Experimental and numerical study on the responses of a transmission tower to skew incident winds

^{a*}H.Z. Deng, ^aH.J. Xu, ^aC.Y. Duan, ^bX.H. Jin, ^bZ.H. Wang

a. Department of Structural Engineering, Tongji University, Shanghai, China, 200092

b. Guangdong Electric Power Design Institute, Guangzhou, China, 510663

Abstract: A study on a lattice suspension tower in a four-span transmission line system subjected to skew incident wind forces is presented via experimental and numerical approaches. The purpose of this work is to investigate the response features of the tower especially to the wind not incoming along the structure's principle axes. At the geometric scaling of 1: 80, the aeroelastic model is manufactured with scaled rigidity, mass, area projection and frequency. Wind tunnel tests on the model, with and without conductors, are conducted at a variety of incident angles. Then numerical simulations are applied quasi-steadily to figure out an analytical way of interpreting and supplementing experimental results. Such quasi-steady method is generally valid since the tested aerodynamic damping ratios, though scattered, roughly meet the quasi-steady formulae. Results show that for the tower without conductors the dynamic accelerations do not vary significantly with the incident angle either in the r.m.s. values or in the spectral contents, as wind forces on the tower lend dependency on both the along wind and the across wind components of fluctuating winds thus have equivalent magnitudes at different wind incidences. For the tower with conductors, the longitudinal forces on the conductor are quite different from the lateral ones according to the simulated mechanism. As the co-effect of the additional loads and extra damping provided by conductors, dynamic

* Corresponding author: Tel (Fax): (86) 21 65981309 E-mail address: denghz@tongji.edu.cn

accelerations of the tower show distinct characteristics in different axes and at different wind incidences.

Keywords: transmission tower; skew incident wind; dynamic response; wind tunnel test; numerical simulation

Nomenclature

a	mass coefficient of Rayleigh model
a'	coefficient of additional damping
A_i	area projection of i th section
b	stiffness coefficient of Rayleigh model
$\mathbf{C}_{damper}$	damping matrix contributed by additional dampers
C_D	drag coefficient
C_L	lift coefficient
d	diameter of conductor and shield wire
D_i	drag force
f	frequency in Hz
F_{xn}^*, F_{yn}^*	n th generalized wind forces in x and y axes
i	i th section
$\mathbf{K}$	stiffness matrix
L_i	lift force
$\bar{m}$	mass per unit length
$\mathbf{M}$	mass matrix
M_{xn}^*, M_{yn}^*	n th generalized mass in x and y axes
$\dot{q}_{xn}, \dot{q}_{yn}$	n th generalized vibration speed in x and y axes
s_0	sag of conductor and shield wire
$\dot{s}_i$	vibration speed
$\bar{U}_i$	mean wind speed
u_i'	along wind fluctuating wind speed

U_i	instantaneous wind speed
U_{Ri}	relative wind speed of U_i against $\dot{s}_i$
$\vec{U}_{Pi}$	projective vector of U_{Ri} on conductors
$\vec{U}_{Ni}$	normal vector of U_{Ri} against conductors
$U_{Pxi}, U_{Pyi}, U_{Pzi}$	component of U_{Pi} in x, y and z axes
$U_{Nxi}, U_{Nyi}, U_{Nzi}$	component of U_{Ni} in x, y and z axes
v_i'	across wind fluctuating wind speed
$\dot{x}_i$	vibration speed in x direction
$\dot{y}_i$	vibration speed in y direction
z_R	reference height
z_0	roughness length
α_N	angle between U_{Ri} and U_{Ni}
β_i	angle between U_i and y axis
$\bar{\beta}_i$	angle between $\bar{U}_i$ and y axis
β_{Ri}	angle between U_{Ri} and y axis
γ_0	adjustment factor for span scaling
γ_i	angle between U_i and $\dot{s}_i$
θ_i	angle between $\dot{s}_i$ and x axis
κ	empirical factor
ρ	air density
ω_{xn}, ω_{yn}	n th circular natural frequency in x and y axes
ϕ_{xni}, ϕ_{yni}	n th mode shape coefficients in x and y axes
ζ	damping ratio
ζ_{axn}, ζ_{ayn}	n th aerodynamic damping ratios of the tower in x and y axes

1 INTRODUCTION

The basic structural utilities in an overhead electrical power transmitting line include conductors, shield wires, insulator strings and supporting towers. Conductors are responsible for transmitting the electricity and they are attached to towers using insulator strings. Shield wires protect the line from lightning strike. For the purpose of changing forward direction or preventing continuous collapse, usually a strain tower will be set every several spans to resist tension in the conductors. Thus two adjacent strain towers, intermediate suspension towers and conductors form a relatively independent section which can bear longitudinal, lateral and vertical loads in operation stage, namely Transmission Line System (TLS).

Structural design of TLS usually treats towers and conductors separately and take conductors' wind loading as nodal force exerting on the joint points. Consequently, earlier studies concentrated on towers or conductors respectively.

Based on a few assumptions and quasi-static theory, GRF method was imported into the expression of tower responses (Holmes, 1994). Then mean, background fluctuating and resonant components of wind loads for freestanding lattice towers were deduced and combined for equivalent static load distribution (Holmes, 1996b). Apart from theoretical research, experiments (e.g. Harikrishna et al., 1999) and field measurements (e.g. Bai et al., 2012) were carried out to study dynamic characteristics and wind-induced vibrations of single tower.

As regards to conductors, Davenport (1979) developed an analytical model to predict conductor response in extreme winds. Multiple wind tunnel tests (Shan, 1993, 1994; Loredou-Souza and Davenport, 1998, 2002) on various conductors subjected to turbulent synoptic wind were carried out to investigate the dynamic tension responses.

With progressive comprehension of tower-conductor interaction and rapid development of wind tunnel test technique, aeroelastic TLS tests (Deng et al., 2003; Zhao et al., 2009; Xie and Yang, 2013) were conducted adopting conductor modelling approach presented by Loredou-Souza and Davenport (2001). Similar to field measurements (e.g. Momomura et al., 1997), test results revealed that conductors have significant influences on wind-induced vibration of towers. Moreover, aerodynamic damping, which explains fluid-solid coupling between wind and structures, was much more investigated and theoretical models were proposed (Holmes, 1996a; Kareem and Gurley, 1996; Macdonald and Larose, 2006; Takeuchi et al., 2010; Raeesi et al., 2013).

Due to difficulties in manufacturing aeroelastic models and high cost in carrying out TLS wind tunnel tests, finite element modelling method developed as an important assistant approach. To accurately simulate TLS' dynamic characteristics, many simplified models were presented, including continuous cable model (Irvine, 1981), spatial truss model (Al-Bremani et al., 1992), model consisting of coupled pendulum model in high frequency regime and multi-mass model in low frequency regime (Ozono and Maeda, 1992) and truss model (Yasui et al., 1999) etc..

Because of the large scale, e.g., several kilometers, in span direction of TLS, the situation that fluctuating wind incomes normal to the conductors is the primary concern in previous researches and engineering practice. However, some wind tunnel tests (Lou et al., 1999; Guo et al., 2007; Li et al., 2008, 2011; Xie et al., 2011; Deng et al., 2013; Liang et al., 2015) and on-site measurements (Ballio et al., 1992; Glanville and Kwok, 1995; Moschas and Stiros, 2014) revealed that the across wind responses, compared to the along wind ones, would be of the same order of magnitude. Accordingly, investigators argued that it is necessary to take account of the combination of responses in both directions. Besides, the transmission lines are usually exposed to skew incident winds.

Wind tunnel tests conducted by Xie et al. (2013) suggested that the largest acceleration responses of TLS occurred at both 60° and 90° incident angle (relative to the span direction). Similar findings were delivered by Deng et al. (2010) according to experimental results for an aeroelastic model of lattice tower without conductors. They reported that wind incidence of 15° was the most unfavorable condition for their objective. Moreover, other issues concerning, say, assessment of response surface or reliability estimation of TLS under different wind events, need considerations of oblique wind directions, like Mara and Hong's work (2013).

As few references can be obtained by the authors from open access, this work focuses on the dynamic responses of lattice tower, with and without conductors, to skew incident winds. A practical method, adopted by design guide (e.g., ASCE, 2010), applies a triangular transformation of the along wind responses. The lattice tower and conductors are assumed to be calculated separately. Gust response factors for tower in principle axes and for conductors in lateral direction are specified. More detailed alternative may take account of three-dimensional wind loads. Since some field monitoring data (e.g., Takeuchi, et al, 2010) and wind tunnel test results (e.g., Duan and Deng, 2014) support the argument that vortex-induced vibrations (VIVs) or self-induced vibrations (SIVs) are not dominant in across wind direction, it seems applicable to model three-dimensional wind loads of lattice tower on the basis of quasi-steady assumption. As to loads on conductors, ESDU (1980) suggested a semi-empirical model for normal and axial aerodynamic forces of an infinite cylinder.

The current work considers the scaled model of a typical section of transmission line including four spans of conductors (see Figure 1). The tower of interest is a suspension tower located at the middle of the whole section, with relatively low stiffness and large vertical extent compared to strain towers. An identical suspension tower is set

at each side of the tower of interest. Thus the conductors are expected to be able to swing in longitudinal and lateral directions. Strain towers are set at ends of the section to restrain rigid body displacement and provide initial tension of the conductors. The scope of this study is limited to synoptic wind events. Wind tunnel tests are conducted on the aeroelastic model in Boundary Layer Wind Tunnel Laboratory TJ3 (BLWTL-TJ3) of Tongji University, and numerical simulations are carried out to study the dynamic responses under three-dimensional wind loads. Section 2 describes the full-scale transmission line system that is modeled. Section 3 describes the procedure of aeroelastic model designing, wind tunnel set-up and test results. Section 4 describes model simulation, wind load calculation and results of numerical simulation. Section 5 delivers brief conclusions and findings from the study.

2 FULL-SCALE SYSTEM

The studied objective is a typical 500kV double-circuit transmission line. The tower of interest is a self-supported suspension tower as illustrated in Figure 2. The vertical square tower shaft extends 98.3m above the base level (ABL) and is assembled from 16 sections. The shaft width is 18.92m at the base level, 4.4m at 66m ABL waist and 2.4m at the top. High strength steel tube members (up to $\Phi 490\text{mm} \times 12\text{mm}$) are arranged at four corners of the shaft and are braced by diagonal cross-bracing which is made up of angle (above 66m ABL) or tube (below 66m ABL) members. Three levels of horizontal cross-arms and one level of upturned ground wire brackets protrude symmetrically from the shaft in both sides, locating at 72m, 84m, 96m and 101m ABL, respectively. The strain tower extends 66m ABL, with cross-arms at the height of 33m, 44.5m, 56m and 66m.

Each tower carries six conductor bundles—four conductors in each bundle connected with 0.5m×0.5m square spacers. The conductor type is aluminum cable steel reinforced (ACSR) that is constructed of 50mm² steel core and 720mm² aluminum strand. The diameter of a conductor is 36.2mm and the mass per unit length is 2.4 kg/m. The elastic modulus is 63.7GPa. The conductor bundles are hung at the termination of cross-arms through composite insulator string pairs with the length of 6.2m. The mid-span sag of the conductor bundle is 28.9m with 40.5kN tension in each cable.

The shield wire is hung at the end of ground wire bracket through a U-plate. The wire diameter is 16mm and the cross section area is 150mm². The mass per unit length is 0.71kg/m. The elastic modulus is 105GPa. The mid-span sag of the conductor bundle is 18.6m with 18.6kN tension.

3 WIND TUNNEL TEST

Similarity scaling of aeroelastic model

The wind tunnel height is 2m and the full-scale height of the lattice tower is 101m. Thus a geometric scaling is chosen to be 1: 80 (reduced-scale: full-scale). As a result, the horizontal length of four-span conductor model would be 31.5m, which exceeds the test section width of the wind tunnel. Rather than modelling the TLS with even smaller scaling ratio, a distorted approach (Loredo-Souza and Davenport, 2001) is utilized to achieve a span scaling of 1: 200 with the adjustment factor $\gamma_0=0.4$. The maximum wind speed achievable in the wind tunnel is about 12m/s, whereas the design wind speed of the full-scale TLS is 30m/s. Consequently, the speed scaling of 1: 3 is applied. Together with the air density scaling of 1: 1, scaling ratios of other physical quantities are derived in Table 1, based on non-dimensional similarity parameters such as Strouhal number,

Cauchy number, damping parameter and inertia parameter. According to similarity theory, values of prototype can be converted from those of scaled model. In view of emphasis on model study, results of wind tunnel test and numerical simulation are discussed from the perspective of scaled model.

Model of lattice tower

The aeroelastic model of lattice tower is manufactured by assembling the reduced-scale members one by one. At the geometric scaling of 1: 80, it is not applicable to scale the member's diameter, thickness and elastic modulus at the same time. The cross sectional dimensions would be millimeter or sub-millimeter. Therefore, the reduced-scale members are modelled by using multi-material in order to provide the tower shaft with equivalent flexural rigidity, mass and area projection.

The flexural rigidity EI of tower shaft is mainly contributed by second moment of axial rigidity EA of leg members about x or y axis (almost identical in this case). Thus capillary copper tubes that have desired EA are used as the skeleton. Consequently, the mass and areas of capillary tubes cannot meet corresponding scaling requirements in Table 1. Thus the aerodynamic figurations are simulated by sticking tube/angle-shaped foams on copper tubes. Fuse wires are also used to adjust the panel weight of the tower shaft. Figure 3 shows the scaled values of flexural rigidity, mass and area projection for the tower of interest and the actual values for aero-elastic model, at the height of centroid of the 16 sections labelled in Figure 2.

Model of conductors

As mentioned above, the span length scaling of conductor and shield wire models are divided by an adjustment factor $\gamma_0=0.4$. Correspondingly, the diameter d and mass

per unit length $\bar{m}$ are enlarged to make sure the total wind area and mass of the whole span unchanged. Nevertheless, the frequency scaling is intentionally kept unchanged for the reason that the distorted model should not have different dynamic characteristics. Thus the sag of the aeroelastic model is determined given the relationship $f^2 \propto 1/s_0$. For the reason that the tangent stiffness of a tensioned cable is mainly contributed by its stress rather than the cross sectional area, the axial rigidity EA of the conductor and shield wire is not strictly modelled. Detailed parameters for the aeroelastic model are listed in Table 2. The reduced-scale conductors and shield wires are manufactured with stainless steel wire as the core and plastic sleeves as the cloths.

Model of insulator strings

As depicted in Figure 4, the insulator string is modelled with a thin-wall steel tube ($\Phi 1.6\text{mm} \times 0.15\text{mm}$) providing approximate mass and sufficient rigidity. 17 circular ABS plates ($\Phi 4.5\text{mm}$) are attached on the steel tube to simulate the aerodynamic figuration. Steel hoops pass through the bores at the ends of the steel tube, and at the cross-arm tips. Thus the insulator string can swing like a pendulum in x and y directions.

The spacer model is manufactured with ABS plate. Laser engraving machine is utilized to carve the plate to square-box shape with short arms at the four corners. Four model conductors pass through those bores so as to form a conductor bundle. The bundled conductor models terminate at the cross-arms of strain towers and are tensioned to reach the required sag in Table 2.

Model instrumentation and characteristics of wind field

BLWTL-TJ3 has a 15m-wide, 2.0m-high and 14m-long test section. As shown in Figure 5, the aeroelastic model of the tower of interest T0 is located at the center of the

test section, about 10 meters downstream of the outlet. Equivalent models of suspension tower T1 are placed adjacent to T0, at a distance of 3.15m on both sides. Then strain tower models T2 stand 3.15m away from T1, next to the side walls of the tunnel.

As shown in Figure 6, six laser transducers (D1~D6) are installed at three heights of the vertical shaft of T0—1.23m, 1.08m and 0.93m ABL, respectively. Two accelerometers (A1 and A2) are installed at the height of 0.9m ABL. The transducers and accelerometers are arranged in pairs, with the one in x direction and the other in y direction. All the data are recorded for 69.63 seconds at a sampling frequency of 250Hz, corresponding to about 30 minutes for the full-scale system. A pitot-static probe is mounted at the reference height $z_R=1.263$ m ABL to monitor the mean wind speed.

The wind tunnel test is conducted in a simulated boundary layer wind environment with mean speed at z_R ranging from 2.53 to 10.25m/s. Rows of 0.074-meter-high wooden elements, 0.1-meter-high concrete cubes, rectangle and triangle spires are set to achieve the desired wind field. The profiles of mean wind speed and turbulence intensity are matched closely to exposure category EC4 in Eurocode (2005) corresponding to an open terrain with full-scale roughness length $z_0=0.05$ m, as depicted in Figure 7.

The coordinate system in this work, shown in Figure 8, defines x axis as the direction normal to conductor span, and y axis parallel to the line. The wind is assumed to income from the negative infinity of x and y axes and is supposed to be “steady” and “broad-front” at the location of the target structure. The wind incidence β is defined as the angle between flow direction and y axis, e.g., $\beta=90^\circ$ for normal-to-line incident wind.

The first stage of the test is for the tower without conductors at seven wind incident angles, ranging from 0 to 90° with an increment of 15° . Then in the second stage, the tower of interest T0, the equivalent suspension towers T1 and the strain towers T2 are

tested together with all the conductors and shield wires installed. Limited by the longitudinal dimension of the wind tunnel, only two wind incidences, 75° and 90° , can be achieved. Other situations will be studied later in numerical simulations.

Eigenvalues of the aeroelastic model

Prior to the wind tunnel test, artificial excitation method is employed to acquire the free vibration records of the aeroelastic tower model, with and without conductors. Natural frequencies are directly read in the power spectrum diagram of the responses. Damping ratios are identified using Eigensystem Realization Algorithm (ERA).

Eigenvalues of the aeroelastic model are listed in Table 3. Only the first flexural mode in x and y axis can be clearly recognized. As the rigidity and mass distributions in the two principle axes are nearly the same, corresponding natural frequencies are quite close to each other. The frequencies of model tower with conductors are slightly scattered among several recognitions, ranging from 33~34Hz (33.4 in average) in x axis and 32~36Hz (34.7 in average) in y axis. The damping ratios of the model tower with conductors are scattered among 3~5% with the averaged value 4.6% in x axis and 3.3% in y axis. Compared to the values of the tower without conductors, it is clear that the conductors offer the tower extra damping which can probably be attributed to their nonsynchronous vibrations.

Wind tunnel test results

(1) Aerodynamic damping of the model tower

The aerodynamic damping of the model tower in wind tunnel test is identified with a variety of wind speed (2.53~10.25m/s) and incident angle (0° ~ 90°). An auto identification

procedure is employed based on Random Decrement Technique (RDT) and ERA, as introduced by Duan and Deng (2014).

As shown in Figure 9, the aerodynamic damping in y axis is generally larger than those in x axis. Most of the identified aerodynamic damping is positive, which would be beneficial to the structure. However, the identified aerodynamic damping ratios are scattered and the dependency of them on incident angle and on wind speed is not very clear. Careful observation on Figure 9 indicates that the x -directional aerodynamic damping varies around 0 at $0\sim 30^\circ$ incident winds but gradually shows positive dependency on wind speed from 45° to 90° , while in y direction this dependency is found under $45\sim 0^\circ$ incidences. Noting that the incident winds of 90° and of 0° respectively income along x axis and y axis, the quasi-steady theory is introduced to interpret the identified aerodynamic damping in Figure 9.

As depicted in Figure 10, fluctuating wind U_i acts on the i th section of the tower at an incident angle β_i . The speed response of the section is $\dot{s}_i = \sqrt{\dot{x}_i^2 + \dot{y}_i^2}$. According to quasi-steady theory, wind forces in x and y axes are dependent on the relative speed U_{Ri} and relative wind incidence β_{Ri} :

$$\begin{aligned} F_{xi} &= \frac{1}{2} \rho A_i U_{Ri}^2 (C_D \sin \beta_{Ri} - C_L \cos \beta_{Ri}) \\ F_{yi} &= \frac{1}{2} \rho A_i U_{Ri}^2 (C_D \cos \beta_{Ri} + C_L \sin \beta_{Ri}) \end{aligned} \quad (1)$$

where ρ , A_i , C_D , C_L denote the air density, area projection of the section, drag coefficient and lift coefficient, respectively. The derivation of Eq. (1) and expressions for U_{Ri} and β_{Ri} are given in Appendix A.

On the basis of quasi-steady wind forces defined in Eq. (1), theoretical expressions for the aerodynamic damping are given in Eq. (B.8) (see Appendix B). Taking the approximation of $U_{Ri} \approx \bar{U}_i$ and $\beta_{Ri} \approx \bar{\beta}_i$ for small amplitude vibrations in a low

turbulent flow, and noting that C_L and derivative terms of C_F ($F = D, L$) can be neglected compared to C_D , Eq. (B.8) reduces to

$$\begin{aligned}\zeta_{axn} &= \frac{\rho C_D \sum_i \phi_{xni}^2 A_i \bar{U}_i / \bar{U}(z_R)}{4M_{xn}^* \omega_{xn}} \bar{U}(z_R) (1 + \sin^2 \bar{\beta}) \\ \zeta_{ayn} &= \frac{\rho C_D \sum_i \phi_{yni}^2 A_i \bar{U}_i / \bar{U}(z_R)}{4M_{yn}^* \omega_{yn}} \bar{U}(z_R) (1 + \cos^2 \bar{\beta})\end{aligned}, \quad (2)$$

where the subscripts i, n, x, y respectively refer to the i th section of the tower, the n th mode, in x axis and in y axis; ζ_a is the aerodynamic damping ratio; ω is the angular natural frequency; M^* is the general mass; ϕ is the mode shape; $\bar{U}(z_R)$ is the mean wind speed at reference height z_R . It is worth to note that the quasi-steady aerodynamic damping defined by Eq. (2) is actually the one adopted in numerical simulations later.

Eq. (2) suggests a possible dependency of the aerodynamic damping on the incoming wind speed and wind incidence. Accordingly, all the identified aerodynamic damping ratios are illustrated in one diagram by designating $U^* = \bar{U}(z_R)(1 + \sin^2 \bar{\beta})/2$ (in x axis) or $U^* = \bar{U}(z_R)(1 + \cos^2 \bar{\beta})/2$ (in y axis) as abscissa, as shown in Figure 11. Generally, although scattered around the quasi-steady lines, the identified aerodynamic damping ratios, both in x axis and y axis, show an obvious trends that the larger values occur at the higher U^* . As a result, the aerodynamic damping tends to reduce the x -directional response at a larger wind incidence and reduce the y -directional response at a smaller wind incidence.

It can also be observed in Figure 11 that the quasi-steady aerodynamic damping ratios and the identified ones are roughly equivalent in value and have the similar trend versus U^* . It indicates that for the lattice structure such as transmission tower the quasi-steady theory seems to be generally valid in evaluation of wind forces under skew

incident winds. Yet between identified aerodynamic damping and the quasi-steady line there exist discrepancies that the former seems to be smaller than the latter at low U^* whereas at high U^* the situation is quite the reverse. It can be attributed to the influence of VIVs and SIVs to some extent. Nevertheless, since there is no obvious negative damping observed at low U^* , VIVs or SIVs are not the dominant contributor in across wind responses for lattice structure like transmission towers.

(2) Mean displacements

The equilibrium position at the top of the model tower under skew winds is illustrated by mean displacement versus wind incidence shown in Figure 12. All results are obtained at the mean wind speed of 7.69m/s for the sake of comparison. Mean displacements of the tower without conductors show the expected relation with wind incidence, as reported by other researches (e.g., Deng, et al., 2010). Limited by the wind tunnel condition, only 75° and 90° incident wind can be realized in the tests on the model of TLS. X-directional displacements of TLS are obviously greater than corresponding values of the tower without conductors, while y-directional displacements are nearly the same (slightly larger). Apparently, one reason is that wind forces acting on the conductors in x axis (normal component) are significantly larger than those in y axis (projective component) according to Eq. (10). Besides, rather than transmitting the y-directional forces to the tower, the insulator string adjusts its swinging so that conductor tensions can make a balance with the longitudinal wind forces.

(3) R.m.s. accelerations

The root-mean-square (r.m.s.) acceleration (see Table 4), obtained at the height $z=0.9\text{m}$ of the model tower, varies with the wind incidence not as greatly as the mean displacement. Test results show that for the tower without conductors the r.m.s. accelerations at 0° are only 19% (in x direction) and 28% (in y direction) larger than

those at 90° . Such results can be partially explained by the quasi-steady wind forces defined in Eq. (1). Neglecting structural vibrations, lift coefficient and the second order terms of fluctuating wind, an approximate evaluation of r.m.s. wind forces yields

$$\begin{aligned}\sigma_{F_{xi}} &= \rho A_i \bar{U}_i C_D (\sigma_{u_i} \sin \bar{\beta}_i + \frac{1}{2} \sigma_{v_i} \cos \bar{\beta}_i) \\ \sigma_{F_{yi}} &= \rho A_i \bar{U}_i C_D (\sigma_{u_i} \cos \bar{\beta}_i + \frac{1}{2} \sigma_{v_i} \sin \bar{\beta}_i)\end{aligned}\quad (3)$$

Accordingly, at 0° incident angle still there are wind forces $\sigma_{F_{xi}} = \rho A_i \bar{U}_i \sigma_{v_i} C_D / 2$ acting in x direction and at 90° incident wind there are $\sigma_{F_{yi}} = \rho A_i \bar{U}_i \sigma_{v_i} C_D / 2$ acting in y direction. The r.m.s. wind forces at the tower tip are qualitatively illustrated in Figure 13(a) according to Eq. (3). The maximum values are 80% (in x direction) and 150% (in y direction) larger than the minimum values. Affected by the asynchronism of fluctuating winds at different points, differences between the maximum and minimum values of the tower's total wind forces will be diminished. And the existence of aerodynamic damping would further reduce the x -directional responses at 90° and y -directional responses at 0° .

The r.m.s. accelerations for the aeroelastic model with conductors, obtained at 75° and 90° incident winds, are found to be different from those for tower without conductors. The x -directional accelerations are greatly enlarged, whilst the y -directional accelerations are reduced. There are two opposite effects of the conductors that are believed to be responsible for such differences. The one is that the extra damping provided by the conductors, evidenced in Table 3, restrains the vibrations of the tower. The other effect refers to the wind loads acting on conductors. Figure 13(b) shows the simulated r.m.s. tension in the upper conductors. Its x -directional component is far larger than the y -directional one at 75° or 90° , which may imply that the enhancement of r.m.s. acceleration in x direction is induced by the growth of wind forces.

The correlation coefficients of the x -directional and the y -directional accelerations (see Table 4) for the tower without conductors have the maximum values of 0.14 at the

incident angle of 60° , while the values at 0° and 90° are relatively small. From Eq. (3) one can figure out that at 0° or 90° the wind forces are related to the along wind fluctuating speed u_i' or the across wind speed v_i' separately but at other incidences are the combination of the both two, which may be in charge of the correlation results. For the tower with conductors, the correlation coefficient at 90° incidence is larger than that at 75° incidence, and they are relatively larger than corresponding values of single tower. It can be attributed to the influence of the wind loads on conductors, as will be discussed in numerical simulations.

(4) Peak factors

The peak factors of displacements at $z=1.263\text{m}$ and of accelerations at $z=0.9\text{m}$ of the model are listed in Table 5. The peak factors of displacements and accelerations are generally among the range of about 4~4.5, with a few samples up to 5.0. No clear evidence implies that there is any difference between the peak factors of any specific wind incidences and of others. As a matter of fact, according to the probabilistic analysis, all the acceleration responses meet the Gaussian distribution. Obviously, the vibrations of the aeroelastic model in the wind tunnel tests conform to the assumption of stationary stochastic process which is the basis of quasi-steady theory.

(5) Spectral contents of accelerations

The spectral characteristics of the acceleration responses of the tower without conductors are rarely changed at different wind incidences. The first flexural mode, either in x or y axis, has the most prominent peak in the power spectrum diagram (see Figure 14).

For the tower with conductors (see Figure 15), the test results show that the first-mode peaks in the spectrum diagram are not as prominent as those for the tower

alone, which could be the consequence of extra damping provided by conductors and complex vibration modes evoked by the conductors.

4 NUMERICAL MODELLING

Towers and conductors

The reduced-scale tower model as well as conductors is simulated via numerical approach. The four-span TLS is simplified as mass-beam-cable system, as illustrated in Figure 16. The tower shaft is simulated by lumped mass at the heights of 16 sections and at the tips of cross-arms, and by elastic beams connecting adjacent mass with three translational degrees of freedom (DOFs) and three rotational DOFs at each end. Each span of conductors or shield wire is simulated by 50 lumped mass and light cables. The insulator strings are simplified as bars that have equivalent mass and large enough rigidity. The coordinate system is consistent with section 3.

The mass matrix of numerical model is assembled by the diagonal element of tower $\mathbf{M}_{11}$ and of conductors $\mathbf{M}_{22}$,

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_{11} & 0 \\ 0 & \mathbf{M}_{22} \end{bmatrix}. \quad (4)$$

The stiffness matrix under averaged wind loading consists of the tower's stiffness $\mathbf{K}_{11}$, conductors' stiffness $\mathbf{K}_{22}$, and their cross-terms $\mathbf{K}_{12}$, $\mathbf{K}_{21}$,

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} \\ \mathbf{K}_{21} & \mathbf{K}_{22} \end{bmatrix}, \quad (5)$$

where $\mathbf{K}_{22}$ includes a linear matrix $\mathbf{K}_{22}^0$ decided by cables' initial state, the nonlinear matrices $\mathbf{K}_{22}^u$ and $\mathbf{K}_{22}^\sigma$ decided by the displacement and stress.

The simulation of damping characteristics is problematic since the natural frequency of model tower ($\sim 34\text{Hz}$) greatly differs from those of conductors ($\sim 1.7\text{Hz}$ the smallest).

Rayleigh damping model cannot provide desired damping ratios for the tower and for the conductors at the same time. As a feasible option, herein a modified Rayleigh model is proposed. Additional dampers are attached to the lumped mass of the tower in x and y directions to adjust Rayleigh damping (see Figure 16). Therefore, the damping matrix of the numerical model can be written as

$$\mathbf{C} = a \cdot \mathbf{M} + b \cdot \mathbf{K} + \mathbf{C}_{damper}, \quad (6)$$

where $\mathbf{C}_{damper}$ is the matrix contributed by additional dampers, a and b are mass and stiffness coefficient of Rayleigh model. As illustrated in Figure 17, the lowest conductor mode and the second flexural mode of the tower are chosen to determine a and b . Corresponding damping ratios of the two modes are assumed to be 4%. Thus most tower modes and conductor modes of interest are included with the damping ratio ranging from 1~4%. Consequently, the damping ratio of the tower's first flexural mode is smaller than the expected value—1.4% for tower without conductors and 3~5% for tower with conductors—as measured from the aeroelastic model. The difference is supplemented with additional dampers by setting $\mathbf{C}_{damper}$ proportional to $\mathbf{M}_{11}$, i.e., $\mathbf{C}_{damper} = a' \cdot \mathbf{M}_{11}$. Thus Eq. (6) yields

$$\mathbf{C} = \begin{bmatrix} (a + a') \cdot \mathbf{M}_{11} + b \cdot \mathbf{K}_{11} & b \cdot \mathbf{K}_{12} \\ b \cdot \mathbf{K}_{21} & a \cdot \mathbf{M}_{22} + b \cdot \mathbf{K}_{22} \end{bmatrix}. \quad (7)$$

Wind load

The wind field simulated in numerical study is illustrated in Figure 18 and Figure 19. The wind speed profile and turbulence intensity (Figure 18) are simulated according to wind tunnel test. The formulae suggested by Kaimal (1972) and Tieleman (1995) are adopted for along wind, across wind and vertical wind velocity spectra (see Eq.(8)). The

turbulence spatial coherence is assumed to obey Shiotani function (Shiotani and Iwatani, 1971) in Eq. (9).

$$\begin{aligned}\frac{fS_u(z,f)}{u_*^2} &= \frac{200x}{(1+50x)^{5/3}} \\ \frac{fS_v(z,f)}{u_*^2} &= \frac{15x}{(1+9.5x)^{5/3}} \\ \frac{fS_w(z,f)}{u_*^2} &= \frac{5.13x}{(1+4.92x)^{5/3}}\end{aligned}\quad (8)$$

$$coh_{ij}^{u_i} = \exp\left\{-\left[\frac{(x_i - x_j)^2}{L_x} + \frac{(y_i - y_j)^2}{L_y} + \frac{(z_i - z_j)^2}{L_z}\right]^{1/2}\right\} = \exp(-\Delta r) \quad (9)$$

where $x = fz/\bar{U}(z)$, f refers to frequency, u_* refers to shear velocity,

$$L_x = L_y = 50, \quad L_z = 60.$$

The spatial wind forces on lattice tower shaft are not fully established in current literature, especially under skew incident winds. In this numerical study, the model of wind forces derived from quasi-steady theory is adopted since no obvious VIVs or SIVs is found in wind tunnel tests and the quasi-steady theory is generally valid in interpreting aerodynamic damping, as discussed in Section 3. Therefore, the wind forces of skew winds on x direction and y direction of the tower shaft are calculated by varying wind incident angle $\bar{\beta}_i$ from 0° to 90° in Eq. (1) and Appendix A.

According to Eq. (1), the drag coefficient C_{Di} and the lift coefficient C_{Li} are important to enhance the accuracy of numerical simulations. However, due to the lack of comprehensive C_{Di} and C_{Li} data for different geometric shape of lattice structure, the global drag and lift coefficients, which were obtained by Deng, et al. (2015) via high frequency force balance test on a similar lattice tower with a variety of wind incidences, are adopted herein for every segment of the tower shaft. Such global coefficients are only valid for calculation of base shear of the tower in strict sense. But for the purpose of

introducing a practicable method to simulate the vibrations under skew winds, using global coefficients would be an expedient effective to obtain general features of dynamic responses. More precise alternative, of course, is to measure the dynamic coefficients of every segment, or to establish the wind force model for shielded members (Prud'homme et al, 2014).

In view of ESDU80025 (1980), wind forces acting on the cable segment of conductors as well as shield wires can be calculated using the suggestions about inclined circular cylinder (see Figure 20). Detailed derivation is given in Appendix C, finally leading to

$$\begin{aligned} F_{xi} &= \frac{1}{2} \rho A_i C_D \left[k \alpha_N \frac{\pi}{180} (2 - \alpha_N \frac{\pi}{180}) \frac{|\vec{U}_{Ri}|^2}{|\vec{U}_{Pi}|} U_{Pxi} + |\vec{U}_{Ni}| U_{Nxi} \right] \\ F_{yi} &= \frac{1}{2} \rho A_i C_D \left[k \alpha_N \frac{\pi}{180} (2 - \alpha_N \frac{\pi}{180}) \frac{|\vec{U}_{Ri}|^2}{|\vec{U}_{Pi}|} U_{Pyi} + |\vec{U}_{Ni}| U_{Nyi} \right], \\ F_{zi} &= \frac{1}{2} \rho A_i C_D \left[k \alpha_N \frac{\pi}{180} (2 - \alpha_N \frac{\pi}{180}) \frac{|\vec{U}_{Ri}|^2}{|\vec{U}_{Pi}|} U_{Pzi} + |\vec{U}_{Ni}| U_{Nzi} \right] \end{aligned} \quad (10)$$

where k is the empirical factor with a suggested value of 0.045 for rough cylinders; $\vec{U}_{Pi}$ is the projective component of $\vec{U}_i$ on the cable segment; $\vec{U}_{Ni}$ is normal component of $\vec{U}_i$ perpendicular to the cable segment; α_N is the angle between $\vec{U}_i$ and $\vec{U}_{Ni}$; drag coefficient $C_D = 1.0$ is adopted according to ESDU80025 (1980).

Steps of the analysis

The following steps are included in the numerical simulations:

(a). Establish the equivalent mass-beam-cable model and conduct numerical tests to obtain the dynamic characteristics and to determine the damping coefficients a , b and a' .

(b). Simulate the instantaneous wind speed based on wind characteristics of the wind tunnel test.

(c). Set the time $t=0$ s at the beginning of the analysis. Read the nodal coordinates of the tower and conductors. Set the vibration speeds of the structure equal to 0.

(d). Calculate wind forces on the tower and conductors according to Eq. (1) and Eq. (10).

(e). Perform non-linear analysis of the model using Newmark- β algorithm. Read the new nodal coordinates and vibration speeds.

(f). Set the time $t=t+0.00375$ s (sampling frequency of 266.7Hz). Return to step (d).

(g). At the time $t=22.5$ s (corresponding to 10 minutes in full-scale) stop the analysis. Change the wind incident angle and return to step (c).

Discussions

(1) Comparison of numerical simulations and wind tunnel tests

As depicted in Figure 12, Section 3, the mean displacements of the numerical simulations are close to those obtained in wind tunnel tests. For the tower with conductors, results at wind incidences of $0\sim 60^\circ$, which cannot be realized in the wind tunnel, are supplemented by numerical simulations. It is observed that displacements in y axis are nearly the same as the tower without conductors. But x -directional displacement greatly increases from $0\sim 90^\circ$.

Table 6 shows the numerical results of acceleration at the height $z=1.263$ m. Compared to tested values in Table 4, the numerical simulation results show a similar trend that in x direction maximum r.m.s acceleration is 10% larger than the minimum and in y direction the maximum is 41% larger. But still there are differences between test and simulation results. The numerical accelerations are generally larger than tested ones and

it may be attributed to the uncertainties of wind forces in wind tunnel tests, such as the actual wind correlations or the wind loading area.

The correlation coefficients of x -accelerations and y -accelerations for the tower without conductors are also similar to those in wind tunnel tests. The values at incidences of 45° and 60° are larger than those at 0° or 90° incident wind. Different trend is found for the tower with conductors. The correlation coefficients gradually increase from 0° to 90° . This trend reflects the influence of wind forces on conductors. According to Eq. (10), the wind forces in x or y direction are always closely related to both u_i' and v_i' components at any wind incidences, and the wind forces on conductors generally increase from 0° to 90° . Nevertheless, this trend needs further validation since the wind tunnel results are not obtained in this study.

(2) The influence of aerodynamic damping under skew winds

As a comparison study, the dynamic responses are also calculated without concerning aerodynamic damping by eliminating the structure's velocity responses in Eq.(A.3) and Eq.(C.2). As shown in Figure 21, the aerodynamic damping leads to 6.9%~16.0% decline of r.m.s. accelerations for the model without conductors in x axis as the wind incidence increases, while in y axis the decline amplitude varies from 15.7%~7.4%. For the model with conductors the influence of aerodynamic damping is nearly the same in y axis both in the trend and in decline amplitude. In x axis the influence of aerodynamic damping is less obvious. But in general, the decline amplitude increases from 0° to 90° .

(3) The influence of conductors under skew winds

Figure 22 shows the numerical simulated r.m.s. tension in the upper conductors. Its x -directional component is far larger than the y -directional one at 75° or 90° , which may imply that the enhancement of r.m.s. acceleration in x direction is induced by the growth

of wind forces. For the skew winds not achieved in the wind tunnel test, numerical results suggest that the conductors' wind loads in x direction rapidly decrease with the incident angle varying from 90° to 0° while y -directional wind loads almost keep the value of zero unchanged (see Figure 22). Therefore, as the co-effect of the extra damping and loads, x -directional r.m.s. accelerations of TLS are larger than those of the tower without conductors at large incident angles ($60^\circ \sim 90^\circ$) but smaller at small incident angles ($0^\circ \sim 30^\circ$). The y -directional r.m.s. accelerations of TLS, however, are always smaller than those of the tower without conductors but they have similar relation with wind incidence.

The spectra of numerical simulated tension in the upper conductors are depicted in Figure 23. Since the y -component of the tension is nearly 0, only x -component spectra and z -component spectra are illustrated in the figure. It is observed that the spectral contents of conductor tension are complicated. Among the range of 2~27Hz there exists a lot of intensive peaks and the corresponding frequencies of those peaks changes with the variation of wind incidences. Generally, as the incident angle varies from 0° to 90° , the spectral peaks move to the higher frequencies. The reason is that as the incident angle increases the average tensions in the conductor would be enlarged and as a result the stress stiffness of the conductor would increase.

5 CONCLUDING REMARKS

In this work wind tunnel tests are carried out on the aeroelastic model consisting of five towers and four-span conductors with the geometric scaling of 1: 80 to investigate the dynamic features under skew incident winds. It is found that for the model tower without conductors the r.m.s. acceleration, which represents the vibration intensity of the model, has roughly equivalent magnitude under skew winds. This feature is quite different from the mean displacement in x axis or in y axis which varies gradually from 0

to the maximum as wind incidence increases or decreases. It is believed that the model is excited in skew winds by the combination of the along wind turbulence u_i' and the across wind turbulence v_i' . For model tower with conductors, the r.m.s. accelerations (at 75° and 90° wind incidences) obtained in wind tunnel tests are enlarged by about 68% in x direction and reduced by 37% in y direction, compared to the tower without conductors. The conductors affect the dynamic responses of the tower at skew winds probably in two ways: (1) the conductors enhancing the structural damping of the tower, and (2) additional wind forces acting on the conductors and finally transmitting to the tower.

The aerodynamic damping of model tower is identified from wind tunnel tests. With the incident angle changing from 0° to 90° , the damping ratio of the model tower generally increases in x axis and decreases in y axis. Comparison of the identified damping ratios and quasi-steady ones shows that the formers are generally in accordance with the latters, supporting the quasi-steady wind loading assumption.

The check of the power spectra of the accelerations indicates that for model tower without conductors the spectral contents are nearly the same at different wind incidences. Both in x and y directions the dominant peaks are corresponding to the first flexural mode in respective axis.

Then based on the wind tunnel test results, numerical simulations are achieved, with the simplification of the reduced-scale model to an equivalent mass-beam-cable system, and with the quasi-steady spatial wind forces on the tower as well as on the conductors. In numerical simulations, the maximum r.m.s. accelerations (at 15°) exceed the minimums (at 90°) by 10% (in x direction) and 41% (in y direction). The quasi-steady spatial wind loading mechanism would be an applicable way of solving such problems. For the tower model with conductors, r.m.s. accelerations increase in x direction and decrease in y direction with the wind incidence changing from 0° to 90° . In spectral

contents, low-frequency vibrations can be observed especially in x -directional accelerations. The first-mode peaks have wider frequency ranges and in low frequency region plenty of sub-peaks are observed. As the wind incidence varies from 0° to 90° , the sub-peaks in low frequency region turn prominent. In y direction, the spectral characteristics are similar to those for the tower without conductors.

The influence of aerodynamic damping on responses is similar for the tower with conductors and for the tower without conductors. In x axis the decline amplitude increases from 0° to 90° and in y axis the decline amplitude decreases from 0° to 90° .

The conductors' tension force in x direction decreases with the incident angle varying from 90° to 0° while y -directional it is around zero. It is the reason why x -directional r.m.s. accelerations of the tower with conductors are larger than those of the tower without conductors at large incident angles but smaller at small incident angles. In the spectra of conductor's tension forces, a lot of intensive peaks are observed in the low-frequency region and the corresponding frequencies of those peaks slightly increase at larger wind incidences.

ACKNOWLEDGEMENTS

The research presented in this paper was supported by the China National Science Foundation under project No. 51578421, which is gratefully acknowledged.

APPENDIX A

As depicted in Figure 10, the incoming wind at the i th section of the tower has the mean speed $\bar{U}_i$, along wind and across wind fluctuating component u_i' and v_i' , and mean incident angle $\bar{\beta}_i$. The vertical fluctuating component is neglected since it has little

influence on the horizontal vibrations of a tower. Then the instantaneous wind speed U_i and the instantaneous incidence are

$$\begin{aligned} U_i &= \sqrt{(\bar{U}_i + u_i')^2 + v_i'^2} \\ \beta_i &= \bar{\beta}_i + \arctan \frac{v_i'}{\bar{U}_i + u_i'} \end{aligned} \quad (\text{A.1})$$

If the vibration speeds of the i th section in x and y axes are respectively $\dot{x}_i$ and $\dot{y}_i$, their vector sum $\dot{s}_i$ and orientation angle θ_i to the x axis follow

$$\begin{aligned} \dot{s}_i &= \sqrt{\dot{x}_i^2 + \dot{y}_i^2} \\ \theta_i &= \arctan \frac{\dot{y}_i}{\dot{x}_i} \end{aligned} \quad (\text{A.2})$$

The relative wind speed U_{Ri} and relative incidence β_{Ri} are given by

$$\begin{aligned} U_{Ri} &= \sqrt{U_i^2 - 2U_i\dot{s}_i \cos \gamma_i + \dot{s}_i^2} \\ \beta_{Ri} &= \bar{\beta}_i \pm \arctan \frac{\dot{s}_i \sin \gamma_i}{U_i - \dot{s}_i \cos \gamma_i} \end{aligned} \quad (\text{A.3})$$

where $\gamma_i = \pm(\beta_i + \theta_i - 90^\circ)$, positive sign is valid for the situation depicted in Figure 10(a) and negative sign for Figure 10(b). According to quasi-steady theory the aerodynamic drag D_i and lift L_i are dependent on U_{Ri} . Hence,

$$\begin{aligned} D_i &= \frac{1}{2} \rho A_i C_D U_{Ri}^2 = \frac{1}{2} \rho A_i C_D (U_i^2 - 2U_i\dot{s}_i \cos \gamma_i + \dot{s}_i^2) \\ L_i &= \frac{1}{2} \rho A_i C_L U_{Ri}^2 = \frac{1}{2} \rho A_i C_L (U_i^2 - 2U_i\dot{s}_i \cos \gamma_i + \dot{s}_i^2) \end{aligned} \quad (\text{A.4})$$

Finally the wind forces in x and y axes are given by

$$\begin{aligned} F_{xi} &= D_i \sin \beta_{Ri} - L_i \cos \beta_{Ri} = \frac{1}{2} \rho A_i U_{Ri}^2 (C_D \sin \beta_{Ri} - C_L \cos \beta_{Ri}) \\ F_{yi} &= D_i \cos \beta_{Ri} + L_i \sin \beta_{Ri} = \frac{1}{2} \rho A_i U_{Ri}^2 (C_D \cos \beta_{Ri} + C_L \sin \beta_{Ri}) \end{aligned} \quad (\text{A.5})$$

APPENDIX B

The aerodynamic damping ratios, ζ_{axn} and ζ_{ayn} , of the n th mode of the tower respectively in x and y axes are given by the derivatives of the generalized wind forces to the generalized vibration speeds (see e.g., Raeesi, et al, 2013):

$$\begin{aligned}\zeta_{axn} &= -\frac{1}{2M_{xn}^* \omega_{xn}} \frac{\partial F_{xn}^*}{\partial \dot{q}_{xn}} = -\frac{1}{2M_{xn}^* \omega_{xn}} \frac{\partial \sum_i \phi_{xni} F_{xi}}{\partial \dot{q}_{xn}}, \\ \zeta_{ayn} &= -\frac{1}{2M_{yn}^* \omega_{yn}} \frac{\partial F_{yn}^*}{\partial \dot{q}_{yn}} = -\frac{1}{2M_{yn}^* \omega_{yn}} \frac{\partial \sum_i \phi_{yni} F_{yi}}{\partial \dot{q}_{yn}},\end{aligned}\quad (\text{B.1})$$

where the n th generalized wind forces are $F_{xn}^* = \sum \phi_{xni} F_{xi}$ and $F_{yn}^* = \sum \phi_{yni} F_{yi}$, the generalized mass are $M_{xn}^* = \sum \phi_{xni}^2 m_i$ and $M_{yn}^* = \sum \phi_{yni}^2 m_i$, ω_{xn} and ω_{yn} are the circular natural frequencies, ϕ_{xni} and ϕ_{yni} are the mode shape coefficients.

Assume the j th and k th generalized vibration speeds are independent of each other, i.e.,

$$\frac{\partial \dot{q}_{xk}}{\partial \dot{q}_{xj}} = \frac{\partial \dot{q}_{yk}}{\partial \dot{q}_{yj}} = 0 \quad (k \neq j). \quad (\text{B.2})$$

The following derivatives are consequently obtained:

$$\frac{\partial \dot{s}_i}{\partial \dot{q}_{xn}} = \phi_{xni} \frac{\dot{x}_i}{\dot{s}_i}, \quad \frac{\partial \dot{s}_i}{\partial \dot{q}_{yn}} = \phi_{yni} \frac{\dot{y}_i}{\dot{s}_i}, \quad (\text{B.3})$$

$$\frac{\partial \gamma_i}{\partial \dot{q}_{xn}} = \mp \phi_{xni} \frac{\dot{y}_i}{\dot{s}_i^2}, \quad \frac{\partial \gamma_i}{\partial \dot{q}_{yn}} = \pm \phi_{yni} \frac{\dot{x}_i}{\dot{s}_i^2}, \quad (\text{B.4})$$

$$\frac{\partial U_{Ri}}{\partial \dot{q}_{xn}} = \phi_{xni} \frac{-U_i \cos(\theta_i \mp \gamma_i) + \dot{x}_i}{U_{Ri}}, \quad \frac{\partial U_{Ri}}{\partial \dot{q}_{yn}} = \phi_{yni} \frac{-U_i \sin(\theta_i \mp \gamma_i) + \dot{y}_i}{U_{Ri}}, \quad (\text{B.5})$$

$$\frac{\partial \beta_{Ri}}{\partial \dot{q}_{xn}} = \phi_{xni} \frac{-U_i \sin(\theta_i \mp \gamma_i) + \dot{y}_i}{U_{Ri}^2}, \quad \frac{\partial \beta_{Ri}}{\partial \dot{q}_{yn}} = \phi_{yni} \frac{U_i \cos(\theta_i \mp \gamma_i) - \dot{x}_i}{U_{Ri}^2}, \quad (\text{B.6})$$

$$\begin{aligned}\frac{\partial C_F}{\partial \dot{q}_{xn}} &= \phi_{xni} \frac{\partial C_F}{\partial \text{Re}_{Ri}} \frac{\text{Re}_i}{U_i} \frac{-U_i \cos(\theta_i \mp \gamma_i) + \dot{x}_i}{U_{Ri}} + \phi_{xni} \frac{\partial C_F}{\partial \beta_{Ri}} \frac{-U_i \sin(\theta_i \mp \gamma_i) + \dot{y}_i}{U_{Ri}^2} \\ \frac{\partial C_F}{\partial \dot{q}_{yn}} &= \phi_{yni} \frac{\partial C_F}{\partial \text{Re}_{Ri}} \frac{\text{Re}_i}{U_i} \frac{-U_i \sin(\theta_i \mp \gamma_i) + \dot{y}_i}{U_{Ri}} + \phi_{yni} \frac{\partial C_F}{\partial \beta_{Ri}} \frac{U_i \cos(\theta_i \mp \gamma_i) - \dot{x}_i}{U_{Ri}^2}\end{aligned}, \quad (\text{B.7})$$

where the symbols are specified in Appendix A and Figure 10. Substituting Eqs. (B.3)~(B.7) into Eq. (B.1), the expressions of aerodynamic damping ratios are given by

$$\begin{aligned}\zeta_{axn} &= \frac{\rho}{4\omega_{xn}M_{xn}^*} \sum_i \phi_{xni}^2 A_i U_i \left[C_D (2 \sin \beta_{Ri} \sin \beta_i + \cos \beta_{Ri} \cos \beta_i) + \frac{\partial C_D}{\partial \beta_{Ri}} \sin \beta_{Ri} \cos \beta_i \right. \\ &\quad + \frac{\partial C_D}{\partial \text{Re}_{Ri}} \frac{U_{Ri}}{U_i} \text{Re}_i \sin \beta_{Ri} \sin \beta_i - C_L (2 \cos \beta_{Ri} \sin \beta_i - \sin \beta_{Ri} \cos \beta_i) \\ &\quad \left. - \frac{\partial C_L}{\partial \beta_{Ri}} \cos \beta_{Ri} \cos \beta_i - \frac{\partial C_L}{\partial \text{Re}_{Ri}} \frac{U_{Ri}}{U_i} \text{Re}_i \cos \beta_{Ri} \sin \beta_i \right] \\ \zeta_{ayn} &= \frac{\rho}{4\omega_{yn}M_{yn}^*} \sum_i \phi_{yni}^2 A_i U_i \left[C_D (2 \cos \beta_{Ri} \cos \beta_i + \sin \beta_{Ri} \sin \beta_i) - \frac{\partial C_D}{\partial \beta_{Ri}} \cos \beta_{Ri} \sin \beta_i \right. \\ &\quad + \frac{\partial C_D}{\partial \text{Re}_{Ri}} \frac{U_{Ri}}{U_i} \text{Re}_i \cos \beta_{Ri} \cos \beta_i + C_L (2 \sin \beta_{Ri} \cos \beta_i - \cos \beta_{Ri} \sin \beta_i) \\ &\quad \left. - \frac{\partial C_L}{\partial \beta_{Ri}} \sin \beta_{Ri} \sin \beta_i + \frac{\partial C_L}{\partial \text{Re}_{Ri}} \frac{U_{Ri}}{U_i} \text{Re}_i \sin \beta_{Ri} \cos \beta_i \right]\end{aligned}. \quad (\text{B.8})$$

With considerations that (a) for lattice tower C_L and the derivatives of C_D and C_L are negligible compared to C_D , (b) at subcritical Reynolds region the aerodynamic coefficients nearly keep unchanged with respect to Re , and (c) u_i' and v_i' are small amounts to $\bar{U}_i$, Eq. (B.8) reduces to

$$\begin{aligned}\zeta_{axn} &= \frac{\rho C_D \sum_i \phi_{xni}^2 A_i \bar{U}_i}{4M_{xn}^* \omega_{xn}} (1 + \sin^2 \bar{\beta}) \\ \zeta_{ayn} &= \frac{\rho C_D \sum_i \phi_{yni}^2 A_i \bar{U}_i}{4M_{yn}^* \omega_{yn}} (1 + \cos^2 \bar{\beta})\end{aligned} \quad (\text{B.9})$$

APPENDIX C

As depicted in Figure 20, the orientation vector $\vec{e}_i$ of the i th section of the conductor is

$$\vec{e}_i = (e_{xi}, e_{yi}, e_{zi}), \quad (C.1)$$

where $\vec{e}_i$ is a unit vector. Therefore the relative wind speed vector

$\vec{U}_{Ri} = (U_{Rxi}, U_{Ryi}, U_{Rzi})$ can be easily obtained in the xyz coordinate, as shown in

$$\begin{cases} U_{Rxi} = (\bar{U}_i + u_i') \sin \bar{\beta}_i + v_i' \cos \bar{\beta}_i - \dot{x}_i \\ U_{Ryi} = (\bar{U}_i + u_i') \cos \bar{\beta}_i + v_i' \sin \bar{\beta}_i - \dot{y}_i, \\ U_{Rzi} = w_i' - \dot{z}_i \end{cases} \quad (C.2)$$

where w_i' is the vertical fluctuating component of wind speed, $\dot{z}_i$ is the vertical vibration speed, other symbols are the same as those in Appendix A.

The projective vector $\vec{U}_{Pi}$ of $\vec{U}_{Ri}$ on $\vec{e}_i$, and the normal vector $\vec{U}_{Ni}$ are

$$\begin{aligned} \vec{U}_{Pi} &= (U_{Pxi}, U_{Pyi}, U_{Pzi}) = (\vec{U}_{Ri} \cdot \vec{e}_i) \vec{e}_i \\ \vec{U}_{Ni} &= (U_{Nxi}, U_{Nyi}, U_{Nzi}) = \vec{U}_{Ri} - \vec{U}_{Pi} \end{aligned} \quad (C.3)$$

Referring to ESDU suggestions, the projective wind force $\vec{F}_{Pi}$ and normal wind force $\vec{F}_{Ni}$ acting on an inclined cylinder are given by

$$\begin{aligned} \vec{F}_{Pi} &= (F_{Pxi}, F_{Pyi}, F_{Pzi}) = \frac{1}{2} \rho A_i C_D k \alpha_N \frac{\pi}{180} (2 - \alpha_N \frac{\pi}{180}) |\vec{U}_{Ri}|^2 \frac{\vec{U}_{Pi}}{|\vec{U}_{Pi}|} \\ \vec{F}_{Ni} &= (F_{Nxi}, F_{Nyi}, F_{Nzi}) = \frac{1}{2} \rho A_i C_D |\vec{U}_{Ni}| |\vec{U}_{Ni}| \end{aligned} \quad (C.4)$$

where the drag coefficient C_D is taken as 1.0 and the empirical factor k is taken as 0.045 for rough strand according to ESDU, α_N is the angle between $\vec{U}_{Ri}$ and $\vec{U}_{Ni}$.

Hence, the three-dimensional wind forces on the conductor are obtained:

$$\begin{aligned}
F_{xi} = F_{Pxi} + F_{Nxi} &= \frac{1}{2} \rho A_i C_D \left[k \alpha_N \frac{\pi}{180} (2 - \alpha_N \frac{\pi}{180}) \frac{|\vec{U}_{Ri}|^2}{|\vec{U}_{Pi}|} U_{Pxi} + |\vec{U}_{Ni}| U_{Nxi} \right] \\
F_{yi} = F_{Pyi} + F_{Nyi} &= \frac{1}{2} \rho A_i C_D \left[k \alpha_N \frac{\pi}{180} (2 - \alpha_N \frac{\pi}{180}) \frac{|\vec{U}_{Ri}|^2}{|\vec{U}_{Pi}|} U_{Pyi} + |\vec{U}_{Ni}| U_{Nyi} \right] \\
F_{zi} = F_{Pzi} + F_{Nzi} &= \frac{1}{2} \rho A_i C_D \left[k \alpha_N \frac{\pi}{180} (2 - \alpha_N \frac{\pi}{180}) \frac{|\vec{U}_{Ri}|^2}{|\vec{U}_{Pi}|} U_{Pzi} + |\vec{U}_{Ni}| U_{Nzi} \right]
\end{aligned} \tag{C.5}$$

REFERENCES

- Al-Bremani, F.G.A., Kitipornchai, S., 1992. Nonlinear Analysis of transmission Towers. Engineering Structures. 14, 139-151.
- ASCE, Manual No. 74, 2010. Guidelines for Electrical Transmission Line Structural Loading. American Society of Civil Engineers.
- Bai, H.F., Yi, T.H., Li, H.N., Ren, L., 2012. Multisensors on-site monitoring and characteristic analysis of UHV transmission tower. International Journal of Distributed Sensor Networks. 2012.
- Ballio, G., Meberini, F., Solari, G., 1992. A 60 years old 100 m high steel tower: limit states under wind action. Journal of Wind Engineering and Industrial Aerodynamics. 43, 2089-2100.
- Davenport, A.G., 1979. Gust response factors for transmission line loading. In: Proceedings of the Fifth International Conference on Wind Engineering. Fort Collins, Colorado, USA.
- Deng, H.Z., Si, R.J., Hu, Y. X., Chen, Q., 2010. Wind tunnel test on aeroelastic model of UHV latticed transmission tower. Journal of Tongji University (Natural Science). 38, 673-678.
- Deng, H.Z., Si, R.J., Hu, X.Y., Duan, C.Y., 2013. Wind tunnel study on wind-induced vibration response of a UHV transmission tower-line system. Advances in Structural Engineering. 16, 1175-1185.
- Deng, H.Z., Si, R.J., Duan, C.Y., Hu, X.Y., 2015. Experimental research on aerodynamic coefficient and aerodynamic admittance for transmission tower. Journal of Vibration and Shock. 34, 188-195.
- Deng, H.Z., Zhu, S.Y., Chen, X. M., Wang, Z.M., 2003. Wind tunnel investigation on model of long span transmission line system. Journal of Tongji University (Natural Science). 31, 132-137.
- Duan, C.Y., Deng, H.Z., 2014. Wind tunnel investigation on the aerodynamic damping of transmission tower based on eigensystem realization algorithm. Journal of Vibration and Shock. 33, 131-136.

- ENV1993-1-1:2004, 2005. Eurocode 3: Design of steel structures –Part 1.1. Brussels: European Committee for Standardization.
- ESDU80025, 1980. Mean forces, pressures and flow field velocities for circular cylindrical structures: single cylinder with two-dimensional flow. London: ESDU International.
- Glanville, M.J., Kwok, K.C.S., 1995. Dynamic characteristics and wind induced response of a steel frame tower. *Journal of Wind Engineering and Industrial Aerodynamics*. 54, 133-149.
- Guo, Y., Sun, B.N., Ye, Y., Shen, G.H., Lou, W.J., 2007. Wind tunnel test on aeroelastic model of long span transmission line system. *Journal of Zhejiang University(Engineering Science)* 41, 1482-1486.
- Harikrishna, P., Shanmugasundaram J., Gomothinayagam, S., Lakshmanan, N., 1999. Analytical and experimental studies on the gust response of a 52m tall steel lattice tower under wind loading. *Computers and Structures*. 70, 149-160.
- Holmes, J.D., 1994. Along-wind response of lattice towers: part I—derivation of expressions for gust response factors. *Engineering Structures*. 16, 287-292.
- Holmes, J.D., 1996a. Along-wind response of lattice towers: part II—Aerodynamic damping and deflections. *Engineering Structures*. 18, 483-488.
- Holmes, J.D., 1996b. Along-wind response of lattice towers: part III—effective load distributions. *Engineering Structures*. 18, 489-494.
- Irvine, H.M., 1981. Cable structures. Cambridge: The MIT Press.
- Kaimal, J.C., Wyngaard, J.C., Izumi, Y., 1972. Spectrum characteristics of surface-layer turbulence. *Journal of Royal Meteorology Society*. 98, 563-589.
- Kareem, A., Gurley, K., 1996. Damping in structures: its evaluation and treatment of uncertainty. *Journal of Wind Engineering and Industrial Aerodynamics*, 59, 131-157.
- Li, Z.L., Xiao, Z.Z., Han, F., 2008. Aeroelastic model design and wind tunnel tests of 1000kV Hanjiang long span transmission line system. *Power System Technology*. 32, 1-5.
- Li, Z.L., Ren, K., Xiao, Z.H., Wang, Z.S., Yu, K., 2011. Aeroelastic model design and wind tunnel tests of UHV transmission line system. *Acta Aerodynamica Sinica*. 90, 102-106.
- Liang, S.G., Zou, L.H., Wang, D.H., Cao, H., 2015. Investigation on wind tunnel tests of a full aeroelastic model of electrical transmission tower-line system. *Engineering Structures*. 85, 63-72.
- Lou, W.J., Sun, B.N., Ye, Y., 1999. Across-wind dynamic response of tall latticed towers. *China Civil Engineering Journal*. 32, 67-71.

- Loredo-Souza, A.M., Davenport, A.G., 1998. The effects of high winds on transmission lines. *Journal of Wind Engineering and Industrial Aerodynamics*. 74-76, 987-994.
- Loredo-Souza, A.M., Davenport, A.G., 2001. A novel approach for wind tunnel modelling of transmission lines. *Journal of Wind Engineering and Industrial Aerodynamics*. 89, 1017-1029.
- Loredo-Souza, A.M., Davenport, A.G., 2002. Wind tunnel aeroelastic studies on the behaviour of two parallel cables. *Journal of Wind Engineering and Industrial Aerodynamics*. 90, 407-414.
- Macdonald, J.H.G., Larose, G.L., 2006. A unified approach to aerodynamic damping and drag/lift instabilities, and its application to dry inclined cable galloping. *Journal of Fluids and Structures*, 22, 229-252.
- Mara, T.G., Hong, H.P., 2013. Effect of wind direction on the response and capacity surface of a transmission tower. *Engineering Structures*. 57, 493-501.
- Momomura, Y., Marukawa, H., Okamura, T., 1997. Full-scale measurements of wind-induced vibration of a transmission line system in a mountainous area. *Journal of Wind Engineering and Industrial Aerodynamics*. 72, 241-252.
- Moschas, F., Stiros, S., 2014. High accuracy measurement of deflections of an electricity transmission line tower. *Engineering Structures*. 80, 418-425.
- Ozono, S., Maeda, J., 1992. In-plane dynamic interaction between a tower and conductors at lower frequencies. *Engineering Structures*. 14, 210-216.
- Prud'homme, S., Legeron, F., Laneville, A., Tran, M.K., 2014. Wind forces on single and shielded angle members in lattice structures. *Journal of Wind Engineering and Industrial Aerodynamics*. 124, 20-28.
- Raeesi, A., Cheng, S., Ting, D.S.K., 2013. Aerodynamic damping of an inclined circular cylinder in unsteady flow and its application to the prediction of dry inclined cable galloping. *Journal of Wind Engineering and Industrial Aerodynamics*. 113, 12-28.
- Shan, L., 1993. Overview of EPRI conductor wind loading experiments. In: *Proceedings of the seventh US national conference on wind engineering*. Los Angeles, California.
- Shan, L., 1994. Evaluation of the results of several full-scale conductor wind loading experiments. Final report[R]. Electric Power Research Inst., Palo Alto, CA (United States); Sverdrup Technology, Inc., Haslet, TX (United States).

- Shiotani, M., Iwatani, Y., 1971. Correlations of wind velocities in relation to the gust loadings. In: Proceedings of the Third International Conference on Wind Effects on Buildings and Structures, Tokyo, Japan, 57-67.
- Takeuchi, M., Maeda, J., Ishida, N., 2010. Aerodynamic damping properties of two transmission towers estimated by combining several identification methods. *Journal of Wind Engineering and Industrial Aerodynamics*. 98, 872-880.
- Tieleman, H.W., 1995. Universality of velocity spectra. *Journal of Wind Engineering and Industrial Aerodynamics*. 56, 55-69.
- Xie, Q., Guan, Z., Yan, C.Y., 2011. Wind tunnel test on across-wind vibration of 1000kV UHV transmission tower. *Power System Technology*. 35, 21-26.
- Xie, Q., Li, J.G., Yan, C.Y., Zhou, Y., 2013. Wind tunnel test on wind load transferring mechanism in the 1000kV UHV transmission tower-line system. *Journal of Chinese Electrical Engineering Science*. 33, 109-116.
- Xie, Q., Yang, J., 2013. Wind tunnel test and numerical simulation on transmission tower-line coupling system. *Power System Technology*. 37, 1157-1164.
- Yasui, H., Marukawa, H., Momomura, Y., Ohkuma, T., 1999. Analytical study on wind-induced vibration of power transmission towers. *Journal of Wind Engineering and Industrial Aerodynamics*. 83, 431-441.
- Zhao, G.F., Xie, Q., Liang, S.G., Li, J., 2009. Wind tunnel test on wind-induced nonlinear vibrations of high-voltage transmission tower-line coupling system. *Journal of Tongji University (Natural Science)*. 37, 1157-1164.

Figure Captions

Figure 1. Outline of the full-scale tower line system

Figure 2. Geometry of the tower of interest

Figure 3. Modelling of the tower shaft, (a) flexural rigidity, (b) mass, (c) area projection

Figure 4. Photo of the insulator string and spacer model

Figure 5. Photos of the tower line system model

Figure 6. Measurement points layout

Figure 7. The experimental wind field, (a) mean wind speed profile, (b) turbulence intensity profile

Figure 8. Coordinate definition

Figure 9. Identified aerodynamic damping, (a) in x axis, (b) in y axis

Figure 10. Wind forces acting on the tower

Figure 11. Aerodynamic damping in, (a) x direction , (b) y direction

Figure 12. Mean displacement at the tower tip, (a) without conductors, (b) with conductors

Figure 13 Wind forces, (a) acting on the tower tip, (b) transmitted from the upper conductor

Figure 14 Acceleration Power spectra of the tower without conductors in, (a) x direction, (b) y direction

Figure 15 Acceleration Power spectra of the tower with conductors in, (a) x direction, (b) y direction

Figure 16. Equivalent model in numerical simulations

Figure 17. Illustration of the damping model in numerical simulations

Figure 18 Numerical wind field, (a) mean wind speed profile, (b) turbulence intensity

Figure 19 Numerical wind spectra, (a) along wind spectrum, (b) along wind coherence, (c) across wind spectrum, (d) across wind coherence, (e) vertical wind spectrum, (f) vertical wind coherence

Figure 20. Wind forces acting on the conductor

Figure 21. The influence of aerodynamic damping, (a) for the model without conductors, (b) for the model with conductors

Figure 22. Tension in the upper conductor, (a) mean forces, (b) r.m.s. forces

Figure 23. Spectra of tension in the upper conductor, (a) in x axis, (b) in z axis

Table Captions

Table 1. Scaling ratios for the aeroelastic model

Table 2. Modelling of the shield wire and conductor

Table 3. Eigenvalues of the aeroelastic model and the numerical model

Table 4. R.m.s. value and correlation of accelerations in wind tunnel tests

Table 5. Peak factors of displacements and accelerations in wind tunnel tests

Table 6. R.m.s. value and correlation of accelerations in numerical simulations

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7

Figure 8

Figure 9

Figure 9b

Figure 10

Figure 11

Figure 12

Figure 13

Figure 14

Figure 15

Figure 16

Figure 17

Figure 18a

Figure 18b

Figure 19a

Figure 19b

Figure 19c

Figure 19d

Figure 19e

Figure 19f

Figure 20

Figure 21a

Figure 21b

Figure 22a

(a)

Figure 22b

Figure 23a

Figure 23b

Table 1. Scaling ratios for the aeroelastic model

Parameter	Scaling ratio
Basic scaling	
Length	$\lambda_L = 1/80$
Velocity	$\lambda_U = 1/3$
Density	$\lambda_\rho = 1$
Derived scaling	
Lattice tower	
Area	$\lambda_A = \lambda_L^2 = 1/6400$
Mass	$\lambda_m = \lambda_L^3 \lambda_\rho = 1/512000$
Axial rigidity	$\lambda_{EA} = \lambda_L^2 \lambda_U^2 \lambda_\rho = 1/57600$
Frequency	$\lambda_f = \lambda_U / \lambda_L = 26.67$
Acceleration	$\lambda_{\ddot{y}} = \lambda_U^2 / \lambda_L = 8.89$
Conductor	
Span length	$\lambda_L^* = \gamma_0 \cdot \lambda_L = 1/200$
Diameter	$\lambda_d^* = \lambda_L / \gamma_0 = 1/32$
Sag	$\lambda_s^* = \lambda_L^2 / \lambda_U^2 = 1/711.1$
Mass per unit length	$\lambda_{\bar{m}}^* = \lambda_L^2 \cdot \lambda_{\rho_f} / \gamma_0 = 1/2560$
Frequency	$\lambda_f^* = \lambda_U / \lambda_L = 26.67$

Table 2. Modelling of the shield wire and conductor

	Parameter	Scaled cable	Aeroelastic model
Shield wire	Axial rigidity EA (N)	111	1617
	Mass per unit length $\bar{m}$ (g/m)	0.28	0.99
	Diameter d (mm)	0.5	1
	Sag s_0 (mm)	26.2	20
Conductor	Axial rigidity EA (N)	343	1617
	Mass per unit length $\bar{m}$ (g/m)	0.94	0.99
	Diameter d (mm)	1.13	1
	Sag s_0 (mm)	40.6	40

Table 3. Eigenvalues of the aeroelastic model and the numerical model

		Without conductors			With conductors		
		X-axis	Y-axis	Torsional	X-axis	Y-axis	Torsional
Aeroelastic model	f (Hz)	32.2	33.0	-	33.4	34.7	-
	ζ (%)	1.4	1.4	-	4.6	3.3	-
Numerical model	f (Hz)	33.7	34.1	67.8	33.1	33.8	65.0
	ζ (%)	1.47	1.52		4.37	3.91	

Table 4. R.m.s. value and correlation of accelerations in wind tunnel tests

	Wind incidence, $\bar{\beta}$ ($^{\circ}$)	R.m.s. value, σ_{ax} (m/s ²), of a_x at $z=0.9$ m	R.m.s. value, σ_{ay} (m/s ²), of a_y at $z=0.9$ m	Correlation coefficient, ρ_{xy} , of a_x and a_y
Without conductors	0	0.869	1.343	0.023
	15	0.806	1.361	0.064
	30	0.777	1.296	0.088
	45	0.750	1.230	0.109
	60	0.762	1.194	0.141
	75	0.753	1.142	0.118
	90	0.731	1.065	0.108
With conductors	75	1.265	0.727	0.168
	90	1.234	0.666	0.236

Table 5. Peak factors of displacements and accelerations in wind tunnel tests

Trial	Wind incidence, $\bar{\beta}$ ($^{\circ}$)	Peak factor, g_x , of x at $z=1.263$ m	Peak factor, g_y , of y at $z=1.263$ m	Peak factor, g_a , of a_x at $z=0.9$ m	Peak factor, g_a , of a_y at $z=0.9$ m
Test results, without conductors	0	5.258	4.461	3.973	4.418
	15	4.547	4.009	3.860	4.360
	30	5.439	4.515	4.948	4.527
	45	4.739	4.107	4.086	4.320
	60	4.351	4.026	4.423	4.290
	75	4.654	4.310	4.015	5.352
	90	4.373	4.286	4.529	5.012
Test results, with conductors	75	4.391	4.052	4.252	4.432
	90	3.986	4.643	5.122	4.375

Table 6. R.m.s. value and correlation of accelerations in numerical simulations

Trials	Wind incidence, $\bar{\beta}$ ($^{\circ}$)	R.m.s. value, σ_{ax} (m/s ²), of	R.m.s. value, σ_{ay} (m/s ²), of	Correlation coefficient, ρ_{xy} , of a_x and a_y
		a_x at $z=0.9$ m	a_y at $z=0.9$ m	
Numerical simulations, without conductors	0	1.654	1.927	-0.041
	15	1.662	2.044	0.031
	30	1.641	2.031	0.111
	45	1.605	1.918	0.158
	60	1.602	1.802	0.174
	75	1.579	1.645	0.161
	90	1.511	1.453	0.125
Numerical simulations, with conductors	0	0.897	1.557	-0.054
	15	1.006	1.608	0.032
	30	1.339	1.526	0.038
	45	1.633	1.384	0.098
	60	1.825	1.243	0.130
	75	1.797	1.103	0.143
	90	1.919	0.959	0.172