Multi-scale structure modeling of damage behaviors of 3D orthogonal woven composite materials subject to quasi-static and high strain rate compressions

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Abstract

We proposed a multi-scale structure modeling scheme to analyze the damage behaviors of three-dimensional orthogonal woven composite materials subject to quasi-static and high strain rate compressions. The multi-scale structure model includes: (1) micro/meso/macro structure model with periodic boundary conditions for homogenizing the heterogeneous fiber/resin system into unit cells, and (2) a macroscopic rate dependent plasticity model combined with critical damage area failure theory that accounts for the compressive deformation and failure strength of the composite material. The numerical results from the multi-scale structure model provide the locations of stress propagation and the progressive failure behavior within the 3D orthogonal woven composite material. The multi-scale model and the numerical simulation results are validated using compression test results at the strain rate range from 0.001 to 2100/s. The methodology we proposed could be applied to understand the microstructure damage mechanisms of 3-D textile composite materials from meso- and macro-structure level in simpler geometrical model and easier design approaches.

Keywords: 3D orthogonal woven composite material; multi-scale structure; strain rates; impact loading; compressive damage; finite element method (FEM)

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Acronyms nomenclature

3DOWC: 3D orthogonal woven composite

3DWCs: 3D woven composites

CDA: critical damage area

FEA: finite element analyses

FEM: finite element method

MMWK: multi-axial multi-layer warp knitting

RUC: representative unit cell

SHPB: split Hopkinson pressure bar

SHTB: split Hopkinson tension bar

UMAT: user-defined material

VUMAT: vectorized user-defined material

VARTM: vacuum assisted resin transfer molding

1 Introduction

Advanced composites have shown a rapid growth in recent years driven by civil aircraft programs such as Boeing 787, Airbus A350XWB and Bombardier C series. Three dimensional (3D) woven performs are particularly attractive because of the reduced part count, their ability to create near-net shapes as well as the presence of through-thickness reinforcement. Such kind of 3D woven fabrics and composites are usually applied to various strain rates occasions which may exceed the quasi-static strain rate range particularly under high velocity impact loading, such as military industries (Foglar and Kovar, 2013; Kulkarni et al., 2013), protective structures (Parsons et al., 2010), and aircrafts (Greenhalgh and Hiley, 2003). Understanding the mechanical behavior of composite materials at various strain rates is important for a wide variety of applications (Chen and Ghosh, 2012; Jia et al., 2012a). The recent years have seen a surge in researches on the 3D woven composite materials and structural responses subject to various strain rate loadings.

The mechanical behaviors of fiber reinforced composite materials under various high strain rates have been reported extensively. For example, the compressive behaviors under different strain rates have been previously investigated (Naik et al., 2010a; Ochola et al., 2004; Oguni and Ravichandran, 2001; Shokrieh and Omid, 2009; Tarfaoui et al., 2009) using hydraulic testing machines (strain rates up to $\sim 100\text{s}^{-1}$) and split Hopkinson pressure bar (SHPB) apparatus for higher strain rates (strain rates up to $\sim 10,000\text{s}^{-1}$). Many materials show strain rate sensitivity that the dynamic strength and stiffness exhibited considerable increasing as compared with those in quasi-static condition (Malvar and Ross, 1998; Walley, 2010). Hosur et al. (MV et al., 2001) and Khan et al. (Khan et al., 2002) also found this result in 2D fiber reinforced resin composite through investigating the compressive stress-strain responses of carbon/epoxy laminated composites at strain rates of 80 s^{-1} to 170 s^{-1} . As for 3D textile preform reinforced composites, most of investigations have been conducted under high strain rates loading conditions (Baozhong et al., 2010; Hao et al., 2008; Ji et al., 2007b; Liu et al., 2006; Shi et al., 2011; Sun and Gu, 2006a, 2007; Sun et al., 2007; Sun and Gu, 2006b; Sun et al., 2005). These studies were mainly focused on the determinations of damage modes and strain rate sensitivity in experimental. In addition, the multi-axial multi-layer warp knitting (MMWK) composite has higher failure stress than the 3D woven composite and the 3D braided composite at the same strain rate. However, the failure strain of the 3D braided composite is higher than that of the 3D woven composite and the 3D knitted

composite at quasi-static compression because of the quasi-isotropic structure feature in the 3D braided composite. The compressive failure modes of the 3D woven composite, the MMWK composite and the 3D braided composite are totally different because of the different preform structures (Sun and Gu, 2007). Experimental studies also include those by Kumar and Garg (Kumar and Garg, 1988), Gerlach (Gerlach et al., 2012) who have tested fiber reinforced composite under dynamic compressive loading conditions to study the effect of fiber orientations on failure modes. They found that fiber orientations have significant influence on the failure modes. Several studies also focused on the high strain rate tensile testing of the composites (Majzoubi et al., 2005; Naik et al., 2010b; Schossig et al., 2008), using a split Hopkinson tension bar (SHTB) apparatus and high speed tensile testing machine. The strain rate dependent strength of the composites was reported to vary with the testing directions. The fracture modes of the composite materials were shown to be affected by the strain rate.

Recent years have also witnessed the increasing interest in the developments of macroscopic mechanical properties of heterogeneous materials by multi-scale and multi-level computational methods. The multi-scale models can be defined as constitutive models in which the global constitutive behavior of composite material is determined simultaneously throughout the analysis based on the behavior of the constituents and their interactions. This is considerably advantageous in the case of evolving microstructure, since the evolution of the microstructure depends on loading history in most practical occasions. Furthermore, it can be analytical or computational, since analytical solutions (Ha-Minh et al., 2013; Naik et al., 2001; Tan et al., 2000) for practical problems are limited to simple geometries and generally do not account for evolutionary damage growth (Souza et al., 2008). Therefore, computational models, especially those based on the finite element method (FEM), have been widely applied.

Several multi-scale 3D finite element models with rate-dependent damage evolution equations have been developed to ascertain the damage due to fiber/matrix debonding, fiber breakage, matrix cracking, and delamination in fiber-reinforced composites under explosions and impact loading (Batra and Hassan, 2007; Chen and Ghosh, 2012; Cuong et al., 2011; Ellyin and Xia, 2001; Nilakantan et al., 2012; Souza et al., 2008). Lee and Pyo (Lee and Pyo, 2007, 2008) proposed a micromechanics based elastic damage model to predict the effective elastic behavior and weakened interface evolution in particle composites. Then a multi-level elastic damage model based on a combination of a micromechanical

formulation and a multi-level damage model was proposed to predict the effective elastic behavior and progressive weakened interface in particulate (brittle) composites. Their multi-level damage model in accordance with the Weibull's probabilistic function described the sequential, progressive weakened interface in the composites. Horstemeyer (Horstemeyer and Bammann, 2010) reviewed the development and the usage of internal state variable theory for polymers, composites and multi-scale modeling which is definitely helpful for use in practical engineering applications.

Generally, the numerical analyses of woven textile composites are often carried out in two scale models (meso-scale and macro-scale) (Cuong et al., 2011; Ha-Minh et al., 2013; Smilauer et al., 2011). The meso-scale model represents detailed fiber tows in a fabric and it can predict the responses of fiber tows: tensile, shear and compression failures. The macro-scale model is considered as a homogeneous material block. Thus the macro-scale model can only describe basic and global responses of composites. However, the meso-scale model is a key model which enables one to take into account the damage mechanism easily as well as to simulate complicated structures used in the industry. A bridge between the micro- and meso-mechanics of delamination for fiber-reinforced composites was described by (Ladeveze et al., 2006). In the cases of meso-model for 3D textile composites, most of the models (Buchanan et al., 2010; Ji et al., 2007a; Luo et al., 2007; Naik et al., 2001; Sun et al., 2009; Tan et al., 2000; Tang et al., 2011) are based on a representative volume element (RVE) homogenized finite elements (Khdir et al., 2013; Nakamachi et al., 2007) to derive the mechanical properties, due to the consequence of their structural complexity and non-linear material constitutive models. The meso-scale finite element modeling of textile composites have been also reviewed and grouped (Lomov et al., 2007).

Recently, a set of constitutive equations for large rate-dependent elastic-plastic-damage (Bruenig and Gerke, 2011) and nonlinear viscoelastic with damage (Jia et al., 2013) materials have been developed. These models are based on the concepts of continuum damage mechanics, and are able to analyze adiabatic, high strain rate dependent deformation processes for a wide range of stress triaxialities. Multi-scale models of textile fabrics based on hybrid element analysis (Nilakantan et al., 2010) and a discretization of the yarn geometry accounting for the yarn-yarn interactions at the yarns crossing points (Assidi et al., 2011) were developed. The predictions of the mesoscopic models regarding the impact of yarn geometries and mechanical properties on the overall behaviors provide a guideline for

the design of woven fabrics. A homogenization theory for non-linear time-dependent materials is rebuilt for periodic elastic-viscoplastic materials with misaligned internal structures, by employing a unit-cell defined for the aligned structure as an analysis domain (Matsuda et al., 2011). The theory is able to deal with arbitrary misalignment using the same unit-cell. Pankow et al. (Pankow et al., 2012) modeled the 3DWCs using a representative unit cell (RUC) model which incorporates rate dependent plasticity to predict the mechanical properties under high rate loading. The results show that the models were able to accurately predict the failure stress and the delamination that occurred in experimental.

To aid the material design, multi-scale mechanical modeling is indispensable for characterizing the mechanical behaviors from different structure scales. Meanwhile, homogenization-based model is proper when the distributions of strain and stress in the fiber tows and resins are required. To respond to the demands of further improvement of the mechanical properties of composite material, both of mechanical and material designs should be performed in parallel. Coupling of them for 3D orthogonal woven composites is achieved by micro-scale, meso-scale to macro-scale computational model.

The purpose of this paper will be to provide a basis for developing a new multi-scale model to investigate the compressive mechanical response of 3D orthogonal woven composite (3DOWC) material under various strain rate loading conditions. In these models, information derived from a subscale is passed in the form of a constitutive law or a homogenized property back into a macro-scale continuum point, typically to the integration point of an element. On the subscale, periodic boundary conditions are an effective common choice. A commercial available finite element method (FEM) code of ABAQUS (Inc, 2010) will be incorporated with a vectorized user-defined material (VUMAT) subroutine to calculate the impact compression of 3D orthogonal woven composites. And the ABAQUS with a user-defined material (UMAT) subroutine will be used to calculate the quasi-static compression of 3D orthogonal woven composites. The multi-scale model and the FEM results are validated using compression test results from strain rate range from 0.001 to 2100/s. The proposed model is also used to understand the effect of microstructure model and unit-cell model on the material response predictions. It should be noted that the continuum formulation presented in this paper is generic in nature and could be extended to model any textile composite material. For the sake of completeness, we will also examine the effect of microstructure on material response predictions along warp, weft and Z-yarn direction respectively.

2 Framework of the numerical simulation methodology

2.1 Multi-scale geometrical model

As the architecture of a fabric is complex in composite, the determination of the effective mechanical behavior of the composite is usually challenging. Continuum models describing the effective mechanical behavior of the textile composite are constructed from the homogenization of an identified repetitive pattern of the textile within a representative unit cell (RUC) (Angioni et al., 2011; Ansar et al., 2011; Fang and Liang, 2011; Goda et al., 2013; Jia et al., 2012c; Luan et al., 2011a; Smilauer et al., 2011). In order to investigate the mechanical properties and damage mechanisms of 3DOWCs under quasi-static and dynamic compression loadings, a methodology of multi-scale unit-cell analysis (Cai and Sun, 2013; Tchalla et al., 2013; Zuo et al., 2013) is developed with application of the periodic boundary conditions. For the multi-scale analyses, the micro-scale unit-cell reveals a uniform, periodically repetitive array of heterogeneities and the macro-scale model is a homogeneous body (Bogdanovich, 2006; Kruch and Chaboche, 2011; Sanahuja, 2013).

2.1.1 Quasi-static simulations

Quasi-static compressive simulations were completed through three scales (micro-scale, meso-scale and macro-scale) as shown in Fig. 1. The yarn mechanical parameters calculated from the micro-scale unit-cell model were transferred to the meso-scale unit-cell model. The mechanical parameters calculated from the meso-scale unit-cell model were then transferred to the macro-scale model.

The resin and fiber in the micro-scale unit-cells were modeled as isotropic elastic material. The matrix impregnated fiber tows in the meso-scale unit-cells were modeled as transversely isotropic material. The elements of the macro-scale model were modeled as orthotropic material. The mechanical properties used in the meso-scale unit-cells and macro-scale model were obtained from the FEM analysis on the sub-scale unit-cell, respectively.

To combine damage analysis into the FEM analysis on the multi-scale analysis models, the user-defined material subroutine was defined in the commercial finite element software package ABAQUS/Standard. This subroutine incorporates an elastic stress-strain relation before the damage,

damage initiation criteria for pure matrix and for the matrix impregnated fiber yarns, and a post-damage stress-strain relation for the damage elements as described in the subsequent sections.

2.1.2 Dynamic simulations

Dynamic compressive simulations were completed through two scales (micro-scale and macro-scale) as shown in Fig. 2. The yarn mechanical parameters calculated from the micro-scale unit-cell model were transferred to the macro-scale model.

The micro-scale unit-cell material model used in the dynamic condition is the same as that of quasi-static condition. The multi-scale material models used in the dynamic condition were defined to be rate-dependent. In a unit-cell of the macro-scale model, fiber tows can be regarded as transverse-isotropic materials.

2.2 The constitutive models

2.2.1 Quasi-static responses

2.2.1.1 Crack initiation criteria

From the experimental observations (Hu et al., 2003; Xia et al., 2003a), the maximum principle strain theory is appropriate to be the crack initiation criterion for the matrix material in current 3DOWC. At each step of FEM analysis, the three principle strain values $\varepsilon_1 \geq \varepsilon_2 \geq \varepsilon_3$ and their directions can be calculated based on the known strain values in the global coordinate system. A local coordinate system is established along the three principal strain direction as shown in Fig. 3. Once the condition $\varepsilon_1 \geq \varepsilon_{cr}$ is satisfied, a crack in the plane perpendicular to the direction of ε_1 in the local coordinate system is assumed to have initiated. Similarly, the maximum principle stress theory is used to describe the fracture of the fiber tows in the micro-scale unit-cell. For matrix impregnated fiber tows in the meso-scale unit-cells, the longitudinal and transverse ultimate strengths were obtained from the analysis of the micro-scale unit-cell and these two values are defined as the failure criteria, i.e. if either of the longitudinal or transverse stress values of the matrix impregnated fiber tows reaches the

corresponding ultimate strength value, a crack in the plane perpendicular to the direction is assumed to have initiated.

2.2.1.2 Post-damage constitutive model

After the initiation of a crack, the stress components σ_1 , σ_{12} and σ_{13} in the local (crack) coordinate system O-123 must be zero. However, the rest of stress components can still exist. Therefore the post-damage constitutive model is defined in the local (crack) coordinate system as

$$\{\Delta\sigma\}^{cr} = [D]\{\Delta\varepsilon\}^{cr} - \chi[B]\{\sigma\}^{cr} \quad (1)$$

For isotropic matrix material, the full form of equation (1) can be written as

$$\begin{Bmatrix} \Delta\sigma_{11} \\ \Delta\sigma_{22} \\ \Delta\sigma_{33} \\ \Delta\sigma_{12} \\ \Delta\sigma_{13} \\ \Delta\sigma_{23} \end{Bmatrix}^{cr} = \begin{bmatrix} \beta Z_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_1 & Z_2 & 0 & 0 & 0 \\ 0 & Z_2 & Z_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta Z_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta Z_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & Z_3 \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_{11} \\ \Delta\varepsilon_{22} \\ \Delta\varepsilon_{33} \\ \Delta\varepsilon_{12} \\ \Delta\varepsilon_{13} \\ \Delta\varepsilon_{23} \end{Bmatrix}^{cr} - \chi \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix}^{cr}$$

where

$$Z_1 = \frac{1-\nu}{(1+\nu)(1-2\nu)} E, Z_2 = \frac{\nu E}{(1+\nu)(1-2\nu)}, Z_3 = \frac{E}{2(1+\nu)} \quad (2)$$

E is the Young's modulus at the instant of damage initiation, β is a small number (such as $\beta=0.001$) to describe the stiffness degradation in these three particular stress directions and the constant χ (such as $\chi=0.1$) allows the three stress components to decrease to a near zero value in a sufficient short time duration.

For the matrix impregnated fiber tows in the meso-scale unit-cell, the full expression of equation (1) is

$$\begin{Bmatrix} \Delta\sigma_{11} \\ \Delta\sigma_{22} \\ \Delta\sigma_{33} \\ \Delta\sigma_{12} \\ \Delta\sigma_{13} \\ \Delta\sigma_{23} \end{Bmatrix}^{cr} = \begin{bmatrix} \beta C_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_2 & C_3 & 0 & 0 & 0 \\ 0 & C_3 & C_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta C_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta C_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_6 \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_{11} \\ \Delta\varepsilon_{22} \\ \Delta\varepsilon_{33} \\ \Delta\varepsilon_{12} \\ \Delta\varepsilon_{13} \\ \Delta\varepsilon_{23} \end{Bmatrix}^{cr} - \chi \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix}^{cr}$$

where

$$\begin{aligned} C_1 &= \frac{E_1(1-\nu_{23}^2)}{1-\nu_{23}^2-2(1+\nu_{23})\frac{E_2}{E_1}\nu_{12}^2}, C_2 = \frac{E_2(1-\frac{E_2}{E_1}\nu_{12}^2)}{1-\nu_{23}^2-2(1+\nu_{23})\frac{E_2}{E_1}\nu_{12}^2} \\ C_3 &= \frac{E_2(\nu_{23}+\frac{E_2}{E_1}\nu_{12}^2)}{1-\nu_{23}^2-2(1+\nu_{23})\frac{E_2}{E_1}\nu_{12}^2}, C_4 = G_{12}, C_6 = \frac{E_2}{2(1+\nu_{23})} \end{aligned} \quad (3)$$

E_1 and E_2 are longitudinal and transverse modulus, respectively, ν_{12} and ν_{23} are the Poisson's ratios.

For the orthogonal anisotropic mesh in the macro-scale model, the full expression of equation (1) can be written as

$$\begin{Bmatrix} \Delta\sigma_{11} \\ \Delta\sigma_{22} \\ \Delta\sigma_{33} \\ \Delta\sigma_{12} \\ \Delta\sigma_{13} \\ \Delta\sigma_{23} \end{Bmatrix}^{cr} = \begin{bmatrix} & 0 & 0 & 0 \\ [L] & 0 & 0 & 0 \\ & 0 & 0 & 0 \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & [M] \\ 0 & 0 & 0 & \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_{11} \\ \Delta\varepsilon_{22} \\ \Delta\varepsilon_{33} \\ \Delta\varepsilon_{12} \\ \Delta\varepsilon_{13} \\ \Delta\varepsilon_{23} \end{Bmatrix}^{cr} - \chi \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix}^{cr} \quad (4)$$

where

$$[L] = \frac{1}{H} \begin{bmatrix} \beta E_1(1-E_3/E_2\nu_{23}^2) & 0 & 0 \\ 0 & E_2(1-E_3/E_1\nu_{13}^2) & E_3(\nu_{23}+E_2/E_1\nu_{12}\nu_{13}) \\ 0 & E_3(\nu_{23}+E_2/E_1\nu_{12}\nu_{13}) & E_3(1-E_2/E_1\nu_{12}^2) \end{bmatrix}$$

$$[M] = \begin{bmatrix} \beta G_{12} & 0 & 0 \\ 0 & \beta G_{13} & 0 \\ 0 & 0 & G_{23} \end{bmatrix}$$

$$H = \frac{E_1 E_2 E_3 - \nu_{23}^2 E_1 E_3^2 - \nu_{12}^2 E_2^2 E_3 - 2\nu_{12} \nu_{13} \nu_{23} E_2 E_3^2 - \nu_{13}^2 E_2 E_3^2}{E_1 E_2 E_3}$$

2.2.1.3 Transformation of stress and strain from local to global coordinate system

The stress/strain components obtained in the local coordinate system through the post-damage constitutive relation must be transformed to the global coordinate system, where the finite element analysis is carried out by user-defined materials (UMATs) subroutines. The transformation matrix between the local and global coordinate systems can be expressed as

$$[T] = \begin{bmatrix} l_1^2 & m_1^2 & n_1^2 & l_1 m_1 & m_1 n_1 & n_1 l_1 \\ l_2^2 & m_2^2 & n_2^2 & l_2 m_2 & m_2 n_2 & n_2 l_2 \\ l_3^2 & m_3^2 & n_3^2 & l_3 m_3 & m_3 n_3 & n_3 l_3 \\ 2l_1 l_2 & 2m_1 m_2 & 2n_1 n_2 & l_1 m_2 + l_2 m_1 & m_1 n_2 + m_2 n_1 & n_1 l_2 + n_2 l_1 \\ 2l_2 l_3 & 2m_2 m_3 & 2n_2 n_3 & l_2 m_3 + l_3 m_2 & m_2 n_3 + m_3 n_2 & n_2 l_3 + n_3 l_2 \\ 2l_3 l_1 & 2m_3 m_1 & 2n_3 n_1 & l_3 m_1 + l_1 m_3 & m_3 n_1 + m_1 n_3 & n_3 l_1 + n_1 l_3 \end{bmatrix} \quad (5)$$

where l_i, m_i, n_i ($i=1,2,3$) are directional cosines of the axes of the local coordinate system in the global coordinate system.

Substituting equation (5) into equation (1), the post-damage constitutive equation in the global coordinate system is

$$\{\Delta \sigma\}^{gl} = [D'] \{\Delta \varepsilon\}^{gl} - \chi [B'] \{\sigma\}^{gl} \quad (6)$$

with

$$\begin{aligned} [D'] &= [T]^T [D] [T] \\ [B'] &= [T]^T [B] [T] \end{aligned}$$

2.2.1.4 Numerical implementation of the constitutive models in UMAT

The constitutive model with crack initiation and propagation is implemented for finite element analysis using the user defined material subroutine (UMAT) in a commercial code ABAQUS/Standard. The essential steps in the UMAT updated algorithm for $t^{(n)}$ and $t^{(n+1)}$ are shown in Fig. 4.

2.2.2 Dynamic responses

2.2.2.1 Rate-dependent constitutive model

As strain rates increase, many materials show an increase in their yield strength. In the dynamic compressive simulations, the materials were defined to be rate-dependent. The description of the rate-dependent properties of the vinyl matrix material and the fiber tow is similar to the formulation of Ellyin and Xia (Ellyin and Xia, 2001) for the simulation of general kinematics problems, due to the rate- and temperature-dependent elastic-plastic behavior of the matrix and the fiber tows.

Yield and memory surfaces: In rate-dependent model, two types of surfaces are defined in the stress space: (a) yield surface which is the locus of the current elastic regime, and (b) stress memory surface which defines the maximum effective stress that the material has been subject to in its previous history.

The yield surface is represented by

$$\phi_y = \frac{3}{2} \bar{s}_{ij} \bar{s}_{ij} - q^2 (\dot{\epsilon}_{eq}, l_p) = 0 \quad (7)$$

where $\bar{s}_{ij} = \bar{\sigma}_{ij} - \delta_{ij} \bar{\sigma}_{kk} / 3$ and $\bar{\sigma}_{ij} = \sigma_{ij} - \alpha_{ij}$. The α_{ij} and q specify the centre and radius

of the yield surface. $\dot{\epsilon}_{eq} = \left(\frac{2}{3} \dot{\epsilon}_{ij} \dot{\epsilon}_{ij} \right)^{1/2}$ is the equivalent strain rate with $\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij} - \delta_{ij} \dot{\epsilon}_{kk} / 3$, and

$l_p = \int \left(\frac{2}{3} d\dot{\epsilon}_{ij}^p d\dot{\epsilon}_{ij}^p \right)^{1/2}$ being the accumulated plastic strain length. An explicit relation for the radius

of the yield surface can be obtained by adopting the conventional logarithmic strain-rate dependency, i.e.

$$q = q \left[\log \left(\frac{\dot{\epsilon}_{eq}}{\dot{\epsilon}_{eq}^0} \right), l_p \right] \quad (8)$$

where $\dot{\epsilon}_{eq}$ is the current von Mises equivalent strain rate, and $\dot{\epsilon}_{eq}^0$ is a reference equivalent strain rate and it is usually taken to be the slowest strain rate in the studied range. The accumulated plastic strain length, l_p , is used to describe kinematic hardening or softening behavior of the material. A characteristic of the yield surface Eqs. (7) and (8) is that its radius q , increases with the increase of the strain rate.

The stress memory surface, ϕ_m , is related to the maximum equivalent stress level,

$\sigma_{eq} = \left(\frac{3}{2} s_{ij} s_{ij} \right)^{1/2}$, experienced in the previous loading history. It is expressed by

$$\phi_m = \frac{3}{2} s_{ij}^* s_{ij}^* - R^2(\sigma_{eq, \max}) = 0 \quad (9)$$

in which $s_{ij}^* = \sigma_{ij} - \beta_{ij}$ with β_{ij} denoting the centre and R the radius of the current memory surface.

The evolution rule of the centre of the yield surface, α_{ij} , and that of the memory surface, β_{ij} , will be specified later on.

Stress-strain relationship: The total strain rate can be decomposed into elastic and plastic parts, the rate formulation of the stress-strain relation is expressed as

$$\dot{\sigma}_{ij} = \frac{E}{1+\nu} \left\{ \delta_{ik} \delta_{jl} + \frac{\nu}{1-2\nu} \delta_{ij} \delta_{kl} - \frac{9g \bar{s}_{ij} \bar{s}_{kl}}{[(1+\nu)/E] + 9\bar{s}_{mn} \bar{s}_{mn}} \right\} \dot{\epsilon}_{kl} \quad (10)$$

where

$$g = \frac{1}{4q^2} \left(\frac{1}{E_t} - \frac{1}{E} \right) \quad (11)$$

In the above, E_t is the current tangent modulus, $E_t = E_t(\sigma_{eq}, \dot{\epsilon}_{eq}, l_p)$ which is both rate- and history-dependent. Note that the radius of the yield surface, q in Eq. (7), is also a function of rate and history of loading.

Depending on the relative position of the current stress point with respect to the yield and the stress memory surface, three types of loading cases are distinguished: (i) elastic loading, (ii) monotonic plastic loading, and (iii) plastic reloading. If the stress point is inside the yield surface or there is elastic unloading, i.e. $s_{ij}\dot{\sigma}_{ij} < 0$, then this is termed the elastic loading case (i). For the plastic loading, $s_{ij}\dot{\sigma}_{ij} > 0$ and the current stress point is on the yield surface. However, if the equivalent stress value increases, then the stress memory surface will expand but it will remain tangent to the current yield surface. This type of loading is defined as the monotonic plastic loading case (ii). The plastic loading in which the current stress point is inside the stress memory surface is classified as the plastic reloading case (iii). Note that for this case the stress memory surface does not change, however, when there is a transition from the elastic to plastic loading, the size of the yield surface would depend on the current equivalent strain rate, Eq. (8). For the latter two plastic loading cases, the evolution rules of the yield surface and the calculation of the tangent modulus, E_t , in Eq. (11) is different and this will be explained below.

Evolution of centre of the yield and memory surface: For the monotonic plastic loading, the evolution rule for the centre of the yield surface, α_{ij} , is determined from

$$\dot{\alpha}_{ij} = \dot{\mu}(\sigma_{ij}^{\text{lim}} - \sigma_{ij}^y) \quad (12)$$

and

$$\sigma_{ij}^{\text{lim}} = \frac{R_{\text{lim}}}{q}(\sigma_{ij}^{\text{lim}} - \alpha_{ij}) \quad (13)$$

in which σ_{ij}^y is the current stress point which is on the yield surface, R_{lim} is a constant, depicting the maximum limit of the stress memory surface radius. When $R_{\text{lim}} \gg q$, then one will approach the well-known Ziegler's rule for the motion of the yield surface, $\dot{\alpha}_{ij} = \dot{\mu}(\sigma_{ij}^y - \alpha_{ij})$.

The value of E_t is calculated from the family of strain-rate dependent stress-strain curves, based on the current equivalent stress value and strain rate.

For the plastic reloading case, the evolution rule of the centre of the yield surface is expressed by

$$\dot{\alpha}_{ij} = \dot{\mu}(\sigma_{ij}^m - \sigma_{ij}^y) \quad (14)$$

where σ_{ij}^m is a point on the memory surface whose exterior normal is parallel to the outward normal of current stress point, σ_{ij}^y . The modulus E_t is related to the ratio of two distances between three stress point, viz. the stress point at which plastic reloading took place, the current stress point, σ_{ij}^y , and the stress point on the memory surface obtained as described earlier.

For a monotonic loading, the centre of the memory surface remains fixed, i.e. $\dot{\beta}_{ij} = 0$.

Numerical implementation of the constitutive model: A sub-routine was written for the numerical implementation of the above described rate-dependent constitutive model. The required material input data are obtained from the rate-dependent uniaxial stress-strain curves. The time-independent elastic constants E and ν are determined from the elastic response curve and rate-dependent parameters $q(\dot{\epsilon}_{eq}, l_p)$ and $E_t(\sigma_{eq}, \dot{\epsilon}_{eq}, l_p)$ are obtained from the rate-dependent part of the stress-strain curves. A 2-D interpolation is used to obtain a continuous description of the dependency of q and E_t on the equivalent strain rate, $\dot{\epsilon}_{eq}$ and the accumulated plastic strain length, l_p .

2.2.2.2 Elastic stiffness and compliance matrices of the unit-cell

Assume $[S]$ is the compliance matrix of basalt tows in the local coordinate.

Then

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{12} & S_{23} & S_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(S_{22} - S_{23}) & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{55} \end{bmatrix} \quad (15)$$

The matrix is isotropic material, the compliance matrix is

$$[S]_m = \begin{bmatrix} 1/E_m & -\nu_m/E_m & -\nu_m/E_m & 0 & 0 & 0 \\ -\nu_m/E_m & 1/E_m & -\nu_m/E_m & 0 & 0 & 0 \\ -\nu_m/E_m & -\nu_m/E_m & 1/E_m & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_m & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_m & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_m \end{bmatrix} \quad (16)$$

For the warp fiber tows in the unit-cell, the local coordinate and the global coordinate is identical. Then

$$[\overline{S}]_{warp} = [S].$$

Generally, for the off-axis compliance matrix transformation, we have: $[\overline{S}] = [T]_{\varepsilon}^{-1} [S] [T]_{\varepsilon}$

where

$$[T]_{\varepsilon} = \begin{bmatrix} l_1^2 & m_1^2 & n_1^2 & 2m_1n_1 & 2l_1n_1 & 2l_1m_1 \\ l_2^2 & m_2^2 & n_2^2 & 2m_2n_2 & 2l_2n_2 & 2l_2m_2 \\ l_3^2 & m_3^2 & n_3^2 & 2m_3n_3 & 2l_3n_3 & 2l_3m_3 \\ 2l_2l_3 & 2m_2m_3 & 2n_2n_3 & 2(m_2n_3 + m_3n_2) & 2(l_2n_3 + l_3n_2) & 2(l_2m_3 + l_3m_2) \\ 2l_1l_3 & 2m_1m_3 & 2n_1n_3 & 2(m_1n_3 + m_3n_1) & 2(l_1n_3 + l_3n_1) & 2(l_1m_3 + l_3m_1) \\ 2l_1l_2 & 2m_1m_2 & 2n_1n_2 & 2(m_1n_2 + m_2n_1) & 2(l_1n_2 + l_2n_1) & 2(l_1m_2 + l_2m_1) \end{bmatrix}$$

The compliance matrix for the weft yarns is $[\overline{S}]_{weft} = [T]_{weft}^{-1} [S] [T]_{weft}$, where $[T]_{weft}$ is strain transformation matrix, and the elements of $[T]_{weft}$ are

$$l_1 = 0, l_2 = -1, l_3 = 0, m_1 = 1, m_2 = 0, m_3 = 0, n_1 = 0, n_2 = 0, n_3 = 1$$

The compliance matrix of Z yarn in global coordinate is: $[\overline{S}]_Z = [T]_Z^{-1} [S] [T]_Z$

The elements of $[T]_Z$ are

$$l_1 = 0, l_2 = 0, l_3 = 1, m_1 = 0, m_2 = 1, m_3 = 0, n_1 = -1, n_2 = 0, n_3 = 0$$

2.2.2.3 Unit-cell homogenization

Under the assumptions of iso-strain, strain can be transformed from the global to the local coordinate

system with the following relation:

$$\boldsymbol{\varepsilon}_k^{local} = [T] \boldsymbol{\varepsilon}^{global} \quad (17)$$

where $[T]$ is a transformation matrix.

Elastically, the global-local stress relations are:

$$\boldsymbol{\sigma}_k^{local} = C_k \boldsymbol{\varepsilon}_k^{local} \quad (18)$$

$$\boldsymbol{\sigma}_k^{global} = [T]^T \boldsymbol{\sigma}_k^{local} \quad (19)$$

In order to apply the elasto-plastic model, incremental stresses and strains must be used with the elasto-plastic compliance S^{ep} and stiffness C^{ep} .

$$[S^{ep}] = [S + S^{vp}] \cdot [C^{ep}] = [C^{ep}]^{-1} \quad (20)$$

$$[C^{ep}] = [[I] + [C] \cdot [S^{ep}]]^{-1} [C] \quad (21)$$

In incremental form:

$$\Delta \boldsymbol{\varepsilon}_k^{local} = [T] \Delta \boldsymbol{\varepsilon}^{global} \quad (22)$$

$$\Delta \boldsymbol{\sigma}_k^{local} = C_k^{ep} \Delta \boldsymbol{\varepsilon}_k^{local} \quad (23)$$

Then:

$$\Delta \boldsymbol{\sigma}_k^{local} = C_k^{ep} \Delta \boldsymbol{\varepsilon}_k^{local} = C_k^{ep} [T] \Delta \boldsymbol{\varepsilon}^{global} \quad (24)$$

$$\text{and: } \Delta \boldsymbol{\sigma}^{global} = \sum_k V_k \Delta \boldsymbol{\sigma}_k^{global} \quad (25)$$

2.2.2.4 Critical damage area (CDA) failure theory

Composite failure is determined by not only the stresses present in the material, but also the area where the stresses exceed critical values. To study the failure mechanisms of textile composites, the damage development and material properties of constituents must be better understood at the microscopic level of RUC. Therefore, a majority of damage models and failure analysis methods have been provided by many scholars in recent years (Campbell et al., 2006; Desrumaux et al., 2001; Huang, 2005; Okabe et al., 2001; Panov et al., 2006; Vignjevic et al., 2012a; Vignjevic et al., 2012b). When 3D textile composites are subjected to the external loading, damage and failure of constituents of tow and matrix within the composites may be happened. Therefore, the damage modes of woven yarn can be

distinguished by the failure criterion for the unidirectional (UD) fibrous reinforced composites. It is well known that the occurrence of damage can be predicted by classical failure criteria such as Tsai–Wu criterion (Wolfram et al., 2012), Chamis criterion (Tang and Zhang, 2012) and Hill criterion (Zhou and Zhuang, 2013) etc. which utilize all stress components and their interactivity to judge material initiation failure. However, several parameters in these criteria need to be determined by biaxial experimental tests. These criteria which give an available failure envelope are widely used to measure damage initiation in majority of commercial finite element analysis packages, while these criteria cannot distinguish the types of different damage modes.

In the previous studies, this critical area used is governed by the composite's interaction length, which can be as low as 1 mm for tape-based composites and up to 100 mm for some 3-D interlock weaves (Cox et al., 1996). The CDA criterion is necessary when considering composite failure because a composite will not necessarily fail when a single fiber or even tow, exceeds its maximum stress. Instead, composite failure will occur only when: (1) stresses exceed the local fiber strength, and (2) a critical damage area has reached. The critical area is calculated by applying Tsai–Hahn fiber-bundle theory (hahn, 1973; Hann and Tsai, 1973), originally proposed by Rosen (Rosen, 1973) to tows to calculate the critical interaction length δ .

The critical damaged length (δ) is defined as

$$\delta = 4\gamma_f \left(\frac{X_f}{4\tau} \right)^{\frac{\kappa}{\kappa+1}} \left[\frac{(\kappa+1)L}{2\gamma_f} \right]^{\frac{1}{\kappa+1}} \quad (26)$$

where γ is the tow radius, X_f is the average fiber tension strength, L is the tow length, typically assumed 1 unit length, τ is the matrix yield stress, and κ is the Weibull parameter, assumed to be 7.6.

The damage of composite must accord with two conditions. One condition is stresses in material beyond the strength of local fibers. The other is the stress area beyond the CDA.

If the CDA is square, it equals δ^2 . Before the damage of composite, fiber degradation increases with the critical damaged area.

The value of δ determined from Eq. (26) is only used as an approximation because of the assumption required for analysis and variations of the material parameters. For instance, the value of δ for E-glass/vinyl ester resin composites was found to be about 27 mm.

As the damage zone grows, the properties of the tows are degraded as

$$E_i = E_0^i \left[1 - \left(\frac{DA - DA_f^i}{CDA} \right)^2 \right] \quad (27)$$

where E_i is the E_{11} Young's Modulus at point i , E_0 is the undamaged modulus, CDA is the Critical Damaged Area, DA is the current damaged area where the longitudinal stress has exceeded the tension strength of the composite X_T , and DA_f^i is the value of DA when point i exceeds its maximum stress.

In the finite element calculation, the fibers and resins are assumed to fail at the same time because all the fibers and resins were simplified into a unit-cell. When the damage area is greater than the critical damage area, the element will be eliminated in calculation.

2.2.2.5 Numerical implementation of the constitutive models in VUMAT

In finite element analyses for impact compression, the multi-scale unit-cell model is implemented in a VUMAT subroutine and connected with a commercial available FEA software package named as ABAQUS/Explicit. This subroutine is compiled and linked with the finite element solver and enables ABAQUS/Explicit to obtain the needed information regarding the state and responses of the composite material during each time step, for each integration point of each element.

The essential features of the coupling between the ABAQUS/Explicit finite element solver and the VUMAT at each time increment at each integration point of each element can be summarized as follows:

- (a) The corresponding previous time increment stresses and material state variables as well as the current time step strain increments are provided by the ABAQUS/Explicit finite element solver to the material subroutine. In the present work, the elastic mechanical properties of the matrix and the fiber

tows, the stress of the matrix and the fiber tows, the strain of the fiber tows, as well as the flags pertaining to the failure status of the yarns and deletion status of the element are used as state variables.

(b) Using the information provided in (a) and the material model, the material stress state as well as values of the material state variables at the end of the time increment are determined within the VUMAT and return to the ABAQUS/Explicit finite element solver. In addition, the changes in the total internal and the inelastic energies (where appropriate) are computed and returned to the solver.

In the computational process, ABAQUS sends a block of strain increment to the VUMAT. Based on iso-strain assumption, the VUMAT transforms global strain increment of the unit-cell into local strain increments of fiber tows and matrix. The assumption means the strain is uniform throughout the unit-cell. With the constitutive relationship, the stress is calculated for each fiber tows and matrix. The failure criterion is used to judge whether the fiber tows and resin matrix fail or not. After that, the global stress increment is calculated according to the homogenization of the stress in fiber tows and resin matrix. Finally the VUMAT returns updated state variables and new stress state to ABAQUS. The computational process of ABAQUS with VUMAT is shown in Fig. 5.

3 Material experimental characterizations

3.1 Three-dimensional (3D) orthogonal woven composite material

The specifications of 3D orthogonal woven basalt fabric used to manufacture composite are listed in Table 1. Fig. 6 shows the top surface and cross-sections of 3D orthogonal woven basalt composite. The unsaturated polyester resin (manufactured by Shanghai Sino Composite Co., Ltd) was injected into the 3D woven fabric (manufactured by Hengdian Group Shanghai Russia & Gold Basalt Fiber Co., Ltd) by vacuum assisted resin transfer molding (VARTM). The thickness of the composite was 5.0mm. The composite coupon was cut to an in-plane size of 9.0mm×9.0mm because the diameter of the SHPB bar is 14.5mm. The diagonal of the test samples can not be greater than the diameter. In addition, considering the periodicity in the material structure and of the FE model, 5 warp yarns and 4 weft yarns were exactly included in the 9.0mm×9.0mm sample which was helpful for meshing of the FE model. The rectangular shaped sample was used in this test. On the one hand, the warp and weft yarns were

perpendicular with each other. Compression tests were conducted in the through-thickness, warp and weft directions for 3D orthogonal woven basalt composites both under quasi-static and high strain rate compression. On the other hand, the unit-cell of 3D orthogonal woven basalt composites was rectangular.

In the 3D orthogonal woven composite, only two components exist: fiber and resin. Part of the resin is filled in the fiber yarns (warp yarns, weft yarns and Z yarns) to form matrix impregnated fiber yarns (Heinrich et al., 2013). Another part fills the spaces between the matrix impregnated fiber yarns. The volume fraction of the fibers in the total composite is about 40% calculated by burning method (L. Tong, 2002). The fiber volume fraction in the matrix impregnated fiber yarn was measured by use of Scanning Electron Micro-scope (SEM) images of tow cross-sections (Pankow et al., 2012). Five regions on the SEM image were randomly selected to calculate the fiber volume fraction in the matrix impregnated fiber yarn, respectively. The average value was then calculated about 73.5%. Based on equivalent fiber volume in 3DOWC, the volume fraction of the matrix impregnated fiber yarns in the total composite was calculated as 54.4% based on the fiber volume fraction in the matrix impregnated fiber yarn (73.5%) and the fiber volume in the total composite (40%). The density of the composite was 1.86 g/cm^3 .

3.2 Test methods

3.2.1 Quasi-static compression tests

Quasi-static compression tests were conducted on MTS 810.23 material testing machine at a constant compression speed of 1mm/min. The strain rate was computed to approximate 10^{-3} s^{-1} corresponding to the loading speed.

3.2.2 High strain rate compression tests

The high strain rate compression tests were performed on a modified SHPB apparatus. The schematic diagram of the apparatus is shown in Fig. 7. It was equipped with a momentum trapping device and a gas gun with an inner diameter of 14.5mm, two maraging steel bars with diameter 14.5 mm and length

600mm. The physical properties of the incident and transmission bars are listed in Table 2. The gas gun operated on compressed nitrogen gas and can propel the strike bar. The length of the strike bar was 200mm. The impact velocity of the strike bar varied with the gas pressure. By changing the gas pressure, we can get different impact velocities and furthermore, different strain rates.

During the impact compression tests on a SHPB apparatus, an elastic stress pulse is imparted to the incident bar by impacting it with a striker bar of the same cross sectional area and modulus. The impact of the striker bar generates an elastic stress wave equal to twice the length of the striker bar and propagates through the incident bar with the velocity of sound in the bar media and passes through the specimen while deforming it. The particle velocity imparted on the incident bar is half the impact velocity of the striker bar. The stress in the bar is given by $\sigma = \rho C_0 v_p$, where ρ is the density of bar material, C_0 is the bar velocity and v_p is the particle velocity. When the elastic wave reaches the specimen-incident bar interface, part of it is reflected back, and part of it is transmitted through the specimen and passes through the transmission bar. To acquire the direct incident pulse, the reflected pulse and the transmitted pulse, strain gages are mounted on both the incident and transmitter bars. One-dimensional wave propagation is assumed to analyze the strain signals from the strain gages. If the modulus, cross section area and density of the bar are denoted by E_b, A_b and ρ_b and those of the specimen are E_s, A_s and ρ_s , the equations for the strain-rate($\dot{\epsilon}$), strain (ϵ) and stress (σ) of the specimen are given by

$$\dot{\epsilon}(t) = -\frac{2C_0}{L_s} \epsilon_R(t) \quad (28)$$

$$\epsilon(t) = -\frac{2C_0}{L_s} \int_0^t \epsilon_R(t) dt \quad (29)$$

$$\sigma(t) = \frac{E_b A_b}{A_s} \epsilon_T(t) \quad (30)$$

where $C_0 = \sqrt{E_b/\rho_b}$ is the longitudinal wave velocity in the bar, L_s is the specimen length, and $\varepsilon_R(t)$ and $\varepsilon_T(t)$ are the strain gage signal of the reflected and the transmitted pulses, respectively. Eqs. (28~30) are based on the assumption that the dynamic forces on both sides of the specimen are equal and can be expressed as:

$$\varepsilon_I + \varepsilon_R = \varepsilon_T \quad (31)$$

Eqs. (29)-(30) give the average stress and strain in the specimen as a function of time. The equations also show that the strain can be obtained by integrating the reflected pulse and the stress in the specimen from the transmitted pulse.

It must be mentioned that the strain rates are not constant during the tests. Wave speed in the considered composite sample is not uniform because of internal stress wave propagation and interaction between the different constituents of the textile architecture (Pankow et al., 2012). Matrix and fibers have significantly different dynamic impedance. The elastic wave speed for fibers and matrix is

$$C_0 = \sqrt{E/\rho} \quad (32)$$

where E is the modulus of the material; ρ is the density of the material.

Fig. 8 presents the strain rate history at three strain rates in warp compression. In the present study, the constant strain rates indicated by the red circle are adopted. The warp and weft directions were subjected to three different strain rates of 800, 1500, 2100 s⁻¹, respectively. The through-thickness direction was 1500, 2800, 3500 s⁻¹. All the tests were performed at room temperature.

3.3 Results presenting illustrative examples of the experiment

Figs. 9, 10 and 11 present the post-mortem photographs of 3D orthogonal woven composite coupons compressed at various strain rates. From Fig. 9, it is observed that the dominant failure pattern is shear failure in through-thickness direction loading. From Figs. 10 and 11, it is found that no delamination

failure modes can be clearly found under quasi-static loading in warp and weft directions because of z-binder reinforcement. The delamination occurs partly on the front face of the sample which is near the incident bar, but the rear of the sample has no delamination as shown in Fig. 12. This is the biggest difference between the quasi-static test and dynamic test in the in-plane direction loading for 3D orthogonal woven composite. (Sun et al., 2010)

4 Numerical analyses

4.1 Connections between constitutive equations and finite element code

A multi-scale structure model ranging from micro level, to meso and macro level has been proposed in this paper. The multi-scale structure model covers from fiber/resin to fiber tows and unit-cells level. The unit-cells have homogenized the heterogeneous fiber/resin system with the periodic boundary conditions. The micro-scale unit-cell model includes elastic brittle fibers in the elastic resin with hexagonal distribution. The meso-scale unit-cell model consists of transversely isotropic resin impregnated fiber bundles (warp yarns, weft yarns and Z yarns) in isotropic resin. The macro-scale model is modeled as orthotropic assembled by the meso-scale unit-cell. In these models, information derived from the simulation of a subscale is passed in the form of a constitutive law or a homogenized property back into a macro-scale continuum point, typically to the integration point of a finite element. Quasi-static compressive simulations were completed through three scales (micro-scale, meso-scale and macro-scale). An elastic stress-strain relation before the damage, damage initiation criteria for pure matrix and for the matrix impregnated fiber yarns, and a post-damage stress-strain relation for the damage elements were included in a user-defined material (UMAT) subroutine as the constitutive equations for quasi-static loading. Dynamic compressive simulations were completed through two scales (micro-scale and macro-scale). A macroscopic rate dependent plasticity model for the woven composite materials combined with critical damage area failure theory that accounts for the compressive deformation and failure strength of the material were defined in a vectorized user-defined material (VUMAT) subroutine to calculate the dynamic compression of 3D orthogonal woven composites.

4.2 Numerical modeling of the test

4.2.1 Micro-scale unit-cell model

For matrix impregnated fiber yarns, each one includes hundreds of fibers impregnated in the matrix with randomly packed pattern in the transverse direction. The tows which consist of matrix and fibers have been utilized for homogenized model as 3D solid. The size of the fibers within a tow was chosen such that the fiber volume fraction and the outer dimension of the tow was equivalent to the dimensions of an actual tow. For the fiber packing patterns in resin, rectangular cross-section model (Haj-Ali and Muliana, 2004; Xia et al., 2003b), column model (Ivanov and Tabiei, 2001), ellipse model (Heinrich et al., 2013; Shunjun et al., 2010; Song et al., 2007), and hexagonal model (Wang et al., 2007) have been investigated. Among them, the hexagonal model is widely applied for modeling transversely isotropic property of matrix impregnated fiber yarns. We assume that the fibers in the matrix impregnated fiber yarns are in a hexagonal array distribution. A micro-scale unit-cell as shown in Fig. 13 can be introduced to analyze the mechanical behaviors and damage evolution of the matrix impregnated fiber yarns. In the micro-scale unit-cell model, the yarns are usually considered as unidirectional fiber reinforced composite with periodic distribution of fibers in resin. The dimensions of the micro-scale unit-cell in Fig. 13 are $3.46 \times 2 \times 0.1$ mm (length $\times$ width $\times$ thickness).

4.2.2 Meso-scale unit-cell model

In the 3D orthogonal woven fabric, the warp and weft yarns provide high in-plane stiffness and strength, and the binder yarns run through the thickness direction to stabilize the woven structure (Jia et al., 2012b). In ideal case, the warp yarns, weft yarns and Z yarns are perpendicular to each other. The unit-cell model of 3D orthogonal woven composite could be simplified as shown in Fig. 14(a). Each fiber tow was modeled as transversely isotropic, while the resin was defined as isotropic. Fig. 14(b) shows the mesh scheme of the unit-cell model. The element type of C3D8R in ABAQUS was chosen for mesh. In this unit-cell, a global Cartesian coordinate, x-y-z, was defined as along longitudinal (warp yarns), transverse (weft yarns) and thickness (Z yarn) direction. The warp, weft and Z yarns' axis are along the global Cartesian coordinate x, y and z, respectively. Each yarn has its own local Cartesian coordinate, 1-2-3, which along the yarn's axis (1-direction), perpendicular to the axis (2-direction) and 3-direction which perpendicular to 1-2 plane. To connect the meso-scale unit-cell model and the

macro-scale model, the size of the meso-scale unit-cell model is same with an element of the macro-scale model. The global Cartesian coordinate, x - y - z , in the meso-scale unit-cell model is the same with the global Cartesian coordinate in the macro-scale model.

4.2.3 Macro-scale model

The macro-scale model was built in the pre-processor of commercial FEA software ABAQUS/CAE as a homogeneous body (Dayyani et al., 2013; Sharma et al., 2014; Uchida and Tada, 2013) which presents the sample used in the test and the mesh scheme is shown in Fig. 16. The element type of C3D8R in ABAQUS was chosen for meshing. The C3D8R element in ABAQUS/CAE is a linear reduced-integration eight-node brick element with hourglass control. The analysis results on the integration point can be gotten easily because the C3D8R element has only one integration point. Furthermore, the shape of C3D8R element is similar to the meso-scale unit-cell model. The element size was the same as that of the meso-scale unit-cell. The macro-scale model size was the same as that of the sample used in the test. There are 5 warp yarns and 4 weft yarns in the sample. As is shown in Fig. 15, in the x - y plane, each intersection point of warp, weft and Z yarn was defined as a unit-cell (Fig. 15(a)). Half thickness of the warp and weft yarn was defined as a unit-cell in through-thickness direction (Fig. 15(b)). In this meshing scheme, 5 elements in warp direction (x direction), 4 elements in weft direction (y direction) and 6 elements in through-thickness direction (z direction) are found in the macro-scale model.

4.3 SHPB testing model

The FE testing model of SHPB is presented in Fig. 16. The bars of the SHPB setups were regarded as elastic and the constitutive relation of the generalized Hooke's Law was adopted. The displacement and rotation freedoms except the displacement along the normal impact direction of the bars were fixed. The length of the bars was 400mm in order to allow for proper strain wave propagation, on other hand, to reduce the calculation time. The input stress waves were derived from the incident bar. The whole model was meshed as 8-node solid elements C3D8R. The contact between the composite coupon and the bar is defined as 'General' contact in the FEM software ABAQUS.

4.4 Periodic boundary conditions for unit-cells

There are several boundary conditions used for the analysis and prediction of the overall behavior of composite materials. In Aboudi's work, homogeneous boundary conditions (Aboudi, 1990) were applied to the RVE or unit cell models. For a shear loading case, many researchers, e.g., Needleman and Tvergaard, Sun and Vaidya, among others, have indicated that the 'plain-remain-plain' boundary conditions (Needleman and Tvergaard, 1993; Sun and Vaidya, 1996) are over-constrained boundary conditions. Actually, these boundary conditions are not only over-constrained boundary conditions but may also violate the stress/strain periodicity conditions.

Xia *et al.* (Xia et al., 2003b; Xia et al., 2006) have developed an explicit unified form of periodic boundary conditions and used it in FEM analyses of unit-cells for unidirectional and angle-ply laminates. Since the 3D orthogonal woven composite materials can also be envisaged as a periodical array of unit-cells, the periodic boundary conditions will be adopted in this analysis. Here, the procedure to implement the periodic boundary conditions to unit-cells in finite element analysis by master-slave node technique is described.

For a 3D periodic structure, the displacement field can be expressed by

$$u_i(x_1, x_2, x_3) = \bar{\varepsilon}_{ik} x_k + u_i^*(x_1, x_2, x_3) \quad (i, k=1, 2, 3) \quad (33)$$

where $\bar{\varepsilon}_{ik}$ are the global (average) strains applied to the periodic body and x_k is the coordinate of a material point in Cartesian system. In the above, $\bar{\varepsilon}_{ik} x_k$ represents a linear distributed displacement field and $u_i^*(x_1, x_2, x_3)$ is the periodic part of the displacements which is repeated from one unit-cell to another.

The displacements on a pair of parallel opposite boundary surfaces denoted as ' j^+ ' and ' j^- ' are expressed as

$$u_i^{j+} = \bar{\varepsilon}_{ik} x_k^{j+} + u_i^* \quad (34)$$

$$u_i^{j-} = \bar{\varepsilon}_{ik} x_k^{j-} + u_i^* \quad (35)$$

For any periodic structure, the $u_i^*(x_1, x_2, x_3)$ is same at two parallel boundaries. Therefore, the difference between the above-mentioned two equations can be written as

$$u_i^{j+} - u_i^{j-} = \bar{\varepsilon}_{ik} (x_k^{j+} - x_k^{j-}) = \bar{\varepsilon}_{ik} \Delta x_k^j \quad (36)$$

where Δx_k^j is a constant for any pair of corresponding points on the two parallel boundary surfaces.

And the right side of equation (36) becomes constant with specified global strains $\bar{\varepsilon}_{ik}$. As a special type of displacement boundary conditions, equation (36) can be easily applied in the finite element analysis as nodal displacement constraint equations. The application of equation (36) can guarantee satisfaction of the continuity of displacement and traction at the opposite parallel boundary surfaces as proved by Kuo (Kuo, 2008).

The equation (36) is applied by master and slave nodes technique as illustrated in Fig. 17. In the cubic unit-cell with periodicity in all three directions, Fig.17, $\xi - \xi'$, $\eta - \eta'$ and $\zeta - \zeta'$ are pairs of periodic nodes locating on the opposite parallel boundary surfaces of the unit-cell and the lines linking them are parallel to the three coordinate axes, respectively. The rigid body translations and rotations of the cubic cell can be constraint by setting the point **A** to be fixed, point **B** to be constraint in x_2 direction and point **D** to be constraint in x_1 and x_2 directions as shown in Fig. 18. Taking the corner points **A'**, **B** and **D** as the master nodes, the equation (36) can be reduced as

$$u_i^\xi = u_i^{\xi'} + u_i^B, u_i^\eta = u_i^{\eta'} + u_i^A, u_i^\zeta = u_i^{\zeta'} + u_i^D \quad (37)$$

In the finite element model, the meshes on the two opposite surfaces of the unit-cell should be same for applying the above equation (37). The implementation of equation (37) can be further divided into three categories for the corner nodes, the nodes on the edges and the rest nodes on the boundary surfaces.

For the corner nodes **C**, **D'**, **B'**, **C'**, equation (37) is reduced to

$$u_i^C - u_i^D = u_i^B, u_i^D - u_i^{D'} = u_i^A, u_i^{B'} - u_i^A = u_i^B, u_i^{C'} - u_i^C = u_i^A \quad (i=1,2,3) \quad (38)$$

For the nodes on the edges of the cubic cell except the corner nodes, the equation (37) would be expressed by equations (39)-(41), respectively

$$u_i^{CD} - u_i^{CD} = u_i^A, u_i^{CD} - u_i^{AB} = u_i^D, u_i^{AB} - u_i^{AB} = u_i^A \quad (i=1,2,3) \quad (39)$$

$$u_i^{CC} - u_i^{DD} = u_i^B, u_i^{CC} - u_i^{BB} = u_i^D, u_i^{BB} - u_i^{AA} = u_i^B \quad (i=1,2,3) \quad (40)$$

$$u_i^{BC} - u_i^{AD} = u_i^B, u_i^{BC} - u_i^{BC} = u_i^A, u_i^{BC} - u_i^{AD} = u_i^B \quad (i=1,2,3) \quad (41)$$

Finally for all the rest nodes on boundary surfaces the equation (37) is applied directly.

5 Results and discussions

5.1 Responses of micro-scale unit-cell and damage evolution

The mechanical behaviors of micro-scale unit-cell were analyzed based on the properties of fiber and resin. For the basalt fiber, the Young's modulus is 71GPa, Poisson's ratio is 0.21 and the failure stress is 1.4GPa. For the vinyl ester resin, the Young's modulus is 2GPa, Poisson's ratio is 0.3 and the failure strain is 4%. For micro-scale unit-cell, the strain-controlled uniaxial normal and shear loading were applied, respectively.

Stress-strain curves of micro-scale unit-cell under different strain loadings are shown in Fig. 18. Before damage initiation, Fig. 18(a) figures a linear stress-strain response under the uniaxial normal loading along the fiber direction. With the increase of strain loading, the matrix impregnated fiber yarn fractured fully at the ultimate strength of 1.055GPa at strain of $\bar{\epsilon}_{11} = 2\%$.

For the loading along the x_2 direction, the maximum stress (ultimate strength in x_2 direction) of $\bar{\sigma}_{22} = 0.2$ GPa is reached at applied load of $\bar{\epsilon}_{22} = 1.8\%$. Similarly, under the transversely loading along x_3 direction, the maximum stress (ultimate strength in x_3 direction) of $\bar{\sigma}_{33} = 0.246$ GPa is reached at applied load of $\bar{\epsilon}_{33} = 2.3\%$ as shown in Fig. 18(b). It is noted that before the damage initiation, the stress-strain curves in the x_2 and x_3 directions are in coincidence, see Fig. 18(b), indicating that the micro-scale unit-cell based on the hexagonal array of fibers does predict a transversely isotropic elastic property. However, when the damage analysis is introduced, different ultimate strength values appear due to different stress/strain distributions and different damage initiation/propagation morphology for the two different loading directions.

The predicted shear stress-shear strain curves for three pure shear loading cases are shown in Fig. 18(c).

It is found that $\bar{\sigma}_{12} - \bar{\varepsilon}_{12}$ and $\bar{\sigma}_{13} - \bar{\varepsilon}_{13}$ relations are the same before the initiation of damage.

However, their responses deviated from each other after the damage, resulting different ultimate shear strengths of $\bar{\sigma}_{12} = 46.4$ MPa, $\bar{\sigma}_{13} = 49.7$ MPa, respectively. The predicted ultimate shear strength in the transverse plane is $\bar{\sigma}_{23} = 43.9$ MPa.

Fig. 19 presents the stress wave propagation in micro-scale unit-cell under uniaxial loading along x_2 direction. At applied load of $\bar{\varepsilon}_{22} = 0.018$ as shown in Fig.19(a), the maximum stress of $\bar{\sigma}_{22} = 0.2$ GPa is reached. With the increase of strain loading, the damage is propagated to the whole resin zone around the fibers as shown in Fig. 19(b).

The stress wave propagation in micro-scale unit-cell under shear strain loading are presented in Fig. 20.

The maximum stress of $\bar{\sigma}_{13} = 49.7$ MPa is reached at the shear loading of $\bar{\varepsilon}_{13} = 0.0108$ as shown in Fig. 20(a). The final failure mode is shown in Fig. 20(b) which is mainly the slip between fiber and resin.

Table 3 summarizes the mechanical parameters calculated from micro-scale unit-cell model. It can be seen that the transversely isotropic elastic properties are well predicted (before damage). As explained above, the current micro-scale unit-cell predicts different ultimate strengths of $\bar{\sigma}_{22} = 0.2$ GPa and $\bar{\sigma}_{33} = 0.246$ GPa. In the later damage analysis on the meso-scale unit-cells, the smaller value will be used as the transverse ultimate strength of the matrix impregnated fiber tows.

5.2 Responses of meso-scale unit-cell and damage evolution

The mechanical behaviors of meso-scale unit-cell are analyzed based on the properties of the resin and the matrix impregnated fiber. As mentioned above, the matrix impregnated fiber yarns are regarded as transversely isotropic. The resin is isotropic. Same as the micro-scale unit-cell, the strain-controlled uniaxial normal and shear loadings were applied to the meso-scale unit-cell.

Fig.21 shows the stress-strain curves of meso-scale unit-cell under different strain loadings respectively. The mechanical parameters calculated from meso-scale unit-cell model based on the properties of the matrix impregnated fiber yarn and resin are listed in Table 4. It can be seen that the meso-scale unit-cells are predicted to be orthogonal anisotropic.

Fig. 22 shows the stress wave propagation in meso-scale unit-cell under compressive loading along warp direction. At global strain of $\bar{\varepsilon}_{xx}=0.007$, one can see from Fig. 22(a) that the warp yarn takes most of the load. The weft yarn and Z yarn take a fraction of the load. The maximum stress in meso-scale unit-cell is reached at strain load of $\bar{\varepsilon}_{xx}=0.036$ as shown in Fig. 22(b).

Fig. 23 displays the stress wave propagation in meso-scale unit-cell under in-plane shear strain loading. Under the strain loading of $\bar{\varepsilon}_{xy}=0.12$ at which the maximum stress of meso-scale unit-cell is reached, the unit-cell has the higher stress concentrated zone to be located by the Z yarn as shown in Fig. 23(a). Upon further loading at $\bar{\varepsilon}_{xy}=0.27$ (Fig. 23(b)), the unit-cell keep similar stress concentrated locations as before. The new stress concentrated area is obvious observed in the warp yarn.

5.3 Responses of macro-scale model and damage evolution

5.3.1 Validation of the multi-scale FE models

In order to validate the multi-scale finite element models, a number of compression tests in warp, weft and through-thickness directions at quasi-static and high strain rates were conducted, respectively. The comparisons of the stress-strain curves of the composites obtained from the experiment and the numerical model were used to verify the validation of FEM as shown in Fig. 24-26. It is noted that all the simulation curves in these figures showed a reasonable agreement with the relative experimental data before the peak load. That suggests the multi-scale finite element models proposed was able to capture the essential features of the composite response at the homogeneous unit-cell level. Further, we can found that the simulation results can fit very well with the experiment in terms of elastic response and failure strain other than the peak stress. The calculated value of peak stress is bigger than those obtained from experiment but within acceptable tolerances. This difference is mainly caused by the

finite element model which does not take the debonding at the interface of composite bearing compression and initial defects (air voids, fiber breakage and micro-crack) into consideration. After the peak load, there are great differences between the experiment and the finite element simulation that all the numerical results suddenly or gradually dropped to a small value due to some of the elements of 3DOWC were damaged or deleted from the model. The damaged or deleted elements can not carry the loads any more, while the damaged composite material can still carry the external loads, especially compression loading. However, in practical, it should be mentioned that the damaged composite material could no longer be applied to engineering structure due to the damaged composite is not safe for design.

5.3.2 Strain rate sensitivity

As depicted by Fig. 24-26, the compressive stress-strain behaviors of 3DOWC along three directions were found to have obvious strain rate sensitivity. With the strain rates ranging from 0.001 to 3500 s⁻¹ in all three compressive directions, its modulus and maximum stress increase while failure strain decreases with the increase of strain rate. While the strain rate sensitivity in in-plane direction was higher than that in through-thickness direction. In this respect, the strain rate sensitivity of maximum stress in three loading directions is used to explain the difference between in-plane direction and through-thickness direction. The maximum stress in Fig. 27 is obtained from Figs. 24-26 as an example to compare the strain rate sensitivity between in-plane direction and through-thickness direction. The strain rate sensitivity in warp, weft and through-thickness direction was calculated as 0.045, 0.034 and 0.03, respectively.

5.3.3 Mechanical responses

To achieve a thorough understanding of the mechanical behavior of the 3DOWC subjected to high strain rate load, the 3D computational analyses were performed to predict the stress propagation, the 3D stress state and the progressive failure behavior. Herein, the rate of 800/s for warp compression, the rate of 2100/s for weft compression and the rate of 2800/s for through-thickness compression are used to explain the warp direction, weft direction and through-thickness direction mechanical responses, respectively.

5.3.3.1 Warp direction

Fig. 28 shows the mechanical responses of the finite element model for 800/s strain rate loading along warp direction. The von Mises stress pictures which are corresponding to the fluctuant points of the stress-strain curve are shown in Fig. 28(a). The degradation of the elements in ellipse (Fig. 28(b)) are the reason of a sudden drop from **a** to **b** in the stress-strain curve. The first element deletion in picture **e** of Fig. 28(a) is corresponding to the point **e** of the stress-strain curve. It can be found that the degradation and failure of the elements both occurred in the transmission surface.

5.3.3.2 Weft direction

Similar to the specimens tested in warp direction, Fig. 29 shows the stress-strain history for the weft FE model at the rate of 2100/s. The von Mises stress pictures **a-h** in Fig. 29(a) are corresponding to the points **a-h**. The first element delete at the transmission surface of picture **h** as shown in picture **i**. A drop from point **a** to point **b** appear because of the element degradation in Fig. 29(b). As the same as above, the degradation of the elements in Fig. 29(c) accounts for the drop from point **e** to point **f**.

5.3.3.3 Z direction

The strain history for the through-thickness finite element model at the rate of 2800/s is shown in Fig. 30. Fig. 30(a) shows the von Mises stress of the fluctuant points (**a, b**) in the stress-strain curve. The first deletion of the elements occurred in the incident surface as shown in Fig. 30(b).

6 Conclusions

In this paper, a multi-scale structure model of 3D orthogonal woven composite material has been proposed. The multi-scale structure mode covers micro structure of fibers and resins, meso structures of resin impregnated fiber tows, macro structures of unit cell model. The periodic boundary conditions and macroscopic rate dependent plasticity model for the woven composite materials have been combined with critical damage area failure theory for calculating the compressive deformation and failure strength of the material.

The quasi-static and high strain rate compressive deformations and damages of 3D orthogonal woven composite material were investigated experimentally and numerically in the warp, weft and through-thickness directions with the multi-scale structure model. In the model, information derived

from the simulation of a subscale is passed in the form of a constitutive law or a homogenized property back into a macro-scale continuum point, typically to the integration point of a finite element. The stress-strain behaviors of the composite along three directions were found to be strain rate sensitive. While the strain rate sensitivity in in-plane direction was higher than that in through-thickness direction. The finite element analyses results provided insight into the locations of mechanical response within the material. A good agreement between experimental results and numerical predictions was observed. It was found that the transmission surface at one end of the composite was first damaged in weft and warp compressive direction, while the incident surface was first damaged under through-thickness compression. The verified multi-scale model can also be extended to analyze the deformations and damages of other types of textile composite materials (or more generally, to fiber reinforced composite material) under quasi-static and high strain rate loading.

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References

- Aboudi, J., 1990. Micromechanical prediction of initial and subsequent yield surfaces of metal matrix composites. *International Journal of Plasticity* 6, 471-484.
- Angioni, S.L., Meo, M., Foreman, A., 2011. A critical review of homogenization methods for 2D woven composites. *Journal of Reinforced Plastics and Composites* 30, 1895-1906.
- Ansar, M., Xinwei, W., Chouwei, Z., 2011. Modeling strategies of 3D woven composites: A review. *Composite Structures* 93, 1947-1963.
- Assidi, M., Ben Boubaker, B., Ganghoffer, J.F., 2011. Equivalent properties of monolayer fabric from mesoscopic modelling strategies. *International Journal of Solids and Structures* 48, 2920-2930.
- Baozhong, S., Zhilin, N., Lvtao, Z., Bohong, G., 2010. Mechanical Behaviors of 2D and 3D Basalt Fiber Woven Composites Under Various Strain Rates. *Journal of Composite Materials* 44, 1779-1795.
- Batra, R.C., Hassan, N.M., 2007. Response of fiber reinforced composites to underwater explosive loads. *Composites Part B-Engineering* 38, 448-468.
- Bogdanovich, A.E., 2006. Multi-scale modeling, stress and failure analyses of 3-D woven composites. *Journal of Materials Science* 41, 6547-6590.
- Bruenig, M., Gerke, S., 2011. Simulation of damage evolution in ductile metals undergoing dynamic loading conditions. *International Journal of Plasticity* 27, 1598-1617.
- Buchanan, S., Grigorash, A., Archer, E., McIlhagger, A., Quinn, J., Stewart, G., 2010. Analytical elastic stiffness model for 3D woven orthogonal interlock composites. *Composites Science and Technology* 70, 1597-1604.
- Cai, Y.M., Sun, H.Y., 2013. Prediction on viscoelastic properties of three-dimensionally braided composites by multi-scale model. *Journal of Materials Science* 48, 6499-6508.
- Campbell, J.C., Vignjevic, R., Mirkovic, J., 2006. A damage model for orthotropic metals. *Journal De Physique Iv* 134, 481-486.
- Chen, Y., Ghosh, S., 2012. Micromechanical analysis of strain rate-dependent deformation and failure in composite microstructures under dynamic loading conditions. *International Journal of Plasticity* 32-33, 218-247.
- Cox, B.N., Dadkhah, M.S., Morris, W.L., 1996. On the tensile failure of 3D woven composites. *Composites Part a-Applied Science and Manufacturing* 27, 447-458.

- Cuong, H.-M., Kanit, T., Boussu, F., Imad, A., 2011. Numerical multi-scale modeling for textile woven fabric against ballistic impact. *Computational Materials Science* 50, 2172-2184.
- Dayyani, I., Friswell, M.I., Ziaei-Rad, S., Flores, E.I.S., 2013. Equivalent models of composite corrugated cores with elastomeric coatings for morphing structures. *Composite Structures* 104, 281-292.
- Desrumaux, F., Meraghni, F., Benzeggagh, M.L., 2001. Generalised Mori-Tanaka scheme to model anisotropic damage using numerical Eshelby tensor. *Journal of Composite Materials* 35, 603-624.
- Ellyin, F., Xia, Z.H., 2001. Rate-dependent constitutive modelling and micro-mechanical analysis of fibre-reinforced metal-matrix composites. *Journal of the Mechanics and Physics of Solids* 49, 2543-2555.
- Fang, G., Liang, J., 2011. A review of numerical modeling of three-dimensional braided textile composites. *Journal of Composite Materials* 45, 2415-2436.
- Foglar, M., Kovar, M., 2013. Conclusions from experimental testing of blast resistance of FRC and RC bridge decks. *International Journal of Impact Engineering* 59, 18-28.
- Gerlach, R., Siviour, C.R., Wiegand, J., Petrinic, N., 2012. In-plane and through-thickness properties, failure modes, damage and delamination in 3D woven carbon fibre composites subjected to impact loading. *Composites Science and Technology* 72, 397-411.
- Goda, I., Assidi, M., Ganghoffer, J.-F., 2013. Equivalent mechanical properties of textile monolayers from discrete asymptotic homogenization. *Journal of the Mechanics and Physics of Solids*.
- Greenhalgh, E., Hiley, M., 2003. The assessment of novel materials and processes for the impact tolerant design of stiffened composite aerospace structures. *Composites Part a-Applied Science and Manufacturing* 34, 151-161.
- Ha-Minh, C., Imad, A., Boussu, F., Kanit, T., 2013. On analytical modelling to predict of the ballistic impact behaviour of textile multi-layer woven fabric. *Composite Structures* 99, 462-476.
- hahn, H.T., 1973. Nonlinear behavior of laminated composites. *Journal of Composite Materials* 7, 257-271.
- Haj-Ali, R.M., Muliana, A.H., 2004. A multi-scale constitutive formulation for the nonlinear viscoelastic analysis of laminated composite materials and structures. *International Journal of Solids and Structures* 41, 3461-3490.
- Hann, H.T., Tsai, S.W., 1973. Nonlinear elastic behaviour of unidirectional composite laminae. *Journal*

of Composite Materials 7, 102-118.

Hao, A., Sun, B.Z., Qiu, Y.P., Gu, B.H., 2008. Dynamic properties of 3-D orthogonal woven composite T-beam under transverse impact. Composites Part a-Applied Science and Manufacturing 39, 1073-1082.

Heinrich, C., Aldridge, M., Wineman, A.S., Kieffer, J., Waas, A.M., Shahwan, K.W., 2013. The role of curing stresses in subsequent response, damage and failure of textile polymer composites. Journal of the Mechanics and Physics of Solids 61, 1241-1264.

Horstemeyer, M.F., Bammann, D.J., 2010. Historical review of internal state variable theory for inelasticity. International Journal of Plasticity 26, 1310-1334.

Hu, Y.F., Xia, Z.H., Ellyin, F., 2003. Deformation behavior of an epoxy resin subject to multiaxial loadings. Part I: Experimental investigations. Polymer Engineering and Science 43, 721-733.

Huang, Z.M., 2005. Efficient Approach to the Structure-Property Relationship of Woven and Braided Fabric-reinforced Composites up to Failure. Journal of Reinforced Plastics and Composites 24, 1289-1309.

Inc, ABAQUS, 2010. ABAQUS Theory Manual. Version V6.10.

Ivanov, I., Tabiei, A., 2001. Three-dimensional computational micro-mechanical model for woven fabric composites. Composite Structures 54, 489-496.

Ji, C., Sun, B., Qiu, Y., Gu, B., 2007a. Impact damage of 3D orthogonal woven composite circular plates. Applied Composite Materials 14, 343-362.

Ji, C.G., Sun, B.Z., Qiu, Y.P., Gu, B.H., 2007b. Impact damage of 3D orthogonal woven composite circular plates. Applied Composite Materials 14, 343-362.

Jia, X., Sun, B., Gu, B., 2012a. A Numerical Simulation on Ballistic Penetration Damage of 3D Orthogonal Woven Fabric at Microstructure Level. International Journal of Damage Mechanics 21, 237-266.

Jia, X., Xia, Z., Gu, B., 2013. Nonlinear viscoelastic multi-scale repetitive unit cell model of 3D woven composites with damage evolution. International Journal of Solids and Structures 50, 3539-3554.

Jia, X.W., Sun, B.Z., Gu, B.H., 2012b. A Numerical Simulation on Ballistic Penetration Damage of 3D Orthogonal Woven Fabric at Microstructure Level. International Journal of Damage Mechanics 21, 237-266.

Jia, X.W., Xia, Z.H., Gu, B.H., 2012c. Micro/meso-scale damage analysis of three-dimensional

orthogonal woven composites based on sub-repeating unit cells. *Journal of Strain Analysis for Engineering Design* 47, 313-328.

Khan, A.S., Colak, O.U., Centala, P., 2002. Compressive failure strengths and modes of woven S2-glass reinforced polyester due to quasi-static and dynamic loading. *International Journal of Plasticity* 18, 1337-1357.

Khdir, Y.K., Kanit, T., Zairi, F., Nait-Abdelaziz, M., 2013. Computational homogenization of elastic-plastic composites. *International Journal of Solids and Structures* 50, 2829-2835.

Kruch, S., Chaboche, J.-L., 2011. Multi-scale analysis in elasto-viscoplasticity coupled with damage. *International Journal of Plasticity* 27, 2026-2039.

Kulkarni, S.G., Gao, X.L., Horner, S.E., Zheng, J.Q., David, N.V., 2013. Ballistic helmets - Their design, materials, and performance against traumatic brain injury. *Composite Structures* 101, 313-331.

Kumar, P., Garg, A., 1988. Failure modes and fractographic study of glass epoxy composite under dynamic compression. *Journal of Materials Science* 23, 2305-2309.

Kuo, C.-M., 2008. Elastic bending behavior of solid orthogonal woven 3-D carbon-carbon composite beams. *Composites Science and Technology* 68, 666-672.

L. Tong, A.P.M., M.K. Bannister, 2002. *3D Fibre Reinforced Polymer Composites* Elsevier

Ladeveze, P., Lubineau, G., Marsal, D., 2006. Towards a bridge between the micro- and mesomechanics of delamination for laminated composites. *Composites Science and Technology* 66, 698-712.

Lee, H.K., Pyo, S.H., 2007. Micromechanics-based elastic damage modeling of particulate composites with weakened interfaces. *International Journal of Solids and Structures* 44, 8390-8406.

Lee, H.K., Pyo, S.H., 2008. Multi-level modeling of effective elastic behavior and progressive weakened interface in particulate composites. *Composites Science and Technology* 68, 387-397.

Liu, Y.W., Lv, L., Sun, B.Z., Hu, H., Gu, B.H., 2006. Dynamic response of 3D biaxial Spacer weft-knitted composite under transverse impact. *Journal of Reinforced Plastics and Composites* 25, 1629-1641.

Lomov, S.V., Ivanov, D.S., Verpoest, I., Zako, M., Kurashiki, T., Nakai, H., Hirosawa, S., 2007. Meso-FE modelling of textile composites: Road map, data flow and algorithms. *Composites Science and Technology* 67, 1870-1891.

Luan, K., Sun, B., Gu, B., 2011a. A Multi-scale geometrical model for finite element analyses of

three-dimensional angle-interlock woven composite under ballistic penetration. *CMES - Computer Modeling in Engineering and Sciences* 79, 31-61.

Luo, Y., Lv, L., Sun, B., Qiu, Y., Gu, B., 2007. Transverse impact behavior and energy absorption of three-dimensional orthogonal hybrid woven composites. *Composite Structures* 81, 202-209.

Majzoobi, G.H., Saniee, F.F., Bahrami, M., 2005. A tensile impact apparatus for characterization of fibrous composites at high strain rates. *Journal of Materials Processing Technology* 162, 76-82.

Malvar, L.J., Ross, C.A., 1998. Review of strain rate effects for concrete in tension. *Aci Materials Journal* 95, 735-739.

Matsuda, T., Kanamaru, S., Yamamoto, N., Fukuda, Y., 2011. A homogenization theory for elastic-viscoplastic materials with misaligned internal structures. *International Journal of Plasticity* 27, 2056-2067.

MV, Hosur, J., Alexander, UK, Vaidya, S, Jeelani, 2001. High strain rate compression response of carbon/epoxy laminate composites. *Compos Struct* 52(3-4), 405-417.

Naik, N.K., Azad, S.N.M., Prasad, P.D., Thuruthimattam, B.J., 2001. Stress and failure analysis of 3D orthogonal interlock woven composites. *Journal of Reinforced Plastics and Composites* 20, 1485-1523.

Naik, N.K., Rao, K.V., Veerraju, C., Ravikumar, G., 2010a. Stress-strain behavior of composites under high strain rate compression along thickness direction: Effect of loading condition. *Materials & Design* 31, 396-401.

Naik, N.K., Yernamma, P., Thoram, N.M., Gadipatri, R., Kavala, V.R., 2010b. High strain rate tensile behavior of woven fabric E-glass/epoxy composite. *Polymer Testing* 29, 14-22.

Nakamachi, E., Tam, N.N., Morimoto, H., 2007. Multi-scale finite element analyses of sheet metals by using SEM-EBSD measured crystallographic RVE models. *International Journal of Plasticity* 23, 450-489.

Needleman, A., Tvergaard, V., 1993. Comparison of crystal plasticity and isotropic hardening predictions for metal-matrix composites. *Journal of Applied Mechanics-Transactions of the Asme* 60, 70-76.

Nilakantan, G., Keefe, M., Bogetti, T.A., Gillespie, J.W., Jr., 2010. Multiscale modeling of the impact of textile fabrics based on hybrid element analysis. *International Journal of Impact Engineering* 37, 1056-1071.

Nilakantan, G., Wetzel, E.D., Bogetti, T.A., Gillespie, J.W., Jr., 2012. Finite element analysis of

projectile size and shape effects on the probabilistic penetration response of high strength fabrics. *Composite Structures* 94, 1846-1854.

Ochola, R.O., Marcus, K., Nurick, G.N., Franz, T., 2004. Mechanical behaviour of glass and carbon fibre reinforced composites at varying strain rates. *Composite Structures* 63, 455-467.

Oguni, K., Ravichandran, G., 2001. Dynamic compressive behavior of unidirectional E-glass/vinylester composites. *Journal of Materials Science* 36, 831-838.

Okabe, T., Takeda, N., Kamoshida, Y., Shimizu, M., Curtin, W.A., 2001. A 3D shear-lag model considering micro-damage and statistical strength prediction of unidirectional fiber-reinforced composites. *Composites Science and Technology* 61, 1773-1787.

Pankow, M., Waas, A.M., Yen, C.F., Ghiorse, S., 2012. Modeling the response, strength and degradation of 3D woven composites subjected to high rate loading. *Composite Structures* 94, 1590-1604.

Panov, V., Vignjevic, R., Bourne, N., Millet, J., 2006. Material failure modelling in metals at high strain rates, in: Furnish, M.D., Elert, M., Russell, T.P., White, C.T. (Eds.), *Shock Compression of Condensed Matter - 2005*, Pts 1 and 2, pp. 646-649.

Parsons, E.M., Weerasooriya, T., Sarva, S., Socrate, S., 2010. Impact of woven fabric: Experiments and mesostructure-based continuum-level simulations. *Journal of the Mechanics and Physics of Solids* 58, 1995-2021.

Rosen, B.W., 1973. Composite materials and the designer. III. Stiffness of fibre composite materials. *Composites* 4, 16-25.

Sanahuja, J., 2013. Effective behaviour of ageing linear viscoelastic composites: Homogenization approach. *International Journal of Solids and Structures* 50, 2846-2856.

Schossig, M., Bieroegel, C., Grellmann, W., Mecklenburg, T., 2008. Mechanical behavior of glass-fiber reinforced thermoplastic materials under high strain rates. *Polymer Testing* 27, 893-900.

Sharma, R., Bhagat, A.R., Mahajan, P., 2014. Finite Element Analysis for Mechanical Characterization of 4D Inplane Carbon/Carbon Composite with Imperfect Microstructure. *Latin American Journal of Solids and Structures* 11, 170-184.

Shi, W.F., Hu, H., Sun, B.Z., Gu, B.H., 2011. Energy absorption of 3D orthogonal woven fabric under ballistic penetration of hemispherical-cylindrical projectile. *Journal of the Textile Institute* 102, 875-889.

- Shokrieh, M.M., Omid, M.J., 2009. Compressive response of glass-fiber reinforced polymeric composites to increasing compressive strain rates. *Composite Structures* 89, 517-523.
- Shunjun, S., Waas, A.M., Shahwan, K.W., Faruque, O., Xinran, X., 2010. Effects of Matrix Microcracking on the Response of 2D Braided Textile Composites Subjected to Compression Loads. *Journal of Composite Materials* 44, 221-240.
- Smilauer, V., Hoover, C.G., Bazant, Z.P., Caner, F.C., Waas, A.M., Shahwan, K.W., 2011. Multiscale simulation of fracture of braided composites via repetitive unit cells. *Engineering Fracture Mechanics* 78, 901-918.
- Song, S., Waas, A.M., Shahwan, K.W., Xiao, X., Faruque, O., 2007. Braided textile composites under compressive loads: Modeling the response, strength and degradation. *Composites Science and Technology* 67, 3059-3070.
- Souza, F.V., Allen, D.H., Kim, Y.R., 2008. Multiscale model for predicting damage evolution in composites due to impact loading. *Composites Science and Technology* 68, 2624-2634.
- Sun, B., Gu, B., 2006a. High strain rate behavior of 4-step 3D braided composites under compressive failure. *Journal of Materials Science* 42, 2463-2470.
- Sun, B., Gu, B., 2007. In-plane Compressive Behaviors of 3-D Textile Composites at Various Strain Rates. *Applied Composite Materials* 14, 193-207.
- Sun, B., Hu, H., Gu, B., 2007. Compressive behavior of multi-axial multi-layer warp knitted (MMWK) fabric composite at various strain rates. *Composite Structures* 78, 84-90.
- Sun, B., Liu, Y., Gu, B., 2009. A unit cell approach of finite element calculation of ballistic impact damage of 3-D orthogonal woven composite. *Composites Part B-Engineering* 40, 552-560.
- Sun, B., Niu, Z., Zhu, L., Gu, B., 2010. Mechanical Behaviors of 2D and 3D Basalt Fiber Woven Composites Under Various Strain Rates. *Journal of Composite Materials* 44, 1779-1795.
- Sun, B.Z., Gu, B.H., 2006b. Shear behavior of 3D orthogonal woven fabric composites under high strain rates. *Journal of Reinforced Plastics and Composites* 25, 1833-1845.
- Sun, B.Z., Gu, B.H., Ding, X., 2005. Compressive behavior of 3-D angle-interlock woven fabric composites at various strain rates. *Polymer Testing* 24, 447-454.
- Sun, C.T., Vaidya, R.S., 1996. Prediction of composite properties, from a representative volume element. *Composites Science and Technology* 56, 171-179.
- Tan, P., Tong, L.Y., Steven, G.P., 2000. Behavior of 3D orthogonal woven CFRP composites. Part II.

- FEA and analytical modeling approaches. *Composites Part a-Applied Science and Manufacturing* 31, 273-281.
- Tang, Y., Sun, B., Gu, B., 2011. Impact Damage of 3D Cellular Woven Composite from Unit-cell Level Analysis. *International Journal of Damage Mechanics* 20, 323-346.
- Tang, Z.W., Zhang, B.M., 2012. Prediction of biaxial failure envelopes for composite laminates based on Generalized Method of Cells. *Composites Part B-Engineering* 43, 914-925.
- Tarfaoui, M., Neme, A., Choukri, S., 2009. Damage Kinetics of Glass/Epoxy Composite Materials Under Dynamic Compression. *Journal of Composite Materials* 43, 1137-1154.
- Tchalla, A., Belouettar, S., Makradi, A., Zahrouni, H., 2013. An ABAQUS toolbox for multiscale finite element computation. *Composites Part B-Engineering* 52, 323-333.
- Uchida, M., Tada, N., 2013. Micro-, meso- to macroscopic modeling of deformation behavior of semi-crystalline polymer. *International Journal of Plasticity* 49, 164-184.
- Vignjevic, R., Djordjevic, N., Campbell, J., Panov, V., 2012a. Modelling of dynamic damage and failure in aluminium alloys. *International Journal of Impact Engineering* 49, 61-76.
- Vignjevic, R., Djordjevic, N., Panov, V., 2012b. Modelling of dynamic behaviour of orthotropic metals including damage and failure. *International Journal of Plasticity* 38, 47-85.
- Walley, S.M., 2010. Historical review of high strain rate and shock properties of ceramics relevant to their application in armour. *Advances in Applied Ceramics* 109, 446-466.
- Wang, X.F., Wang, X.W., Zhou, G.M., Zhou, C.W., 2007. Multi-scale analyses of 3D woven composite based on periodicity boundary conditions. *Journal of Composite Materials* 41, 1773-1788.
- Wolfram, U., Gross, T., Pahr, D.H., Schwiedrzik, J., Wilke, H.-J., Zysset, P.K., 2012. Fabric-based Tsai-Wu yield criteria for vertebral trabecular bone in stress and strain space. *Journal of the Mechanical Behavior of Biomedical Materials* 15, 218-228.
- Xia, Z.H., Hu, Y.F., Ellyin, F., 2003a. Deformation behavior of an epoxy resin subject to multiaxial loadings. Part II: Constitutive modeling and predictions. *Polymer Engineering and Science* 43, 734-748.
- Xia, Z.H., Zhang, Y.F., Ellyin, F., 2003b. A unified periodical boundary conditions for representative volume elements of composites and applications. *International Journal of Solids and Structures* 40, 1907-1921.
- Xia, Z.H., Zhou, C.W., Yong, Q.L., Wang, X.W., 2006. On selection of repeated unit cell model and

application of unified periodic boundary conditions in micro-mechanical analysis of composites. *International Journal of Solids and Structures* 43, 266-278.

Zhou, L.D., Zhuang, Z., 2013. Strength analysis of three-dimensional braided T-shaped composite structures. *Composite Structures* 104, 162-168.

Zuo, Z.H., Huang, X.D., Rong, J.H., Xie, Y.M., 2013. Multi-scale design of composite materials and structures for maximum natural frequencies. *Materials & Design* 51, 1023-1034.

Table 1. Specifications of 3D orthogonal woven fabric

Parameters	Value
Fiber diameter (μm)	7
Warp number (Tex)	400
Weft number (Tex)	400 $\times$ 2
Z-binder number (Tex)	45
Warp density (ends/layer/10cm)	50
Weft density (ends/layer/10cm)	36
Warp layer number	3
Weft layer number	4

Table 2. Physical properties of incident and transmission bars

Material	Stainless steel
Density (kg/m^3)	7800
Young's modulus (GPa)	210
Ultimate tensile strength (MPa)	1965
Brinell hardness	580

Table 3. Mechanical parameters calculated from micro-scale unit-cell model/(GPa)

E_{11}	E_{22}	E_{33}	G_{12}	G_{13}	G_{23}	ν_{12}	ν_{13}	ν_{23}	σ_{11}	σ_{22}	σ_{33}	σ_{12}	σ_{23}
52.7	11.1	11.1	4.4	4.4	4.29	0.22	0.22	0.30	1.05	0.2	0.24	0.04	0.04
4	9	9	6	6	6	8	8	3	5		6	6	4

Note: 1-fiber direction, 2,3-transversely cross-section directions

Table 4. Mechanical parameters calculated from meso-scale unit-cell model/(MPa)

E_{11}	E_{22}	E_{33}	G_{12}	G_{13}	G_{23}	ν_{12}	ν_{13}	ν_{23}	σ_{11}	σ_{22}	σ_{33}
3737	5225	3357	451	451	352	0.014	0.047	0.061	135.433	202.925	413.734

Fig. 1 Quasi-static multi-scale models

Fig. 2 Dynamic multi-scale models

Fig. 3 Local coordinate system for matrix crack

Fig. 4 Flow chart of UMAT in ABAQUS/Standard

Fig. 5 Flow chart of interaction between ABAQUS and VUMAT

Fig. 6 Cross-section of 3-D orthogonal woven composite material coupon

Fig. 7 Schematic diagram of split Hopkinson pressure bar

Fig. 8 Strain rate vs. time curves in compression tests along warp direction

Fig. 9 Compressive failure of 3-D orthogonal woven composite in through-thickness direction loading

Fig. 10 Compressive failure of 3-D orthogonal woven composite in warp direction loading

Fig. 11 Compressive failure of 3-D orthogonal woven composite in weft direction loading

Fig. 12 A profile of the post-mortem sample in warp direction loading for 3D orthogonal woven composite

Fig. 13 Micro-scale unit-cell model for fiber yarn

Fig. 14 Simplified unit-cell model of 3D orthogonal woven composite

Fig. 15 Unit-cell definition method

Fig.16 Sketch diagram of SHPB testing system

Fig.17 Application of periodic boundary conditions by master and slave nodes technique

(a)

(b)

(c)

Fig. 18 Stress-strain curves of micro-scale unit-cell under different strain loadings respectively

(a) $\bar{\varepsilon}_{22} = 0.018$

(b) $\bar{\varepsilon}_{22} = 0.028$

Fig.19 Stress wave propagation in micro-scale unit-cell under uniaxial loading along x_2 direction

Fig. 20 Stress wave propagation in micro-scale unit-cell under shear strain loading

Fig. 21 Stress-strain curves of meso-scale unit-cell under different strain loadings respectively

Fig. 22 Stress wave propagation in meso-scale unit-cell under compressive loading along warp direction

Fig. 23 Stress wave propagation in meso-scale unit-cell under shear strain loading

Fig. 24 Comparison of stress-strain curves in warp direction between finite element model and experimental

Fig. 25 Comparison of stress-strain curves in weft direction between finite element model and experimental

Fig. 26 Comparison of stress-strain curves in through-thickness direction between finite element model and experimental

Fig. 27 Comparison of strain rate sensitivity between in in-plane direction and through-thickness direction

(a)

(b)

Fig. 28 Strain history for the warp finite element model at the rate of 800/s

(a)

Fig.29 Strain history for the weft finite element model at the rate of 2100/s

Fig. 30 Strain history for the through-thickness finite element model at the rate of 2800/s