B_2^∞ and μ_c have a small contributions to the crack tip mode-I stress intensity factor K_I/K_{I0} , however, the influence of κ_c on K_I/K_{I0} is very large. It is also found that the effect of crack surfaces electrostatic tractions on K_D/K_{D0} and K_B/K_{B0} cannot be neglected when a large magnetic induction loads B_2^∞ is applied, and both K_D/K_{D0} and K_B/K_{B0} increase with the increasing B_2^∞ by considering the electrostatic tractions σ_{22}^c from Figs. 10-13. Finally, it is important to note that the electrostatic tractions on the crack surfaces always have the tendency to close the crack thus retard the crack propagation,

and the traditional traction-free crack model overestimates the effect of applied magnetoelectric loads on the crack tip field intensity factors based on the numerical results obtained from Figs. 2-13.

5. Conclusions

The fracture mechanics problem of magnetoelectroelastic material subjected to remote mechanical, electric and magnetic loads is successfully studied by extending Stroh formalism and continuous distribution generalized dislocation method in this paper. Magnetoelectric semi-permeable crack model is adopted to investigate the effect of crack surfaces electrostatic tractions on crack tip fields. Explicit and closed-form expressions of electric displacement and magnetic induction inside the crack, and stress, electric displacement and magnetic induction intensity factors are obtained. Numerical results show that the crack surfaces electrostatic tractions should be considered for large applied magnetic or electric loads as well as small mechanical loads, dielectric constant and magnetic permeability of the crack interior.

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