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Numerical Simulation of Multiple Interacting Wind Turbines on a Complex Terrain

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Abstract

Modern wind farms are subjected to significant aerodynamic interference due to unsteady wakes of individual turbines as well as the complex terrains on which they are erected. The present study uses a new mixed basis formulation of the *Navier-Stokes* equations to numerically simulate turbines on a complex terrain. The turbines are modeled using a distribution of momentum sources. A finite-volume procedure (SIMPLER algorithm) is used to solve the incompressible *Reynolds Averaged Navier-Stokes* equations on body-fitted grids to obtain the flow-field. Three different turbulence models, the standard, RNG, and realizable $\mathcal{K} - \epsilon$ are implemented and compared. Results validating the ability of the numerical procedure to simulate flows over complex terrains and interacting wind turbines are presented. Applications providing insights into the performance and loading on wind turbines on complex terrains are studied. The evolution and interaction of the turbine wakes over the complex terrain are also analyzed.

Keywords: Wind farm, Complex terrain, Turbine-Terrain interaction, Turbine-Turbine interaction.

1 Introduction

The United States Department of Energy (DOE) has set an ambitious goal of meeting 10, 20, and 35% of the annual energy consumption in the year 2020, 2030, and 2050, respectively, through wind energy [1]. To put things into perspective, only 0.5% of the total energy demand was met by wind during the year 2005. It has risen to 4.5% for the year 2013. Among other factors, the growing demand for electricity, increasing concerns over climate change, favorable economics, and the reliability of modern wind turbines are only expected to further accelerate the growth of the wind energy production. One of the challenges facing the wind energy industry is the design of the wind farm layout. Commercial wind farms have multiple wind turbines operating relatively close to each other. Owing to their proximity, the wakes of the upstream turbines interact with the downstream turbines, leading to significant aerodynamic interference. Such interference between the turbines is referred to as turbine-turbine interaction. Additionally, in the case of an onshore wind farm, the surfaces on which the turbines are erected may be complex. The terrain may have features like hills, trees, buildings, etc., and the effect from such topographical features on the wind turbines is referred to as turbine-terrain interaction. The interference from the terrain adds to the complexity of the flow-field. As a result of turbine-turbine and turbine-terrain interference, all turbines in a wind farm do not face a free-stream, leading to sub-optimal power output. The presence of terrain also causes high wind shear and turbulence, resulting in increased fatigue loads on the turbines [2]. In the design of a modern wind farm, there is an increased emphasis to understand the effects of the turbine-terrain interaction. In the initial design phase, numerical models are a suitable tool for optimizing the layout of the turbines on the terrain. Such optimization will minimize the turbine-terrain interference losses and result in a better overall performance. The objective of the present research is to develop a computational tool capable of simulating a wind farm, including the aerodynamic interference of turbine-turbine and turbine-terrain interactions.

Wind turbines have been an active area of research in the past three decades. A diverse range of literature, focusing on different aspects of the current problem of interest is available. Baker and Walker [3] used primitive kite anemometers to measure the wakes behind MOD-2 turbines at the Goodnoe Hills in Washington, USA. They found the wake profiles behind various turbines were not similar and deduced the reason to be topographical effects. Elliott and Barnard [4] performed more extensive measurements at the same location with bi-vane anemometers to measure the wake properties and found a linear relationship between velocity deficit and downstream distance. The effects of the roughness caused by the trees upwind of

the site were also studied. Subramanian et al. [5] used an autonomous, sophisticated drone to measure the near-wake properties behind a single turbine located in the Mont Crossin wind farm in Switzerland. They observed the pitch between subsequent tip vortices increased in the near-wake as it evolved. Kim et al. [6] used meteorological towers before and after construction of the Yeongheung wind farm in the mountainous island of South Korea. A significant increase in the wind shear and turbulence intensity was observed after the construction of the wind farm. The downside of field measurements in a wind farm is that they are useful for post construction analysis and not directly useful in the initial design phase. Additionally, they are often expensive, time consuming, require massive logistics, and result in turbine downtime.

Migoya et al. [7] numerically studied the Sotavento wind farm located in the Galicia region of Spain. The turbine wake effects were taken into account using the UPMPARK model, which is based on the UPMWAKE model proposed by Crespo and co-authors [8,9]. The terrain was accounted using Migoya et al.'s linearized UPMORO model and the linearized WAsP model [10]. These linearized models were found to be suitable only for attached flows and they had to resort to non-linear computational fluid dynamics (CFD) in the case of separated flows. Ivanell et al. [11] made use of an existing in-house CFD code, EllipSys3D, with actuator disk and actuator line turbine models to perform Large Eddy Simulations (LES) of turbine-wake interactions at the offshore Horns-Rev wind farm (with no complex terrain). An inverse relationship between the turbine wake length and the turbulence intensity was found from the numerical studies. Porté-Agel et al. [12] found the direction of the incident wind angle had a dramatic impact on the turbine-wake interactions and a small change of 10° in the wind angle from the worst case scenario caused a 43% increase in the total power output at Horns-Rev. As a part of the Upwind WP8 [13] project, Prospathopoulos et al. [14] studied the influence of terrain on turbines using two different non-linear CFD solvers and the linearized WAsP [10] model. The CFD solvers were body-fitted RANS solvers with various $\mathcal{K}-\epsilon$ and $\mathcal{K}-\omega$ turbulence models. A simplistic actuator disk model without the swirl effects is used to model the turbines. The actuator disk model extracted axial momentum using a single thrust coefficient (C_t) value known a priori. Moreover, the C_t value is an input parameter, obtained from an analytical or experimental thrust-velocity correlation for the specific turbine. This simple actuator disk model does not consider the turbine blade sectional properties or the effects of turbine-induced wake-swirl. Using these codes, flow through a turbine on a flat, quasi-3D, 3D Gaussian terrain [14], and a complex wind farm site in Spain [15] were simulated. Makridis and Chick [16] used the commercial CFD software Fluent, with unstructured grids to simulate wind turbines on complex terrains. The RANS

equations along with the *Reynolds* stress turbulence model are used for flow solution. The wind turbines are modeled using the Fluent’s virtual blade model developed by Ruith [17], based on the momentum source model for vertical axis wind turbines and helicopters by Rajagopalan and co-authors [18–21]. Makridis and Chick performed simulations of flow over the Askervein Hill without turbines, single turbine on a flat terrain, and a coastal terrain wind farm site. Castellani et al. [22, 23] used an existing RANS solver, WindSim, with the RNG $\mathcal{K} - \epsilon$ turbulence model to study two wind farm sites located in southern Italy. The wind turbines are modeled with the actuator disk model and the swirl effect is ignored. Also in this analysis, the power estimation is based on the pressure differential across the turbine disk and not the mechanical torque on the blades. Numerical estimation of wind speed-up, wind directional shifts, turbulence intensity, and power were compared against the turbines’ Supervisory Control and Data Acquisition (SCADA) data sets.

The present work involves studying multiple wind turbines on a complex terrain using a new RANS-based computational tool capable of simulating turbine-turbine and turbine-terrain interactions. For proper resolution of geometry in flows involving complex terrains, there is a need to use an unstructured or a structured body-conforming grid. Within the scope of structured body-fitted grids using general curvilinear coordinates, there are a variety of methods that differ in their equation formulation [24–32]. Recently, Murali and Rajagopalan [33, 34] proposed a new mixed basis *Navier-Stokes* formulation for simulating convection-dominated flows over complex geometry. The new mixed-basis *Navier-Stokes* formulation uses structured, general curvilinear, non-orthogonal body-fitted coordinates for accurate terrain geometry incorporation. The flow-field is solved using the full *Reynolds Averaged Navier-Stokes* equations. Three different turbulence closure models including the standard [35], RNG [36], and realizable $\mathcal{K} - \epsilon$ [37] are implemented and compared. The momentum source method of Rajagopalan and co-authors [18–21, 38] is used to model the wind turbines. The incompressible *Navier-Stokes* equations are solved using the pressure-based SIMPLER algorithm by Patankar [39]. Results validating the ability of the current CFD solver to simulate flows over complex terrains and wind turbines are presented first. Cases exploring the turbine-wake and complex terrain interactions are studied. Results relating to the turbine performance, blade loading, and wake interactions with the terrain are also analyzed. It is noted here the present literature overview only addresses prior work relating to wind turbines on complex terrains. A more detailed overview relating to other aspects of current problem of interest, including wind flow prediction over a complex terrain and wind turbine wakes are presented in [34].

2 Governing Equations and Methodology

2.1 Mixed basis *Reynolds Averaged Navier-Stokes* equations

This section briefly introduces the equations that govern the fluid flow. The mixed basis *Reynolds* averaged mass and momentum conservation equations are given by,

$$\frac{\partial \rho}{\partial t} + \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^i} \left(\frac{\rho \sqrt{g} g^{ij} v_j}{h^j} \right) = 0, \quad (1)$$

$$\begin{aligned} \frac{\partial \rho v_i}{\partial t} + \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^j} \left(\frac{\sqrt{g} t_i^j}{h_j} \right) + h^i \frac{\partial p}{\partial x^i} + h^i \frac{\partial}{\partial x^i} \left(\frac{2}{3} \rho \mathcal{K} \right) \\ = \left(\Gamma_{ij}^k h^i + \delta_{ik} \frac{\partial h^k}{\partial x^j} \right) \frac{t_k^j}{h^k h_j} + s_{i-WT}. \end{aligned} \quad (2)$$

The mixed form representation of the convection-diffusion flux is $t_i^j = \rho v_i v^j - 2(\mu + \mu_t) s_i^j$, where the components, s_i^j , of the mean strain rate tensor are given by,

$$s_i^j = \frac{1}{2} \left\{ h^i h_j \left(\frac{\partial(v_i/h^i)}{\partial x^k} - \frac{v_l}{h^l} \Gamma_{ik}^l \right) g^{kj} + h^j h_i \left(\frac{\partial(v_j/h^j)}{\partial x^k} - \frac{v_l}{h^l} \Gamma_{jk}^l \right) g^{ki} \right\}. \quad (3)$$

Here ρ , v_i , v^i , p , μ , μ_t , $\mathcal{K}$, and s_{i-WT} are the density, physical covariant velocity, physical contravariant velocity, pressure, molecular viscosity, turbulent eddy viscosity, turbulent kinetic energy per unit mass, and force per unit volume of fluid due to the presence of wind turbines, respectively. Equations (1), (2), and (3) also contain geometric terms like g^{ij} , Γ_{ij}^k , h_i , h^i , and g which are the components of the contravariant metric tensor, *Christoffel symbols of the second kind*, covariant scale factor, contravariant scale factor, and the determinant of the covariant metric tensor, respectively. No summation is implied on the repeated indices when used with scale factors. Additional details on the mixed-basis governing equations, formulation, and its advantages are explained in [33,34].

2.2 Turbulence closure models

For the purpose of turbulence closure, various two-equation $\mathcal{K} - \epsilon$ models including the standard, RNG, and realizable $\mathcal{K} - \epsilon$ are used in the present study. The $\mathcal{K} - \epsilon$ transport equations for the three models are,

$$\frac{\partial \rho \mathcal{K}}{\partial t} + \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^i} \left(\frac{\rho \sqrt{g} v^i \mathcal{K}}{h_i} \right) = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^i} \left[\left(\mu + \frac{\mu_t}{\sigma_{\mathcal{K}}} \right) \sqrt{g} g^{ij} \frac{\partial \mathcal{K}}{\partial x^j} \right] + P_{\mathcal{K}} - \rho \epsilon, \quad (4)$$

$$\frac{\partial \rho \epsilon}{\partial t} + \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^i} \left(\frac{\rho \sqrt{g} v^i \epsilon}{h_i} \right) = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^i} \left[\left(\mu + \frac{\mu_t}{\sigma_{\epsilon}} \right) \sqrt{g} g^{ij} \frac{\partial \epsilon}{\partial x^j} \right] + S_{\epsilon} \quad (5)$$

The production term, $P_{\mathcal{K}}$, in Eq. (4) is modeled following Kato [40] and is expressed as,

$$P_{\mathcal{K}} = \mu_t S \Omega. \quad (6)$$

S and Ω are the modulus of the strain rate and rotation rate tensors and they are given by $S = \sqrt{2\tilde{S} : \tilde{S}}$ and $\Omega = \sqrt{2\tilde{\Omega} : \tilde{\Omega}}$, respectively. The components, s_i^j , of the strain rate tensor, $\tilde{S}$, are given by Eq. (3). The components, Ω_i^j , of the rotation rate tensor, $\tilde{\Omega}$, are given by,

$$\Omega_i^j = \frac{1}{2} \left\{ h^i h_j \left(\frac{\partial(v_i/h^i)}{\partial x^k} - \frac{v_l}{h^l} \Gamma_{ik}^l \right) g^{kj} - h^j h_i \left(\frac{\partial(v_j/h^j)}{\partial x^k} - \frac{v_l}{h^l} \Gamma_{jk}^l \right) g^{ki} \right\}. \quad (7)$$

The calculation of the modulus of a tensor requires the use of the double dot product operator and it is evaluated as follows.

$$S = \sqrt{2\tilde{S} : \tilde{S}} = \sqrt{2 \frac{s_i^j}{h^i h_j} \frac{s_m^n}{h^m h_n} g^{im} g_{jn}}. \quad (8)$$

The modulus of the rotation rate tensor is obtained in a similar manner and the production term is evaluated using Eq. (6). The turbulent eddy viscosity is given by $\mu_t = C_\mu \rho \mathcal{K}^2 / \epsilon$. The various closure constants and the parameters for the three $\mathcal{K} - \epsilon$ models are summarized in Table 1. The various other closure parameters are $C_{1R} = \eta(1 - \eta/4.38)/(1 + 0.015\eta^3)$, $\eta = S\mathcal{K}/\epsilon$, $U^* = \sqrt{S^2/2 + \Omega^2/2}$, $A_0 = 4.04$, $A_s = \sqrt{6} \cos \phi$, $\phi = 1/3 \cos^{-1}(\sqrt{6}W)$, and $W = ((\tilde{S} \cdot \tilde{S}) : \tilde{S})/S^3$.

Table 1: Closure constants and parameters for the three $\mathcal{K} - \epsilon$ models.

Parameter	Standard	RNG	Realizable
S_ϵ	$\left(C_{1\epsilon} \frac{\epsilon}{\mathcal{K}} P_{\mathcal{K}} - C_{2\epsilon} \rho \frac{\epsilon^2}{\mathcal{K}} \right)$	$\left(C_{1\epsilon} \frac{\epsilon}{\mathcal{K}} P_{\mathcal{K}} - C_{2\epsilon} \rho \frac{\epsilon^2}{\mathcal{K}} \right)$	$\rho C_{1\epsilon} S_\epsilon - \frac{\rho C_{2\epsilon} \epsilon^2}{\mathcal{K} + \sqrt{\nu \epsilon}}$
$\sigma_{\mathcal{K}}$	1.0	0.7179	1.0
σ_ϵ	1.3	0.7179	1.2
C_μ	0.09	0.085	$1/(A_0 + A_s \mathcal{K} U^* / \epsilon)$
$C_{1\epsilon}$	1.44	$1.42 - C_{1R}$	$\max [0.43, \eta/(\eta + 5)]$
$C_{2\epsilon}$	1.92	1.68	1.9

2.3 Momentum source model for wind turbines

For engineering applications using RANS-based simulations, the momentum source model developed earlier for vertical axis wind turbines [18] and helicopter applications [19–21, 38]

by Rajagopalan and co-authors provides a simple, robust, and efficient method that strikes a balance between computational complexity and accuracy. In the present research, horizontal axis wind-turbines are modeled by the addition of a source term, s_{i-WT} , in the momentum Eq. (2), leading to the term-momentum source model for wind turbines. In literature, turbines are often modeled using the actuator disk model. In its simplest form, the turbine forces are modeled only along the axial direction (z) using a single thrust coefficient value and this method is known as the actuator disk model with no rotation. As the name suggests, the turbine rotational effects are ignored. In the case of the actuator disk model with rotation, the turbine swirl effects are considered. The turbine is assumed to be a flat disk, the flow-field to be axisymmetric [41, 42], and the two-dimensional blade element theory is used to model the axial (z) and tangential (θ) forces. The present momentum source method also uses blade element theory similar to the 2D actuator model but requires no assumption about flow axisymmetry or a flat turbine disk. The current model can account for more realistic geometric representations, including blade deflections, blade cone angle, tilt angle, and pitching harmonics. The resulting turbine forces are decomposed into radial, tangential, and axial directions, and hence this model is considered to be a 3D model. If the turbine were to be perfectly flat with no tangential variation in its properties, then the momentum source model and the actuator disk model with rotation would become one and the same.

To calculate the forces due to the spinning turbines, the blades are discretized into a number of span-wise sections. Blade sectional properties like chord, twist, deflection, and thickness are treated as constant over the individual sections. Among other factors, the magnitude of the force on the turbine blade is a function of the geometry, sectional aerodynamic properties, and the local relative velocity at the blade element.

$$\vec{s}_{WT} = \vec{s}_{WT}(C_l, C_d, \alpha, \vec{V}, \vec{\omega}, x, y, z, \rho, \mu, M, N_b, c). \quad (9)$$

where $C_l, C_d, \alpha, \vec{V}, \vec{\omega}, x, y, z, \rho, \mu, M, N_b$, and c are the coefficient of lift, coefficient of drag, angle of attack, flow velocity vector, blade rotational velocity vector, blade locations, density, viscosity, sectional mach number, number of blades, and blade chord, respectively.

Figure 1 shows the various velocity and force components on an arbitrary cross-section of the turbine blade. From the solution of the RANS equations, the absolute flow velocity can be decomposed into the normal and tangential components at the blade section. The relative velocity components seen by the blade is obtained by vectorially subtracting the blade velocity from the absolute velocity. The computation of the blade relative velocities from the absolute

Figure 1: Aerodynamic forces at a blade section.

velocities are as follows.

$$v'_n = v_n \quad (10)$$

$$v'_\theta = v_\theta - r\omega \quad (11)$$

The total relative velocity, v' , seen by the airfoil section is

$$v' = \sqrt{v_n'^2 + v_\theta'^2}, \quad (12)$$

and the inflow angle is $\beta = \arctan(-v'_n/v'_\theta)$. If ϕ is the sectional twist angle, then, the effective angle of attack is, $\alpha = \phi - \beta$. The local Mach number is $M' = v'/a$, where a is the speed of sound. Once the angle of attack and the local Mach number are known, the aerodynamic lift coefficient, C_l , and the drag coefficient, C_d , are obtained from an airfoil table look-up. The aerodynamic lift force, L , and the drag force, D , are calculated using,

$$L = \frac{1}{2} \rho v'^2 C_l c \, ds, \quad (13)$$

$$D = \frac{1}{2} \rho v'^2 C_d c \, ds, \quad (14)$$

where c is the sectional chord and ds the span-wise length of the blade element. From the lift and drag forces, the forces on the turbine blade are computed in the normal and tangential

directions as follows.

$$F_n = L \cos \beta - D \sin \beta. \quad (15)$$

$$F_\theta = L \sin \beta + D \cos \beta. \quad (16)$$

If $\vec{F} = F_n \hat{e}_n + F_\theta \hat{e}_\theta$ is the vector force on the turbine blade, then by *Newton's* third law, the force on the fluid due to the turbine is $-\vec{F}$. In the case of N_b number of turbine blades, each subtending an angle, $\Delta\theta$, as it traverses through a certain control volume, the force per unit volume of the fluid can be time averaged using the following equation.

$$\vec{s}_{WT} = \frac{N_b \Delta\theta}{2\pi} \left(\frac{-\vec{F}}{\Delta\forall} \right), \quad (17)$$

where $\Delta\forall$ is the size of the control volume. Once this vector force, $\vec{s}_{WT}$, is calculated, it can be decomposed to obtain the corresponding physical covariant components, s_{i-WT} , and added to Eq. (2). The turbine source term is added only to those control volumes intersected by the blades. The preceding turbine force computations are based on the blade element theory and works well over most of the blade section. However, with the three-dimensional nature of the flow near the blade tip, application of the two-dimensional blade element theory results in a non-zero angle of attack, and erroneous blade load prediction. To account for three-dimensional effects, a tip correction procedure is applied beyond a reference radial point (usually $r_{ref}/R = 0.9$). The angle of attack is corrected beyond this point, such that the tip angle of attack becomes zero. The equation for the tip correction is as follows.

$$\alpha_c = (\alpha - \alpha_0) \left| \frac{\Delta p_r}{\Delta p_{ref}} \right| \left(\frac{R - r}{R - r_{ref}} \right) + \alpha_0, \quad (18)$$

where α_c is the corrected angle of attack, Δp_r is the difference in the pressure between the front and back of the turbine blade at the radial location, r , Δp_{ref} is Δp_r at $r = r_{ref}$, and α_0 is the zero lift angle of attack. More details on the momentum source model and its implementation can be found in [18–21, 38].

2.4 Numerical method

A new, RANS-based, three-dimensional CFD code has been developed using the mixed-basis *Navier-Stokes* formulation and the momentum source model. The governing equations are discretized on a structured body-conforming grid. The velocities are stored at the control volume

cell faces and all other variables are stored at the cell centers to avoid checkerboard pressure oscillations. The incompressible *Navier-Stokes* equations are solved using the pressure-based SIMPLER algorithm of Patankar [39]. The discretized equations for the mass and momentum conservation are manipulated to obtain an exact equation for the pressure and an approximate equation for the pressure correction. Power law scheme is used for flux interpolation and a fully-implicit *Euler* method is used for time integration. The discretized equations form a set of simultaneous algebraic equations and are solved using the Tri-Diagonal Matrix Algorithm. Appropriate under-relaxation factors and source term linearization are used for ensuring stability and convergence acceleration. During the computational process, the RANS equations are solved first. The momentum source model is used to include the turbine source term at every iteration of the RANS solution. Multiple iterations of pressure, momentum, and pressure-correction are performed successively until convergence is achieved in each time-step. The turbulence equations are solved after the primary equations for pressure and velocity have converged. Figure 2 shows the steps involved in solving the flow-field for one time-step. Detailed information relating to the discretization and the numerical implementation can be found in [33,34].

3 Results

3.1 Complex terrain flow validation - Askervein Hill

The first objective is to validate the ability of the present CFD solver to simulate flows over complex terrains. Towards this end, the flow over Askervein Hill is considered. The Askervein Hill has been studied extensively through field measurements, experiments, and simulations by many researchers [16,43–54]. Detailed topographic data and surface roughness are also easily available. The Askervein Hill is an isolated hill, located along the west coast of the island of South Uist, in the outer Hebrides of Scotland, United Kingdom. The hill is nearly elliptical in shape with the major and minor axes approximately 2 *Km* and 1 *Km*, respectively. The highest point of the hill is 125 *m* above the mean sea level and the surrounding areas nearly 8 *m* above the mean sea level.

3.1.1 Computational setup

The topographic data from the derived map B [45] are used in this simulation. Figure 3 shows the contours of Askervein terrain elevation at intervals of 10 *m*. The horizontal extents of the domain are 6000 *m* $\times$ 6000 *m* and the vertical extent of the domain is 1500 *m* above the

Figure 2: Flow-chart depicting the solution procedure for one time-step.

hill. The center point (CP) of the terrain is located at $(0,0)$. The point corresponding to the Hill Top (HT) is located at $x = -304.6875 \text{ m}$ and $y = 281.25 \text{ m}$. Field measurements of the mean wind speed are recorded with a series of cup anemometers at an elevation of $\zeta = 10 \text{ m}$ above the surface along three distinct lines $A - A$, $AA - AA$, and $B - B$. The lines $A - A$ and

Figure 3: Topographic contours of the Askervein Hill.

AA – AA are along the minor axis of the hill and oriented at an angle 227° clockwise from the north. The line A – A passes through the HT and the line AA – AA passes through the CP. The line B – B connects HT and CP, and is aligned at an angle 313° clockwise from the north, also the major axis of the hill. The incoming velocity, V , is at angle 210° clockwise from the north. A non-uniform surface grid of size 101×101 is constructed. Figure 4 shows the surface grid in which the resolution is finer in the hill region and coarser in the surrounding regions. The surface mesh is exaggerated in the z -direction to show the details of the terrain. The vertical direction contains 74 cells, which are finer in the near surface region and gradually stretch at higher elevations.

The hill surface is set as a viscous no-slip wall boundary. In accordance with Walmsley and Taylor [45], the surface roughness height, z_o , is set at 0.03 m . The top surface of the domain is set as a slip boundary condition. The west and south boundaries of the computational domain are the inlet boundaries. The inlet velocity and turbulence profiles are same as those of

Figure 4: An exaggerated surface mesh on the Askervein Hill showing the terrain features.

Montavon [46] and based on the formulation proposed by Alexandrou [55]. The inlet velocity profile is given by

$$\frac{|\vec{V}|}{u^*} = \frac{1}{\kappa} \left[\ln \left(\frac{\zeta'}{\zeta' + c} \right) - \left(\frac{c}{1+c} + a \right) \frac{\zeta'^2}{2} + a\zeta' + b - \ln \kappa \right], \quad (19)$$

where the constants are $c = 0.183$, $b = 7.91$, $a = 1/c$, and $\kappa = 0.42$. The wall friction velocity, u^* , is 0.654 m/s . The non-dimensional distance is $\zeta' = \zeta/h$, where ζ is the elevation above the surface in m and $h = 1090 \text{ m}$ is the height of the boundary layer. The inlet turbulence profiles are ([46])

$$\epsilon = \frac{u^{*3}}{\kappa\zeta} \exp \left[- \left(\frac{c}{1+c} + a \right) \frac{\zeta'^2}{2} \right], \quad (20)$$

$$\mathcal{K} = \left(\frac{\kappa\zeta}{C_\mu u^*} \right)^{\frac{1}{2}} \epsilon \left(\frac{\partial u}{\partial \zeta} \right)^{\frac{-1}{2}}, \quad (21)$$

where

$$\frac{\partial u}{\partial \zeta} = \frac{u^*}{h\kappa} \left[\frac{c}{\zeta'(\zeta' + c)} - \left(\frac{c}{1+c} + a \right) \zeta' + a \right]. \quad (22)$$

C_μ is the closure constant from the turbulence model. The north and east boundaries are set as the outflow regions. The inlet condition is set as the initial condition throughout the domain. Standard atmospheric conditions of $\rho = 1.225 \text{ Kg/m}^3$ and $\mu = 1.812 \times 10^{-5} \text{ Nsm}^{-2}$

are used. The simulation is marched in time until the solution attains a steady-state. Besides the three turbulence models, the simulation is performed also in the laminar mode.

3.1.2 Velocity and turbulence profiles comparison

Figure 5: Velocity and turbulence profiles for flow over Askervein Hill.

The horizontal wind speed-up ratio at an elevation of $\zeta = 10$ m above the hill surface is plotted along the lines $A-A$, $AA-AA$, and $B-B$ in Figs. 5a - 5c, respectively. The speed up ratio is given by

$$\text{Speed-up ratio} = \frac{\sqrt{u^2 + v^2}}{\sqrt{u_{ref}^2 + v_{ref}^2}} - 1, \quad (23)$$

where u and v are the velocities in the x and y -directions at an elevation of 10 m above the surface, while u_{ref} and v_{ref} are the inflow reference velocities from Eq. (19) at $\zeta = 10$ m. The

figures include the speed-up profiles from the laminar, standard $\mathcal{K} - \epsilon$, realizable $\mathcal{K} - \epsilon$, and RNG $\mathcal{K} - \epsilon$ models as well as the field measurement values. From Figs. 5a and 5b, one can see the overall trend of decrease in wind speed in the upwind, acceleration near the hill peak, and deceleration in the leeside of the peak are well predicted by these simulations. Along both lines $A - A$ and $AA - AA$, in the far upstream region, the laminar simulation compares well with the measured values, while the three $\mathcal{K} - \epsilon$ models are nearly identical and under-predicted. In the near upstream region, between -375 m and 0 m , the profile from the laminar and the various turbulent models are identical and compare well with the measured values. The peak value is slightly under-predicted in the case for line $A - A$. It is in the leeside of the hill where distinct differences between the various simulations are observed. In general, the laminar simulation over-predicts the wind speed and is far from the measured values. Along line $A - A$, the standard and RNG values are identical and capture the decrease in wind speed remarkably well. The realizable model over-predicts the wind speed values in comparison to standard and RNG models. The performance of the turbulence models is reversed in the case of line $AA - AA$. Here, the realizable model compares better with the measured values than the standard and RNG models. In the case of $B - B$ line, there is a slight under-prediction of the wind speed by all models, although the laminar simulation is closer to the measurements. In Fig. 5d, the vertical profile of the normalized turbulent kinetic energy, defined as $\mathcal{K}/(u_{ref}^2 + v_{ref}^2)$, is plotted at the Hill Top (HT) point and compared against the measured values obtained from [56]. There is a good qualitative agreement in the trend of the turbulent kinetic energy between the measurements and the simulations. Among the various turbulence models, the realizable model compares closest with the measurements, while the standard and RNG models over-estimate the turbulent kinetic energy closer to the surface. The stochastic nature of the wind and the stratification in the density and temperature of the atmosphere influence the field observations. Such features are not modeled in the current simulations. Taking into account the overall features for the wind speed and turbulence profiles, the realizable model correlates better with the field measurements and is the choice of turbulence model for this research henceforth. This test case establishes the current numerical method is able to reproduce the features of complex terrain flows.

3.2 Wind turbine validation - Isolated NREL HAWT

Once the CFD solver has been validated for complex terrain flows, the next objective is to validate the momentum source model for wind turbines. For this purpose, the NREL combined

phase II experimental wind turbine described by Butterfield et al. [57, 58] is simulated. This turbine is well-documented and is used by many researchers, including Duque et al. [59] and Sorensen and Michelsen [60] for validation studies. Here, the turbine is studied in isolation without the effects of the tower, nacelle, or boundary layer. The NREL phase II turbine is a downwind, horizontal axis wind turbine with a rated power capacity of 20 KW. It has three blades of rectangular plan-form, *i.e.*, constant chord. This fixed pitch turbine has non-twisted blades and rotates at a constant rotational velocity of 72 RPM. The turbine directly faces the free-stream and there is no yaw angle. The specifications for the turbine are reported in [34].

3.2.1 Computational setup

The domain size is $21.5D \times 13D \times 13D$ in the x , y , and z directions, respectively. It is discretized with a Cartesian grid of size $120 \times 100 \times 100$. The center of the wind turbine is located at $14.25D$, $6.5D$, and $6.5D$. As a downwind turbine, the free-stream velocity, u_∞ , is set in the negative x -direction. Thus, there is distance of $7.25D$ and $14.25D$ in the upstream and downstream directions of the turbine, respectively. The near turbine region is discretized with $21 \times 42 \times 42$ cells of size $0.025D \times 0.025D \times 0.025D$, respectively. Standard atmospheric conditions are used for density and viscosity of air. Free-stream velocity is varied from 5 m/s to 25 m/s in steps of 5 m/s. All boundaries, except the inlet and outlet planes, are set as slip walls. No turbulence model is used in this problem and the laminar simulation is marched in time until the solution reaches a steady-state.

3.2.2 Power comparison

Thrusts produced by the turbine are -0.304 , 1.04 , 2.12 , 3.03 , and 4.43 KN at 5, 10, 15, 20, and 25 m/s, respectively. To account for the turbine shaft mechanical efficiency, the numerically-obtained values for the mechanical power output are related to the generator power using the following curve fit as described in Duque et al. [59].

$$P_{generator} (in KW) = 0.9036 P_{mechanical} (in KW) - 0.847. \quad (24)$$

Figure 6 presents a good correlation between the generator power from the present simulation and the measured values obtained from IEA Annex XIV database by Schepers et al. [61]. Due to wind speed variations and yaw error, the power values from the IEA database were unsteady. Duque et al. [59] processed these unsteady power values and extracted the steady-state power.

Figure 6: Comparison of the generator power for the isolated NREL downwind turbine. Present simulation (■) and measurements (■) from Schepers et al. [61] processed by Duque et al. [59].

The processed steady-state power values from Duque et al. [59] are used for comparison in Fig. 6. Note, the turbine does not produce any useful power at free-stream speeds below 7 m/s. Instead, the turbine requires an external power to keep it spinning. With the flow solver for the turbines validated, wind turbines on various complex terrains are studied next.

3.3 Nibe HAWT on flat terrain

While the previous validation case analyzed an isolated turbine's power production, the present case focuses on validation of momentum source wind turbine model by considering the velocity wake profiles from a turbine on flat terrain. Towards this end, the Nibe wind turbine described in Pedersen and Nielsen [62], Nielsen [63], and Taylor et al. [64] is simulated. Field measurements of the wake profiles from the Nibe turbine were documented by Taylor [65]. Several researchers, including Sørensen and Myken [66], El Kasmi and Masson [67], Réthoré [68], Prospathopoulos et al. [69], Makridis and Chick [16], and Laan et al. [70] have studied the performance and wake evolution from the Nibe wind turbines. Two turbines—Nibe A and B, are located on the westerly shoreline at Limfjord, near Aalborg, in the Northern Jutland of Denmark. The Nibe A turbine is located 5D downstream of the Nibe B turbine. Four meteorological

masts equipped with cup anemometers, vortex anemometers, directional vanes, and pressure barometers were used to measure the wake characteristics. The masts 1 through 4 are located at $2.5D$, $4D$, $6D$, and $7.5D$ downstream of the Nibe B turbine, respectively. Thus, masts 1 and 2 are located in between turbine B and A; whereas, masts 3 and 4 are located beyond turbine A. At the time of the experiments described in Taylor [65], only Nibe B was in operation. While the idle Nibe A turbine did not result in any wake profile of its own, it did affect the wake from Nibe B, measured at masts 3 and 4. Similar to the computational studies in [16, 66–70], only Nibe B turbine is considered here, while Nibe A is not modeled. The influence of tower and nacelle is also ignored. Thus, only the turbine B and the ground plane are considered. Nibe B turbine is an upwind, horizontal axis wind turbine with a rated power capacity of 630 KW at an operating wind speed of 13 m/s. The specifications for the turbine are detailed in [34]. The turbine blade airfoil section is NACA 4412, whose aerodynamic characteristics are generated with the RotCFD software [71].

3.3.1 Computational setup

The domain extents are $-20D \leq x \leq 85D$, $-25D \leq y \leq 25D$, and $0D \leq z \leq 26.25D$. The turbine center is located at $(0D, 0D, 1.125D)$. The domain is discretized with a Cartesian grid of size $90 \times 70 \times 80$ in the x , y , and z directions, respectively. Standard atmospheric conditions are used for density and viscosity of air. For turbulence, the realizable $\mathcal{K} - \epsilon$ model is used. The z_{min} ground plane has a surface roughness of $z_o = 0.01$ m and is set as a viscous no-slip wall boundary. The z_{max} , y_{min} , and y_{max} planes set as slip boundaries. At the inlet plane, an atmospheric boundary layer profile described in Makridis and Chick [72] is set as the inflow condition and is given by,

$$u = u_{ref} \frac{\ln(\zeta/z_o)}{\ln(H/z_o)}, \quad (25)$$

where ζ is the elevation above the terrain in m. The reference velocity, u_{ref} , is 13 m/s at the reference hub height of $H = 45$ m. The inflow profiles for $\mathcal{K}$ and ϵ are as follows.

$$\mathcal{K} = \frac{u^{*2}}{\sqrt{C_\mu}}, \quad \epsilon = \frac{u^{*3}}{\kappa\zeta}, \quad (26)$$

where $C_\mu = 0.09$, $u^* = \kappa u_{ref} / \ln(H/z_o)$, and $\kappa = 0.42$. At outlet, negligible gradient of velocity in the stream-wise direction is assumed and the boundary condition is enforced in accordance with the global mass conservation principle. The turbine directly faces the incoming wind and there is no yaw angle. The simulation is marched in time until the solution reaches a steady-

Figure 7: Lateral profiles of wake velocity from Nibe B turbine at hub height, $z/D = 1.125$. Present simulation (—), Taylor [65] measurements (•), and Makridis and Chick [16] simulation (---).

state.

3.3.2 Wake profiles comparison

Lateral and vertical profiles of the wake from Nibe B at downstream distances $2.5D$, $4D$, $6D$, and $7.5D$ are used for comparison. The lateral profiles are obtained at hub height, $z/D = 1.125$. The vertical profiles are obtained from the symmetry plane, $y/D = 0$. Figure 7 shows the lateral profiles of wake velocity from the present simulations with comparisons against the field measurements from Taylor [65] and numerical prediction using Fluent's virtual blade model by Makridis and Chick [16] which used *Reynolds* stress transport turbulence model.

Figure 8: Vertical profiles of wake velocity from Nibe B turbine at symmetry section, $y/D = 0$. Present simulation (—), Taylor [65] measurements ($\bullet$), and Makridis and Chick [16] simulation

($\bullet$ —).

At mast 1, the present simulation compares well with the measurements and predictions, both in terms of wake profile and the maximum velocity deficit. There is a slight under-prediction of the maximum velocity deficit from the present simulation, possibly due to the difference in the turbulence models between the present simulation and simulation of Makridis and Chick [16]. At mast 2, while the wake width is predicted accurately, the velocity deficit is not. The simulations tend to over-estimate the velocity deficits. In the case of mast 3, the peak deficit velocity is predicted satisfactorily. However, the measured wake profile has diffused and become more wider when compared to the simulated profiles. At mast 4, the present simulation is less diffuse,

with a greater velocity deficit than Makridis and Chick's [16]. Both the simulated profiles are symmetric, while the measured wake profile is not. The wake width is estimated accurately and the peak velocity deficit is over-estimated. According to Taylor [65] and Makridis and Chick [16], the increase in wind speeds away from the centerline, $y/D = 0$, is due to the errors in the field measurement of the reference velocity, u_{ref} . The mismatch between the field measurements and predictions at masts 3 and 4 is attributable to the nacelle as well as the idle Nibe A turbine. In general, the present simulation results are as good as the simulation from Makridis and Chick [16] and captures the trends of the field measurements from Taylor [65]. An interesting observation from Fig. 7 is that the wake profiles obtained from the present simulation using the realizable $\mathcal{K} - \epsilon$ model is almost comparable to the profile with Reynolds stress model from [16]. In an earlier study by Cabezon et al. [73], with a different turbine, the wake from the realizable model showed a much lesser deficit than the one with Reynolds stress model. The turbine used in that case corresponds to Sexbierum experiments, which is a 300 KW upwind 3 bladed HAWT. It is worth noting the turbine in the simulations of [73] was modeled with the one-dimensional actuator disk model using a single thrust coefficient value. Further research is needed to ascertain whether the differing behavior of the turbulence models between the present study and that of [73] is caused due to the difference in the actual turbine or the turbine modeling technique. A comparative study on a single turbine using both turbulence models and both turbine modeling modeling techniques will illustrate whether the one dimensional actuator disk approach is more sensitive to the choice of turbulence model.

Figure 8 shows the vertical profiles of wake velocity at masts 1 through 4. At mast 1, the measurements and predictions are in good agreement at the hub height and above, although the velocity is over-estimated near the ground plane. This over-estimation is likely due to the effect of the tower, which is not modeled in the simulations. The inclusion of tower will result in lower velocity tower wake near the ground plane. It is noted here that tower and nacelle can be readily included in the present simulation either as a solid geometry or through modeled drag forces. For lack of geometry information and consistency with prior simulations [16,66–70], the tower and nacelle are ignored here. At masts 2 through 4, the simulation captures the trend of the velocity profile qualitatively, while there are quantitative disagreements. In general, the velocities are over-estimated near the ground and under-estimated above the hub.

3.4 Two HAWTs in isolation, on a flat, and quasi-3D Gaussian terrains

For the next problem, two horizontal axis wind turbines in isolation, on a flat terrain, and quasi-3D Gaussian terrain are studied. The objectives of this case are three-fold: (1) to study the near-ground effects on the performance of the turbines, (2) interaction between the turbine wakes and the ground plane, and (3) the effect of the terrain on the evolution of the turbine wakes. Gaussian terrains have been used in the past for simulations of turbine on terrains. Notably, Prospathopoulos et al. [14] studied a single turbine on quasi-3D and 3D Gaussian terrains. They used $\mathcal{K} - \epsilon$ and $\mathcal{K} - \omega$ RANS-based solvers with an actuator disk model where the turbines are simulated through axial momentum extraction using a single thrust coefficient value known *a priori*. As such, their actuator disk model neither considered any blade sectional characteristics nor the effect of the turbine induced wake-swirl. Makridis and Chick [72] studied two horizontal axis wind turbines on flat and quasi-3D Gaussian terrains using the virtual blade model of the commercial Fluent CFD software. Their results focused on the qualitative nature of the wake and not many details were presented with regards to the turbine performance or the terrain-wake interaction. The present work investigates the effects of the ground plane on the turbine performance as well as the wake evolution on different terrains. The turbine chosen for the present simulation is the NREL 5 MW HAWT described in detail in Jonkman [74]. It is a three-bladed, upwind HAWT, with a blade diameter of 126 m. Further details on the turbine, including the hub diameter, height, rotational velocity, direction, blade cone, pitch, twist angle, chord distribution, and airfoil sections are detailed in [34]. The turbines directly face the on-coming wind and there is no yaw angle.

3.4.1 Computational setup

The geometry for the quasi-3D Gaussian hill is given by,

$$z = h \exp \left(-0.5 \left(\frac{x}{\sigma} \right)^2 \right), \quad (27)$$

where $h = 700 \text{ m}$ is the height of the hill and $\sigma = 1750/1.7774 \text{ m}$, where 1750 m is the distance at which height of the hill is half its peak value. Table 2 summarizes the domain size, locations of the turbine centers, and the grid size used in each of the three simulations. The cells are clustered around the turbines with the near turbine region discretized using $21 \times 42 \times 42$ cells of size $0.025D \times 0.025D \times 0.025D$, respectively. Figure 9 depicts the geometry and the turbine

locations for the Gaussian terrain case. The geometries in the isolated turbine and turbine on flat terrain cases are very simple and they are not shown. Standard sea level conditions are used for the density and viscosity of air. In cases involving terrain, an atmospheric boundary layer is set as the inlet velocity and turbulence profiles according to Eqs. (25) - (26). The reference velocity, u_{ref} , is 10 m/s at the reference hub height of $H = 90$ m. The terrain surface roughness is $z_o = 0.0445$ m. For the isolated turbines simulation, a uniform free-stream velocity is set as the inflow boundary condition. The free-stream velocity, u_∞ , is evaluated from Eq. (25) at an elevation of $\zeta = 80D$, which is also the maximum elevation in the two terrain simulations. Thus, the isolated turbines have a uniform inflow velocity whose magnitude is the same as the free-stream velocity outside the boundary layer in the case of the terrain simulations. The free-stream velocity is $u_\infty = 16.1987$ m/s. The free-stream $\mathcal{K}$ and ϵ for the isolated turbines are estimated, using an inlet turbulent intensity of 13% and length scale of $0.07D$. The simulations are marched in time until the solution reaches a steady-state.

Table 2: Domain size, turbine centers, and the grid size used in the three simulations.

Case	Domain size (x_{min}, x_{max}) (y_{min}, y_{max}) (z_{min}, z_{max})	Turbine center (WT1) (WT2)	Grid size N_x, N_y, N_z
Isolated	($-10D, 50D$) ($-10D, 10D$) ($0D, 20D$)	($0D, -1D, 10D$) ($0D, 1D, 10D$)	90, 124, 74
Flat	($-75D, 75D$) ($-75D, 75D$) ($0D, 80D$)	($0D, -1D, 0.7143D$) ($0D, 1D, 0.7143D$)	100, 132, 87
Quasi-3D Gaussian	($-75D, 75D$) ($-75D, 75D$) ($0D, 80D$)	($0D, -1D, 6.2698D$) ($0D, 1D, 6.2698D$)	100, 132, 99

3.4.2 Power comparison

The effect of the terrain on the turbine performance can be discerned by comparing the power produced by the two turbines in the three scenarios, which is summarized in Table 3. The steady-state average mechanical power output from the isolated turbines is 2.91 MW; whereas, the turbines on the flat terrain produce 1.528 MW. In contrast to the isolated turbines, the turbines on the flat terrain face a boundary layer profile. The terrain retards the velocity, thus reducing the amount of extractable power. In the absence of any change in the incoming wind-speed, the

Figure 9: Geometry for simulating two HAWTs on the quasi-3D Gaussian terrain.

Table 3: Mechanical power produced by the two HAWTs in isolation, on flat, and quasi-3D Gaussian terrains.

Case	WT1 (MW)	WT2 (MW)	Average (MW)
Isolated	2.910	2.910	2.910
Flat	1.533	1.523	1.528
Gaussian	2.984	2.979	2.982

turbines on the flat terrain may have to operate at an increased pitching angle to produce the same amount of power as the isolated turbines. In the case of the quasi-3D Gaussian hill, the turbines still face a boundary layer profile. However, the terrain geometry causes an accelerated boundary layer flow at the turbine locations, leading to an enhanced power output. In-fact, the accelerated boundary layer has a velocity greater than the free-stream. Thus, the turbines' power output have a marginal increase over the isolated turbines. To illustrate this further, the profiles of u -velocity at the turbine locations are plotted in Fig. 10. $\zeta' = (\zeta - H)/D$ is the non-dimensional vertical distance from the hub center and the u -velocity is normalized with the free-stream velocity, u_∞ . The hub location corresponds to $\zeta' = 0$ and the turbine blades span from $\zeta' = -0.5$ to $\zeta' = 0.5$, a region in which velocity defect is observed. Compared to the free-stream, the reduced and increased velocities in the flat and quasi-3D Gaussian boundary layers,

respectively, explain the difference in the performance of the turbines in the three scenarios. It is worth noting the inflow profiles and the corresponding free-stream kinetic energies in the isolated turbines and turbines on terrains are different. However, the turbines on the flat and quasi-3D Gaussian terrains face similar inflow profiles and the difference in performance of the turbines is only on account of the terrain.

Figure 10: Velocity profile at the two wind turbines in isolation (—), on the flat (—), and quasi-3D Gaussian terrains (—).

3.4.3 Evolution of the wakes

The stream-wise evolution of the wakes from the two HAWTs in isolation, on the flat, and quasi-3D Gaussian terrains are shown in Figs. 11 - 13, respectively. To show the finer details of the wake structure, the visualization is limited only to the regions of interest and not the entire flow-field. A shaded ground plane, if any, indicates the presence of terrain. The axial u -velocity is normalized with the reference velocity, u_{ref} .

In the case of the isolated HAWTs, the uniform incoming velocity is visible from the section, $x/D = -2.5$, in Fig. 11. The turbines are present at $x/D = 0$. Downstream of this section, the wakes start to appear. A pair of co-rotating vortices, induced by the two spinning turbines, start

Figure 11: Wakes from the two isolated HAWTs.

Figure 12: Wakes from the two HAWTs on the flat terrain.

Figure 13: Wakes from the two HAWTs on the quasi-3D Gaussian terrain.

to merge near $x/D = 10$. At $x/D = 17.5$, the two vortices have formed a common region, while still retaining their individual cores, indicating the merger is still in progress. The prominent flow-feature in the case of the isolated turbine wakes is their merger.

With the two HAWTs on the flat terrain, stratification in the incoming axial velocity owing to the boundary layer effect is visible from the section, $x/D = -2.5$, in Fig. 12. Downstream of these turbines, near $x/D = 2.5$, the wakes start to interact and coalesce into the lower velocity region near the ground plane. In addition to this coalescing, the two wakes start to merge near $x/D = 7.5$. At $x/D = 17.5$, the wakes and the boundary layer have almost merged into a single lower velocity zone. The prominent feature in this case is the interaction and coalescing of the wakes with the near-ground region rather than the simple merger seen in the case of the isolated HAWTs.

In the case of two HAWTs on the quasi-3D Gaussian terrain, the incoming velocity is accelerated at $x/D = -2.5$ compared to the flat terrain case. This acceleration is caused by the hill contour. The leeside of the hill is dominated by the wakes from the two turbines and the hill-induced re-circulation zone. The turbine-wake regions can be idealized as stream-tubes. The expanding area behind the hill causes overall flow deceleration, which, in turn, results in the con-

tinuous vertical expansion of the wake stream-tubes. The two cores of the wakes are convected downstream approximately at an elevation equal to the hub-height, where the interaction with the ground plane is less dominant than the flat terrain case. As evident from Fig. 13, identifiable turbine wakes are present only until $x/D = 7.5$, beyond which, the hill-induced re-circulation zone is dominant. Thus, in the case of the quasi-3D Gaussian hill, the prominent feature seems to be the vertical expansion of the wakes as against the ground interaction in the case of the flat terrain or the wake-merger in the case of the isolated turbines.

3.5 Four HAWTs on the Askervein Hill

For the next problem, four horizontal axis wind turbines on top of the Askervein Hill are simulated. The purpose of this problem is to analyze the turbine performance, flow-field features like wakes, and the blade loading in the case of turbines on a real terrain as against an analytical one like quasi-3D or 3D Gaussian terrains.

3.5.1 Computational setup

Figure 14: Orientations of the ξ , η , ridge, and turbine disk planes on Askervein Hill.

Looking at the hill layout in Figs. 3 and 4, the terrain has a sharp ridge-like feature along

the major axis line, $B - B$. From the hill profile in Fig. 5c, this ridge approximately extends to a distance of 1000 m, *i.e.*, from -750 to 250 m along the line $B - B$. Four turbines are chosen to be equally spaced along this ridge, thereby fixing their x and y locations. To determine their z positions, the terrain elevations are needed for these x and y points. To create a grid of suitable size and resolution for the present simulation, a terrain grid generator program is developed. First, a parametric NURBS surface is constructed that interpolates the 257×257 grid points from the derived map B using the global surface interpolation algorithm by Piegl and Tiller [75]. With this parametric surface, the hill elevation z points are obtained at the desired horizontal x and y points using the point inversion algorithm by Piegl and Tiller. The x and y points are chosen independently to obtain the corresponding z points. Once the x , y , and z coordinates of the terrain are obtained, the turbine centers are determined by adding a further elevation corresponding to the hub-height. The coordinates of the centers of the four HAWTs thus obtained are listed in Table 4. Additional details on the grid generation are described in [34]. The problem geometry is described in Fig. 14. The domain size and the boundary conditions are same as those of the earlier Askervein terrain case. Even though the turbines are located on the ridge line $B - B$, they are oriented such that they directly face the inflow velocity. There is an angle of 13° between the orientations of the ridge plane and the turbine disk planes. For the purpose of presenting the results, a rotated ξ , η coordinate system is used. ξ and η have the same origin location as x and y , and are rotated from them by an angle of 60° in the counter-clockwise direction. Thus, ξ is oriented along the inflow velocity, $\vec{V}$, while η is normal to it. The $\xi = \text{constant}$ planes are also parallel to the turbine disk planes. The turbine characteristics are same as the 5 MW NREL HAWT from the previous problem. The domain is discretized using a body-fitted grid of size $185 \times 181 \times 68$ in the x , y , and z directions, respectively. To estimate the baseline performance, a single isolated HAWT is simulated. The free-stream velocity for the isolated HAWT is estimated beyond the boundary layer from Eq. 19. Substituting for the boundary layer height $h = 1090$ m, $|\vec{V}_\infty|$ is found to be 17.54 m/s. The inlet turbulent intensity and length scale are set to 13% and $0.07D$, respectively. The simulation is marched in time until the solution attains a steady-state.

3.5.2 Power production

The steady-state power produced by the single isolated turbine, four turbines on the ridge of the Askervein Hill, average power of the four turbines, and their percentage difference from the isolated turbine are summarized in Table 5. The four turbines on the Askervein Hill produce

Table 4: Centers for the four HAWTs on Askervein Hill.

HAWT	x (m)	y (m)	z (m)
WT1	-365.676	341.0	212.739
WT2	-182.838	170.5	210.352
WT3	0.0	0.0	205.363
WT4	182.838	-170.5	191.936

almost similar amounts of power. While the isolated turbine produces 2.948 MW, the average power produced by the four turbines on the Askervein Hill is 3.046 MW, a marginal increase of 3.324%. Despite the presence of the terrain and its accompanied flow retardation, it is the acceleration due to the hill profile that causes the turbines on the ridge of the Askervein Hill to produce slightly more power than the isolated turbine.

Table 5: Mechanical power produced by the HAWT in isolation and the four HAWTs on the Askervein Hill.

Case	Power (MW)	% difference
Isolated WT	2.948	0%
WT1	3.049	3.426%
WT2	3.046	3.324%
WT3	3.043	3.223%
WT4	3.044	3.256%
Average	3.046	3.324%

3.5.3 Flow-field features and turbine wakes

In Figure 15, the flow-field is visualized with contours of velocity magnitude and streamlines at $z = 205.1$ m ($z/D = 1.628$) plane, which is the average height of the four turbine centers. The conforming of the streamlines to the Askervein Hill geometry is clearly visible. Upstream from the turbines (also upstream from the hill), the streamlines originate beneath $z/D = 1.628$ plane. Near the turbines (ridge of the hill) and the downstream regions (leeside of the hill), the streamlines rise above and go beneath the $z/D = 1.628$ plane, respectively. The flow is accelerated outside the turbine stream-tubes. Since the turbines retard the incoming air, the flow squeezes itself into the non-turbine regions, causing them to accelerate. Acceleration is also caused by the hill geometry. The wakes from the turbines are convected individually for a short downstream distance, beyond which, they evolve into an indistinguishable common region.

To elucidate interaction with the terrain, the wakes are visualized between $\eta = -500 \text{ m}$ and $\eta = 750 \text{ m}$ at $\xi/D = 0, 1, 2, 3, 4, 5, 6$, and 7 in Fig. 16. At $\xi = 0D$, the turbine-induced wakes are circular in shape. As they travel downstream, they become distorted, due to interactions with the terrain as well as the terrain-induced wake. Around $\xi = 2D$, the wake of the hill appears. Between $\xi = 3D$ and $6D$, the wakes from WT2 and WT3 coalesces with the terrain-induced wake. At $\xi = 7D$, the wakes from all turbines and the hill have evolved into one common wake region.

Figure 15: Streamlines and contours of velocity magnitude at $z/D = 1.628$.

3.5.4 Blade loading

The load on the blade sections is calculated as follows.

$$\text{Load} = \left(\sqrt{L^2 + D^2} \right) / ds, \quad (28)$$

where L and D are the aerodynamic lift and drag forces, and ds is the span-wise length of the blade element. The lift and drag forces are computed using Eqs. (13) and (14), respectively. While the magnitudes of the loads on the various turbines are similar to each other, their distribution is not. Figure 17 shows the distribution of the sectional loads on the isolated turbine and the four turbines on top of the Askervein Hill. The loads are shown between the

(a) $\xi = 0D$

(b) $\xi = 1D$

(c) $\xi = 2D$

(d) $\xi = 3D$

(e) $\xi = 4D$

(f) $\xi = 5D$

(g) $\xi = 6D$

(h) $\xi = 7D$

Figure 16: Terrain-wake interaction with 4 HAWTS on the Askervein Hill.

Figure 17: Sectional blade load on the isolated WT and 4 WTs on Askervein Ridge.

blade root and the tip over the entire range of the azimuth angle, ψ . The azimuth angle starts from $\psi = 0^\circ$ at the 12 o' clock position and increases in the clockwise direction until it reaches $\psi = 360^\circ$ and the 12 o' clock position again. As expected, the isolated turbine has a perfectly symmetrical load distribution. It is characterized by a region of lower load near the hub. The blade sections in this region are made of airfoils that predominantly produce drag over lift force. Outside this region, the load increases radially and attains the peak value just before the outer edge of the turbine disk. At the outer periphery, the blade load then reduces. Overall, the simulated load distribution is in line with the expected loading behavior of an isolated turbine. The azimuthal symmetry is no longer present in the case of the turbines on the Askervein Ridge, especially for WT2, WT3, and WT4. A broader radial section of the blades experiences higher loads in the region $\psi = 0^\circ$ to $\psi = 270^\circ$ than $\psi = 270^\circ$ to $\psi = 360^\circ$.

3.5.5 Effect of inflow wind direction on the turbine performance

Figure 18: Normalized power of the individual turbines and the average normalized power of the four turbines for various inflow wind directions.

The effect of the inflow wind direction on the turbine performance is studied by carrying out

several simulations over the entire range of possible wind directions, from $\theta = 0^\circ$ to 360° . These wind directions encompass the full-wake, partial-wake, and no-wake conditions, in which some of the turbines are completely affected, partially affected, and unaffected by other turbines. Figure 18 shows the simulated mechanical power outputs of the individual wind turbines as well as the average power of the four turbines, for the various incoming wind directions. The power in each case is non-dimensionalized with the power of the isolated wind turbine from Table 5. From the average power curve, it can be seen the power produced is significantly lower when the wind flow direction results in full-wake condition, at $\theta = 135^\circ$ and 315° , *i.e.*, south-east and north-west directions, respectively. At $\theta = 135^\circ$ and 315° , the average power produced is just 31.41 and 20.53%, respectively. The average power produced is maximum when the wind direction is $\theta = 0^\circ$ (also 360°), *i.e.*, north. In this case, the power produced is 107.87% of the isolated turbine. The turbines 1 and 4 become downstream turbines, subjected to full-wake conditions, and their performance is degraded the most for the south-east ($\theta = 135^\circ$) and north-west ($\theta = 315^\circ$) wind directions, respectively. The turbines 2 and 3 are affected for both $\theta = 135^\circ$ and 315° directions. The performance of the turbine 2 is degraded the most for $\theta = 135^\circ$, while it occurs at $\theta = 315^\circ$ for the turbine 3.

4 Discussion

4.1 Summary

1. The broader objective of the present research is to analyze wind turbines subjected to turbine-turbine and turbine-terrain interactions. For accurate geometry resolution of the complex terrains, general curvilinear non-orthogonal body-fitted coordinates are used. A new mixed-basis *Navier-Stokes* formulation that uses the physical covariant velocity for the primary solution variable and the physical contravariant velocity for flux computations is used. This novel solution procedure based on the mixed-basis formulation is shown to be capable of simulating flow over complex terrain and multiple wind turbines placed on the complex terrain. The incompressible and turbulent flow-field is solved using the *Reynolds Averaged Navier-Stokes* equations. Three different turbulence closure models, the standard, RNG, and realizable $\mathcal{K} - \epsilon$ are implemented and compared.
2. A novel 3D momentum source method capable of obtaining integrated turbine performance, loading on the blades, and turbine wakes is introduced for HAWTs. This wind turbine modeling technique (which requires no *a priori* knowledge of turbine performance

or wake) combined with the flow solver can be used to study individual turbines as well as near and far-wake interference effects from multiple turbines.

3. The ability of the present numerical method to simulate flows over complex terrains and wind turbines is validated by simulating the Askervein terrain case, isolated NREL phase II experimental turbine, and the Nibe B turbine on a flat terrain. The validation is carried out by comparing the present numerical results against field measurements and numerical results from published literature. To analyze the turbine-wake interference, wake evolution, turbine performance, blade loading, and the influence of wind direction, multiple HAWTs on various terrains including a flat terrain, quasi-3D Gaussian terrain, and the Askervein Hill are studied.

4.2 Specific conclusions

The specific conclusions from this study and from the various cases are summarized below.

1. The Askervein complex terrain, isolated NREL turbine, and the Nibe B turbine cases establish the capability of the present numerical procedure to capture the features of interest, including the near surface velocity speedups, turbulence profiles, turbine power output, and the wake profiles. Among the standard, RNG, and realizable $\mathcal{K}-\epsilon$ turbulence closure models, the latter is recommended, as it compares closest to measurements.
2. The effect of the terrain on the turbine performance and the wake evolution is studied by considering two NREL 5 MW HAWTs in isolation, on a flat, and quasi-3D Gaussian terrains. Compared to the isolated ones, the turbines on the flat and quasi-3D Gaussian terrains experience a decelerated and an accelerated boundary layer, which explains the enhanced and reduced power production, respectively. The dominant feature of the wakes in the case of the turbines in isolation, flat terrain, and quasi-3D Gaussian terrain are their merger, coalescing with the boundary layer, and the vertical stretching, respectively.
3. The terrain-wake interaction is studied in a more realistic complex terrain by considering four HAWTs on the ridge of the Askervein Hill. The wakes from the turbines travel individually for a short distance downstream, beyond which they distort and evolve into a common deficit zone on account of the interference with the ground plane and the hill-induced re-circulating region. On average, the turbines on the Askervein ridge produce 3.324% more power than an isolated turbine. The blades of the turbines on the Askervein ridge also face an asymmetrical loading as against the perfectly symmetrical loading of an isolated turbine. Based on several numerical simulations, wind blowing from the north

($\theta = 0^\circ$) is found to be the most favorable and north-west ($\theta = 315^\circ$) the most unfavorable direction resulting in 107.87% and 20.53% power of the isolated turbines.

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