Micromechanical modelling of the progressive failure in unidirectional composites reinforced with bamboo fibres

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Abstract: A multiscale approach based upon the micromechanical analysis of multiple damage events at the fibre/matrix scale is developed to investigate the mechanical behaviour of unidirectional composites subjected to tensile loading. The composites are reinforced with bamboo fibres under conditions of fibre breakage and matrix transverse cracking, by considering longitudinal splitting derived from the weak interfacial bonding between fibre and matrix. Due to the statistical nature, an improved Weibull model is employed to describe fibre strength distributions affected by the diameter change in the fibres. A simulation scheme coupled with the Monte Carlo method is performed to characterise the progressive failure in the composite. Compared with experimental results, the validity of the model for composite strength prediction is examined as well.

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1. Introduction

Demands on high performance materials for use in a spectrum of structural and non-structural applications have increased to the point that monolithic materials may not fully satisfy multi-functional requirements. However, most resins and fibres made from petroleum are non-degradable and non-sustainable. There has been an increasing interest in the past few decades regarding the development of natural fibre-based biocomposites that have properties tailored for industrial applications (Dicker et al., 2014). Bamboo fibre, for example, is derived from plant sources that are abundantly available in Asia, South America and many other regions. The material is a unidirectional reinforcing composite and is called ‘nature’s glass fibre’ (Li et al., 1995; Abdul Khalil et al., 2012) and has many techno-ecological advantages such as a high growth rate, cost effectiveness, natural biodegradability and acceptable specific strength, which make it possible to replace synthetic fibres (Joshi et al., 2004; Huang and Netravali, 2009; Hebel et al., 2014). This natural fibre is widely accepted to be one of the most favoured reinforcements and has a great potential to be used in the polymer composite industry. Composite materials are composed of reinforcements and matrices, and their mechanical properties can be greatly influenced by their components. For natural fibre reinforced composites that possess structural diversity and complex

damage modes, it is essential to understand their underlying failure mechanisms from the view of micro-structures for industrial applications of composite materials.

Damage and failure in fibrous composites evolve at several different length scales. At the smallest scale, pre-existing defects in fibres grow. The fibre strength exhibits a large variability due to the statistical distribution of such defects within the volume. When an increasing load is applied to the composite, the fibre with the lowest strength will break first at an early loading stage; thus the load carried by the broken fibre is redistributed among the remaining unbroken fibres and matrix, as determined by the intrinsic properties of the fibres, matrix and interface. Subsequent damage may occur in other fibres near the failure site, causing them to fail and shedding more of the load to intact fibres. Therefore, local damage evolution in composites will determine their fracture toughness, stiffness loss and ultimate failure. It is obliged to admit the fact that the connection between the microstructural scale and the macroscopic scale is nontrivial and involves mechanics, stochastics, and volume scaling (Landis and McMeeking, 1999; Zhang et al., 2009; Redenbach and Vecchio, 2011).

Reliability concerns regarding utilising fibrous composites in structural applications have motivated the development of many numerical and analytical failure models in the presence of interior damage. The micromechanics of damage and fracture has been studied by many investigators for the case of fibre-reinforced polymer matrix composites (Landis et al., 2000; Bouaziz et al., 2007; Okabe et al., 2014). However, the inhomogeneity and anisotropy of the microstructure of plant-based fibrous composites lead to a more complex procedure for modelling damage evolution and failure behaviour. For example, due to their growth in natural environments, plant fibres are

affected by growth environment, treatment conditions, and species variations (Espert et al., 2004; Fuentes et al., 2011). The large variability in fibre strength, which is derived from between-fibre and within-fibre diameter variations, is a typical structural character of such composites. On the other hand, many natural fibres have disadvantages, such as high moisture absorption and poor interface compatibility with some polymeric matrices (Chen et al., 2009; Fuentes et al., 2013) and is the reason that these fibres have not completely replaced conventional fibre materials in high-load applications (Zhang et al., 2015). It is expected that there is a desirable fibre-matrix bonding that effectively transfers the load from one fibre to another through the neighbouring matrix. In the past, extensive studies mainly concentrated on mechanical properties in plant-based fibrous composites (Wong et al., 2010; Prasad and Rao, 2011; Lu et al., 2013), but very few of the models can take into account the intricate nature of the onset and interaction of damage mechanisms, and can describe damage accumulation correlated with material property degradation. Therefore, prediction of mechanical properties in such a structural composite under severe loads is important, starting from the scientific significance.

The aim of the present work is to propose a new numerical model for simulating the tensile failure process of bamboo fibre reinforced composites (BFRP). An experimental investigation using scanning electron microscopy (SEM) is conducted to determine potential fracture characteristics of the composites suffering longitudinal tensile loading. In order to obtain the stress profiles for any configuration of breaks in composites, an analytical method is performed by using a shear-lag model that can account for matrix transverse cracking and interfacial longitudinal splitting around broken fibres. In addition, this paper is concerned with the random variation of fibre

strength based on a modified two-parameter Weibull model that exhibits the non-uniform geometrical structure of bamboo fibre. The statistical parameters of the Weibull distribution are calculated using the Maximum Likelihood Estimation (MLE) with multiple data sets of fibre lengths. It should be emphasized that the bamboo fibres obtained herein are extracted using an explosion mechanical method rather than being reconstituted from cellulose by a viscose process (Bamboo and the FTC, 2009).

2. Experimental

2.1. Materials

The composite materials used in this study were a unidirectional continuous system, fabricated with bamboo fibre as the reinforcement and CYDF-175 epoxy resin as the matrix. The bamboo fibre belongs to one of the most common bamboo species, Moso bamboo, *Phyllostachys heterocycla var. pubenses*, which was imported from Ban Ltd. of Japan. All of the technical fibres, rather than regenerated “bamboo viscose”, were mechanically extracted from bamboo culms of at least three years of age using a blasting extraction process that uses neither chemical nor high temperature (Zakikhani et al., 2014). The cross-section of bamboo fibre appears irregular but approximately circular in shape (Wang et al., 2015). In this batch of samples, the fibres were found to range normally from 150 to 600 μm in diameter. A detailed description of diameter variations was provided in literature (Wang et al., 2015). Fibre strength statistics were obtained through tensile tests. To align the fibres as straight as possible between the test

clamps for mechanical measurements, each fibre specimen was glued onto a stiff paper frame that would be carefully cut in the middle after being clamped by the jaws of the testing machine. The opening of the frame determined the gauge length of the fibre samples.

The selected laminate was a single lamina with a ply angle of 0° . The composite had a fibre volume fraction of 32%. For the purpose of composites preparation, the fibres that have an average length of 170 mm were used as reinforcement through artificial selection. Since bamboo fibres are hygroscopic and thus absorb moisture from the surrounding environment, the fibres were dried at 80°C until a constant weight was reached. The dried fibres were stored in a desiccator with a sealed plastic bag to avoid atmospheric moisture contamination prior to use. After these steps, the fibres were further uniformly laid in a mould after being fully dipped in the epoxy solution. The size of the mould was $175 \times 170 \times 2 \text{ mm}^3$, and the inner surfaces for upper and lower plates were coated with a polystyrene stripper (Dongheng Chemical Industry Co., Ltd., Shanghai, China). A vulcanization moulding machine (XLB-0050, Jinrunqi, China) was adopted to preheat the samples for 10 min at 80°C without pressure to allow permeation of the polymer through the fibres, and then hot pressing was conducted on the samples for 5 h at 100°C and 15 MPa. After curing at room temperature for 11 h and then at 80°C at atmospheric pressure for 8 h in the moulding machine, the composite plate was removed. The straight-bar samples were rectangular coupons cut from this plate to the required dimensions of length $2L_s = 150 \text{ mm}$, width $W_s = 20 \text{ mm}$, and thickness $t_s = 2 \text{ mm}$ by using a dedicated cutting machine with a diamond-coated cutting blade. Bracing cleats (prepared with dimensions of $35 \times 20 \times 2 \text{ mm}^3$) made of aluminium were glued

onto each end for clamping. The geometry and size of the laminate specimens for tensile testing are presented in Fig. 1.

2.2 Tensile test

To investigate the variability in fibre strength, bamboo fibres were used to produce sets of samples at three different gauge lengths of 20, 40 and 50 mm. At least 40 individual fibre samples were mounted for testing at each of these fibre lengths, because samples broken close to the inner edges of the clamps (about 10 mm) had not been considered in the analysis. In order to obtain all qualified specimens per test size, a total of 152 fibre tests were conducted on a WDW3050 computer-controlled universal testing machine (Kexin Testing Instrument Co., Ltd., Changchun, China) fitted with a Celtron 2 kN load cell (model PSD-200kgSJTH, serial number 58232), according to the preparation procedure described in ASTM D3379-75 (1989). The cross-head speed was 0.5 mm/min, and all of the quasi-static tests were conducted at room temperature of 25 °C and average humidity of 50%, under atmospheric pressure. Load cell output voltage was automatically converted to force in Newtons. Strain and load data acquired from each specimen test were logged at a rate of 10 Hz and saved to disk for subsequent processing in the EXCEL spreadsheet (Pearce et al., 1998).

Five composite specimens were tested on the above testing machine fitted with a Celtron 50 kN load cell (model PSD-5tSJTH, serial number 33766) in accordance with ASTM D638-10 (2010) under the same external conditions. During the tensile test, all of the specimens were clamped at both ends in the chuck to guarantee the

synchronisation of the strain increasing in the fibre phase and matrix phase. They were held under a quasi-static monotonic tensile loading until failure occurred, the time-to-failure being defined as the time at which the laminate could no longer support the externally applied load. Therefore, the tensile strength was taken as the maximum load divided by the cross sectional area of the specimen. The tensile tests performed on the composite specimens were conducted at a gauge length of 80 mm and a cross-head speed of 0.5 mm/min. The mechanical properties obtained from the tests are provided in Table 1.

Several representative stress-strain curves from the tensile tests are presented in Fig. 2. It can be seen that the stress-strain relationship is linear at the initial loading stage, the characteristic of which is that only a small quantity of broken fibres are in the composite. Subsequently, the irregular curve shows a nonlinear behaviour with increasing strain before ultimate failure, which reflects the occurrence and accumulation of other potential damage during the tensile test. As seen in Table 1, it is seen that the maximum strain to failure for all of the specimens is less than 4%, indicating a brittle fracture.

Through an image of the fracture profile in Fig. 3, it is shown that there are other damage modes besides fibre fracture. For example, a longitudinal splitting is also observed along the fibre axis.

2.3 Damage characterisation

To study the damage mechanism in a detailed manner, the fractured surface of the

composite reinforced by the bamboo fibre is observed by scanning electron microscope (S-4800, Hitachi, Japan).

The SEM photomicrograph of Fig. 4(a) represents the local morphology and structure of the material, which depicts that the tensile fracture of the specimen under the quasi-static loading is due mainly to fibre breakage, matrix transverse cracking and longitudinal splitting. As seen from Fig. 4(b), the fibre is not a single filament but rather a bundle of single aggregated elementary fibres. Thus, fibre separation in the clusters is observed. Few cavities left by the extruding fibres and fibres surfaces that were infiltrated with epoxy indicate that the specimen has relatively good interfacial bonding. In Fig. 4(c), the lateral surface of the fracture shows that a platform of splitting appeared in the sample, which indicates the source and subsequent propagation of splitting. Longitudinal cracking propagates preferentially along the fibre/matrix interface. Then, splitting occurs when the accumulated stress caused by shear deformation of the interface reaches the ultimate bearing capacity of local shear failure but does not reach the fracture strength of the constituents adjacent to the broken fibre. Such a case can be also observed in Fig. 4(a). Fig. 4(d) shows that matrix transverse cracking occurred in the composite. The matrix cracking in the epoxy-rich zone exhibits straight river markings on the fracture surface, which are parallel to the crack propagation direction (Gabr et al., 2014).

3. Modelling

3.1 Damage model

Consider evenly spaced bamboo fibres embedded in a matrix material. The fibres and matrix regions are serially numbered in two-dimensional (2D) unidirectional composites, as shown in Fig. 5(a). The lamina is loaded in tension along the fibre direction. The width of the fibres is D , and the width of each matrix region W can be related to the fibre volume fraction, V_f (Zhang et al., 2002). To analyse the complex combination of microdamage modes, we propose a micromechanical model in which a broken fibre, accompanied by its transverse cracking matrix and local shear splitting interface, is referred to as a damage event, as shown in Fig. 5(b).

Since the fibre strength exhibits large variability due to the statistical distribution of defects, the first fibre break usually takes place at an early loading stage. This break causes the severe induction of shear zones that are staggered in different directions in the matrix, forming numerous deformation expansion areas, which were hampered by the reinforcement fibres and accumulated around them (Hedgepeth and Van Dyke, 1967). As a consequence, stress concentrations induce local transverse cracking in the matrix regions. After increasing some loading levels, N fibre breaks occur in the composite. For the purpose of analysis, the coordinates of the n -th fibre break, which is located in the k_n -th fibre at vertical position ξ_n , are given by (k_n, ξ_n) ($n = 1, 2, \dots, N$). With continuous mechanical loading and damage accumulation, the shear zone of the interface will extend forward from the fibre break tip until the splitting around the fibre break occurs within a certain length, $2L_n$, which is associated with the n -th fibre break. It must be pointed out that $2L_n$ differs from the total length of each fibre.

3.2 Shear-lag analysis

In multifibre composites of practical interest, failure behaviour is, in principle, a progressive process that involves multiple forms of damage and is controlled by the localised stress transfer between the fibre and matrix. Therefore, accurate prediction of the mechanical properties of the composite materials requires a detailed knowledge of the micromechanical stress state in and around broken fibres as a function of the constituent material properties (Xia et al., 2002).

Recently, considerable research has been devoted to evaluate stress profiles in fibres and matrix using various methods (Phoenix and Beyerlein, 2000; Hanan et al., 2003; Iarve et al., 2007). Shear-lag models with varying degrees of complexity have been widely used for stress and failure analysis of fibre-reinforced composites because they are computationally more efficient than finite element models and yet predict many primary features relevant to composite failure (Zhang and Wang, 2010). To obtain solutions for the stress in the fibre and matrix under multiple damage events, a theoretical model utilising the framework of 2D shear-lag arguments (Cox, 1952; Nairn, 1997; Beyerlein and Landis, 1999) will be used to derive stress profiles for any configuration of breaks in the presence of matrix cracking and interface shear splitting. It is assumed that the tensile load parallel to the fibre-reinforcement direction is carried mainly by the fibre and matrix regions, while the axial stress in the fibre and matrix is transferred through the fibre/matrix interface. The interface is considered to be in a state of pure shear behaviour in a very thin layer (Zhang and Wang, 2010), and thus the material parameters for the interface are assumed to be the same as those of the matrix.

As the load increases, N fibre breaks appear where each fibre break is accompanied by the interface splitting of length, $2L_n$, and where M breaks ($M \leq N$) cause simultaneous associated matrix transverse cracking. When a fibre breaks, it expands radially to create friction with the neighbouring components, and the load released from the broken site is transferred by the shear stress that appears at the interface between the fibre and the matrix. An increment of the shear stress close to the fibre breakage may lead to interfacial splitting when the shear stress reaches the shear strength τ_m . Once the splitting occurs, the shear stress will reduce to the constant sliding friction τ_f instead of τ_m (Goda, 1999), as shown in Fig. 6.

For the purpose of analysis, τ_f is considered to be equal to half of the shear strength (Wang and Zhao, 2001). Therefore, the interfacial shear stress within the splitting zone $0 \leq |\xi - \xi_n| \leq L_n/l_c$, associated with the n -th fibre break, has to be equal to the sliding friction stress, and it follows that

$$\tau(k_n, \xi) = \tau_f \quad \text{for } \xi = \xi_n + L_n/l_c \quad \text{and} \quad n = 1, 2, 3, \dots, N. \quad (1)$$

where $\tau(k_n, \xi)$ is the interface shear stress at the location (k_n, ξ) , and $l_c = \sqrt{\frac{WE_m}{K}}$ is a length scale of the composite system. Here, E_m is the Young's modulus of the matrix and K is the shear-lag parameter associated with the fibre/matrix interface.

Since the shear stress is assumed to be constant (τ_f) within the interface splitting region, the fibre stress can be obtained from the equilibrium condition of the fibre, i.e.,

$$\sigma^f(k_n, \xi) = \frac{2\tau_f}{D} l_c (\xi - \xi_n), \quad \text{for } 0 \leq |\xi - \xi_n| \leq L_n/l_c \quad \text{and} \quad n = 1, 2, 3, \dots, N. \quad (2)$$

where $\sigma^f(k_n, \xi)$ is the fibre stress at the location (k_n, ξ) .

In another case matrix cracking around the fibre break will occur due to the stress

concentration when the axial stress reaches strength σ_m . It is assumed that M breaks ($M \leq N$) cause simultaneous matrix transverse cracking under the current loading level.

At this time, the axial stress of the matrix at the cracking site has to be equal to

$$\sigma^m(k_i, \xi_i) = 0, \quad i = 1, 2, 3, \dots, M. \quad (3)$$

If the coordinates of the n -th fibre break, which is located in the k_n -th fibre with vertical position ξ_n , are given by (k_n, ξ_n) ($n = 1, 2, \dots, N$), the stress boundary condition at the broken sites is given as

$$\sigma^f(k_n, \xi_n) = 0, \quad \text{for } n = 1, 2, 3, \dots, N. \quad (4)$$

which is a special case of Eq. (2) when $\xi = \xi_n$.

The fundamental concepts behind *influence superimposition techniques* are employed to obtain first, analytical solutions for the influence of a ‘unit’ failure on the stress and displacement fields, and next, use a weighted superposition technique to obtain the solution to the full problem. Indeed, the unit solutions for a single fibre break and a single matrix break, called the influence functions, are used as building blocks for the solutions for fibre stresses and matrix stresses in the composite. By this method, the axial stresses in the fibre and matrix regions and the interfacial shear stress at any position (k, ξ) are expressed, respectively, by

$$\sigma^f(k, \xi) = \bar{\sigma}_f + \sum_{j=1}^N \int_{\xi_j - L_j/l_c}^{\xi_j + L_j/l_c} w_j^f(\xi') p_{k-k_j}^f(\xi - \xi') d\xi' + \sum_{i=1}^M w_i^m q_{k-k_i}^f(\xi - \xi_i) \quad (5)$$

$$\sigma^m(k, \xi) = \bar{\sigma}_m + \sum_{j=1}^N \int_{\xi_j - L_j/l_c}^{\xi_j + L_j/l_c} w_j^f(\xi') p_{k-k_j}^m(\xi - \xi') d\xi' + \sum_{i=1}^M w_i^m q_{k-k_i}^m(\xi - \xi_i) \quad (6)$$

$$\tau(k, \xi) = \sum_{j=1}^N \int_{\xi_j - L_j/l_c}^{\xi_j + L_j/l_c} w_j^f(\xi') s_{k-k_j}(\xi - \xi') d\xi' + \sum_{i=1}^M w_i^m t_{k-k_i}(\xi - \xi_i) \quad (7)$$

where $\bar{\sigma}_f$ and $\bar{\sigma}_m$ denote the remote stresses in the fibre and the matrix,

respectively. The influence functions $p_{k-k_j}^f(\xi - \xi_j)$, $p_{k-k_j}^m(\xi - \xi_j)$ and $s_{k-k_j}(\xi - \xi_j)$ represent the fibre stress, matrix stress and interface shear stress, respectively, at position (k, ξ) due to a unit opening displacement applied at the fibre break at position (k_j, ξ_j) . The influence functions $q_{k-k_i}^f(\xi - \xi_i)$, $q_{k-k_i}^m(\xi - \xi_i)$ and $t_{k-k_i}(\xi - \xi_i)$ represent the fibre stress, matrix stress and interface shear stress, respectively, at position (k, ξ) due to a unit opening displacement applied at the matrix break (k_i, ξ_i) . The undetermined coefficient functions w_j^f ($j = 1, 2, \dots, N$), the unknown coefficient constant w_i^m ($i = 1, 2, \dots, M$) and the interface splitting length L_j ($j = 1, 2, \dots, N$) can be obtained by the stress conditions given by Eqs. (1-3). Substitution of Eqs. (5-7) into Eqs. (1-3) gives

$$\sum_{j=1}^N \int_{\xi_j - L_j/l_c}^{\xi_j + L_j/l_c} w_j^f(\xi') s_{k_n - k_j}(\xi_n + L_n/l_c - \xi') d\xi' + \sum_{i=1}^M w_i^m t_{k_n - k_i}(\xi_n + L_n/l_c - \xi_i) = \tau_f$$

for $n = 1, 2, 3, \dots, N$. (8)

$$\bar{\sigma}_f + \sum_{j=1}^N \int_{\xi_j - L_j/l_c}^{\xi_j + L_j/l_c} w_j^f(\xi') p_{k_n - k_j}^f(\xi - \xi') d\xi' + \sum_{i=1}^M w_i^m q_{k_n - k_i}^f(\xi - \xi_i) = \frac{2\tau_f}{D} |l_c(\xi - \xi_n)|$$

for $0 \leq |\xi - \xi_n| \leq L_n/l_c$ and $n = 1, 2, 3, \dots, N$. (9)

$$\bar{\sigma}_m + \sum_{j=1}^N \int_{\xi_j - L_j/l_c}^{\xi_j + L_j/l_c} w_j^f(\xi') p_{k_t - k_j}^m(\xi_t - \xi') d\xi' + \sum_{i=1}^M w_i^m q_{k_t - k_i}^m(\xi_t - \xi_i) = 0$$

for $t = 1, 2, 3, \dots, M$. (10)

By solving the integral Eqs. (8-10) one can obtain the shear splitting length, L_n , the coefficient functions w_j^f and the coefficients w_i^m . Physically w_j^f and w_i^m represent the crack opening displacement of the j -th fibre break and the i -th matrix crack.

3.3 Simulation

The catastrophic failure of the fibre-reinforced composites is primarily dominated by the accumulation and interaction of fibre breakages (Landis et al., 2000; Rousset et al., 2009; Wang et al., 2010). For natural fibres, there is a considerable variability in fibre strength due to the non-uniform geometrical structure (Trujillo et al., 2014); thus it is important to employ a statistical model for accurately describing the strength distribution of such brittle fibre. There is growing evidence that the power-law Weibull model, which is capable of explaining the effect of its variable geometrical structure, is the best candidate for characterising the strength distribution and its size effect (Wang and Shao, 2014). Compared with the linear-law model (Wang et al., 2016), the power-law model that introduces an exponential parameter can improve the predictive level with respect to experimental data (Ricks et al., 2014), the cause of which is a consideration of within-fibre and between-fibre diameter variations. Consequently, the variation of the fibre strength σ_f for length L can be characterised by the probability density function, $P(\sigma_f)$, that is

$$P(\sigma_f) = 1 - \exp \left\{ - \left(\frac{L}{L_0} \right)^\kappa \left(\frac{\sigma_f}{\sigma_0} \right)^\beta \right\} \quad (11)$$

where $P(\sigma_f)$ is the failure probability less than or equal to σ_f . σ_0 represents the scale parameter of the material, and L_0 is the reference length when the characteristic fibre strength σ_0 is measured. The shape parameter β controls the scatter of the fibre tensile strength in the distribution and κ is the exponential parameter that can represent the fibre diameter variations. It should be mentioned that since the transverse deformation in the fibre and matrix regions is neglected in the shear-lag

model, the influence of cross-section variations on the axial stress along a fibre length is not considered in this study.

In the approach that we use here, it is assumed that the total length of the composite specimen to be modeled, $2L_T$, is divided into N_T segments of equal length Δx such that $\Delta x = 2L_T / N_T$. The composite specimen to be modelled contains N_f fibres and N_m matrix elements, respectively. For a given failure probability P , the strength of the fibre segments with length Δx can be derived from the inversion of Eq. (11),

$$\sigma_f = \sigma_0 \left\{ \left(\frac{L_0}{\Delta x} \right)^\kappa \ln \left(\frac{1}{1-P} \right) \right\}^{1/\beta} \quad (12)$$

in which the failure probability P is a uniform random number in the range from 0 to 1.

The Monte Carlo simulation technique coupled with the proposed analytical model is under the conditions of overall composite strain control. At the beginning of each simulation, all of material segments are assumed to be intact. Fig. 7 shows the detailed simulation procedure as a flowchart.

4. Results and Discussion

A bamboo-fibre-reinforced epoxy-matrix composite is examined in this paper. The mechanical properties used in the calculations to follow are summarised in Table 2. A numerical simulation is executed to demonstrate the capability of prediction of the present theory.

4.1 Estimation for Weibull parameters

The Weibull parameters obtained for each of the gauge lengths of 20, 40 and 50 mm are presented in Table 3. It can be seen that the average strength decreases when the gauge length increases. The results can be attributed to the fact that with larger sizes there are more flaws in the fibres that make the chance of failure larger (Sutherland et al., 2009; Wang et al., 2015). Virk et al. (2009) conducted a comprehensive investigation of the weak-link scaling analysis for fibre strength predictions, which stressed that the use of multiple data set weak-link scaling predictions can provide better correlation with the experimental data than the standard single data set method (a point estimate). Therefore, the optimized Weibull parameters estimates for strength are computed from the strength data in 20, 40 and 50 mm gauge length, which can bound the entire range of test lengths. Table 3 also gives the computed parameters κ , β and σ_0 estimated by the MLE method.

4.2 Progressive failure

Fig. 8 shows a simulated failure process of the composite under tensile loading, indicating the damage configurations under different instantaneous stages, including approximately 0%, 34%, 78% and 96% of the composite failure stress. Due to the large variability in fibre strength, the weakest fibre fractured at an early loading level, as shown in Fig. 8(b). The high stress concentrations induced by the broken fibre

accumulate in other intact fibres or in the matrix, so that matrix cracking or new fibre breakage occurs when the accumulated stress on the plane reaches the associated bearing capacity. On the other hand, the epoxy used is a hard and stiff thermosetting resin with poor ductility. The difference between the modulus of the fibre and the matrix is so large that the fibre/matrix interface cannot prevent the shear deformation on the interfacial plane when a neighbouring fibre or matrix region fails. Consequently, longitudinal cracking prefers to propagate along the fibre/matrix interface, and the interfacial splitting appears when the accumulated stress reaches the ultimate bearing capacity for shear failure. As the applied load increases, more fibres undergo breakage within the whole specimen, which produce more severe damage, as shown in Fig. 8(c). Severe damage accumulation can be observed in Fig. 8(d).

Fig. 9 illustrates the stress-strain response corresponding to the example in Fig. 8. It is clearly observed through this Monte Carlo simulation that the composite stress shows a tendency to generally increase with the applied strain ε . Due to the occurrence and accumulation of longitudinal splitting and matrix transverse cracking, however, the curve begins to deviate from linear behaviour beyond the elastic range, but continues to increase staggeringly up to the ultimate stress. After this turning point, the composite stress decreases. This behaviour attributes to the microdamage mechanism in which a dominant ‘critical cluster’ of breaks is shaped, which leads to the failure of the rest of the fibres without any further increasing composite strain (Thionnet et al., 2014).

To show the accumulation of damage in the composite, Fig. 10 is a plot of the evolution of the fibre break number, matrix cracking number and total interface splitting length. In the initial loading level, only the fibre element with the lowest strength breaks.

Stress concentrations are generated in the matrix, which most likely lead to the transverse cracking. Because of the large variability in the fibre strength, the growth rate of fibre breakage is small at this stage. At a later stage of loading, the accumulation of interfacial splitting will disperse or propagate the stress concentrations derived from the fibre breakage, which in turn stimulates a rapid increase in the number of broken fibres with increasing applied load. It is clearly observed that the microfailure mechanism associated with the tensile behaviour is characterised by a fibre-dominated fracture, with evidence of a smaller amount of matrix cracking.

4.3 Strength prediction

An appropriate selection for the fibre segment length is critical for characterising fibre strength variability and predicting composite strength distributions. Some work has been devoted to investigate the influence of discretisation lengths on the composite strength. Results indicate that the discretisation size must be chosen small enough to minimize discretisation errors (Landis et al., 2000). Therefore, an initial series of simulations are performed to determine the appropriate choice of the discretisation length, Δx , for a single segment. In Table 4, eight discretisation sizes are compared, $N_T = 40, 60, 80, 100, 120, 150, 180$, and 200 , to illustrate the influence of fibre segment length on the composite length. It is evident that as Δx is decreased, the predicted strength converges to the same average value, which suggests that the composite strength is independent of the element length. The details of results from multiple simulations are postponed until the Appendix. Accordingly, an efficient discretisation

length of $\Delta x = 1.0$ mm, which leads to convergent composite strength, is used in our subsequent simulation.

Fig. 11 shows the predicted tensile strength of 50 simulations for a different number of fibres embedded in the composite. Note that there is a consistent size effect on the average strength of the composite, where the composite strength decreases as the number of fibres in the composite increases. Compared with the strength of the 30-fibre composite, the composite strength decreases by 6% for 160 fibres. This suggests that when the specimen is large enough, the ultimate strength will tend to be 144 MPa, which is close to the experimental value of 147 MPa. Thus, the size effect on the strength disappears when the specimen is sufficiently large.

5. Conclusions

This study has presented a numerical micromechanical investigation of the mechanical behaviour and damage evolution of bamboo fibre reinforced composites. An attempt has been made to develop a multiscale approach that can provide a direct connection between the detailed damage modes at the micro-structural (fibre-matrix) level and the macroscopic material performance. The local matrix transverse cracking and interfacial shear splitting around fibre breaks have been taken into account and reflect the characteristic damage modes in polymer matrix composites reinforced by plant-based fibres. An analytical method using the framework of a shear-lag model is presented to obtain stress profiles for any configuration of multi-damage events and closed-form solutions are derived. A simulation scheme combined with the Monte Carlo

method has been proposed to predict the mechanical properties. In addition, the statistical variation of the fracture strength of bamboo fibre is described by the power law Weibull model. It is important to note that the statistical parameters of the distribution are estimated using the MLE method with multiple data sets of fibre lengths and can be used to perform scaling predictions for the tensile strength at different fibre lengths. The results obtained are useful for developing a comprehensive understanding of the mechanical performance of plant-based fibre reinforced composites.

As a final remark, it should be mentioned that composite dimensionality has a significant effect on the tensile strength distribution (Curtin, 2000). Indeed, there are larger local stress concentrations on fibres neighbouring existing fibre breaks in the 2D model. Thus, inner damages are severely localised compared with actual 3D materials, and give rise to a lower tensile strength of composites. In any case, the 2D modeling presented here does generalize the governing equations to include interactions with multiple damage events and develop the basic mechanics necessary to understanding failure modes of the composite produced by the failure of one or more of its components from a micromechanics perspective. Thus the proposed model could be considered an approximate, but mechanically justifiable one. A more precise calibration of the model, capable of explaining such effects as 3D fibre arrays and transverse stress disturbance on composite strength, would have to be considered in further study.

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Appendix A. Details of the results from multiple simulations

References

- Abdul Khalil, H.P.S., Bhat, I.U.H., Jawaid, M., Zaidon, A., Hermawan, D., Hadi, Y.S., 2012. Bamboo fibre reinforced biocomposites: a review. *Mater. Des.* 42, 353-368.
- ASTM D3379-75(1989)e1, 1975. Standard test method for tensile strength and Young's modulus for high modulus single-filament materials. West Conshohocken, PA, USA.
- ASTM D638-10, 2010. Standard test method for tensile properties of plastics. West Conshohocken, PA, USA.
- Bamboo and the FTC. URL <<https://oecotextiles.wordpress.com/category/fibers/bamboo>>, accessed on August 19th 2009.
- Beyerlein, I.J., Landis, C.M., 1999. Shear-lag model for failure simulations of unidirectional fiber composites including matrix stiffness. *Mech. Mater.* 31 (5), 331-350.
- Bouaziz, A., Zaïri, F., Naït-Abdelaziz, M., Gloaguen, J.M., Lefebvre, J.M., 2007. Micromechanical modeling and experimental investigation of random discontinuous

glass fiber polymer-matrix composites. *Compos. Sci. Technol.* 67 (15-16), 3278-3285.

Chen, H.Y., Miao, M.H., Ding, X., 2009. Influence of moisture absorption on the interfacial strength of bamboo/vinyl ester composites. *Compos. Part A* 40 (12), 2013-2019.

Cox, H.L., 1952. The elasticity and strength of paper and other fibrous materials. *Br. J. Appl. Phys.* 3 (1), 72-79.

Curtin, W.A., 2000. Dimensionality and size effects on the strength of fiber-reinforced composites. *Compos. Sci. Technol.* 60 (4), 543-551.

Dicker, M.P.M., Duckworth, P.F., Baker, A.B., Francois, G., Hazzard, M.K., Weaver, P.M., 2014. Green composites: a review of material attributes and complementary applications. *Compos. Part A* 56, 280-289.

Espert, A., Vilaplana, F., Karlsson, S., 2004. Comparison of water absorption in natural cellulosic fibres from wood and one-year crops in polypropylene composites and its influence on their mechanical properties. *Compos. Part A* 35 (11), 1267-1276.

Fuentes, C.A., Tran, L.Q.N., Dupont-Gillain, C., Vanderlinden, W., De Feyter, S., Van Vuure, A.W., Verpoest, I., 2011. Wetting behaviour and surface properties of technical bamboo fibres. *Colloids Surf. A* 380 (1), 89-99.

Fuentes, C.A., Tran, L.Q.N., Van Hellemont, M., Janssens, V., Dupont-Gillain, C., Van Vurre, A.W., Verpoest, I., 2013. Effect of physical adhesion on mechanical behaviour of bamboo fibre reinforced thermoplastic composites. *Colloids Surf. A* 418, 7-15.

Gabr, M.H., Phong, N.T., Okubo, K., Uzawa, K., Kimpara, I., Fujii, T., 2014. Thermal and mechanical properties of electrospun nano-celulose reinforced epoxy nanocomposites. *Polym. Test* 37, 51-58.

Goda, K., 1999. The role of interfacial debonding in increasing the strength and reliability of unidirectional fibrous composites. *Compos. Sci. Technol.* 59 (12), 1871-1879.

Hanan, J.C., Üstündag, E., Beyerlein, I.J., Swift, G.A., Almer, J.D., Lienert, U., Haeflner, D.R., 2003. Microscale damage evolution and stress redistribution in Ti-SiC fiber composites. *Acta Mater.*, 51 (14): 4239-4250.

Hebel, D.E., Javadian, A., Helsel, F., Schlesier, K., Griebel, D., Wielopolski, M., 2014.

Process-controlled optimization of the tensile strength of bamboo fiber composites for structural applications. *Compos. Part B* 67, 125-131.

Hedgepeth, J.M., Van Dyke, P., 1967. Local stress concentrations in imperfect filamentary composite materials. *J. Compos. Mater.* 1 (3), 294-309.

Huang, X.S., Netravali, A., 2009. Biodegradable green composites made using bamboo micro/nano-fibrils and chemically modified soy protein resin. *Compos. Sci. Technol.* 69 (7), 1009-1015.

Iarve, E.V., Kim, R., Mollenhauer, D., 2007. Three-dimensional stress analysis and Weibull statistics based strength prediction in open hole composites. *Compos. Part A* 38 (1), 174-185.

Joshi, S.V., Drzal, L.T., Mohanty, A.K., Arora, S., 2004. Are natural fiber composites environmentally superior to glass fiber reinforced composites?. *Compos. Part A* 35 (3), 371-376.

Landis, C.M., Beyerlein, I.J., McMeeking, R.M., 2000. Micromechanical simulation of the failure of fiber reinforced composites. *J. Mech. Phys. Solids* 48 (3), 621-648.

Landis, C.M., McMeeking, R.M., 1999. A shear-lag model for a broken fiber embedded in a composite with a ductile matrix. *Compos. Sci. Technol.* 59 (3), 447-457.

Li, S.H., Zeng, Q.Y., Xiao, Y.L., Fu, S.Y., Zhou, B.L., 1995. Biomimicry of bamboo bast fiber with engineering composite materials. *Mater. Sci. Eng. C* 3 (2), 125-130.

Lu, T.J., Jiang, M., Jiang, Z.G., Hui, D., Wang, Z.Y., Zhou, Z.W., 2013. Effect of surface modification of bamboo cellulose fibers on mechanical properties of cellulose/epoxy composites. *Compos. Part B* 51, 28-34.

Nairn, J.A., 1997. On the use of shear-lag methods for analysis of stress transfer unidirectional composites. *Mech. Mater.* 26 (2): 63-80.

Okabe, T., Sasayama, T., Koyanagi, J., 2014. Micromechanical simulation of tensile failure of discontinuous fiber-reinforced polymer matrix composites using Spring Element Model. *Compos. Part A* 56, 64-71.

Pearce, N.R.L., Guild, F.J., Summerscales, J., 1998. An investigation into the effects of fabric architecture on the processing and properties of fibre reinforced composites produced by resin transfer moulding. *Compos. Part A* 29 (1-2), 19-27.

- Phoenix, S.L., Beyerlein, I.J., 2000. Statistical strength theory for fibrous composite materials. *Compr. Compos. Mater.* 1, 559-639.
- Prasad, A.V.R., Rao, K.M., 2011. Mechanical properties of natural fibre reinforced polyester composites: Jowar, sisal and bamboo. *Mater. Des.* 32, 4658–4663.
- Redenbach, C., Vecchio, I., 2011. Statistical analysis and stochastic modeling of fibre composites. *Compos. Sci. Technol.* 71 (2), 107-112.
- Ricks, T.M., Lacy, T.E., Bednarczyk, B.A., Arnold, S.M., Hutchins, J.W., 2014. A multiscale progressive failure modeling methodology for composites that includes fiber strength stochasticity. *Comput. Mater. Con.* 40 (2), 99-129.
- Rousset, G., Martin, E., Lamon, J., 2009. In situ fibre strength determination in metal matrix composites. *Compos. Sci. Technol.* 69 (15-16), 2580-2586.
- Sutherland, L.S., Shenoi, R.A., Lewis, S.M., 2009. Size and scale effects in composites: I. Literature review. *Compos. Sci. Technol.* 59 (2), 209-220.
- Thionnet, A., Chou, H.Y., Bunsell, A., 2014. Fibre break processes in unidirectional composites. *Compos. Part A* 65, 148-160.
- Trujillo, E., Moesen, M., Osorio, L., Van Vuure, A.W., Ivens, J., Verpoest, I., 2014. Bamboo fibres for reinforcement in composite materials: Strength Weibull analysis. *Compos. Part A* 61, 115–125.
- Virk, A.S., Hall, W., Summerscales, J., 2009. Multiple Data Set (MDS) weak-link scaling analysis of jute fibres. *Compos. Part A* 40 (11), 1764–1771.
- Wang, F., Chen, Z.Q., Wei, Y.Q., Zeng, X.Q., 2010. Numerical modeling of tensile behavior of fiber-reinforced polymer composites. *J. Compos. Mater.* 44 (19), 2325-2340.
- Wang, F., Shao, J.X., 2014. Modified Weibull distribution for analyzing the tensile strength of bamboo fibers. *Polymers* 6 (12), 3005-3018.
- Wang, F., Shao, J.X., Keer, L.M., Li, L., Zhang, J.Q., 2015. The effect of elementary fibre variability on bamboo fibre strength. *Mater. Des.* 75, 136-142.
- Wang, F., Shao, J.X., Li, X., 2016. Statistics on the fracture strength of bamboo fibers. *Polym. Compos.* <http://onlinelibrary.wiley.com/enhanced/doi/10.1002/pc.23173>.
- Wang, X.F., Zhao, J.H., 2001. Monte-Carlo simulation to the tensile mechanical

behaviors of unidirectional composites at low temperature, *Cryogenics* 41 (9), 683-691.

Wong, K.J., Zahi, S., Low, K.O., Lim, C.C., 2010. Fracture characterisation of short bamboo fibre reinforced polyester composites. *Mater. Des.* 31, 4147–4154.

Xia, Z., Okabe, T., Curtin, W.A., 2002. Shear-lag versus finite element models for stress transfer in fiber-reinforced composites. *Compos. Sci. Technol.* 62 (9), 1141-1149.

Zakikhani, P., Zahari, R., Sultan, M.T.H., Majid, D.L., 2014. Extraction and preparation of bamboo fibre-reinforced composites. *Mater. Des.* 63, 820–828.

Zhang, F., Lisle, T., Curtin, W.A., Xia, Z., 2009. Multiscale modeling of ductile-fiber-reinforced composites. *Compos. Sci. Technol.* 69 (11), 1887-1895.

Zhang, J.Q., Wang, F., 2010. Modeling of damage evolution and failure in fiber-reinforced ductile composites under thermomechanical fatigue loading. *Int. J. Damage Mech.* 19 (7), 851-875.

Zhang, J.Q., Wu, J., Liu, S.L., 2002. Cyclically thermomechanical plasticity analysis for a broken fiber in ductile matrix composites using shear lag model. *Compos. Sci. Technol.* 62 (5), 641-654.

Zhang, X.P., Wang, F., Keer, L.M., 2015. Influence of surface modification on the microstructure and thermo-mechanical properties of bamboo fibers. *Materials* 8 (10), 6597-6608.

Figure 1: Geometry and size of composite specimen.

Figure 2: Typical stress-strain curves for some specimens under tensile loading.

Figure 3: Fracture profile of unidirectional composite.

Figure 4: SEM photograph for tensile fracture of composite: (a) fracture surface; (b) fibre breakage; (c) longitudinal splitting; (d) matrix cracking.

Figure 5: Schematic view of (a) the composite arranged by fibres and matrix; and (b) a damage event.

Figure 6: Stress-strain curve of interface.

Figure 7: Flow chart for the analysis of a progressive failure.

Figure 8: Simulated damage evolution of bamboo/epoxy unidirectional composite: (a) 0% of failure stress; (b) 34% of failure stress; (c) 78% of failure stress; (d) 96% of failure stress.

Figure 9: Simulated stress-strain response of the composite.

Figure 10: Damage evolution in the fibre, matrix and interface during loading.

Figure 11: Relationship between the predicted composite strength and number of bamboo fibres embedded.

Table 1: Details of the properties of composite specimens.

Specimen number	H_s (mm)	W_s (mm)	t_s (mm)	$P_{\max}$ (kN)	Tensile Strength (MPa)	Failure strain (%)
1	83.2	21.4	1.9	5.95	149.0	3.8
2	82.5	21.3	1.9	6.39	155.3	3.3
3	80.8	21.4	2.0	6.11	142.2	2.8
4	82.9	21.5	1.8	5.43	141.5	2.5
5	82.8	21.6	2.0	6.36	145.5	3.0
Average	82.5	21.4	1.9	6.05	146.7	3.1

Note: The thickness of each layer of bracing cleats, δ_s , is 2 mm.

Table 2: Mechanical properties of fibre and matrix.

Property	Fibre	Matrix
Tensile strength, (MPa)	200-800	65
Young's modulus, E (GPa)	12	2.7
Shear modulus, G (GPa)	---	1.3
Volume faction, V (%)	32	68
Shear strength, τ_s (MPa)	---	130
Friction stress, τ_f (MPa)	---	65
Shear-lag parameter, K (GPa / mm)	---	37.1

Table 3: Weibull statistics for tensile strength of bamboo fibres.

Gauge length, L	Scale parameter, σ_0 (MPa)	Shape parameter, β	Average strength, $\bar{\sigma}$ (MPa)	κ	Estimation
20	625	3.98	555	--	LR
40	567	3.42	507	--	LR
50	497	4.17	451	--	LR
Entire data	1085	3.98	504	0.70	MLE

Note: LR, Linear Regression. MLE, Maximum Likelihood Estimation.

Table 4: Influence of fibre element length on composite strength.

Discretisation number, N_T	Simulated length of component element, $\Delta x = 2L_s / N_T$ (mm)	Average predicted strength (MPa)
40	3.75	183
60	2.50	134
80	1.88	136
100	1.50	143
120	1.25	150
150	1.00	144
180	0.83	144
200	0.75	144

Table A1: The predicted strength of bamboo/epoxy unidirectional composite.

No.	$N_T = 40$	$N_T = 60$	$N_T = 80$	$N_T = 100$	$N_T = 120$	$N_T = 150$	$N_T = 180$	$N_T = 200$
1	196	138	141	133	152	150	148	151
2	187	135	130	149	157	139	152	148
3	181	134	143	146	156	135	143	139
4	165	118	127	135	140	151	149	141
5	188	141	131	135	136	147	146	138
6	193	128	147	135	141	142	142	145
7	180	137	140	156	160	146	153	140
8	178	128	137	148	153	137	149	148
9	201	134	139	148	142	152	146	145
10	187	128	140	148	148	146	151	142
11	170	134	127	148	144	139	139	140
12	187	127	150	137	148	143	148	143
13	200	128	138	146	157	143	146	144
14	190	152	142	145	148	145	149	153
15	173	139	120	152	158	151	145	139
16	184	126	136	146	155	136	150	142
17	194	128	140	145	157	138	151	144
18	190	135	141	142	154	146	146	141
19	162	128	137	150	156	150	144	146

20	186	142	151	151	155	145	140	138
21	173	144	130	152	161	144	141	147
22	186	140	147	139	154	140	146	150
23	171	138	110	143	147	148	139	146
24	184	125	144	153	155	139	144	143
25	198	132	136	134	143	150	148	149
26	180	137	153	149	149	153	142	136
27	188	127	135	132	153	148	153	152
28	200	115	127	136	144	150	140	143
29	181	136	119	148	154	148	138	149
30	195	147	144	139	147	146	144	138
31	200	155	138	149	144	150	147	144
32	178	146	142	139	149	132	141	137
33	179	136	129	137	157	146	136	149
34	186	129	128	145	153	142	151	142
35	206	141	133	134	151	138	149	150
36	175	128	139	140	152	150	134	148
37	184	148	145	145	154	144	151	145
38	175	130	142	145	149	138	140	151
39	183	131	138	146	147	140	133	142
40	176	134	138	151	151	135	146	136

41	185	120	137	144	145	134	136	150
42	174	151	142	139	159	146	138	140
43	177	121	130	146	153	139	140	142
44	173	142	137	142	146	150	148	135
45	183	124	117	139	155	142	152	148
46	168	135	142	147	140	151	138	143
47	162	142	148	144	152	148	145	153
48	170	131	133	134	142	147	142	148
49	190	122	130	145	141	138	139	146
50	172	122	133	142	154	145	143	142

Note: The unit is MPa.