Calendering of Metal/Polymer Composites: An Analytical Formulation

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Abstract

The process of calendering, or cold rolling, is relevant to geologic processes and industrial applications, ranging from food processing to thin film transistor manufacturing. When dissimilar materials are sandwich stacked and then subjected to extrusion-like deformation between rollers, several factors influence the outcome of the product composite. In this work, Newtonian and Power Law models are developed to predict the pressure and stresses experienced by the materials during calendering. Experiments with polymer/metal sandwiched layers inform the parameters used in the models and allow for validation. A parametric analysis is conducted to further explore the influence of material and processing parameters. The pressure profiles predicted by the models are compared to experimental results of the pressure sensor data and the parameters leading to inclusion fracture in the six composite material combinations explored. The Newtonian model is found to have the best correlation with experimental findings.

Keywords: Calendering; polymer matrix composites; lubrication approximation; layered materials; cold rolling

1. Introduction

Cold rolling is a process that reduces the thickness of sheets of materials by extruding them through hard rotating rollers. Cold rolling can also be combined with folding to induce deformation and structural transformations of metallic multilayers, induce grain refinement (Crone et al., 2004; Crone et al., 2002; Perepezko et al., 2004), and to produce particle disperse composite from multilayer material (Antoun, 2003; Calcagno, 2008). Cold rolling is also known as calendering when the material processed is paper, plastic or rubber (Gaskell, 1950).

A variety of industries use calendering to process materials and components. In food processing, sheeting/rolling is used for forming pastas, bread, and biscuit dough. Cereal grains go through the flaking process to produce cereal flakes. Work has also been conducted to find the process conditions to shape cheese without fracturing by use of rolling (Levine, 1996; Levine and Levine, 1997; Mitsoulis and Hatzikiriakos, 2009; Peck et al., 2006). Calendering is also extensively used to improve paper and fabric's surface smoothness (Forseth and Helle, 1998; Hardman, 1994; Retulainen et al., 1997). For electronic applications, rolling processes are used to control grain orientation of materials to produce thin film transistors on steel and plastic foil (Shen, 2008; Suo et al., 1999).

Gaskel developed a model that uses the momentum and continuity equations to analyze the process of calendering of Newtonian and Non-Newtonian viscous liquids was developed (Gaskell, 1950). This model assumes unidirectional, incompressible, isothermal and steady state flow through a very small nip gap. These assumptions together with neglecting of the inertial effects are known as the lubrication approximation (Deen, 1998). McKelvey, and Chong worked on a more complete analysis of Newtonian and power law fluids which used three constitutive equations to analyze the calendering of thermoplastics: a power law, a

three-constant Oldroyd, and modified second-order Rivlin-Ericksen fluid equations (Chong, 1968; McKelvey, 1962). An isothermal model accounting for slip at the calender surface was developed by Vlachopoulos and Hrymak compared to experimental values obtained for PVC resins with good agreement (Vlachopoulos and Hrymak, 1980). A two dimensional, isothermal, and non-isothermal analysis of calendering without the lubrication approximation was performed using a finite element program for Newtonian, power law, and viscoelastic fluids (Mitsoulis et al., 1985). In their work, the results were compared with those obtained with the lubrication approximation, showing that the latter underestimates the temperature profiles in the nip gap but gives similar pressure, and moment distributions.

Levine and coworkers used the lubrication approximation to develop a model for a two dimensional flow of power-law fluids for the calendering of finite width sheets (Levine et al., 2002). Results obtained with finite element method were compared with experimental data showing that the model predicted reasonably well the flow and pressure fields of the material in the z direction. Sofou and Mitsoulis used the lubrication approximation theory to produce numerical results for the calendering of pseudoplastic and viscoplastic sheets of finite thickness for a specified exit thickness (Sofou and Mitsoulis, 2004). The same authors reported later work that included a slip coefficient to account for slip over the roller surfaces for the calendering of a sheet from an infinite reservoir (Mitsoulis and Sofou, 2006). The results reported by most of the prior works published include the pressure gradient and distribution of the pressure inside the roller's gap, the maximum pressure reached, the forces exerted on the rollers, and the power input to the rollers.

In a previous work, which explored the calendering of a sandwich composite incorporating metallic inclusion, we reported fracture and deformation behavior similar to

those exhibited in geological formations seen in nature (Calcagno et al., 2012). Effects such as boudinage and folding are observed in both systems (Lloyd et al., 1982; Masuda and Kuriyama, 1988; Zhao and Ji, 1997). The potential for an analog system with closer materials property match which is testable in a laboratory setting would hold significant value for geophysicist. Moreover, the development of a mathematical description that could model behavior for a range of scientific and industrial applications has significant value.

In this work we develop an approximate analytical formulation to determine the pressure and tensile stresses profiles in the inclusion during calendering, and the role that material properties, and processing parameters play in the fracture event. In particular, we extend the equations developed for a traditional calendering process for a composite compressed between two rotating cylinders to model the load transferred by a polymeric matrix to a metal strip inclusion embedded within the composite.

This model is based on the lubrication approximation, and at this stage the metal inclusion is modeled as a fluid with a much larger viscosity than the polymer matrix. We are not considering strains in the metal inclusion but the model permit us to find a first estimate of the pressure and stresses on the inclusion, which may cause deformation and ultimately its fracture. Both, a Newtonian and a Power law approaches are used to find the pressure profiles, which are compared against experimental values measured with a pressure sensor. Some of the experimental data used for comparison was taken from controlled rolling experiments which aimed to obtain fracture of the metal strip with the minimum rolling passes (Calcagno et al., 2012). In addition a parametric analysis is performed to analyze the effect of some processing parameters on the pressure profile.

2. Analytical Model

An analytical solution using dimensionless variables was derived using the continuity and momentum equations to determine the velocity profiles in a sandwich composite sample in the zone between the rollers during a calendering process. The sample modeled consists of two layers of a material with properties identified by subscript (1), and a thin layer of a second material sandwiched between those layers with properties identified by subscript (2). Following the shear lag or load transfer theory (Lloyd et al., 1982), a strong interfacial adhesion was assumed to exist throughout the process. The flow of the sample through the rollers was assumed unidirectional, isothermal, steady state, and incompressible. Following the work of Gaskell [1950] and Sofou and Mitsoulis [2004], friction between the polymer and the roller is neglected in this analysis. Because the highest stress regions within the nip occur at the roller/polymer interface, prior work has shown that the polymer behaves much more like a liquid than a solid locally. This is consistent with the lubrication approximation and is relevant to calendering of polymeric materials like those at the interface of the roller in the experiments reported here, as in contrast to cold rolling of metallic materials (Karabin and Smelser, 1990). Further, following prior assumptions of the lubrication approximation (Deen, 1998), the momentum equations were simplified under the assumption of having a very small nip gap between the rollers compared to the radius of the calenders, $2h_0 \ll R$ (see Figure 1). Additional assumptions similar to those used in the lubrication approximation are: a) the pressure gradient inside the nip gap is a function of x only, thus $\frac{\partial P}{\partial y} \ll \frac{\partial P}{\partial x}$, b) the viscous forces are much larger than the inertial forces, and c) very low Reynolds number. In addition, the change in the shear force with x was assumed to be much smaller than the change with y , and the inclusion much thinner than the sample.

Figure 1: Schematic diagram of the cold rolling system.

Model Governing Equations:

The general continuity equation is given by

$$0 = \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}), \quad (1)$$

where $\vec{v}$ is the fluid velocity, and ρ the density. For an incompressible fluid, unidirectional and steady state flow, the continuity equation simplifies to

$$\frac{\partial v_x}{\partial x} = 0. \quad (2)$$

The equation of motion for an incompressible fluid is given by

$$\rho \frac{D\vec{v}}{Dt} = -\nabla p - \nabla \cdot \bar{\bar{\sigma}} + \rho \vec{g} , \quad (3)$$

where $\bar{\bar{\sigma}}$ is the deviatoric symmetric stress tensor and $\vec{g}$ the force due to gravity.

Then, for steady state, unidirectional flow where gravity is neglected, and applying the lubrication approximation assumptions, the equation of motion is

$$\vec{0} = -\nabla p - \nabla \cdot \bar{\bar{\sigma}} . \quad (4)$$

The generalized Newtonian fluid (GNF) constitutive equation for incompressible fluids (Morrison, 2001) was used to specify the stress tensor $\bar{\bar{\sigma}}$ as

$$\bar{\bar{\sigma}} = -\eta(\dot{\gamma})\bar{\bar{\dot{\gamma}}} , \quad (5)$$

where $\eta(\dot{\gamma})$ is a scalar function and $\dot{\gamma} = |\dot{\gamma}|$, and the rate of strain tensor, $\bar{\bar{\dot{\gamma}}}$, is a symmetric tensor described by the following relation:

$$\bar{\bar{\dot{\gamma}}} = \left[\nabla \vec{v} + (\nabla \vec{v})^T \right] . \quad (6)$$

In the following subsections the solution for the continuity and momentum equations together with the boundary conditions are presented in a dimensionless form for both the cases of Newtonian and Power Law behavior.

2.1 Newtonian Case

The constitutive equation for a Newtonian fluid in steady shear flow predicts a linear behavior, thus $\eta(\dot{\gamma}) = \mu$ where μ is the fluid viscosity. Given that $v_x = v_x(y)$, and $p = p(x)$ as stated in the assumptions, there is only one nonzero component of the equation of motion, the x-component. Using U as the speed of the rollers and μ_l as the viscosity of the outside polymer layers, and h_0 as half the nip gap space, the following dimensionless parameters were defined:

$$\tilde{y} = y/h_0, \quad \tilde{x} = x/R, \quad \tilde{p} = \frac{ph_0}{\mu_1 U}, \quad \tilde{v} = \tilde{v}(\tilde{x}, \tilde{y}) = v_x / U, \quad \tilde{\sigma} = \frac{\sigma h_0}{\mu_1 U}, \text{ and } \alpha = h_0 / R.$$

The equation of motion is given by,

$$\alpha \frac{d\tilde{p}}{d\tilde{x}} = \frac{\partial^2 \tilde{v}}{\partial \tilde{y}^2} \quad (7)$$

The spacing between the rollers h , depends on x , and can be approximated as follows

(Calcagno, 2010; Gaskell, 1950):

$$\tilde{h}(\tilde{x}) = 1 + \frac{\tilde{x}^2}{2\alpha} \quad (8)$$

Model Boundary Conditions

From Figure 1, the boundary conditions are:

- (i) The no-slip condition at the surface of the rollers, gives for the polymer

$$\tilde{v}^{(1)} = 1 \quad \text{for} \quad \tilde{y} = \pm 1.$$

- (ii) The symmetry of two rollers moving at the same speed, and the symmetry of the polymer/metal sandwich establishes

$$\frac{\partial \tilde{v}_x^{(2)}}{\partial \tilde{y}} = 0 \quad \text{at} \quad \tilde{y} = 0.$$

- (iii) The same surface tractions are present at the surface interface:

$$\tilde{\sigma}_{xy}^{(1)} = \tilde{\sigma}_{xy}^{(2)} \quad \text{at} \quad \tilde{y} = \pm \tilde{t}.$$

- (iv) The no-slip condition at the surface interface between polymer and metal is given by

$$\tilde{v}^{(1)} = \tilde{v}^{(2)} \quad \text{for} \quad \tilde{y} = \pm \tilde{t}$$

where $\tilde{t} = \frac{t}{2h_0}$ is the dimensionless metal strip thickness.

Integrating equation (7) and using the appropriate boundary conditions, the polymer velocity is,

$$\tilde{v}^{(1)}(\tilde{x}, \tilde{y}) = \frac{\alpha}{2} \left(\frac{d\tilde{p}}{d\tilde{x}} \right) [\tilde{y}^2 - \tilde{h}^2] + 1. \quad (9)$$

where $\tilde{h}$ depends on $\tilde{x}$ according to Equation (8), and the velocity profile for the metal is,

$$\tilde{v}^{(2)}(\tilde{x}, \tilde{y}) = \frac{\alpha}{2} \left(\frac{\mu_1}{\mu_2} \right) \left(\frac{d\tilde{p}}{d\tilde{x}} \right) [\tilde{y}^2 - \tilde{t}^2] + \frac{\alpha}{2} \left(\frac{d\tilde{p}}{d\tilde{x}} \right) [\tilde{t}^2 - \tilde{h}^2] + 1. \quad (10)$$

where μ_2 is the viscosity of the metallic layer. All the variables and parameters in equations (9) and (10) are known but $d\tilde{p}/d\tilde{x}$, thus a mass balance is used to solve for the pressure gradient.

The inconsistency of having a rigid body moving with different velocities at the surface is similar to that found with Bingham fluids when applying the lubrication approximation (Lipscomb and Denn, 1984). Such phenomena known as the “lubrication paradox” arises in the modeling when an unyielded material component is assumed to be rigid. In our system, the solid metal inclusion deforms significantly less than the surrounding polymer. The inclusion is assumed to be of infinite length in the axial direction, but in reality is very small and under certain calendering conditions fracture occurs when the composite passes through the testing region. In this first approximation, we are assuming that the inclusion has a much larger viscosity than the polymer layers, and in doing so, we are able to estimate the stresses acting on its surface.

Mass Balance

In order to obtain the pressure gradient, $d\tilde{p}/d\tilde{x}$, the continuity equation was applied to the control volume shown in Figure 1.

The following assumptions were established:

- The velocity of the polymer, strip and roller are the same at the exit of the rollers.

This location is identified as (x_f, y_f) .

- The width of the strip and polymer are assumed to be the same.
- The dimensions in the xz plane are assumed to be infinite.

The continuity equation for steady state and incompressible materials in a dimensionless form is given by,

$$\rho_1 \int_{\tilde{t}}^{\tilde{h}} \tilde{v}_x^{(1)} d\tilde{y} + \rho_2 \int_0^{\tilde{t}} \tilde{v}_x^{(2)} d\tilde{y} = \rho_1 (\tilde{h}_f - \tilde{t}) + \rho_2 \tilde{t}, \quad (11)$$

where $\tilde{v}_1$ and $\tilde{v}_2$ are the polymer and metal velocity profiles described by equations (9) and

(10), ρ_1 and ρ_2 are the polymer and metal densities, $\tilde{t} = \frac{t}{2h_0}$, and $\tilde{h}_f = \frac{h_f}{h_0}$ with h_f

representing half of the dimensionless sample thickness where the sample exits the rollers.

Solving for $\frac{d\tilde{p}}{d\tilde{x}}$ the following equation is obtained:

$$\frac{d\tilde{p}}{d\tilde{x}} = \frac{2}{\alpha} \left(\frac{\rho_1}{\rho_2} \right) \frac{(\tilde{h}_f - \tilde{h})}{\left[\frac{\rho_1}{\rho_2} \left(\tilde{h}^2 \tilde{t} - \frac{\tilde{t}^3}{3} - \frac{2\tilde{h}^3}{3} \right) + \tilde{t}^3 \left(1 - \frac{2\mu_1}{3\mu_2} \right) - \tilde{h}^2 \tilde{t} \right]}, \quad (12)$$

where $\tilde{h}$ and $\tilde{h}_f$ are function of $\tilde{x}$. Equation (12) is integrated as follow,

$$\int_{\tilde{p}(\tilde{x}_{in})}^{\tilde{p}(\tilde{x})} d\tilde{p} = \int_{\tilde{x}_{in}}^{\tilde{x}} \left\{ \frac{2}{\alpha} \left(\frac{\rho_1}{\rho_2} \right) \frac{(\tilde{h}_f - \tilde{h})}{\left[\frac{\rho_1}{\rho_2} \left(\tilde{h}^2 \tilde{t} - \frac{\tilde{t}^3}{3} - \frac{2\tilde{h}^3}{3} \right) + \tilde{t}^3 \left(1 - \frac{2\mu_1}{3\mu_2} \right) - \tilde{h}^2 \tilde{t} \right]} \right\} d\tilde{x}. \quad (13)$$

The lower limit of the right hand integral, $\tilde{x}_{in}$, represents the location where the sample touches the rollers at entrance. The pressure at this location is equal to the atmospheric pressure; therefore the pressure profile is given by,

$$\tilde{p}(\tilde{x}) = \int_{\tilde{x}_{in}}^{\tilde{x}} \left\{ \frac{2}{\alpha} \left(\frac{\rho_1}{\rho_2} \right) \frac{(\tilde{h}_f - \tilde{h})}{\left[\frac{\rho_1}{\rho_2} \left(\tilde{h}^2 \tilde{t} - \frac{\tilde{t}^3}{3} - \frac{2\tilde{h}^3}{3} \right) + \tilde{t}^3 \left(1 - \frac{2\mu_1}{3\mu_2} \right) - \tilde{h}^2 \tilde{t} \right]} \right\} d\tilde{x}. \quad (14)$$

The lower limit in Equation (14), $\tilde{x}_{in}$ in dimensionless form is obtained using the relation between the roller dimension variables given by,

$$\tilde{x}_{in} = \left[\tilde{T}(1 + \alpha) - 2\alpha - \alpha^2 - \frac{\tilde{T}^2}{4} \right]^{1/2}, \quad (15)$$

with $\tilde{T} = \frac{T_{in}}{R}$, and T_{in} the thickness of the sample where it enters the rollers.

At this point the viscosity of the metal, μ_2 , is assumed to be much larger than the viscosity of the polymeric matrix, μ_1 , hence the term $\left(-\tilde{t}^3 \frac{2\mu_1}{3\mu_2} \right)$ is dropped from Equation (14). See the

Supplemental Materials on the Viscosity Term Assumption for a detailed justification. The

pressure at the exit of the roller is atmospheric pressure, thus the condition $\tilde{p}(\tilde{x}_f) = 0$ is

established, and the value of $\tilde{x}_f$, where the sample leaves the rollers, is found solving the

following integral:

$$\tilde{p}(\tilde{x}_f) = \int_{\tilde{x}_{in}}^{\tilde{x}_f} \left\{ \frac{2}{\alpha} \left(\frac{\rho_1}{\rho_2} \right) \frac{(\tilde{h}_f - \tilde{h})}{\left[\frac{\rho_1}{\rho_2} \left(\tilde{h}^2 \tilde{t} - \frac{\tilde{t}^3}{3} - \frac{2\tilde{h}^3}{3} \right) + \tilde{t}^3 - \tilde{h}^2 \tilde{t} \right]} \right\} d\tilde{x} = 0. \quad (16)$$

Once the value of $\tilde{x}_f$ is known, Equation (8) is used to calculate $\tilde{h}_f$, the expression $d\tilde{p}/d\tilde{x}$ is found with Equation (12), and the velocity profiles for both constituents are completely defined.

In the following sections, the equation for the velocity of the polymer is used to find the shear stresses on the interface between the polymer and metal, and the tensile stresses profile inside the metal strip.

Shear and Tensile Stresses Profiles

For the Newtonian case, the shear stress on the metal strip surface in a dimensionless form can be expressed as,

$$\tilde{\sigma}_{xy}|_{\tilde{t}} = \frac{\partial \tilde{v}^{(1)}}{\partial \tilde{y}} \Big|_{\tilde{t}} = \frac{\mu_2}{\mu_1} \frac{\partial \tilde{v}^{(2)}}{\partial \tilde{y}} \Big|_{\tilde{t}}. \quad (17)$$

The shear stress profile on the surface of the metal is determined by taking the derivatives of the polymer velocities, and evaluating it at $\tilde{y} = \tilde{t} = \frac{t}{2h_0}$.

The tensile stress profile in the metal strip was found assuming that the thickness of the metal was very small compared to the metal width, thus the tensile stress is a function only of x .

Then, the dimensionless tensile stress can be determined using the x component of force equilibrium as follows:

$$d\tilde{\sigma}_{xx} = \frac{\tilde{\sigma}_{xy}|_{\tilde{t}}}{\alpha \tilde{t}} d\tilde{x}. \quad (18)$$

After the material exits the rollers it is no longer under external load, thus $\tilde{\sigma}(\tilde{x}_f) \approx 0$. Then, integrating equation (18), the tensile stress profile is given by:

$$\tilde{\sigma}_{xx}(\tilde{x}) = \frac{2}{\alpha} \left(\frac{\rho_1}{\rho_2} \right) \int_{\tilde{x}_f}^{\tilde{x}} \frac{(\tilde{h}_f - \tilde{h})}{\left[\tilde{t}^3 \left(1 - \frac{\rho_1}{3\rho_2} \right) + \tilde{h}^2 \tilde{t} \left(\frac{\rho_1}{\rho_2} - 1 \right) - \frac{2\rho_1}{3\rho_2} \tilde{h}^3 \right]} d\tilde{x}. \quad (19)$$

2.2 Power Law Approach

The constitutive equation used for the polymer is the power law Generalized Newtonian Fluid (GNF), while for the metal strip, a linear constitutive relation is assumed.

The constitutive equation that relates the stress tensor, $\bar{\bar{\sigma}}$, and the velocity components using the power law GNF equation is,

$$\bar{\bar{\sigma}} = -\eta(\dot{\gamma}) \bar{\bar{\gamma}},$$

where the viscosity, η , depends on the magnitude of the strain rate, $\dot{\gamma}$, as follows:

$$\eta(\dot{\gamma}) = K \dot{\gamma}^{m-1},$$

with K and m are parameters determined experimentally, and

$$\dot{\gamma} = |\bar{\bar{\gamma}}| = +\sqrt{\frac{\bar{\bar{\gamma}} : \bar{\bar{\gamma}}}{2}} = +\sqrt{\left(\frac{\partial v_x}{\partial y} \right)^2} = \pm \frac{\partial v_x}{\partial y}.$$

Since the magnitude of a tensor is positive, the sign of $\partial v_x / \partial y$ depends on whether the

velocity gradient is positive or negative. Assuming that $\frac{\partial P}{\partial y} \ll \frac{\partial P}{\partial x}$, and that the change in

the shear force with x will be assumed to be much smaller than the change with y , only one

non-zero component of the equation of motion for the polymer is obtained. This equation is

$$\frac{dp}{dx} = \frac{\partial}{\partial y} \left[\left(\eta(\dot{\gamma}) \frac{\partial v_x^{(1)}}{\partial y} \right) \right], \quad (20)$$

and substituting for $\eta(\dot{\gamma})$ gives

$$\frac{dp}{dx} = \frac{\partial}{\partial y} \left[K \left(\pm \frac{\partial v_x^{(1)}}{\partial y} \right)^{m-1} \left(\frac{\partial v_x^{(1)}}{\partial y} \right) \right]. \quad (21)$$

To find a solution for the velocity profiles of both constituents, two different regions need to be considered. These regions depend on the sign of the polymer velocity gradient, $dv_x^{(1)}/dy$, and they are labeled as *Region I* and *Region II* for positive and negative gradient respectively. *Region I* goes from $-x_{in}$, the location where the sample comes in contact with the rollers at entrance, to $-x_f$, where the average velocity is equal to the rollers velocity, and the pressure is maximum. *Region II* goes from $-x_f$ to x_f , where the sample leaves or separates from the rollers, and the pressure is atmospheric.

Region I: For $-x_{in} \leq x < -x_f$ the velocity gradient of the polymer is positive, $\frac{\partial v_x^{(1)}}{\partial y} > 0$,

thus $\eta = K \left(\frac{\partial v_x^{(1)}}{\partial y} \right)^{m-1}$.

Region II: For $-x_f \leq x < +x_f$ the velocity gradient, $\frac{dv_x^{(1)}}{dy} < 0$, therefore $\eta = K \left(-\frac{\partial v_x^{(1)}}{\partial y} \right)^{m-1}$

For the metal strip a linear relation between the stress tensor and the velocity component is assumed, hence the x -component of the equation of motion for the Newtonian Approach, Equation (7) is used.

Defining the dimensionless pressure parameter as $\tilde{p} = \frac{p}{K} \left(\frac{h_0}{U} \right)^m$, and the dimensionless shear

stress as $\tilde{\sigma}_{xy} = \frac{\sigma_{xy}}{K} \left(\frac{h_0}{U} \right)^m$, the equations for the polymer are:

$$\text{Region I:} \quad \frac{d\tilde{p}}{d\tilde{x}} = \frac{1}{\alpha} \frac{\partial}{\partial \tilde{y}} \left(\frac{\partial \tilde{v}^{(1)}}{\partial \tilde{y}} \right)^m \quad (22)$$

$$\text{Region II:} \quad \frac{d\tilde{p}}{d\tilde{x}} = -\frac{1}{\alpha} \frac{\partial}{\partial \tilde{y}} \left(-\frac{\partial \tilde{v}^{(1)}}{\partial \tilde{y}} \right)^m \quad (23)$$

The dimensionless equation for the metal is the same for both, *Region I*, and *II*, and is given by

$$\frac{d\tilde{p}}{d\tilde{x}} = \frac{\mu_2}{\alpha K} \left(\frac{h_0}{U} \right)^{m-1} \frac{\partial^2 \tilde{v}^{(2)}}{\partial \tilde{y}^2}. \quad (24)$$

The boundary conditions for both regions are:

- (i) $\tilde{v}^{(1)} = 1 \quad \tilde{y} = \pm 1$
- (ii) $\frac{\partial \tilde{v}_x^{(2)}}{\partial \tilde{y}} = 0 \quad \tilde{y} = 0$
- (iii) $\tilde{\sigma}_{xy}^{(1)} = \tilde{\sigma}_{xy}^{(2)} \quad \text{for} \quad \tilde{y} = \pm \tilde{t}$
- (iv) $\tilde{v}^{(1)} = \tilde{v}^{(2)} \quad \text{for} \quad \tilde{y} = \pm \tilde{t}$

To find the velocity profiles for both constituents, equations (22), (23) and (24) are integrated using the boundary conditions (i) through (iv), while assuming that the pressure is only a function of $\tilde{x}$, i.e. $\tilde{p} = \tilde{p}(\tilde{x})$. The equations for the polymer and metal for the two regions are then given by:

Region I:

$$\tilde{v}^{(1)} = \frac{m}{(m+1)} \alpha^{\frac{1}{m}} \left(\frac{d\tilde{p}}{d\tilde{x}} \right)^{\frac{1}{m}} \left[\tilde{y}^{\frac{m+1}{m}} - \tilde{h}^{\frac{m+1}{m}} \right] + 1 \quad (25)$$

$$\tilde{v}^{(2)} = \frac{\alpha K}{2\mu_2} \left(\frac{U}{h_0} \right)^{m-1} \left(\frac{d\tilde{p}}{d\tilde{x}} \right) [\tilde{y}^2 - \tilde{t}^2] + \frac{m}{(m+1)} \alpha^{\frac{1}{m}} \left(\frac{d\tilde{p}}{d\tilde{x}} \right)^{\frac{1}{m}} \left[\tilde{t}^{\frac{m+1}{m}} - \tilde{h}^{\frac{m+1}{m}} \right] + 1 \quad (26)$$

Region II:

$$\tilde{v}^{(1)} = -\frac{m}{(m+1)} \alpha^{\frac{1}{m}} \left(-\frac{d\tilde{p}}{d\tilde{x}} \right)^{\frac{1}{m}} \left[\tilde{y}^{\frac{m+1}{m}} - \tilde{h}^{\frac{m+1}{m}} \right] + 1 \quad (27)$$

$$\tilde{v}^{(2)} = \frac{\alpha K \left(\frac{U}{h_0} \right)^{m-1}}{2\mu_2} \left(\frac{d\tilde{p}}{d\tilde{x}} \right) [\tilde{y}^2 - \tilde{t}^2] - \frac{m}{(m+1)} \alpha^{\frac{1}{m}} \left(-\frac{d\tilde{p}}{d\tilde{x}} \right)^{\frac{1}{m}} \left[\tilde{t}^{\frac{m+1}{m}} - \tilde{h}^{\frac{m+1}{m}} \right] + 1 \quad (28)$$

To find the pressure gradient, $d\tilde{p}/d\tilde{x}$, the continuity equation is applied to the control volume shown in Figure 1 following the same procedure used for the mass balance of the Newtonian case. For the Power Law case, the polymer and metal velocity profiles, $\tilde{v}_1$ and $\tilde{v}_2$, in Equation (11), would be substituted by equations (25) through (28) depending on the region that is being analyzed.

Solving for $d\tilde{p}/d\tilde{x}$ the following equations are obtained:

Region I:

$$\frac{d\tilde{p}}{d\tilde{x}} = \frac{1}{\alpha} \left\{ \left(\frac{\rho_1}{\rho_2} \right) \frac{(\tilde{h}_f - \tilde{h})}{\left[\frac{\rho_1}{\rho_2} \frac{m}{(m+1)} \left[\tilde{h}^{\frac{m+1}{m}} \tilde{t} - \frac{m}{(2m+1)} \tilde{t}^{\frac{2m+1}{m}} \right] - \frac{\rho_1}{\rho_2} \frac{m}{(2m+1)} \tilde{h}^{\frac{2m+1}{m}} + \frac{m}{(m+1)} \left[\tilde{t}^{\frac{2m+1}{m}} - \tilde{h}^{\frac{m+1}{m}} \tilde{t} \right] \right]} \right\}^m \quad (29)$$

Region II:

$$\left(-\frac{d\tilde{p}}{d\tilde{x}} \right) = \frac{1}{\alpha} \left\{ \left(\frac{\rho_1}{\rho_2} \right) \frac{(\tilde{h}_f - \tilde{h})}{\left[\frac{\rho_1}{\rho_2} \frac{m}{(m+1)} \left[\frac{m}{(2m+1)} \tilde{t}^{\frac{2m+1}{m}} - \tilde{h}^{\frac{m+1}{m}} \tilde{t} \right] + \frac{\rho_1}{\rho_2} \frac{m}{(m+1)} \tilde{h}^{\frac{2m+1}{m}} - \frac{m}{(m+1)} \left[\tilde{t}^{\frac{2m+1}{m}} - \tilde{h}^{\frac{m+1}{m}} \tilde{t} \right] \right]} \right\}^m \quad (30)$$

As in the Newtonian case, the value of $\tilde{h}_f$, is unknown. To calculate $\tilde{h}_f$, the condition

$\tilde{p}(\tilde{x}_f) = 0$ is established and the value of $\tilde{x}_f$, where the sample leaves the rollers, is found

solving the following integral:

$$\tilde{p}(\tilde{x}_f) = \int_{-\tilde{x}_m}^{\tilde{x}_f} \frac{1}{\alpha} \left\{ \left(\frac{\rho_1}{\rho_2} \right) \frac{(\tilde{h}_f - \tilde{h})}{\left[\frac{\rho_1}{\rho_2} \frac{m}{(m+1)} \left[\tilde{h}^{\frac{m+1}{m}} \tilde{t} - \frac{m}{(2m+1)} \tilde{t}^{\frac{2m+1}{m}} \right] - \frac{\rho_1}{\rho_2} \frac{m}{(2m+1)} \tilde{h}^{\frac{2m+1}{m}} + \frac{m}{(m+1)} \left[\tilde{t}^{\frac{2m+1}{m}} - \tilde{h}^{\frac{m+1}{m}} \tilde{t} \right] \right]} \right\}^m d\tilde{x} - \quad (31)$$

Once the value of $\tilde{x}_f$ is known, Equation (8) is used to calculate $\tilde{h}_f$, the expression $\frac{d\tilde{p}}{d\tilde{x}} = 0$ is found, and the velocity profiles for both constituents are completely defined.

Shear and Tensile Stresses Profiles

For the Power Law case, the dimensionless shear stress on the metal strip surface is given by

$$\tilde{\sigma}_{xy} \Big|_{\tilde{t}} = \left(\pm \frac{\partial \tilde{v}^{(1)}}{\partial \tilde{y}} \right)^m \Big|_{\tilde{t}}. \quad (32)$$

The x-component of the force balance in dimensionless form for the power law approach gives the same expression obtained for the Newtonian behavior in Equation (18), namely

$$d\tilde{\sigma}_{xx} = \frac{\tilde{\sigma}_{xy} \Big|_{\tilde{t}}}{\alpha \tilde{t}} d\tilde{x}.$$

To get the tensile stress on the complete region between the rollers, the integration should be performed as

$$\tilde{\sigma}_{xx} = \frac{1}{\alpha \tilde{t}} \int \tilde{\sigma}_{xy}^{(I)} \Big|_{\tilde{t}} d\tilde{x} + \frac{1}{\alpha \tilde{t}} \int \tilde{\sigma}_{xy}^{(II)} \Big|_{\tilde{t}} d\tilde{x}. \quad (33)$$

For *Region I*, equations (32), and (25) give

$$\tilde{\sigma}_{xy}^{(I)} \Big|_{\tilde{t}} = \alpha \tilde{t} \left(\frac{d\tilde{p}}{d\tilde{x}} \right)^{(I)}. \quad (34)$$

For *Region II*, equations (32), and (27) give

$$\tilde{\sigma}_{xy}^{(II)} \Big|_{\tilde{r}} = \alpha \tilde{\alpha} \left(-\frac{d\tilde{p}}{d\tilde{x}} \right)^{(II)}. \quad (35)$$

The pressure gradients for both regions are given by equations (29), and (30). Then, the tensile stress in the metal inclusion is,

$$\tilde{\sigma}_{xx} = \int \left(\frac{d\tilde{p}}{d\tilde{x}} \right)^{(I)} d\tilde{x} + \int \left(-\frac{d\tilde{p}}{d\tilde{x}} \right)^{(II)} d\tilde{x}. \quad (36)$$

The maximum tensile stress, which occurs at, $\tilde{x} = -\tilde{x}_f$ is calculated as,

$$\tilde{\sigma}_{xxMax} = \int \left(-\frac{d\tilde{p}}{d\tilde{x}} \right)^{(II)} d\tilde{x}. \quad (37)$$

This value will be used for comparison with the ultimate strength of the metallic inclusion.

3. Experimental Methods

A key parameter that allows prediction of the stress state in the metal inclusion is the pressure exerted by the rollers. In order to validate the model pressure profiles, a pressure measurement system able to record in real-time the contact pressure on the interface between the sandwich sample composite and the rotating cylinders was used (Matikas, 2007). These calendering tests were done setting the nip-gap between rollers at spacing greater than 80% of the original sample thickness to protect the sensors.

In previous work calendering tests were performed to quantify the maximum tensile stress in the metal strip (Calcagno, 2010; Calcagno et al., 2012). These tests consisted in rolling sandwich composites made of a NiTi or Ti strip embedded in a polymeric matrix with the goal of fracturing the metal inclusion with a minimum of rolling passes. The polymers used were PC, PP, and HDPE. The values corresponding to the rolling experiments for the PC/NiTi sandwich composites are used here to validate the stresses transferred to the metallic inclusion within the model and compared to the ultimate strength of the NiTi.

The shear and tensile stresses profiles in the metal strip for the model proposed depends greatly on the ability of the polymers to flow or deform during the calendering process. The contribution of this behavior on the inclusion stress profile is shown in the definition of the dimensionless form of the tensile stress for both, the Newtonian and the Power Law models and is represented by the viscosity of the polymer in the Newtonian model, μ , and by K and m in the Power Law. These parameters were determined using models that take into account the strain rates applied during the rolling. The details are provided in the Supplemental Materials on Rheological Parameters.

3.1 Experimental Pressure Measurements

The industry standard for pressure measurement during calendering involves inserting a sensor between the rollers during operation (I-scan System, Tekscan Inc). The data acquisition of the pressure over the composite was done through a sensor array, which was connected to the computer via a “handle” and a USB cable. The sensor, model 5051, consisted of a flexible thin-film 0.1 mm thick, with a 55.9 mm x 55.9 mm sensing area with a 44 x 44 sensel array with 1.27 mm spacing between sensels.

Sample preparation and Experimental Method

Sandwich composites with two layers of polymer and one metallic strip in between were made following the procedure described in a previous publication (Calcagno et al., 2012). The materials used as matrices were PC, PP, and HDPE, and etched Ti and NiTi strip as inclusions. The samples dimensions were 50 mm x 25 mm, and the length of the metallic strips was 20 mm. The sensor was calibrated with compression testing using an Instron 5566 testing machine.

A 5471 150 New Durston rolling mill (model D2-120, 30 mm roller radius) was used to cold roll the samples. During this process, each sample was placed over the sensor with a thin Teflon film in between to minimize damage on the sensor, however the use of Teflon did have the drawback of allowing some slip to occur during data collection, which will be discussed further below. The film and the sensor added 0.15 mm to the initial thickness of the sample. Data was collected at 40 frames per second. The rolling tests were performed at machine speeds that range, from 1.16 to 9.30 mm/s. The nip gap between the rollers was set to values higher than 80% of the thickness of the sample plus 0.15 mm. Smaller spacing and higher roller speeds caused permanent damage to the sensors. Typically during a rolling test, thousands of frames were recorded.

3.2 Fracture of Metal Inclusion Using Calendering

Sandwich composites of NiTi or Ti strips embedded in a PC, PP, or HDPE matrix were made by the same technique used for pressure measurements and rolled in the same rolling mill. In these experiments the goal was to produce a fracture in the inclusion with a minimum number of rolling passes (Calcagno, 2010; Calcagno et al., 2009). For this purpose a smaller nip gap than the thickness of the composite was chosen. The following parameters were recorded: the speed of the rollers, thickness of the strip and the sample, the nip gap between rollers, and the composite stretch. The values corresponding to PC/NiTi composites where fracture occurred were input into the model to calculate the maximum tensile stress produced in the inclusion and compared to the ultimate stress as a means of validating of the model.

3. Results

The cold rolling tests were conducted with the purpose of fracturing the metal inclusion within a sandwich composite. The goal was to induce fracture with a minimum of rolling passes. Under the conditions described below, one or two fractures were obtained in the inclusion.

Relationships between the overall composite stretch and the ratio between the roller nip-gap and the initial thickness of the composite were obtained for two roller speeds. These relations show that for the same roller speed and initial sample composite thickness, NiTi embedded in PP needed smaller nip-gap to fracture than in a PC matrix. Fracture of NiTi in HDPE occurred at extremely small nip-gap and speeds six times higher than with PP and PC. For Ti composites with the same initial sample thickness and roller speed, fracture of Ti in PP occurred at smaller nip-gaps than those needed for the NiTi in PP. Ti in PC fractured at the same roller speed but with very small nip-gaps. Fracture of the Ti in the HDPE composites required higher speeds but occurred at bigger nip-gaps than NiTi in the HDPE (Calcagno et al., 2009). Some of the data obtained in these cold rolling tests are used for comparison to the pressure profiles obtained with the analytical model.

When solving the equations developed for the Newtonian and Power Law models presented in Section 2, the following variables were incorporated: sample and metal strip thicknesses, T and t , densities of both constituents, ρ_1 and ρ_2 ; roller radius, R ; the cold rolling machine speed, U ; and the nip gap, $2h_0$. For the power law model, the viscous flow exponent, m , for the polymeric matrix was also provided. The results include the locations where the sample leaves the rollers, and where the maximum pressure occurs, the pressure and the pressure gradient profiles, and the shear and tensile stresses profiles. All results were obtained in a dimensionless form. These results were used to calculate the velocity profiles

between the rollers along the flow direction. In addition, a parametric analysis was performed for the Newtonian model to analyze the effect of the following parameters on the pressure profile: density ratio, inclusion thickness, sample initial thickness, and nip gap. The effect of an inclusion with a much larger viscosity than the matrix was also studied.

Under the assumptions of the lubrication approximation, the solution obtained for the pressure and tensile stress have the same form, thus only the pressure profiles were used for analysis. Comparison was made between the normalized dimensionless pressure profiles obtained for the Newtonian and Power Law model. The normalized Newtonian pressure profile was also compared with the normalized curve corresponding to experimental pressures measured with the pressure sensor system.

When the inclusion was caused to fracture by a given set of process parameters, it is known that the ultimate strength of the inclusion material, $\tilde{\sigma}_U$, was exceeded. For each set of conditions that produced fracture of the NiTi inclusion in the PC/NiTi composite, the maximum tensile stress, $\tilde{\sigma}_{xx \max}$, was calculated. These values were compared to the ultimate strength of the NiTi as one means to validate the models.

4.1 Velocity Profiles in the sandwich composite in the nip gap between rollers

As an illustration, the equations for the velocity profiles were solved using the data obtained from a cold rolling test performed on a sample composite of Ti strip embedded in a HDPE matrix. Table 1 displays the data corresponding to the sample HDPE/Ti used for the graphs.

Table 1: Data of the HDPE/Ti sample used in Figures 2 to 8.

Roller radius R (mm)	Strip thickness, t (mm)	Sample thickness, T_{in} , (mm)	Nip gap height, $2h_0$, (mm)	Matrix density, ρ_1 , (g/cm ³)	Inclusion density, ρ_2 , (g/cm ³)	Roller speed (mm/s)
30	0.11	1.6	0.6	0.94	4.5	93

Figure 2: Diagram showing the dimensionless polymer velocity profiles between the rollers during the calendaring process for the Newtonian model.

Figure 3: Diagram showing the dimensionless polymer velocity profiles between the rollers during the calendaring process for the Power Law model.

For both cases the sample of HDPE/Ti touches the roller at $\tilde{x} = -\tilde{x}_{in} = -0.1818$, and exits at $\tilde{x} = \tilde{x}_f = 0.0598$ for the Newtonian model, and at $\tilde{x} = \tilde{x}_f = 0.049$ for the Power Law. The roller is rotating at a speed $U = 0.093$ m/s. Figures 2 and 3 show the velocity profiles for the polymer during the calendaring process for the Newtonian and the Power Law models.

4.2 Parametric Analysis for the Newtonian Approach

The equations obtained for the Newtonian model were used to analyze the effect of several parameters on the inclusion through the use of the pressure profiles. These dimensionless parameters are: the nip-gap ratio α , the sample initial thickness $\tilde{T}$, the inclusion thickness $\tilde{t}$, and the density ratio ρ_1/ρ_2 . Table 2 displays the maximum

dimensionless pressure for two different values of these parameters using the data shown in Table 1.

Table 2: Maximum pressure obtained for two representative values of the following parameters: the nip-gap ratio α , the sample initial thickness $\tilde{T}$, the inclusion thickness $\tilde{t}$, and the density ratio ρ_1/ρ_2 .

Maximum Pressure	α		$\tilde{T}$		$\tilde{t}$		ρ_1/ρ_2	
	0.005	0.010	0.027	0.053	0.092	0.183	0.150	0.209
$\tilde{P}_{\max}$	3.19	1.97	0.47	1.97	0.43	1.97	1.61	1.97

With the exception of the nip gap ratio, an increase in each one of these parameters produces a higher pressure. However, in order for a higher pressure to be produced through the nip gap parameter, the nip gap must be decreased.

Effect of parameter α

The parameter α was defined as the ratio between the half nip gap and the radius of the rollers. Figure 4 shows the pressure profiles in dimensionless form in the nip gap region for two α values. For $\alpha = 0.005$, which corresponds to a smaller nip gap, the location where the sample touches the rollers at entrance is $\tilde{x}_{in} = -0.2070$, where the sample leaves the rollers is $\tilde{x}_f = 0.0497$, and the maximum pressure is $\tilde{P}(\tilde{x}) = 3.1868$. For $\alpha = 0.01$, $\tilde{x}_{in} = -0.1818$, $\tilde{x}_f = 0.0598$, and the maximum pressure is $\tilde{P}(\tilde{x}) = 1.9655$. Thus, as the nip gap shrinks, the sample experiences a significantly higher peak pressure.

Figure 4: Dimensionless pressure profile vs dimensionless location for a HDPE/Ti sample.

$\tilde{P}(\tilde{x})$ corresponds to $a = 0.005$, and $\tilde{P}1(\tilde{x})$ to $a = 0.01$.

Effect of parameter $\tilde{T}$

$\tilde{T}$ is the ratio between the initial thickness of the sample and the radius of the roller.

Figure 5 shows the pressure profiles in dimensionless form in the nip gap region as a function of $\tilde{T}$. For $\tilde{T} = 0.0267$, which corresponds to a thinner sample, the location where the sample touches the rollers at entrance is $\tilde{x}_{in} = -0.0816$. $\tilde{x}_f$, where the sample leaves the rollers is 0.0364, and the maximum pressure $\tilde{P}(\tilde{x})$ is 0.4667.

Figure 5: Dimensionless pressure profile vs dimensionless location for a HDPE/Ti sample.

$\tilde{P}(\tilde{x})$ corresponds to $\tilde{T} = 0.0267$, and $\tilde{P}1(\tilde{x})$ to $\tilde{T} = 0.0533$.

For the thicker sample with $\tilde{T} = 0.0533$, the sample touches earlier the roller attaining a much higher pressure profile with a maximum, $\tilde{P}1(\tilde{x})$ of 1.965.

Effect of parameter $\tilde{t}$

This parameter $\tilde{t}$ was defined as the ratio of the inclusion thickness and the nip gap between the rollers. The pressure profiles in the nip gap region for two $\tilde{t}$ values are displayed in Figure 6. For a strip with $\tilde{t} = 0.0917$, the location where the sample touches the rollers at entrance is $\tilde{x}_{in} = -0.1153$, the location where the sample leaves the rollers is $\tilde{x}_f = 0.0512$, and the maximum pressure is $\tilde{P}(\tilde{x}) = 0.4274$. For a thicker strip with $\tilde{t} = 0.1833$, $\tilde{x}_{in} = -0.1818$, $\tilde{x}_f = 0.0598$, and the maximum pressure is $\tilde{P}1(\tilde{x}) = 1.9655$. These results show that as the parameter $\tilde{t}$ decreases, the dimensionless pressure increases.

Figure 6: Dimensionless pressure profile vs dimensionless location for a HDPE/Ti sample.

$\tilde{P}(\tilde{x})$ corresponds to $\tilde{t} = 0.0917$, and $\tilde{P}1(\tilde{x})$ to $\tilde{t} = 0.1833$.

Effect of density ratio ρ_1/ρ_2

The parameter ρ_1/ρ_2 is defined as the ratio of the densities of the polymeric matrix and the metallic strip inclusion. Figure 7 shows the pressure profiles in dimensionless form in the nip gap region for two ρ_1/ρ_2 values. For $\rho_1/\rho_2 = 0.15$, which corresponds to a lower density ratio, the location where the sample touches the rollers at entrance is $\tilde{x}_{in} = -0.1818$, $\tilde{x}_f = 0.0608$, where the sample leaves the rollers is 0.0608, and the maximum pressure is $\tilde{P}(\tilde{x}) = 1.6107$. For $\rho_1/\rho_2 = 0.2089$, the maximum pressure attained is $\tilde{P}(\tilde{x}) = 1.9659$. This graph shows an increase of the pressure between the rollers with an increase in the density ratio.

Figure 7: Dimensionless pressure profile vs dimensionless location for a HDPE/Ti sample. $\tilde{P}(\tilde{x})$ corresponds to $\rho_1/\rho_2 = 0.15$ and $\tilde{P}(\tilde{x})$ to $\rho_1/\rho_2 = 0.2089$

Pressure profiles on NiTi strips embedded in different polymeric matrices

The pressure profiles calculated for a sample with a NiTi strip embedded in three different polymers: PC, PP, and HDPE are displayed in Figure 8. The input data used for this plot correspond to the values given in Table 1. The value of $\tilde{x}_{in}$, is the same for the three composites, and is equal to -0.1818. The location for the exit is $\tilde{x}_f = 0.0601$ for the PC/NiTi,

$\tilde{x}_f = 0.0609$ for the PP/NiTi, and $\tilde{x}_f = 0.0608$ for the HDPE/NiTi. The maximum pressures are: $\tilde{P}1 = 1.845$ for the PC/NiTi, $\tilde{P}2 = 1.6109$ for the HDPE/NiTi, and $\tilde{P} = 1.5744$ for the PP/NiTi.

Figure 8: Dimensionless pressure profile vs dimensionless location for a NiTi strip embedded in different polymeric matrices. $\tilde{P}2(x)$ corresponds to the pressure profile of the HDPE/NiTi composite; $\tilde{P}1(x)$ to PC/NiTi, and $\tilde{P}(x)$ to PP/NiTi.

These values follow the trend of an increase in the pressure profile as the density ratio of the constituents increases. The PC/NiTi composite with a density ratio equal to 0.1846 displays the highest pressures, followed by the HDPE/NiTi with a lower pressure and a density ratio of 0.1446. The lowest pressure is exhibited by the PP/NiTi composite, which has a density ratio equal to 0.1391. Similar results were obtained for the Ti composites.

4.3 Comparison of the pressure profiles obtained with the Newtonian and Power Law models

The dimensionless pressure parameter, $\tilde{P}$, was defined differently for the Newtonian and Power Law models (see Sections 2.1 and 2.2), therefore to compare the shape of the profiles obtained, normalization was performed using the maximum pressure for each case.

Figure 9 shows the normalized dimensionless pressure profiles for the Newtonian and Power Law models. The plots show that for the Power Law the exit location and therefore the maximum location, occur earlier than for the Newtonian.

Figure 9: Diagram showing the normalized dimensionless pressure profiles for Newtonian and Power Law models.

The entrance location is the same for both because it depends on the thickness of the sample, the nip gap, and the radius of the rollers. Additionally, the profiles have similar shapes for the leading half of the contact with the pressures predicted by the Power Law lower than the Newtonian for the trailing half. McKelvey also predicts lower pressures for Power Law behavior (McKelvey, 1962).

4.4 Validation of model using pressure profiles experiments

The pressure profiles obtained with the Newtonian approach were compared with experimental pressure values obtained with the pressure sensor array. To obtain more points for the experimental profile, the data from several frames was extracted. Each frame contained information from 4 sensels with a spacing of 1.27 mm. The sensel location from the frames used were superimposed, and translated to locate the maximum pressure at $x = 0$. The distances and pressures were normalized in order to compare the experimental pressure and the model pressure profiles. The combined results for a PP/NiTi sample deformed by calendaring at a speed of 2.32×10^{-3} m/s, and nip gap $2h_0 = 1.4$ mm are plotted in Figure 10. This figure shows that the maximum pressure occurs at the same location for both the theoretical and experimental curves. The right side of the pressure curves have similar shape however there is a difference between the left sides. During the rolling tests, the sensor was observed to slip for every data set. This slipping occurred due to the Teflon film that was used to protect the integrity of the sensor as it went through the rollers.

Figure 10: Comparison between the normalized pressure profiles from pressure sensor measurements and Newtonian model results.

4.5 Validation of model using predicted ultimate stress values

The composite samples of NiTi or Ti strips embedded in a polymeric matrix (PC, PP, or HDPE) were subjected to cold rolling with the goal of producing fracture of the metallic inclusion with a minimum number of rolling passes to minimize plastic deformation in the matrix (Calcagno, 2010). In this work we use the data corresponding to the PC/NiTi composite of representative samples to calculate the maximum tensile stress $\tilde{\sigma}_{xx \max}$ in the metallic strip, and the location where the sample leaves the rollers, $\tilde{X}_f$, for both the Newtonian and the Power Law models. These values were calculated using the definition of

the dimensionless tensile stress parameter as $\tilde{\sigma}_{xx} = \frac{\sigma_{xx} h_0}{\mu_1 U}$ for the Newtonian, and

$$\tilde{\sigma}_{xx} = \frac{\sigma_{xx}}{K} \left(\frac{h_0}{U} \right)^m \text{ for the Power Law.}$$

The results for the calculated values of the maximum tensile stresses, exit roller locations, and ratio of the maximum tensile stress and ultimate strength of the NiTi are shown in Table 3 for both models. The dimensionless parameters are included for an easier comparison of the results. The table also shows the speed of the rollers, U , and the nip-gap, $2h_0$, as the processing parameters; the thickness of the inclusion, t , and the initial and final thicknesses of the sandwich composite, T_0 and T_f .

Table 3: Calculated values for the maximum tensile stress, exit location, and ratio between the maximum tensile stress and inclusion ultimate strength using the Newtonian and the Power Law for representative sample PC/NiTi composites.

Processing Parameters			Sample Thicknesses		Newton law					Power law				
U (m/s)	$2h_0$ (10^{-3} m)	t (mm)	T_0 (mm)	T_f (mm)	$\tilde{\sigma}_{xx \max}$	$\tilde{X}_f$	$\sigma_{xx \max}$ (Mpa)	X_f (mm)	$\frac{\sigma_{xx \max}}{\sigma_U}$	$\tilde{\sigma}_{xx \max}$	$\tilde{X}_f$	$\sigma_{xx \max}$ (Mpa)	X_f (mm)	$\frac{\sigma_{xx \max}}{\sigma_U}$
0.012	0.60	0.10	1.48	0.94	1.772	0.058	390	1.74	0.52	8.428	0.043	153	1.29	0.21
0.012	0.70	0.10	1.45	1.00	1.399	0.058	264	1.74	0.35	7.108	0.042	129	1.26	0.17
0.012	0.85	0.11	1.54	1.19	1.089	0.059	169	1.77	0.23	5.888	0.042	107	1.26	0.14
0.012	1.20	0.11	1.94	1.48	0.800	0.064	88	1.92	0.12	4.494	0.045	81	1.35	0.11
0.035	0.90	0.10	1.51	1.18	0.888	0.057	380	1.71	0.51	5.368	0.041	98	1.23	0.13
0.035	1.00	0.10	1.47	1.29	0.609	0.053	234	1.59	0.31	4.436	0.037	81	1.11	0.11
0.058	0.55	0.11	1.34	0.92	1.763	0.056	2045	1.68	2.75	8.751	0.041	162	1.23	0.22
0.058	0.67	0.11	1.30	0.97	1.268	0.055	1207	1.65	1.62	6.966	0.039	128	1.17	0.17
0.058	0.70	0.12	1.47	1.05	1.374	0.059	1252	1.77	1.68	7.164	0.042	132	1.26	0.18
0.058	0.80	0.10	1.46	1.12	1.074	0.058	857	1.73	1.15	6.101	0.041	112	1.23	0.15
0.058	0.90	0.11	1.50	1.21	0.846	0.057	600	1.71	0.80	5.335	0.040	98	1.20	0.13
0.058	0.95	0.10	1.55	1.28	0.854	0.058	574	1.74	0.77	5.095	0.041	94	1.23	0.13
0.058	1.31	0.11	2.18	1.62	0.818	0.068	398	2.04	0.53	4.442	0.049	81	1.47	0.11

The ratio between the maximum tensile stress and the ultimate strength of the metallic inclusion was included as a means of comparison for the results of the two models. The values of the location where the sample touches at the entrance, $\tilde{X}_{in}$, are the same for both models since they depend only on the thickness of the composite, roller radius and nip gap separation (see Equation 15), therefore they were not included. The locations where the samples leave the rollers are different as shown in the table. If the parameters were those just extreme enough to cause fracture, then the ratio $\sigma_{xx \max} / \sigma_U$ should equal one. However, for most cases the process parameters produced an overstress causing fracture of the inclusion, so values greater than one are expected. The ratios of the maximum tensile stresses are significantly lower than one for the Power Law model. While the ratios calculated with the Newtonian model don't follow a consistent pattern, they are higher than those obtained with the Power Law model.

5. Discussion

An analytical approach was undertaken to establish the key factors that contribute to the fracture of the inclusion of multiple combinations of polymer/metal-alloy composite during rolling. The goal of this work is to enhance fundamental understanding of the process

mechanics. The calendering process was modeled by using the continuity and momentum equations and the lubrication approximation. The analytical solution was developed for both an incompressible Newtonian flow and a Power Law flow in the polymer/metal sandwich layers, which provided independent predictions of the tensile stress distribution in the metallic inclusion.

Because the shear stress is intrinsically linked to the gradient of the velocity profile, calculating the velocity values was the first step to obtain information about the stress state of the composite during calendering. Figure 2 shows the velocity profiles for the polymer during the calendering process for the Newtonian model. The average velocity increases progressively as the sample goes between the rollers until it reaches, $-\tilde{x}_f$ where for all points within the cross section the velocity is equal to the velocity of the rollers, U . At this point the pressure reaches a maximum value. After that location, the velocity keeps increasing until after the nip gap and then decreases until point $\tilde{x}_f$ where the velocity is again equal to that of the rollers, and the polymer detaches from the rollers.

In the Newtonian model we assume that the viscosity is constant. However, as the strain rate increases, the viscosity decreases and this has an influence in the shear stress that develops. Another approach, which does not require this assumption, is the Power Law model. Figure 3 shows the velocity profiles for the polymer during the calendering process using the equations obtained with the Power Law model. The dimensionless polymer profiles for the Power Law model change progressively in a similar way as was observed with the Newtonian model. The average velocity increases as the sample goes between the rollers until it reaches, $-\tilde{x}_f$ where the velocity is equal to the velocity of the rollers, U , and the pressure reaches a maximum value. After that location, the velocity increases until the nip

gap and then decreases until point $\tilde{x}_f$ where the velocity profile is flat and of a magnitude equal to the velocity of the rollers. At this point the polymer detaches from the rollers. The velocity in the inclusion strip is a function only of $\tilde{x}$ when assuming $\mu_1 \ll \mu_2$.

When the lubrication approximation is used in modeling viscoplastic materials that flow in confined domains, the solution shows that the region that has not yielded moves with a velocity that varies along the axis of motion contradicting the assumption that this region behaves as a rigid body (Lipscomb and Denn, 1984). This is observed in the velocity profiles of both approaches, the Newtonian and Power Law. Analytical and computational works have demonstrated that unyielded zones can exist in complex geometries (Szabo and Hassager, 1992; Walton and Bittleston, 1991).

Two main methods have been used to reproduce the flow of these viscoplastic fluids with a plug region and the surface that separates them and the yielded material. These are the Augmented Lagrangian and the Regularization methods. The first uses an optimization algorithm to find a solution of the Navier-Stokes equations (Glowinski and Wachs, 2011). In the second method, the infinite viscosity at the yield surface is replaced by a very large viscous fluid (Frigaard and Nouar, 2005; Putz et al., 2009). In our simplified model where we assume that the inner inclusion has a much higher viscosity, we calculate an approximation of the stresses acting on the surface of the inclusion and compared them to actual observations where the metallic layer fractures when the sandwich composite passes through the rollers nip gap (see Table 3).

The velocity profile develops as the polymer goes through the nip gap. When comparing the two models, however, the velocity approaches a constant value a very short distance away

from the roller surface for the Power Law model in contrast to the velocity profile calculated for the Newtonian model and shown in Figure 2.

A parametric analysis on the pressure profile (which was calculated from the velocity profile discussed above) was undertaken to better understand the influence of variables involved in the calendering process. The parameters explored were chosen because of their influence on the pressure profile, and thus the stress state during calendering. Table 2 displays the parameters values and the maximum pressures obtained using the Newtonian model. The pressure profiles in dimensionless form in the nip gap region for two values of the ratio between the half nip gap and the radius of the roller, α , were analyzed. For $\alpha = 0.005$, which corresponds to a smaller nip gap, a maximum pressure of $\tilde{P}_{\max} = 3.1868$ was obtained. For $\alpha = 0.01$, $\tilde{P}_{\max} = 1.9655$. As expected, higher values of pressure are obtained in the region between the rollers at smaller α . Additionally, smaller α values assures one of the fundamental assumptions of the lubrication approximation (Deen, 1998).

The thickness of the composite divided by the radius of the roller, $\tilde{T}$, was also explored for its influence. For $\tilde{T} = 0.0267$, which corresponds to a thinner sample, the maximum pressure is 0.4667. The profile of a thicker sample with $\tilde{T} = 0.0533$ shows a much higher pressure profile with a maximum, of 1.965. This result was expected since for a larger value of $\tilde{T}$ the sample touches the roller earlier widening the interval where the integration of the pressure is performed.

In addition to the overall composite dimension, the thickness of the embedded metal strip divided by the nip gap between the rollers, $\tilde{t}$, is also a parameter of interest. For $\tilde{t} = 0.0917$, which corresponds to a thinner strip, the sample reaches a maximum pressure $\tilde{P} = 0.4274$.

For $\tilde{t} = 0.1833$, the maximum pressure is 1.9655. These results show that the dimensionless pressure increases as the parameter $\tilde{t}$ decreases.

Given the experimental evaluation of composites containing both polymer and metal components, the density difference of the two materials included in the composite is significant. Although the study of the velocity profile and shear stress in layers of fluid with different viscosity is a classic problem, there is no prior work published which considers rolling of multilayers of solids with different densities. The pressure profiles in dimensionless form in the nip gap region are shown for two ρ_1/ρ_2 values. For the lower value of $\rho_1/\rho_2 = 0.15$, the sample reaches a maximum pressure of $\tilde{P}(\tilde{x}) = 1.6107$. For $\rho_1/\rho_2 = 0.2089$, the maximum pressure is $\tilde{P}(\tilde{x}) = 1.9659$. This represents an increase of pressure of 22%. Thus the pressure between the rollers increases when the density ratio increases as expected given the form of Equation 14.

The actual densities of the materials used in the experimental studies were used to assess their influence correlated with the trends observed for each material combination. Figure 8 shows the pressure profile calculated for a sample with a NiTi strip embedded in three different polymers: PC, PP, and HDPE. The maximum pressures are: $\tilde{P}_1 = 1.845$ for the PC/NiTi, $\tilde{P}_2 = 1.6109$ for the HDPE/NiTi, and $\tilde{P} = 1.5744$ for the PP/NiTi. These values meet expectations given that the only difference between the three composites in the solution is the density ratio. For the PC/NiTi composite which displays the highest pressures, the density ratio is equal to 0.1846, followed by the HDPE/NiTi with a density ratio of 0.1446. The lowest pressure is exhibited by the PP/NiTi composite, which has a density ratio equal to 0.1391. Similar results were obtained for the Ti composites.

Experimental results on the cold rolling of sandwich composites with metal inclusion embedded in a polymeric matrix show that for the same initial sample thickness and roller speed, fracture of NiTi strips occurred at smaller nip-gaps in PP composite than PC composites (Calcagno, 2010). This would confirm the results of the model that higher pressures are obtained for higher density ratio as it is the case of NiTi/PC. However fracture for the NiTi in HDPE occurred at even smaller nip-gaps and with speeds six times higher, which indicates that other parameters are involved in the load transfer between the matrix and the inclusion. Furthermore, when results of fracture of Ti in composites with the same polymeric matrices are analyzed, it is found that for the same initial sample thickness and roller speed, fracture of Ti in PP occurred at smaller nip-gaps than those needed for NiTi in PP. In order to induce fracture in the Ti inclusion in PC composites, the nip-gap had to be decreased considerably to the point that crazing and fracture occurred in the PC. Fracture of the Ti strips embedded in HDPE required higher rollers speeds as in the case of NiTi in the same polymer, with the difference that for the Ti bigger nip-gaps were needed when compared to those for NiTi. However, interpretation of the overall trends is challenged by the fact that adhesion between the matrix and inclusion breaks down during the rolling process for some material combinations under certain processing conditions (Calcagno et al., 2012).

Table 3 shows the results for calculated values of the maximum tensile stresses, exit locations, and the ratio between the maximum tensile stress and the ultimate strength of the inclusion for both, the Newtonian and the Power Law models. The data used from the calendering tests correspond to samples where the NiTi strip embedded in PC matrix fracture with the minimum number of rolling passes. Assuming that the maximum tensile stresses

obtained from the models produced this fracture event, the ultimate strength, σ_U of the inclusion was used as a means of comparison for the results of the two models. The results obtained show different values for the locations where the samples exit the rollers. The values calculated with the Power Law model predict that the samples exit approximately 0.5 mm earlier than the value given by the Newtonian model. This would imply a thinner composite sheet for the Power Law model for a shear thinning behavior or $n < 1$. Similar results have been reported for the calendering of a finite polymer sheet for a power law behavior at ratios of entrance sheet thickness to nip gap height smaller than 20 (Middleman, 1977; Sofou and Mitsoulis, 2004).

Using the exit locations obtained with the model, the corresponding thicknesses of the samples at the exit can be estimated using Equation 8 and the definitions for dimensionless parameters involved. As expected, the measured thickness of samples is closer in value to the Newtonian value prediction. Because the polymer undergoes recovery after exiting the rollers, which is not incorporated into either model, both models under predict the final thickness. Based on the results, the Newtonian model has better correlation with the experimental results.

To compare the pressure profiles obtained with the Newtonian and Power Law models, the pressure values were normalized using the maximum pressure for each case. Figure 9 shows that the exit location and therefore the maximum location occur earlier for the Power Law than for the Newtonian. The entrance location is the same for both because it depends on the thickness of the sample, the nip gap, and the radius of the rollers. Both pressure profiles have similar shapes and show that the pressures predicted by the Power Law are lower than the Newtonian. This behavior is also predicted by Mackelvey when using the

Power Law constitutive equation (McKelvey, 1962). The normalized pressure data from experiments were plotted with results calculated using the Newtonian model in Figure 10. This graph shows similar shapes for both profiles. The right side of both profiles has excellent agreement while the left side shows a gap between the pressures. This is attributed to slipping of the sample due to the presence of the Teflon film located between the sample and the sensor.

The calculated ratios of the maximum tensile stress to the ultimate strength of the inclusion were expected to be greater than one, however approximately half of the values obtained with the Newtonian model are lower than one, while for the Power Law model are significantly lower than one (see Table 3). These differences are attributed to the assumptions made in the model and the estimation of the rheological properties. In contrast to the actual sample dimensions, the width of both inclusion and matrix layers are assumed to be the same and infinite in the model. Although an assumption of uniaxial flow would then follow, the samples show significant deformation of the polymer in the z direction.

For both models, assumptions are made in the choice of the viscosity parameter: a constant zero-shear viscosity for the Newtonian model, and elongational viscosity parameters for the Power Law model that were determined from tensile tests performed through a limited range of strain rates. Furthermore, in the experimental conditions, the strain rate changes continuously as the sample goes through the nip gap during the rolling process and the viscosity is dependent on the strain rate. The viscosity is highly dependent on the strain rate, which in the case of calendering changes continuously as the sample goes through the nip gap.

Currently we are working on a numerical solution using the Herschel-Bulkley model which includes a yield stress and a rate dependent viscosity in the form of a power law. A slip coefficient is added to account for the case of slip at the roll surface.

6. Conclusions

A dimensionless solution was developed to describe the calendering of a sandwich composite to predict the load transferred from a polymeric matrix to a metallic inclusion. The momentum and continuity equations were used to model the process as an incompressible Newtonian flow and separately for a Power Law flow, assuming unidirectional, isothermal, and steady state behavior, through the rollers. The model takes into account the speed of the rollers, the nip gap, the densities and viscosities of the constituents, and the dimensions of the sample composite. The influence of several dimensionless parameters on the stress profiles were analyzed to understand how these parameters could be used to predict the fracture behavior of the different composite sandwich combinations of a metal strip and polymer layers when subjected to calendering.

The normalized pressure profiles obtained with the model and the measured pressure profiles were found to be similar. The parametric analysis showed that the density ratio, defined as the polymeric matrix density to the metallic inclusion, greatly influences the magnitude of the dimensionless pressure on the surface of the strip. Thus, an increase in density ratio produces an increase in the pressure. There are other parameters of importance in addition to the density ratio, such as α , defined as the ratio of half the nip gap to the radius of the roller. A decrease in this parameter produces an increase in the dimensionless pressure. The other two parameters analyzed, the ratio of the sample thickness to radius of the roller

and the ratio of the inclusion thickness to the nip gap height, produce a decrease in the dimensionless pressure when either of them decreases in value.

Although the Power Law and Newtonian models have similar profiles, the velocity of the material moving through the rollers approaches a constant value a very short distance from the roller surface for the Power Law model. The outcome is a difference in where the sample is predicted to exit the rollers. The Power law model predicts earlier exit, and this correspondingly a thinner composite after processing than the Newtonian model. The experimental measurements of final thickness are consistently closer in value to the Newtonian model prediction for all composite combinations tested.

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