Perturbation Methods for the Reliability Analysis of Wind-Turbine Blade Failure due to Flutter

Pariya Pourazarm¹, Luca Caracoglia², Matthew Lackner¹, Yahya Modarres-Sadeghi¹

¹Department of Mechanical and Industrial Engineering

University of Massachusetts, Amherst, MA 01003, USA

²Department of Civil and Environmental Engineering

Northeastern University, Boston, MA 02115, USA

Abstract

A new reliability framework is proposed to study the influence of uncertainty in various system parameters on the onset of coupled-mode flutter for large wind turbine blades. This study is the first comprehensive example of application of reliability analysis to wind turbine blade instability, affected by various sources of uncertainty. The randomness in flow forces, through a random lift coefficient, and in the structural properties is investigated. The probability of flutter is estimated using a recursive search procedure and four reliability models: First Order Reliability Method (FORM), First Order Reliability Method including the effect of unsteadiness in the aeroelastic loads (FORM-C), Second Order Reliability Method (SORM) and Weighted Average Reliability Method (WARM). Flutter probabilities are compared against the results of Monte Carlo simulations. All methods provide accurate probability approximation in regions close to the mean

value of the critical flutter speed or flutter frequency. Among the four methods, the WARM provides the closest approximation to the Monte Carlo simulations in all regions of the variables. It is shown that failure probabilities can be found through an iterative procedure, which could be converted to a simplified formula in future standards for the design of wind turbine blades.

1. Introduction

For more than a decade, wind power has been the world's fastest growing source of energy. Offshore wind energy has been growing rapidly due to its capacity for utilizing much larger turbines and thus higher power generation compared to onshore. Long wind turbine blades are susceptible to experiencing flow-induced dynamic instabilities. Among the various instabilities, coupled-mode flutter is a major problem (e.g., Hansen, 2015, Lobitz, 2004, Riziotis et al., 2004, Riziotis et al., 2008) and a possible cause of destructive failures.

Wind turbine blade structural properties may vary among blades of the same design. It has been suggested to treat these variations as random variables (Ernst and Sueme, 2013). The presence of stochastic parameters in systems experiencing flow-induced instabilities can influence the onset of dynamic instability (Caracoglia, 2013, Hracov et al., 2005, Náprstek, 1996, Náprstek, 2001, Pourazarm et al., 2015a). The aim of the present study is to introduce a new reliability framework to evaluate flutter probability. Four different reliability methods are considered, including both existing approaches and a novel technique. In the context of long-span bridge aeroelasticity, these methods have been widely employed to investigate the critical speed probability for the deck flutter, which leads to structural collapse (e.g., (Cheng et al., 2005, Dragomirescu et al., 2003, Ge et al., 2000, Ge and Xiang, 2005, Prenninger et al., 1990)). A recent review of several of these methods, along with a critical analysis of advantages and limitations, is

discussed in Canor et al. (2015). In the case of wind turbine structures, reliability methods have been considered in recent years mostly to investigate structural integrity near the failure of the tower at high wind speeds and when the blades are in a parked configuration (e.g., (Taflanidis et al., 2013)). Nevertheless, these methods often rely on simplified models to simulate the blade aeroelasticity, and moreover, very limited attention has been paid to the complex fluid-structure interaction of the blades in the case of serviceability and normal operational conditions close to the rated speed.

A number of numerical models can be used to predict flutter onset for wind turbine blades in a deterministic setting. A comprehensive review can be found in Hansen (2002), Hansen (2004) and Lobitz, (2004). Lobitz (2005) used a flutter prediction tool to investigate the effect of torsional and flapwise natural frequencies on the critical flutter speed. Hansen (2007) predicted the onset of flutter for the NREL 5MW blade (Jonkman et al., 2009), assuming zero incoming flow and zero blade pitch. Resor et al. (2012) studied the influence of mass on the onset of flutter stability of the SNL 100-00 blade. Owens et al. (2013) estimated the onset of flutter for the WindPACT 1.5MW and SNL 100-00 blades. Pourazarm et al. (2015b) showed that high flexibility in the torsional direction drastically influences the critical wind speed for instability in MW size blades.

The reliability of wind turbine blades is affected by the uncertainties in composite material properties. A deterministic analysis of wind turbine blade dynamics is not sufficient, as variability in structural properties (i.e., uncertainty) can influence the response and performance of the blade. Griffith et al. (2006) quantified the variability in the natural frequencies and damping ratios of two identical CX-100 and TX-100 wind turbine blades. Resor and Paquette (2011) found a 14% discrepancy between the estimated critical flutter speeds depending on the modeling assumptions of a wind turbine blade, represented as an equivalent beam. Ernst and Sueme (2013) assumed

various sources of uncertainty, as a result of the production process: flapwise stiffness, edgewise stiffness and the mass density of the NREL 5MW blade were randomly distributed around their mean values with coefficient of variation of 0.1. They found the deviation of natural frequencies to be around 10% from the reported baseline values. Recently, stochastic flutter of a NREL 5MW blade, subjected to random flow forces and structural properties, has been studied by Pourazarm et al. (2015a). Monte Carlo simulations were employed to approximate the failure probability. It was observed that the randomness in flow forces reduces the flutter safety margin from 70% to 40%, and randomness in the torsional natural frequencies results in no safety margin.

In the present paper, a new reliability framework is introduced to study the influence of randomness in the blade parameters on the blade flutter onset. Monte Carlo simulation results, derived from the previous work (Pourazarm et al., 2015a), usually return an “exact” flutter probability but they require very large computing time. As a practical design alternative, four alternative reliability methods are proposed and examined herein to solve this problem. These methods provide approximate flutter probability estimation which could result in a computationally efficient procedure or formula to be included in wind-turbine design standards. The NREL 5 MW blade (Jonkman et al., 2009) is chosen as the model blade in this investigation (length of 61.5 meters, total mass of 17,740 kg and rated rotor speed of 12.1 rpm). Randomness in the manufacturing process is accounted for by simulating uncertainty in both random structural and aerodynamic properties. Due to coupled-mode flutter instability, the influence of any structural randomness can be studied by focusing on the randomness in the structural natural modes, which are contributing to the blade’s flutter mode.

2. Equations of motion

The equations of motion used in the present study are the same as the ones used previously (Pourazarm et al., 2015a). These are based on the equations originally derived by Hodges and Dowell (1974). Since the focus of the current study is on the coupled-mode flutter instability, only the deflections in the flapwise and torsional directions are considered. Theodorsen's theory (Theodorsen, 1935) is used to model the unsteady aerodynamic loading acting on each blade element. The dimensionless equations in the torsional and flapwise directions for the generic cross-section located at a distance x from the root are:

$$\begin{aligned}
 & -\left\{\gamma\bar{J} + \kappa_m^2 \Omega^2 \int_{\xi}^1 \bar{A}\hat{\xi}d\hat{\xi}\right\}\varphi'' - \left\{\gamma\bar{J}' + \Omega^2 \left[-\bar{A}\xi\kappa_m^2 + 2\kappa_m\kappa_m' \int_{\xi}^1 \bar{A}\hat{\xi}d\hat{\xi}\right]\right\}\varphi' \\
 & + \Omega^2 \bar{A}(\kappa_{m2}^2 - \kappa_{m1}^2)\varphi + \bar{A}\kappa_m^2\ddot{\varphi} - \Omega^2 \varepsilon\eta'' \int_{\xi}^1 \bar{A}\hat{\xi}d\hat{\xi} + \Omega^2 \varepsilon\bar{A}\eta'\xi + \varepsilon\bar{A}\ddot{\eta} \\
 & = -C_{L\alpha} \frac{\rho_{\infty}}{\rho} \left\{ 2\nu\bar{b}^2 (\xi\Omega) \left(\frac{1}{2} + a\right) C(k)\dot{\eta} + \nu\Lambda\bar{b}^3 (\xi\Omega) \left(\frac{1}{2} - a\right) [1 - (1+2a)C(k)]\dot{\varphi} \right. \\
 & \quad \left. - \nu\Lambda\bar{b}^2 (\xi\Omega)^2 (1+2a)C(k)\varphi + \bar{b}^3 a\ddot{\eta} + \nu\Lambda\bar{b}^4 \left(a^2 + \frac{1}{8}\right)\ddot{\varphi} \right\}, \tag{1}
 \end{aligned}$$

$$\begin{aligned}
 & \bar{I}''\eta'' + 2\bar{I}'\eta''' + \bar{I}\eta^{(4)} + \Omega^2 (\bar{A}\xi\eta' - \eta'' \int_{\xi}^1 \bar{A}\hat{\xi}d\hat{\xi}) + \bar{A}\ddot{\eta} + \Omega^2 (\bar{A}\varepsilon'\xi)\varphi + \Omega^2 (\bar{A}\varepsilon\xi - 2\varepsilon' \int_{\xi}^1 \bar{A}\hat{\xi}d\hat{\xi})\varphi' \\
 & - \varepsilon\Omega^2 \varphi'' \int_{\xi}^1 \bar{A}\hat{\xi}d\hat{\xi} + \varepsilon\bar{A}\ddot{\varphi} \\
 & = -C_{L\alpha} \frac{\rho_{\infty}}{\rho} \left\{ 2\frac{\nu}{\Lambda}\bar{b} (\xi\Omega)C(k)\dot{\eta} - \nu\bar{b}^2 (\xi\Omega)[1 + (1-2a)C(k)]\dot{\varphi} - 2\nu\bar{b} (\xi\Omega)^2 C(k)\varphi \right. \\
 & \quad \left. + \frac{\nu}{\Lambda}\bar{b}^2 \ddot{\eta} + \nu\bar{b}^3 a\ddot{\varphi} \right\}, \tag{2}
 \end{aligned}$$

in which:

$$\begin{aligned}
 \eta &= \frac{w}{L}, \xi = \frac{x}{L}, \Omega = \frac{\omega}{\beta}, \gamma = \frac{GJ_0}{EI_0}, \bar{A} = \frac{A}{A_0}, \bar{J} = \frac{J}{J_0}, \bar{I} = \frac{I}{I_0}, \kappa_m = \frac{K_m}{L}, \\
 \kappa_{m1} &= \frac{K_{m1}}{L}, \kappa_{m2} = \frac{K_{m2}}{L}, \varepsilon = \frac{e}{L}, \bar{b} = \frac{b}{b_0}, \Lambda = \frac{b_0}{L}, \nu = \frac{b_0^3}{A_0 L}, \tag{3}
 \end{aligned}$$

where, “dot” denotes the derivative with respect to dimensionless time and “prime” denotes the

derivative with respect to ξ ; $\bar{L}$ and $\bar{M}$ are the resultant dimensionless lift force and pitching moment at a generic cross-section of the blade due to the aeroelastic interactions; A is the cross-sectional area; L the length of the blade; ρ the material density; G is the shear modulus of elasticity; J is the torsional stiffness constant; K_m is the polar radius of gyration of the cross-section about the elastic axis; e is the distance between the mass (and area) centroid and the elastic axis (mass offset); E is the Young's modulus of elasticity; I is the bending moment of inertia; ω is the angular rotor speed. The quantities A_0, J_0, I_0 are the torsional stiffness constant and the bending moment of inertia at the root of the blade; Ω is the dimensionless angular velocity and $\beta = \sqrt{EI_0 / \rho A_0 L^4}$, with ρA_0 being the blade cross sectional mass at the root; ρ_∞ is the density of air; $C_{L\alpha}$ is the slope of the lift coefficient curve (which may vary depending on the blade profile at each cross section) versus the angle of attack in the attached flow region; $C(k)$ is the Theodorsen complex circulation function, which depends on the reduced frequency $k = (2\pi f_f b) / u$, in which f_f is the airfoil frequency of oscillation, $u = x\omega$ is the rotational velocity at each cross-section at a distance x from the root and b is the half chord length; b_0 is the half-chord at the root of the blade; and $(1/2 + a)$ is the distance between the elastic axis and the aerodynamic center, divided by b . The derivation of these equations is discussed in more details in (Pourazarm et al., 2015a). The coupled partial differential equations of motion are solved using the Galerkin method as discussed in more detail in (Pourazarm et al., 2015b).

3. Expansion (or perturbation) methods for reliability analysis of wind turbine blades

In (Pourazarm et al., 2015a) the flutter probability was estimated using Eqs. (1-2) by Monte Carlo method to find the variability in flutter onset due to randomness in structural or aerodynamic parameters. As outlined in Section 1, this study examines an alternative reliability framework to

approximate the flutter probability. Perturbation methods (Nayfeh, 2008) are usually efficient (Canor et al., 2015), at least as a first approximation, to investigate long-span bridge flutter under the influence of various random parameters. It is therefore valuable to investigate the applicability of these methods to the current problem.

The flutter point of instability is defined as the point where one of the structural modes obtains a negative real part (negative damping). To find the variability of the flutter onset, the variability of eigenvalues should be studied. The perturbation methods are based on the first order or second order expansion of the real and imaginary parts of the eigenvalue (and eigenmode) that obtains a negative damping (shown in red in Figure 1). Using this approximation, the distribution of damping (real part of eigenvalue $\Re[\lambda_i]$) is first found at each rotor speed, Ω (with $\Omega_{flutter}$ denoting the random variable rotor speed at flutter or flutter speed). Subsequently, flutter probability “failure” is calculated as the complementary probability of non-failure (Canor et al., 2015):

$$\Pr_{flutter} = 1 - \Pr[\Re[\lambda_i] < 0 | \Omega_{flutter} = \Omega] = 1 - F_{\lambda_i, \Re}(0 | \Omega_{flutter} = \Omega). \quad (4)$$

In Eq. (4), the function $F_{\lambda_i, \Re}$ is a conditional cumulative density function (CDF). The process is then repeated for other rotor speeds and the probability of failure is obtained for each case. Finally using all the data, the probability of flutter due to the random parameter is plotted with respect to the rotor speed.

Different expansion (or perturbation) methods are examined to estimate wind-turbine blade flutter by Eq. (4). Each method is described in one of the following sections and used to study the influence of randomness in both the flow forces and the torsional natural frequencies on flutter.

4. First Order Reliability Method (FORM)

Equation (4) requires estimating the probability that the real part of the flutter eigenvalue becomes positive as a function of the rotor speed Ω . The generic unstable branch eigenvalue is denoted as λ_i in the following treatment. The quantity λ_i (complex) depends on a generic random input parameter g (which will be later specified). The functional relationship between λ_i and g is usually not known. If a reference “design point” is selected through the variable g_0 (approximately the mean value of g), an approximate relationship may be established by expanding in Taylor series, truncated at the first order of the derivative, the eigenvalue λ_i in terms of g . This approach is widely used in reliability analysis of engineering structures and it is often referred to as the First Order Reliability Method (FORM) (Der Kiureghian, 2005, Haldar and Mahadevan, 2000). In the specific case under investigation, eigenvalues at flutter are nonlinear functions of structural and aerodynamic properties such as modal frequencies or lift force aerodynamic parameters.

Using the first order approximation, the unstable-branch eigenvalue λ_i (or eigenvalues) can be linearized about g_0 as:

$$\lambda_i(g) \approx \lambda_i(g_0) + \left. \frac{d\lambda_i}{dg} \right|_{g_0} (g - g_0). \quad (5)$$

If g_0 is the mean value of the random variable g ($E[g] = g_0$), then the expected value (with $E[]$ denoting the expectation operator) and the variance (var) of the linearized $\lambda_i(g)$ are:

$$E[\lambda_i(g)] = \lambda_i(g_0), \quad (6)$$

and

$$\text{var}[\lambda_i(g)] \approx \left(\left. \frac{d\lambda_i}{dg} \right|_{g_0} \right)^2 E[(g - g_0)^2] = \left(\left. \frac{d\lambda_i}{dg} \right|_{g_0} \right)^2 \text{var}[g]. \quad (7)$$

Using Eqs. (6) and (7), the mean value and variance of eigenvalues are found at each rotor speed Ω (or flow velocity). Assuming that the eigenvalues have the same distribution as the random parameter, the distribution of the frequency (imaginary part of λ_i) and damping of each mode (real part of λ_i , $\Re[\lambda_i]$) can be obtained. The first two statistical moments of $\Re[\lambda_i]$ are found by using the real parts of Eqs. (6) and (7) (or $E[\Re[\lambda_i(g)]] = \Re[\lambda_i(g_0)]$ $\text{var}[\Re[\lambda_i(g)]] = \left\{ \Re \left[d\lambda_i / dg \big|_{g_0} \right] \right\}^2 \text{var}[g]$), i.e., the variations of damping evaluated at each rotor speed Ω . The probability of failure in Eq. (4) is estimated by using the same cumulative density function as g . The process is repeated at each velocity Ω .

In Eq. (7) the derivative of the eigenvalues with respect to the generic parameter g can be found either numerically or analytically. The procedure to determine the first derivative of the eigenvalue analytically is based on a series of studies of eigenvalue sensitivity in engineering mechanics and engineering modal analysis applications (Adhikari and Friswell, 2001, Adhikari and Friswell, 2004, Fox and Kapoor, 1968). After applying the Galerkin method (Pourazarm et al., 2015b), the equations of motion (1) and (2) can be rewritten in the following general matrix form:

$$\mathbf{B}\dot{\mathbf{X}} + \mathbf{D}\mathbf{X} = \mathbf{0}, \quad (8)$$

The state vector $\mathbf{X}$ collects generalized coordinates of the flapwise and torsional modes, and their first derivatives with respect to time $\dot{\mathbf{X}} = d\mathbf{X}/dt$. If $\mathbf{X} = \boldsymbol{\psi}_i e^{-\lambda_i t}$ is postulated, then the equation can be written as a function of the generic eigenvalue, λ_i , and the corresponding right eigenvector, $\boldsymbol{\psi}_i$, as:

$$-\mathbf{B}\lambda_i \boldsymbol{\psi}_i + \mathbf{D}\boldsymbol{\psi}_i = \mathbf{0}. \quad (9)$$

After defining $\mathbf{A} = \mathbf{B}^{-1}\mathbf{D}$, Eq. (9) can be rewritten as:

$$(\mathbf{A} - \lambda_i \mathbf{I})\boldsymbol{\psi}_i = \mathbf{0}, \quad (10)$$

where λ_i and $\boldsymbol{\psi}_i$ are the eigenvalue and right eigenvector of the matrix $\mathbf{A}$, respectively, and $\mathbf{I}$ is the identity matrix. The left eigenvector of matrix $\mathbf{A}$, $\boldsymbol{\chi}_i$, is defined as:

$$(\mathbf{A}^T - \lambda_i \mathbf{I}) \boldsymbol{\chi}_i = \mathbf{0}, \quad (11)$$

where, $\mathbf{A}^T$ is the transpose of matrix $\mathbf{A}$. Taking the transpose of Eq. (11) results in:

$$\boldsymbol{\chi}_i^T (\mathbf{A} - \lambda_i \mathbf{I}) = \mathbf{0}, \quad (12)$$

where, $\boldsymbol{\chi}_i^T$ is the transpose of $\boldsymbol{\chi}_i$. Taking the derivative of Eq. (12) with respect to g leads to:

$$\left[\frac{d\mathbf{A}}{dg} - \frac{d(\lambda_i \mathbf{I})}{dg} \right] \boldsymbol{\psi}_i + (\mathbf{A} - \lambda_i \mathbf{I}) \frac{d\boldsymbol{\psi}_i}{dg} = \mathbf{0}. \quad (13)$$

After pre-multiplication of Eq. (13) by $\boldsymbol{\chi}_i^T$, the following equation is found:

$$\boldsymbol{\chi}_i^T \left[\frac{d\mathbf{A}}{dg} - \frac{d(\lambda_i \mathbf{I})}{dg} \right] \boldsymbol{\psi}_i + \boldsymbol{\chi}_i^T (\mathbf{A} - \lambda_i \mathbf{I}) \frac{d\boldsymbol{\psi}_i}{dg} = 0. \quad (14)$$

Owing to the vanishing of Eq.(12), the second part of Eq. (14) is equal to zero, which after some manipulations leads to the following scalar equation:

$$\frac{d\lambda_i}{dg} = \frac{\boldsymbol{\chi}_i^T \left(\frac{d\mathbf{A}}{dg} \right) \boldsymbol{\psi}_i}{\boldsymbol{\chi}_i^T \boldsymbol{\psi}_i}. \quad (15)$$

In Eq. (15), the derivative of the matrix $\mathbf{A} = \mathbf{B}^{-1} \mathbf{D}$ is found as (Petersen and Pedersen, 2008)

$$\frac{d\mathbf{A}}{dg} = \frac{d(\mathbf{B}^{-1} \mathbf{D})}{dg} = \left[\frac{d(\mathbf{B}^{-1})}{dg} \right] \mathbf{D} + \mathbf{B}^{-1} \frac{d\mathbf{D}}{dg} = \left[\mathbf{B}^{-1} \frac{d(\mathbf{B})}{dg} \mathbf{B}^{-1} \right] \mathbf{D} + \mathbf{B}^{-1} \frac{d\mathbf{D}}{dg}. \quad (16)$$

The derivative of the eigenvalue is a complex function of the right and the left eigenvectors and the derivative of the matrix $\mathbf{A}$. The method assumes that the derivative of $\mathbf{A}$ with respect to g can be found in closed form. A numerical procedure is implemented to inspect the real part of the

unstable branch eigenvalue at each Ω and calculate the flutter probability using Eqs. (8), (9) and (15) after inserting Eq. (16) using real values $\Re[\lambda_i(g_0)]$ and $\text{var}[\Re[\lambda_i(g)]]$.

Since the system is a self-excited Fluid-Structure Interaction system, in which all the system parameters are functions of the system's response, we have only considered system uncertainties. First, the cross-sectional lift coefficients are assumed to be stochastic parameters as originally proposed and examined in a previous study (Pourazarm et al., 2015a). In this case, the random parameter is $g=C_{L\alpha}$. Two random distributions are used to describe $C_{L\alpha}$: the normal distribution and the uniform distribution. These two distributions are chosen to replicate two extreme cases, i.e., the normal distribution case corresponds to a scenario in which the fidelity in the experimental results of $C_{L\alpha}$ or in the local geometry is very high, and the uniform case corresponds to a low fidelity or limited information on the experimental $C_{L\alpha}$. For each distribution, two values are considered for the coefficient of variation (cov): 0.1 and 0.2. In the Monte Carlo simulations, used in this section and in the following ones, the sample population is based on 10^4 realizations of the random variables. The geometry and properties of the NREL 5MW wind turbine blade are described in a previous study (Pourazarm et al., 2015a) and are not reported in this paper. We have chosen the cov values such that they have physical significance for the system that we study. A cov of 0.1 results in 10% deviation in the natural frequencies. For example, in the NREL 5 MW blade that is considered here, the original torsional natural frequency, is 5.4 Hz, and cov of 0.1 means that the natural frequency could be anywhere from 3.7 Hz to 7.1 Hz (99.8% confidence). This is already a very wide range, much larger than the uncertainty that is normally observed in the design of wind turbine blades. To test the reliability of the method, we have also taken the analysis to an extreme case of $cov=0.2$, which implies that the natural frequency could be anywhere from 2 Hz to 8.7Hz (99.8% confidence).

In the previous study (Pourazarm et al., 2015a), the randomness in the lift coefficient slope was modeled through 6 uncorrelated random sets corresponding to 6 different airfoils located along

the span of the blade. This choice was dictated by the need for replicating the effects of manufacturing errors. In the current study, for simplicity and to enable the use of Eq. (16), the randomness in the lift coefficient is modeled by a single normalized scalar random variable that is simultaneously applied to all the 6 random sets. Figure 2 shows the CDFs of the normalized critical angular velocity assuming 6 uncorrelated (Pourazarm et al., 2015a) and fully correlated random sets with $cov=0.2$. These distributions are evaluated by Monte Carlo sampling. Both assumptions result in almost identical probability distributions confirming that the current assumption causes no loss of generality and that the approach based on Eq. (16) can be employed to examine flutter probability.

Figure 3 shows the CDFs of the normalized angular velocity Ω^* , estimated using the FORM for uniform and normal distributions as well as Monte Carlo simulations. The normalized variable $\Omega^* = \Omega_{flutter} / \Omega_{flutter,0}$ is defined as the ratio between the rotor angular velocity in the presence of uncertainty ($\Omega_{flutter}$) and the reference solution ($\Omega_{flutter,0}$), where the reference solution is the mean flutter angular velocity found using the deterministic method. It can be seen that for cases with the lower cov value, in general, the FORM provides a very close approximation to Monte Carlo simulations. The estimations at the upper and lower tails become less accurate for the higher cov value.

To investigate the effect of structural imperfection or material uncertainties, the most critical structural parameter, the torsional natural frequency, is considered to be a random parameter (Pourazarm et al., 2015a). The random parameter in this case is chosen to be the torsional rigidity (GJ), and therefore $g = GJ$. Both normal and uniform distributions are considered with either $cov = 0.1$ or $cov = 0.2$. Figure 4 shows the CDFs of the normalized critical flutter angular velocity, Ω^* , estimated using Monte Carlo simulations and FORM for uniformly and normally distributed GJ .

It must be noted that, in this second case and in all subsequent simulations involving a random GJ , the variability in the input is established by pre-selecting the cov of the fundamental torsional mode angular frequency, labeled as ω_T in the figure. Consequently, the standard deviation (and cov) of the random variable GJ is calibrated in each simulation to match the desired value of $cov(\omega_T)$. The inspection of Figure 4 reveals that the CDFs show a wider distribution compared to the previous case, suggesting that the torsional natural frequency has a larger influence on the flutter onset. Moreover, estimations of the flutter probability using FORM in this case are not adequate since several discrepancies are noted in the figure in comparison with the “exact” Monte Carlo simulations. The performance of the FORM progressively deteriorates for higher cov values, indicating the need for a better expansion method.

5. First Order Reliability Method including the effect of $C(k)$ variation (FORM-C)

In the expansion method discussed in the previous section, it was implicitly assumed that the Theodorsen function, $C(k)$, is independent from the input random variable (C_{La} and GJ). When structural or aerodynamic parameters are altered, however, flutter characteristics are altered as well, which results in a change in the Theodorsen function. In this section, a modified 1st order expansion method is derived as an attempt toward improving the estimation of the flutter distribution function and reducing some of the discrepancies observed in Figure 4. In the modified method, the variation of $C(k)$ with respect to the random parameter is directly taken into account. The derivative of the eigenvalue, λ_i , considering the variation of $C(k)$, is defined in terms of the total or material derivative operator with respect to a generic variable, g , denoted as $D()/Dg$, as follows:

$$\frac{D\lambda_i}{Dg} = \frac{\partial\lambda_i}{\partial g} + \frac{\partial\lambda_i}{\partial[C(k)]} \frac{d[C(k)]}{dk} \frac{dk}{dg} . \quad (17)$$

In Equation (17), partial derivatives are used to highlight the functional relationship between λ_i and $C(k)$. It is observed that the reduced frequency, k , is in turn a function of g through the imaginary part of the eigenvalue, $\Im[\lambda_i]$, i.e., the frequency of oscillation at flutter. This term can therefore be expanded using the derivative dk/dg and thus Eq. (17) is rewritten as:

$$\frac{D\lambda_i}{Dg} = \frac{\partial\lambda_i}{\partial g} + \frac{\partial\lambda_i}{\partial[C(k)]} \frac{d[C(k)]}{dk} \frac{d[\Im(\lambda_i)]}{dg} \frac{\bar{b}}{\xi\Omega}. \quad (18)$$

The first-order expansion of the real part of the eigenvalue is then found as:

$$\Re\left[\frac{D\lambda_i}{Dg}\right] = \Re\left[\frac{\partial\lambda_i}{\partial g}\right] + \Re\left[\frac{\partial\lambda_i}{\partial[C(k)]} \frac{d[C(k)]}{dk}\right] \Im\left[\frac{d\lambda_i}{dg}\right] \frac{\bar{b}}{\xi\Omega}. \quad (19)$$

The new derivative of the eigenvalue has two terms: the first term is equal to Eq. (15) and excludes the effect of $C(k)$ variation, and the second term reflects the indirect effect of $C(k)$ variation on the eigenvalues.

In Eq. (18), the term $d[C(k)]/dk$ is needed. The Theoderson function, $C(k)$, is defined as (Theodorsen, 1935):

$$C(k) = \frac{H_1^{(2)}(k)}{iH_0^{(2)}(k) + H_1^{(2)}(k)}, \quad (20)$$

where, $H_v^{(2)}(k)$ is the Hankel function of the second kind of order v ($v=1$ or $v=0$) and i is the imaginary unit. The function $H_v^{(2)}(k) = J_v(k) - iY_v(k)$ depends on J_v and Y_v , Bessel functions of the first and second kind of order v , respectively. The derivative $d[C(k)]/dk$ is therefore found analytically as:

$$\frac{d[C(k)]}{dk} = i \frac{\frac{d[H_1^{(2)}(k)]}{dk} H_0^{(2)}(k) - H_1^{(2)}(k) \frac{d[H_0^{(2)}(k)]}{dk}}{[iH_0^{(2)}(k) + H_1^{(2)}(k)]^2}. \quad (21)$$

Equation (21) can be expanded using the properties of the Bessel functions (Abramowitz and Stegun, 1964) to find:

$$\frac{d[H_1^{(2)}(k)]}{dk} = \frac{J_0(k) - J_2(k)}{2} - i \frac{Y_0(k) - Y_2(k)}{2}, \quad (22)$$

and

$$\frac{d[H_0^{(2)}(k)]}{dk} = -J_1(k) + iY_1(k). \quad (23)$$

Then, the partial derivative of the eigenvalue with respect to $C(k)$ in Eq. (18) can be determined in a manner similar to Eq. (15) as:

$$\frac{\partial \lambda_i}{\partial [C(k)]} = \frac{\chi_i^T \frac{\partial \mathbf{A}}{\partial [C(k)]} \Psi_i}{\chi_i^T \Psi_i}, \quad (24)$$

where (as in Eq. (16)),

$$\frac{d\mathbf{A}}{d[C(k)]} = \left[\mathbf{B}^{-1} \frac{d(\mathbf{B})}{d[C(k)]} \mathbf{B}^{-1} \right] \mathbf{D} + \mathbf{B}^{-1} \frac{d\mathbf{D}}{d[C(k)]}. \quad (25)$$

Figure 5 shows the CDFs of normalized angular velocity, Ω^* , estimated using 1st order expansion method with random flow forces (random $C_{L\alpha}$) with and without the $C(k)$ effect as well as the results based on the Monte Carlo simulations. It can be seen that including the effect of $C(k)$ provides some improvements. For example, in the case of a uniformly distributed $C_{L\alpha}$ with $cov = 0.2$ the approximate CDF better reconstructs the “exact” function even around $\Omega^* = 0.9$, close to the lower values of the flutter speed distribution. However, the use of a more accurate first-order expansion is still not sufficient, which is believed to be related to the fact that the lift coefficient has a moderate influence on the modal natural frequencies and thus k . In contrast, structural properties such as the mass, and the flapwise and torsional stiffness values tend to have a greater

effect on the natural frequencies (and k) compared to the added mass and stiffness, which are controlled by $C_{L\alpha}$.

Figure 6 shows a plot similar to Figure 5 when the uncertainty is related to the first-mode torsional frequency ω_T (and therefore the torsional rigidity GJ). The method provides a much better estimation of the CDF function by more closely replicating the Monte Carlo results, in comparison with the 1st order solution, which excluded the $C(k)$ effect. This improvement is more significant for large angular velocities, Ω^* (above the design point $\Omega^*=1$), while for low Ω^* the representation of flutter probability is still unacceptable, for example, around $\Omega^*=0.7$ in the case of a uniform distribution of GJ with $\text{cov}(\omega_T)=0.2$ (bottom right panel in Figure 5).

6. Second Order Reliability Method (SORM)

As discussed in Sections 4 and 5, the 1st order expansion method provides an acceptable representation of the flutter probability, except in the upper and lower tails of the CDFs, where it exhibits non-negligible differences and inaccuracies. The Second Order Reliability Method (SORM), which uses the 2nd order expansion of eigenvalues in addition to the first order, provides a more accurate representation of the eigenvalue variation due to stochasticity in the aeroelastic system through the random parameter ($g=C_{L\alpha}$ or $g=GJ$). Both the first and the second derivatives of the eigenvalues with respect to a generic g are needed:

$$\lambda_i(g) \approx \lambda_i(g_0) + \left. \frac{d\lambda_i}{dg} \right|_{g_0} (g - g_0) + \frac{1}{2} \left. \frac{d^2\lambda_i}{dg^2} \right|_{g_0} (g - g_0)^2. \quad (26)$$

The ordinary derivative operator is used in this equation and in further derivations below since the effect of $C(k)$ is neglected (refer to Eq. (5)). Similar expressions may be derived if this effect

is included (Eq. (18)). The first and the second derivatives of the eigenvalues can be found either numerically or analytically. The mean and variance of the eigenvalues are then derived as:

$$E[\lambda_i(g)] = \lambda_i(g_0) + \frac{1}{2} \frac{d^2 \lambda_i}{dg^2} \bigg|_{g_0} E[(g - g_0)^2], \quad (27)$$

and

$$\begin{aligned} \text{var}[\lambda_i(g)] \approx & \left(\frac{d\lambda_i}{dg} \bigg|_{g_0} \right)^2 E[(g - g_0)^2] + \frac{1}{4} \left(\frac{d^2 \lambda_i}{dg^2} \bigg|_{g_0} \right)^2 E[(g - g_0)^4] \\ & + \left(\frac{d\lambda_i}{dg} \bigg|_{g_0} \right) \left(\frac{d^2 \lambda_i}{dg^2} \bigg|_{g_0} \right) E[(g - g_0)^3] - \frac{1}{4} \left(\frac{d^2 \lambda_i}{dg^2} \bigg|_{g_0} \right)^2 \left(E[(g - g_0)^2] \right)^2. \end{aligned} \quad (28)$$

To calculate the variance of the eigenvalue, other than the variance, the skewness and kurtosis of random parameter are also needed. These are characteristics of the random distribution. For both uniform and normal distributions, the skewness is equal to zero. For the normal distribution, the kurtosis is equal to $3\{\text{var}[g]\}^2$ and for the uniform distribution, the kurtosis is equal to $1.8\{\text{var}[g]\}^2$ (Mood and Graybill, 1974). The second derivative of λ_i is also needed to calculate the variance of the eigenvalue. If the eigenvectors are normalized such that $\chi_i^T \Psi_i = 1$, this expression is derived from Eq. (15) as:

$$\frac{d}{dg} \left(\frac{d\lambda_i}{dg} \right) = \frac{d}{dg} \left(\chi_i^T \frac{d\mathbf{A}}{dg} \Psi_i \right), \quad (29)$$

or

$$\frac{d^2 \lambda_i}{dg^2} = \frac{d}{dg} \left(\chi_i^T \right) \frac{d\mathbf{A}}{dg} \Psi_i + \chi_i^T \frac{d^2 \mathbf{A}}{dg^2} \Psi_i + \chi_i^T \frac{d\mathbf{A}}{dg} \frac{d}{dg} (\Psi_i). \quad (30)$$

In Equation (30) the first and the second derivatives of $\mathbf{A}$ are needed as well as the first derivatives of the right and left eigenvectors. The second derivative of $\mathbf{A}$ is (Eq. (16)):

$$\frac{d^2 \mathbf{A}}{d g^2} = \frac{d^2 (\mathbf{B}^{-1} \mathbf{D})}{d g^2} = \left[\frac{d^2 (\mathbf{B}^{-1})}{d g^2} \right] \mathbf{D} + 2 \frac{d (\mathbf{B}^{-1})}{d g} \frac{d \mathbf{D}}{d g} + \mathbf{B}^{-1} \frac{d^2 \mathbf{D}}{d g^2}. \quad (31)$$

Closed-form calculation of the first derivatives of the right and left eigenvectors is a complex procedure since it necessitates the use of the inverse of the expression $(\mathbf{A} - \lambda_i \mathbf{I})$ (for example found from Eq. (15)), which is singular. A closed-form method for systems with proportional damping is suggested by Liu (2013). Choi et al. (2004) introduced a series of methods, requiring the extended definition of the quadratic eigenvalue problem, which is not employed in this paper. There are the methods that require the knowledge of all other eigenvectors for expanding the unknown derivative of $\boldsymbol{\psi}_i$ or $\boldsymbol{\chi}_i$ (Adhikari and Friswell, 2001, Adhikari and Friswell, 2004). All the above methods appeared to be difficult to implement due to the inherent complexity in matrix $\mathbf{A}$. As an alternative, iterative methods have been proposed to estimate the derivative of eigenvectors (Andrew and Tan, 1996, Tan and Andrew, 1989). Preliminary implementation of the iterative methods in the case of the aeroelastic system matrix $\mathbf{A}$ has exhibited potential convergence issues. Therefore, in the present study, the second derivative of eigenvalues is calculated numerically by approximating the first derivatives of the left and right eigenvectors through finite differences.

Figure 7 and Figure 8 illustrate the CDFs of normalized angular velocity, Ω^* , estimated using the 2nd order expansion (SORM) and Monte Carlo simulations, for random flow forces (C_{La} parameter) and random torsional natural frequencies (GJ parameter, as explained earlier), respectively. In comparison with the 1st order expansion methods, SORM provides an estimation of CDF curves closer to Monte Carlo simulations, with especially better estimation for higher *cov* values of the input variables and in regions of Ω^* away from the design point. This is expected since the new approximation utilizes both the 1st and the 2nd order expansions to represent the eigenvalue variation, while FORM entirely depends on the first order expansion. However, the

upper and the lower tails of the CDFs are not estimated accurately, especially for higher cov values. Inspection of Figure 8 also reveals that another effect of adding the 2nd order term is the appearance of small non-zero regions in CDFs before and after the start and end of the lower and upper limits of the Ω^* distribution, which is bounded since the input random variable is uniform.

7. Weighted Averaging Reliability Method (WARM)

None of the three methods discussed in Sections 4, 5 and 6 was able to provide a complete representation of the Monte Carlo simulations, especially for high cov values. In this section an averaging method based on FORM is introduced to improve the estimation of flutter probability by reliability methods.

In Section 4, the flutter probability function was approximated by expansion about the original design parameter value, and it was shown that the method does not provide a good representation of the lower and upper tails of the CDFs obtained from Monte Carlo simulations. It is noteworthy that the first-order expansion only returns accurate flutter probability distributions for cases with randomly distributed C_{La} with $cov = 0.1$, while for all other cases, it is only accurate for values close to the original design point (the mean flutter angular velocity) and becomes less and less accurate as this parameter gets farther from the original design value. In order to overcome this issue, the Weighted Averaging Reliability Method (WARM) is proposed.

In WARM, instead of using one approximation of the eigenvalues about the original design point (or parameter), a number of first-order approximations are constructed about “altered” design points. The altered points or parameters should be selected in order to span the random parameter domain (and consequently the domain of Ω) with sufficient accuracy, from a minimum value to a maximum value. The parameter range of variation depends on the random distribution of g and its

corresponding cov. For example, in the case of a random GJ , Figure 9 presents various CDFs reconstructed using various initial expansion points. Each curve refers to a different expansion point, with $\overline{\omega}_t^*$ the normalized altered torsional natural frequency. The curve with $\overline{\omega}_t^* = 1.0$ corresponds to the 'unaltered' case, used in previous sections. Since most discrepancies between the Monte Carlo and the previously proposed reliability methods were observed when the torsional natural frequency was chosen as the stochastic term, application of the WARM is exclusively carried out in the case of the a random GJ .

After obtaining the CDFs about the altered design parameters or expansion points, a weighted averaging among the set of CDF curves is used to combine them to obtain the approximation of the flutter angular velocity CDF. The averaging method depends on the distribution of the input random parameter ($g=GJ$ in this example). A normalized probability density function (PDF) of each random parameter is used to determine the weightings. Figure 10 shows the normalized PDFs for both the uniform and the normal distributions with $cov=0.1$ and $cov=0.2$. The CDFs are weighted based on the distance of the altered parameter or expansion point from the original design parameter. This approach is in part inspired by the kernel density estimation method, used for non-parametric probability distribution assessment and smoothing of probability density functions (e.g., (Hwang et al., 1994)). For the uniform distribution, all CDFs have the same unit weight and in order to integrate the curves into one CDF, it is sufficient to take the weighted average among the various curves, point by point. For the normal distribution, however, the weighting factor decreases as the altered random parameter gets farther from the original design point. For example, if the random input parameter, torsional frequency, has a normal distribution with $cov=0.2$, a CDF with $\overline{\omega}_t^* = 1$ is weighted as 1 and a CDF with $\overline{\omega}_t^* = 0.8$ is weighted as 0.6.

Figure 11 and Figure 12 show the CDFs of normalized angular velocity, Ω^* , estimated using WARM, FORM and Monte Carlo simulations for both the normal and uniform distributions for flow forces and natural frequencies, respectively. In the case of a random torsional natural frequency, the discrepancy between the approximate solution and the Monte Carlo simulation almost vanishes when WARM is used. The upper and lower tails (or limits) of the angular flutter velocity CDFs are better estimated. The expansion method and reliability approximation, based on WARM, gives the closest match to the Monte Carlo simulation among all other methods. Examination of Figure 12 suggests that the proposed methodology, yielding excellent results at moderate *cov* values, is less efficient when the *cov* is large (*cov*=0.2), and the eigenvalue variability, used to detect flutter becomes broader and nonlinear. For large *cov* values, the brute-force Monte Carlo method still remains the preferable approach. In any case, it must also be noted that the range of *cov*=0.2 is less compatible with actual parameter variability, and was primarily selected to verify the procedure.

Another parameter that plays a role in WARM is the selection of the number of altered design points or additional “runs” needed to approximate the curve. Figure 13 shows CDFs of the normalized flutter critical angular velocity, Ω^* , for a normally distributed *GJ* with *cov*=0.2 using different numbers of additional runs (N_R). It can be observed that even when using four additional runs at altered design points or parameters, very good approximation of the CDF curve is obtained. By increasing the number of additional runs to 6 and 12 the resulting CDF becomes smoother and closer to the Monte Carlo approximation.

8. Conclusions

In this paper, four different reliability methods are introduced to model the influence of the blade's inherent uncertainties on the onset of flutter. The NREL 5MW blade is considered for this study. The variability in the flow forces is modeled through the cross-sectional lift coefficient and the variability in the structural properties is modeled through the torsional natural frequency. Both normal and uniform distributions are considered for each random parameter.

The four reliability methods are the First Order Reliability Method (FORM), the First Order Reliability Method including the effect of $C(k)$ variation (FORM-C), the Second Order Reliability Method (SORM) and the presently proposed Weighted Average Reliability Method (WARM). Both FORM and SORM provide acceptable approximations but they cannot model reliability away from the original design point (deterministic value of the angular flutter velocity). In order to improve estimation the method (FORM-C) is employed, which includes the effect of $C(k)$ function variation on flutter probability; the FORM-C returns a closer match to the exact solution, especially in the upper tail ($\Omega^* > 1$), but not for the lower tail ($\Omega^* < 1$). Therefore the WARM is introduced to deal with the poor estimation of the probability tails. In the WARM other design points are included, covering the random parameter variability range, in addition to the original design point.

Variability in the torsional frequency usually has the most influence on the onset of flutter. For cases in which the lift coefficient is set as the random variable with $cov=0.1$, the FORM returns a very close approximation to the Monte Carlo simulations, while for all other cases WARM is the most accurate method. The computing time is also drastically reduced with WARM in comparison with the Monte Carlo method, which usually requires a large sample size to ensure adequate estimation of flutter occurrence probabilities. The WARM method can also be used together with other approaches for solving fluid-structure interaction problems. The key is to develop a linear relationship between the parameter of interest and the random input parameter.

Acknowledgements

This study was completed while Luca Caracoglia was on sabbatical leave at the University of Massachusetts – Amherst (UMass) in the spring of 2015. The collaboration of the Department of Mechanical and Industrial Engineering of UMass is gratefully acknowledged. The work was supported in part by the National Science Foundation of the United States, Collaborative Awards CMMI-1462646 and CMMI-1462774.

References

- Abramowitz, M., Stegun, I.A. 1964. Handbook of mathematical functions: with formulas, graphs, and mathematical tables. Courier Corporation.
- Adhikari, S., Friswell, M. 2004. Random eigenvalue problems in structural dynamics. Proceedings of the 45th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics & Materials Conference.
- Adhikari, S., Friswell, M.I. 2001. Eigenderivative analysis of asymmetric non-conservative systems. International Journal for Numerical Methods in Engineering 51, 709-733.
- Andrew, A.L., Tan, R.C. 1996. Computation of derivatives of repeated eigenvalues and corresponding eigenvectors by simultaneous iteration. AIAA Journal 34, 2214-2216.
- Canor, T., Caracoglia, L., Denoël, V. 2015. Application of random eigenvalue analysis to assess bridge flutter probability. Journal of Wind Engineering and Industrial Aerodynamics 140, 79-86.

- Caracoglia, L. 2013. An Euler–Monte Carlo algorithm assessing Moment Lyapunov Exponents for stochastic bridge flutter predictions. *Computers & Structures* 122, 65-77.
- Cheng, J., Cai, C., Xiao, R., Chen, S. 2005. Flutter reliability analysis of suspension bridges. *Journal of Wind Engineering and Industrial Aerodynamics* 93, 757-775.
- Choi, K., Jo, H., Kim, W., Lee, I. 2004. Sensitivity analysis of non-conservative eigensystems. *Journal of Sound and Vibration* 274, 997-1011.
- Der Kiureghian, A. 2005. First-and second-order reliability methods. *Engineering design reliability handbook*, 14-11.
- Dragomirescu, E., Miyata, T., Yamada, H., Katsuchi, H. 2003. Probabilistic approach of structural reliability applied to bridge components 1. *Proceeding of the 11th International Conference on Wind Engineering*, Lubbock, USA, 789-796.
- Ernst, B., Sueme, J.R. 2013. Modelling structural uncertainty of wind turbine rotor blades for aeroelastic investigations. *The Seventh MIT Conference on Computational Fluid and Solid Mechanics*.
- Fox, R., Kapoor, M. 1968. Rates of change of eigenvalues and eigenvectors. *AIAA Journal* 6, 2426-2429.
- Ge, Y., Xiang, H. 2005. Probability-based assessment of aerodynamic vibration of long-span bridges. *The 9th International Conference on Structural Safety and Reliability, ICOSSAR*, 3856-3863.

- Ge, Y., Xiang, H., Tanaka, H. 2000. Application of a reliability analysis model to bridge flutter under extreme winds. *Journal of Wind Engineering and Industrial Aerodynamics* 86, 155-167.
- Griffith, D.T., Casias, M., Smith, G., Paquette, J., Simmermacher, T.W. 2006. Experimental uncertainty quantification of a class of wind turbine blades. The 24th International Modal Analysis Conference, St. Louis, MO.
- Haldar, A., Mahadevan, S. 2000. Reliability assessment using stochastic finite element analysis. John Wiley & Sons.
- Hansen, M.O. 2015. Aerodynamics of wind turbines. Routledge.
- Hansen, M. 2002. Vibrations of a three-bladed wind turbine rotor due to classical flutter. ASME 2002 Wind Energy Symposium, Reno, NV, 256-266.
- Hansen, M.H. 2007. Aeroelastic instability problems for wind turbines. *Wind Energy* 10, 551-577. DOI:10.1002/we.242.
- Hansen, M. 2004. Aeroelastic stability analysis of wind turbines using an eigen value approach. *Wind Energy* 7, 133-143. DOI:10.1002/we.116.
- Hodges, D.H., Dowell, E. 1974. Nonlinear equations of motion for the elastic bending and torsion of twisted nonuniform rotor blades. NASA report, NASA TN D-7818, 1974.
- Hracov, S., Náprstek, J., Pospíšil, S. 2005. Flutter stability condition of bluff body with respect to stochastic approach. The 10th Americas Conference on Wind Engineering (10ACWE), Baton Rouge, LA.

- Hwang, J., Lay, S., Lippman, A. 1994. Nonparametric multivariate density estimation: a comparative study. *Signal Processing, IEEE Transactions on* 42, 2795-2810.
- Jonkman, J.M., Butterfield, S., Musial, W., Scott, G. 2009. Definition of a 5-MW reference wind turbine for offshore system development. National Renewable Energy Laboratory, Golden, CO,.
- Liu, X. 2013. A new method for calculating derivatives of eigenvalues and eigenvectors for discrete structural systems. *Journal of Sound and Vibration* 332, 1859-1867.
- Lobitz, D. 2005. Parameter sensitivities affecting the flutter speed of a MW-sized blade. *Journal of Solar Energy Engineering-Transactions of the ASME* 127, 538-543. DOI:10.1115/1.2037091.
- Lobitz, D. 2004. Aeroelastic stability predictions for a MW-sized blade. *Wind Energy* 7, 211-224. DOI:10.1002/we.120.
- Mood, A., Graybill, F., Boes, D. 1974. *Introduction to the Theory of Statistics*. McGraw-Hill, New York, NY.
- Náprstek, J. 1996. Stochastic exponential and asymptotic stability of simple non-linear systems. *International Journal of Non-Linear Mechanics* 31, 693-705.
- Náprstek, J. 2001. Stability domains of wind-excited random nonlinear systems through Lyapunov function. *Journal of Wind Engineering and Industrial Aerodynamics* 89, 1499-1512.
- Nayfeh, A.H. 2008. *Perturbation methods*. John Wiley & Sons, New York, NY.

Owens, B.C., Griffith, D.T., Resor, B.R., Hurtado, J.E. 2013. Impact of Modeling Approach on Flutter Predictions for Very Large Wind Turbine Blade Designs. Proceedings of the American Helicopter Society 69th Annual Forum, Phoenix, AZ.

Petersen, K.B., Pedersen, M.S. 2008. The matrix cookbook. Technical University of Denmark 7, 15.

Pourazarm, P., Caracoglia, L., Lackner, M., Modarres-Sadeghi, Y. 2015a. Stochastic analysis of flow-induced dynamic instabilities of wind turbine blades. Journal of Wind Engineering and Industrial Aerodynamics 137, 37-45. DOI:10.1016/j.jweia.2014.11.013

Pourazarm, P., Modarres-Sadeghi, Y., Lackner, M. 2015b. A parametric study of coupled-mode flutter for MW-size wind turbine blades. Wind Energy. DOI:10.1002/we.1847

Prenninger, P., Matsumoto, M., Shiraishi, N., Izumi, C., Tsukiyama, Y. 1990. Reliability of bridge structures under wind loading: consideration of uncertainties of wind load parameters. Journal of Wind Engineering and Industrial Aerodynamics 33, 385-394.

Resor, B.R., Owens, B.C., Griffith, D.T. 2012. Aeroelastic Instability of Very Large Wind Turbine Blades. European Wind Energy Association, Copenhagen, Denmark.

Resor, B., Paquette, J. 2011. Uncertainties in Prediction of Wind Turbine Blade Flutter. The 52nd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, Denver, Co

Riziotis, V., Voutsinas, S., Politis, E., Chaviaropoulos, P. 2008. Assessment of passive instability suppression means on pitch-regulated wind turbines. Wind Energy 11, 171-192.

Riziotis, V., Voutsinas, S., Politis, E., Chaviaropoulos, P. 2004. Aeroelastic stability of wind turbines: the problem, the methods and the issues. *Wind Energy* 7, 373-392.

Taflanidis, A.A., Loukogeorgaki, E., Angelides, D.C. 2013. Offshore wind turbine risk quantification/evaluation under extreme environmental conditions. *Reliability Engineering & System Safety* 115, 19-32.

Tan, R.C., Andrew, A.L. 1989. Computing Derivatives of Eigenvalues and Eigenvectors by Simultaneous Iteration. *IMA journal of numerical analysis* 9, 111-122.

Theodorsen, T. 1935. General theory of aerodynamic instability and the mechanism of flutter. NACA Report 496, Washington, D.C.

Figure 1. (a) Frequency (imaginary part of eigenvalues) and (b) damping (real part of eigenvalues) of different modes versus the rotor speed for the NREL 5MW blade. In the bottom panel a positive value of damping corresponds to stable vibration. Specific designation of the various modes is indicated in the legend.

Figure 2. CDFs of the normalized critical angular velocity for flutter, Ω^* , for a normally and a uniformly distributed $C_{L\alpha}$ using MC simulations with uncorrelated and fully correlated random sets for the NREL 5-MW wind turbine.

Figure 3. CDFs of the normalized critical angular velocity for flutter, Ω^* , for a normally and a uniformly distributed $C_{L\alpha}$ using FORM and MC simulations for the NREL 5-MW wind turbine.

Figure 4. CDFs of the normalized critical angular flutter velocity (Ω^*) of the NREL 5-MW wind turbine for a normally and a uniformly distributed GJ – comparison between FORM (1st order) and MC simulations.

Figure 5. CDFs of the normalized critical angular flutter velocity (Ω^*) of the NREL 5-MW wind turbine for a normally and a uniformly distributed $C_{L\alpha}$ – comparison between FORM (1st order), FORM-C with the $C(k)$ effect (1st order with $C(k)$) and MC simulations.

Figure 6. CDFs of the normalized critical angular flutter velocity (Ω^*) of the NREL 5-MW wind turbine for a normally and a uniformly distributed GJ – comparison between FORM (1st order), FORM-C with the $C(k)$ effect (1st order with $C(k)$) and MC simulations.

Figure 7. CDFs of the normalized critical angular flutter velocity (Ω^*) of the NREL 5-MW wind turbine for a normally and a uniformly distributed $C_{L\alpha}$ - comparison between FORM (1st order), SORM (2nd order) and MC simulations for the NREL 5-MW wind turbine.

Figure 8. CDFs of the normalized critical angular flutter velocity (Ω^*) of the NREL 5-MW wind turbine for a normally and a uniformly distributed GJ using FORM (1st order), SORM (2nd order) and MC simulations.

Figure 9. CDFs of the normalized critical angular flutter velocity (Ω^*) for a normally distributed GJ using FORM with altered torsional natural frequencies for the NREL 5-MW wind turbine.

Figure 10. Weighting functions for normal and uniform random distributions.

Figure 11. CDFs of the normalized critical angular flutter velocity (Ω^*) of the NREL 5-MW wind turbine for a normally and a uniformly distributed $C_{L\alpha}$ using – comparison between FORM (1st order), WARM and MC simulations.

Figure 12. CDFs of the normalized critical angular flutter velocity (Ω^*) of the NREL 5-MW wind turbine for a normally and a uniformly distributed GJ - comparison between FORM (1st order), WARM and MC simulations.

Figure 13. CDFs of the normalized critical angular flutter velocity (Ω^*) of the NREL 5-MW wind turbine for a normally distributed GJ using WARM and different number of additional altered design points and runs (N_R).