A Critical Review of Arctic Pack Ice Driving Forces: New Sources of Data¹

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Key Words

Sea ice, driving force, pack ice, limit force, multi-year ice

Abstract

This paper provides the details of ice loading events that can be used to further the understanding of pack ice driving forces in the Beaufort Sea. Several methods have been reviewed and employed including *in situ* stress measurements, loads on the Molikpaq offshore caisson, shoreline pile-up events, pile-ups and rubble fields on offshore shoals and relic berms, analysis of shear walls on offshore rubble fields, and analysis of deep ridge keels. Over 50 different events are identified with 33 suitable for a pack ice analysis. The data are considered in terms of both the ISO 19906 (2010) Arctic Structures Standard and pack ice pressures that can be exerted across various widths. A new approach is proposed, in which the calculated and measured values from past pack ice pressure events are used to predict limit force.

Introduction

What determines the force that an ice sheet or ice floe can exert on an offshore platform? Croasdale (1984) summarized this situation by viewing it as the minimum force value from three different limit situations. That is, the force on a structure placed in ice will be limited by one of three mechanisms:

- the strength of the ice sheet or floe acting on the total width of the structure (*limit stress*).
- the energy of the floe interacting with the structure (*limit energy or momentum*). This is usually associated with an isolated floe that impacts a structure and comes to a rest after its momentum is dissipated by the ice failure.
- the force available to drive an ice floe against a structure (*limit force*).

The highest ice forces generally result from ice-structure interactions that were controlled by *limit stress* (see e.g. Jeffries and Wright, 1988; Wright and Timco, 1994; Johnston *et al.*, 1999; Timco and Johnston, 2003, 2004). In these cases there was sufficient force generated by the moving pack ice to cause the advancing ice sheet to fail across the total width of the offshore structure. This occurred even when the multi-year ice was several meters thick. But in a few recorded cases, the pack ice itself failed away from the platform (Wright *et al.*, 1992; Wright and Timco, 2000). This is a *limit force* situation where the force that can be generated across the width of an ice floe or feature in pack ice was not sufficient to drive the feature to fail against the structure. But the important question is: What is this force level? Answering this question would help determine the maximum thickness and width of a multi-year ice floe or extreme ice feature that can be driven by the pack ice and fail across the full width of a given size and shape of offshore platform.

Unfortunately there is very little information on the situation of a first-year ice sheet failing and building a ridge against a thick multi-year floe. This is a complex situation where the dynamics of ridge-building are little known. One could speculate the important mechanisms based on observations of ridge

building from the interaction of two first-year sea ice floes, or from rubbing and piling up of first-year sea ice against an offshore platform, grounded ice or a shoreline. In these situations, the first-year ice can fail by bending (flexure), buckling, crushing, splitting or a combination of these modes (i.e. mixed mode). With continued ice movement, the first-year ice begins to pile-up and this can cause changes in the mode of failure. Then the ice can ride-up the face of the pile-up, or fail against the front face of the pile-up with different failure modes, or can penetrate into the rubble pile. With floating first-year ice, the weight of the pile-up can cause a bending or flexural failure in one of the ice sheets and the broken ice can re-distribute itself before the process continues. The failure of first-year ice against a multi-year ice floe would likely be different due to the higher thickness and strength of the multi-year ice. The whole rubbing process has both spatial and temporal variations associated with it. Factors that one would expect to be important include the thickness of the first-year ice, the duration of the interaction process, the details of the mechanics of the rubbing process, the width of the interacting ice, the interaction geometry, and the failure or non-failure of the multi-year ice floe. Since there is little known about these processes and forces due to the interaction of first-year ice with a multi-year ice floe, other approaches must be used to gain insight into this problem.

The concept of pack ice driving force has been described by Croasdale *et al.* (1992, 1996) and the state of knowledge in this area was recently summarized by Croasdale (2009, 2012a, 2012b). This approach has been adopted as part of the informative section in the 19906 ISO Arctic Offshore Structures Standard (ISO 19906, 2010). In the ISO 19906 (2010) standard it is referred to as the ridge-building action and is given by Equation A.8-53:

Equation 1
$$p_D = R (h^{1.25}) (D^{-0.54})$$

where p_D is the ridge building action (line load) imposed by pack ice on a thicker ice feature per unit width [MN/m], h is the thickness of the pack ice [m], D is the width of the thicker ice feature [m], and R is a coefficient. Both h and D are geometric values related to the loading situation. Results from early *in situ* stress tests, discussed in detail later in this paper, indicated that the pack ice line load depends on the width D of the thicker ice feature. The exponent applied to D in Equation 1 was chosen to fit this empirical data. The pack ice thickness h also has an exponent applied; this factor of 1.25 is used to 'normalize' the pack ice pressure to 1 m of ice (Croasdale, 2009; Sandwell and Canatec, 1994). The normalized line load is given by $p_D / h^{1.25}$. Finally, R and the exponent on D are coefficients derived by curve fitting to the data. R can be thought of as a 'material' property related to failure mode of the pack ice, and it incorporates some of the uncertainties in the values of h and D and the exponents applied to them. ISO 19906 (2010) also states that the frozen-in condition can be considered by application of a multiplicative factor of 1.5 to the ridge building action.

The R value of 2 was chosen to fit to the data from five *in situ* stress tests. Subsequent measurements on the Molikpaq offshore platform resulted in an additional five data points that indicated higher R values, on the order of 10. The ISO standard states that an upper bound estimate at the 99% confidence level is given by $R = 10$, and that $R = 6$ can be used for a 50 % confidence level. It also states that if a probabilistic approach is used, the parameter R should be uniformly distributed between 2 and 10. There is little basis for this assumption of a uniform distribution for the R value, largely because of the limited data. This large range of possible R values leads to an uncertainty of the pack ice force by a factor of five. Setting too low a value for R , or too high a probability of a low value of R , can lead to an under prediction of forces from the interaction of multi-year floes with a structure.

Both industry and regulators have indicated that pack ice driving forces remain a large unknown when it comes to design of offshore structures for ice-covered water. Methods for their determination are inconclusive, which complicates the reliability of load estimates for engineering design and may be an impediment to development in these regions. For example, in an Ice Experts Workshop (Timco, 2012), leading ice engineering experts were asked to predict the loads due to a multi-year ice floe embedded in pack ice interacting with a vertical structure in the shear zone of the Beaufort Sea. All used the pack ice driving force as the determination of the force, yet there were still very large discrepancies (a factor of over 3.5) for predicting loads. This large uncertainty is unacceptable to operators and regulators. Many past determinations of pack ice forces were based on a limited, field measured data set and rather crude analysis techniques.

What can be done to try to narrow this uncertainty in the pack ice driving force? Barker and Timco (2012) took a critical look at various methods that could be used to reduce the uncertainty in pack ice driving forces in the Beaufort Sea, in an effort to better understand the R value and how it impacts engineering design. They identified and ranked a number of approaches that had not yet been evaluated in the context of pack ice driving forces. They showed that data mining of well-documented extreme ice features and ice pile-ups could be useful as input into analytical and numerical models of ice forces. This marriage of large-scale data and force calculation methods may produce new information to refine pack ice forces. The present paper continues this effort to narrow the uncertainty in pack ice driving forces, by re-examining the approach given in ISO 19906 (2010) and the data (from the *in situ* field measurements and the force measurements on offshore platforms) that were used to derive the ISO approach. The paper then explores the forces that can be derived from observations of extreme ice pile-up features and events to try to obtain additional data on the forces that Arctic pack ice can generate.

The simple approach

At first glance it would seem that determining the pack ice driving force should be quite straightforward. In its simplest form, the external driving forces are comprised of wind and current shear on the ice. For a large area of pack ice, the total stress (S_t) is the sum of the wind (air) shear stress (S_a) and the water (current) shear stress (S_w) and is given by

$$\text{Equation 2} \quad S_t = S_a + S_w = \rho_a * C_a * v_a^2 + \rho_w * C_w * v_w^2$$

where ρ is the density ($\rho_a = 1.2 \text{ kg/m}^3$; $\rho_w = 1020 \text{ kg/m}^3$), C is the drag coefficient ($C_a \sim 0.002$, $C_w \sim 0.005$; see e.g. Kubat *et al.* 2010) and v is the speed. For the Beaufort Sea region, a typical current speed is on the order of 0.15 m/s. A wind speed of 20 m/s is a reasonable value for a sustained wind speed in this region. The question is: What fetch length should be used to calculate the force from this shear stress? If a fetch length of 100 km is assumed for a continuous pack ice cover (with 10/10ths concentration), this gives a force per unit width of 107 kN/m. If a fetch length of 200 km is assumed for these conditions, a force per unit width of 214 kN/m is calculated. Thus estimates can be made if there is a fetch-limited situation. But for the Arctic, the fetch length is effectively unlimited. Thus, this simple calculation can only give an estimate of the range of pack ice driving force values for the Arctic by assuming a range of nominal fetch lengths, but is not helpful in providing a unique value for the driving force there.

It is useful to examine the implications of Equation 2 with respect to pack ice driving forces. The equation shows that the shear stress and the resulting forces are a function largely of the wind speed and to a lesser extent the current speed. Thus it would be expected that if the wind speed is low, the

pack ice forces would also be low. This is not a concern for an offshore platform but it could be for vessels operating in these conditions. Even at lower wind speeds, in-plane pressure in the pack ice can impede vessel movement. Kubat *et al.* (2016) have developed models to predict pressured ice conditions for the Beaufort Sea which appear to work well (see also Kubat *et al.*, 2015). For an offshore platform, the influence of the pack ice pressure becomes relevant during high wind and storm conditions. To gain a better understanding of the pack ice forces during these conditions, it is necessary to examine the more “extreme” ice features in the Arctic since they were very likely created during high wind or storm events. It is important for the reader to understand that the features described in this paper should not be taken as being representative of the range of ice pile-up features in the Arctic since only the larger features were chosen to obtain information on pack ice forces during high loading events.

Approaches to refining pack ice driving forces

To date, there have been only a few methods employed for determining the pack ice driving force. These methods require a number of assumptions, both explicit and implicit, to calculate the pack ice driving force. In this paper, past methods for determining pack ice forces are re-examined to help to identify these assumptions and provide guidance on the reliability of each method. Also, as noted above, a number of new approaches are discussed that may provide additional data on pack ice forces. As will be seen, all of the approaches have to make a number of assumptions, largely because the analysis is based on post mortems of events not observed in real time. Because of this, a large number of events and different approaches are required to try to refine the pack ice driving force value. The following sections describe these approaches, which can be divided into three categories:

1. *In situ* stress measurements in isolated multi-year ice floes;
2. Force measurements on an offshore platform;
3. Observations of extreme ice pile-up features with subsequent post analysis.

***In Situ* stress measurements**

The form of the ISO 19906 (2010) expression for determining the ridge building or pack ice driving force, given in Equation 1, was derived from *in situ* measurements of ice stress. Croasdale (1984) proposed an approach to relate pack ice driving forces to ridge building forces in pack ice. This concept involved instrumenting an ice floe, measuring stresses in it, and relating them to pack ice driving forces at the edge of the floe. A number of different field programs were conducted in the Beaufort Sea and around Katie’s Floeberg, and are discussed in the following sections.

Beaufort Sea in situ stress measurements

A pilot project to test the concept and its application was conducted in 1986 in the Beaufort Sea (Croasdale *et al.*, 1987). This was followed by two more years of measurements, 1989 and 1991, in the Beaufort Sea (Comfort and Ritch, 1990, 1992). Figure 1 shows the locations of the three field studies.

The basic idea of this field project was to try to infer the ridge-building forces due to first-year ice failing against a multi-year ice floe by placing stress sensors in the centre of a thick multi-year floe and monitoring the stresses as the floe drifted with the polar pack. The stresses in the floe are an integration of all the forces acting on the boundary of the floe as a result of ridge building at the perimeter, interaction with adjacent floes, bending stresses and those internal to the floe such as wind and current

drag. Thus, the stresses measured in the floe can be taken as an indication of pack ice driving forces. Two key requirements have to be met: first, the sensors must be capable of accurately and reliably measuring *in situ* stresses in the ice floe; and second, a calculation method is required to relate measured stresses in the floe to boundary forces. The projects involved periodic site visits to the floes to collect data and observe changes in ridging around the periphery of the floe.

Figure 1: Location of field projects in the Canadian Beaufort Sea (after Croasdale *et al.*, 1992).

The 1986 field measurement project extended from April 12 to May 4 (Croasdale *et al.*, 1986). The *in situ* stress sensors were placed in a multi-year floe 2.5 km by 4.5 km in size and 1.8 to 4.5 m thick. Two 120-degree rosettes were installed at a depth of about 1 m in the floe. Five loading events were noted – all between April 20 and April 27, an interval when the wind was blowing onshore, pressing the pack up against the landfast ice. One of the rosettes registered a maximum increase in principal stress of 20 kPa on April 21. A site visit on April 24 confirmed that new ridging had occurred along part of the perimeter of the floe. Assuming a uniform vertical stress distribution, a simple interpretation of the measured results gave an average pack ice force about 50 kN/m with a range of 25 to 62 kN/m. This segment of stress record is illustrated in Figure 2.

Figure 2: Pressure events in 1986 field program (after Croasdale *et al.*, 1986)

The 1989 field measurement project extended from March 15 to May 16 (Comfort and Ritch, 1990). Five rosettes of *in situ* stress sensors were placed in a multi-year floe and two pressure events were identified as shown in *Table 1*. The maximum principal stress from one of the rosettes for the May 2 event was 40 kPa and a later site visit indicated ridging 500 m away. The second event, on May 12, recorded a stress increase of 240 kPa and a ridge formed about 50 m distant.

The 1991 field measurement project was much more extensive, and ran from March 16 to May 26 (Comfort and Ritch, 1991). Three multi-year floes were instrumented, each with 6 rosettes. The most significant events were identified in Comfort and Ritch (1991) in late March and are shown in *Table 1*. A maximum principal stress increase of 300 kPa was measured for a ridge building event only 95 m from the instrumentation. More typical stress increases measured were 40 kPa for ridge building 750 m distant.

Table 1 summarizes the results of the three years of Beaufort Sea ice pressure measurements in ice floes. Some of the information about the Beaufort pack ice events has been extracted from Croasdale *et al.* (1992). The analysis process outlined in Croasdale *et al.* (1992) is as follows: The maximum measured stress in the floe (typical peak stress) is multiplied by the floe thickness to get the force per unit width at the measurement location. This force is set equal to the ice loading at the edge of the floe (deduced edge loading based on later observations of the floe edge). The edge loading is related to the thickness of the ice which is failing against the edge of the floe (ridge building). Since different thicknesses of ice will generate different magnitudes of edge loading, the edge loading was normalized with respect to ice thickness. Normalizing was done using ice thickness to the power of 1.25, as derived from an expression relating force to ice thickness for 2-D ridge building. To this point, ridge building force at the edge of the floe is apparently independent of the horizontal scale. In the field projects, attempts were made to estimate the length over which ridge building had occurred, but this was not successful. For a horizontal size factor, it was assumed that the floe length over which ridge building occurred was equivalent to the distance between the instrument location and the floe edge where ridge building was occurring. Croasdale *et al.* (1992) summarized the results of edge loading force across the corresponding width of ridge building and suggested an equation of the form

Equation 3 $w = 2 h^{1.25} D^{-0.54}$

where w is pack ice line load [MN/m], h is thickness of ice [m] which is ridging against the floe, and D is the width [m] over which ridging is occurring. Note that Equation 3 is the basis for the ISO 19906 (2010) Equation A.8-53 for ridge building or pack ice driving force (Equation 1 in this paper). Equation 3 was developed by Croasdale *et al.* (1992) using only five data points from the *in situ* measurements, which gave a value of $R = 2$ and exponent on D of -0.54.

It is necessary to recognize that there are a number of uncertainties in using this type of approach to predict pack ice driving forces:

1. The assumption that width of influence is equivalent to the distance to the location of ridge building is unproven;
2. The assumption that the ice is uniform without cracks between the instrumentation and the location of the ridge building and that there is a uniformity of stresses within the floe;
3. It is an inverse problem to assign a distribution of boundary forces around the edge of the floe to the measured stresses at a point in a floe;
4. Without direct observation of ridge building while floe stresses are measured, it is difficult to assign the length of influence;
5. Stress measurements in the polar pack do not provide an upper bound of pack ice forces unless ridge building is occurring.

The basis for the first assumption (or uncertainty) is an analysis that assumes that the width of influence (or the length over which ridging takes place) is the same as the distance of the sensor to the floe edge. Croasdale *et al.* (1992) explain their reasoning as follows: *“The distance of the sensors to the floe edge has also been called the “width of influence”, implying that the distance to the floe edge is roughly equal to the width over which the ridging forces have been averaged by the sensors. This is intuitively correct when one thinks of an isolated thick floe surrounded by thinner pack ice which implies spatially varying pack ice pressures around its perimeter. In this situation, one would expect sensors placed in the middle of the floe to be measuring the spatially-averaged pack ice pressure across the floe diameter, whereas the sensors placed close to the floe edge would measure the highest pack ice pressure peaks local to the width of the perimeter and roughly equal to the distance of the sensors to the floe edge.”* Although there is logic in this argument, in the authors view this does not appear to be an overly convincing argument that the quoted “width of influence” is correct from these pack ice experiments. The width dependence of the ISO 19906 (2010) ridge-building equation (Equation 1) is based solely on this assumption.

These *in situ* field measurement programs made use of the type of geometry (i.e. first-year ice failing against a multi-year floe) which is of importance to pack ice analysis on an offshore platform in the Arctic. As such, they provide useful information on pack ice driving forces. But it should be noted that these events were between two floating bodies and as such the relative interaction speed was likely quite low. This may have resulted in lower ice forces than if the first-year ice was pushing against a multi-year floe anchored by an offshore platform.

In Situ measurements of pack ice pressure at Katie’s Floeberg

An alternative to determining pack ice driving forces by measuring stresses in drifting floes, as described in the previous section, is measuring stresses in floes that have come up against an obstacle. One such obstacle is Katie’s Floeberg, a naturally grounded ice feature that has formed over the 20-25 m deep Hanna Shoals in the Chukchi Sea about 160 km off the coast of Alaska near 160°W, 72°N. The massive

grounded ice rubble field feature at Hanna Shoals changes throughout the year and can be quite different from year-to-year. It lies in the path of a flux of drifting polar pack ice, including many multi-year floes. In April 1985, BP carried out a field program to investigate pack ice driving forces at Katie's Floeberg (Hallam *et al.*, 1985). Seven multi-year floes in reasonable proximity to and with a good likelihood of impacting Katie's were identified, assessed and instrumented with rosettes of ice pressure gauges and ice strain gauges. Floe thicknesses ranged from 1.8 m to 5 m. General meteorological conditions, wind speed and direction and air temperature were measured as well as drift trajectories. Unattended data loggers recorded stresses every 30 seconds. As an indication of the difficulty of making such measurements, only two of the floes yielded results and data from only one were considered acceptable for analysis.

The most noteworthy event was from Floe 6, which experienced stresses as a result of the floe impacting the Floeberg as described here. After principal stress peaked at 350 kPa early on April 16, an average stress of 200 kPa was maintained for about four hours. Ice thickness of the floe at the instrumentation site was 3.2 m, and active ridging was occurring about 100 m from the instrument location and across a front of 400 m. The thickness of the ice actively ridging was estimated to be about 3.1 m.

Hallam *et al.* (1985) used two approaches for the analysis of measurement results leading to pack ice forces. Here, as in the Beaufort Sea measurements, the primary problem is how to relate the measured stress to a floe perimeter ridge building force. The first approach was to use a pressure-area relation with an exponent of -0.5 on area to transfer the measured stress to a pack ice force on the floe perimeter. The average pressure of 0.2 MPa was related to the 400 m front of active ridging (400 m by 3.2 m thick = 1280 m²). This was scaled to the 8.6 km total width of the floe (8600 m by 3.2 m = 27,520 m²) and yielded a pressure of $0.2 \times (27520/1280)^{-0.5} = 0.043$ MPa. This corresponds to a pack ice driving force of 0.14 MN/m (0.043 MPa x 3.2 m thick floe). Unfortunately, the thickness of the pack ice on the far side of the floe is not known, so it cannot be normalized to a 1 m thickness. The second approach was to average measured stress over a longer time period and apply it to the Floeberg as a whole. In this case an average ice stress of 0.1 MPa over about a 35-hour period was obtained, which yielded a pack ice force of 0.32 MN/m (0.1 MPa x 3.2 m). The conclusion of the BP work was that pack ice driving forces at Katie's Floeberg were probably between 0.14 and 0.32 MN/m. *Table 1* summarizes these approaches.

Note that the approaches used by BP are different from the one used by Croasdale *et al.* (1992) described above (i.e. Equation 3). Applying the Croasdale approach to the BP measurements would give a pack ice force of 0.16 MN/m on a 100 m-long front, normalized to 1 m ice thickness force (using a typical stress of 200 kPa measured in ice 3.2 m thick, and 100 m from active ridge building in ice 3.1 m thick). This event has been added to *Table 1* as Event S6.

Table 1: Summary of the in situ stress measurement events

Event	Location & Date	Floe Size	Floe thickness	Edge Length	Ridge building thickness	Typical peak stress	Deduced Edge Loading	Edge Loading normalized to 1 m thickness by $h^{1.25}$	Pack Ice Pressure	Source
		km x km	m	m	m	kPa	MN/m	MN/m	kPa	
Beaufort Sea										
S1	1986 Apr 22	4.5 X 2.5	1.8	1500	1.3	29	0.05	0.04	38	Croasdale <i>et al.</i> (1992)
S2	1989 May 2	3 km diameter	3	500	1.5	40	0.12	0.07	80	Croasdale <i>et al.</i> (1992)
S3	1989 May 12	3 km diameter	3	50	3	240	0.72	0.18	240	Croasdale <i>et al.</i> (1992)
S4	1991 March	4.5 X 2.5	2.5	95	2.5	285	0.75	0.24	300	Croasdale <i>et al.</i> (1992)
S5	1991 April	4.5 X 2.5	2.5	750	1.5	37	0.09	0.05	60	Croasdale <i>et al.</i> (1992)
Katie's Floeberg										
S6	1985, April 16		3.2	400	3.1	200	0.14		45	Hallam <i>et al.</i> (1985)
S6	1985, April 16		3.2	8600	3.1	200	0.32		100	Hallam <i>et al.</i> (1985)
S6	1985, April 16		3.2	100	3.1	200	0.15	0.16	48	Croasdale <i>et al.</i> (1992) approach

Force measurements on an offshore platform

The second approach for estimating pack ice forces uses results from ice force measurement panels that were placed on the outer face of an offshore platform. These force panels were used to record forces on the platform due to a multi-year ice floe that was in contact with the platform and pushed by the pack ice.

Molikpaq old ice loading events

Wright *et al.* (1992) have analyzed five old ice (multi-year and second-year ice) loading events that occurred on the Molikpaq Mobile Arctic Caisson structure in the Beaufort Sea in 1986. The Molikpaq structure was developed by Gulf Canada Resources and operated by Beaudril, a subsidiary of Gulf. It was used for exploration drilling for four winter seasons in the Canadian Arctic. The Molikpaq consists of a continuous steel annulus on which sits a self-contained deck structure. The core of the annulus was filled with sand, which provided over 80% of the horizontal resistance. When deployed at a set down draft of 20 m as it was for the events described by Wright *et al.* (1992), the caisson had a waterline diameter of 90 m and an above water freeboard of 13.5 m. At this draft, the “long sides” of the octagonal caisson were 60 m in length at the waterline while the “shorter sides” were 22 m long. The angle of the outer face was 8° from vertical for these events. The Molikpaq’s extensive instrumentation systems have been well documented (Jeffries and Wright, 1988; Wright *et al.*, 1992; Timco and Johnston, 2003).

During the 1985/86 winter season, the Molikpaq was deployed on a berm in the 32 m water depth at the Amauligak I-65 site. In early March, strong northerly winds moved the pack ice and a number of large multi-year ice floes to the drilling location. This resulted in several interactions of multi-year ice being driven by pack ice against the Molikpaq. These interactions occurred between March 5 and March 8 1986, whereupon the ice in the region became quasi-stationary. During that time period, there were three major multi-year ice interactions which lasted for periods of minutes to tens-of-minutes. After a month of mostly stationary conditions, strong easterly winds on April 11 caused movement of the ice in a westerly and north westerly direction. This resulted in two more multi-year ice floe interactions with the Molikpaq structure. Following these loading events, there was both areal and on-ice reconnaissance to determine the relevant geometry of the old ice floes and the thickness of both these floes and the pack ice. It is important to emphasize that the geometry of these loading events are identical to the geometries of interest for loading of an offshore platform in ice-covered waters. That is, in each case the

pack ice was pushing a large multi-year floe against the offshore platform. Since the floe was wider than the platform, the wider fetch of the floe was “catching” a wider width of pack ice forces and concentrating these loads onto the platform. These were quite well documented events and important ones for understanding pack ice driving forces.

Wright *et al.* (1992) analyzed the loads on the Molikpaq during these five events (Events M1 to M5, *Table 2*), which represent these interaction scenarios where the pack ice pressure was a key driving force. In this case, large multi-year floes of 3 to 7 m thickness were imbedded in moving pack ice and the multi-year feature interacted with the stationary offshore platform. The loads in *Table 2* were estimated by Wright *et al.* (1992) to have an accuracy of 20%. Subsequent analysis by Frederking *et al.* (2011) brought the “historical” analysis of the Molikpaq loads into question, and suggested that these loads could be reduced by up to one-half. For the present analysis, the original data presented by Wright *et al.* (1992) is used since the ISO 19906 (2010) Standard uses the original data values from the Molikpaq in applicable analysis in the Standard. As can be seen from *Table 2*, in four of the five events, the pack ice did not fail but remained intact when pushing the multi-year ice floe. Then, there was continuous crushing of the multi-year ice against the Molikpaq and the pack ice was strong enough to maintain these crushing failures. In event M1 there was ridging of the pack ice behind the multi-year floe; i.e. the pack ice “failed”. This event represented an upper bound value of the pack ice pressure since the failing pack ice could not exert a higher pressure on the multi-year ice floe. Note that the other four events where the pack ice did not fail were less than the upper bound pressure (since it could be argued that higher pressures would be possible before the pack ice would fail). It should be noted that the multi-year floes were firmly frozen into the first-year pack ice matrix and there was no open water or leads in the immediate vicinity of the Molikpaq. Thus these events are relevant to a frozen-in “break-out” condition which is required before ridge building could occur behind the multi-year floe. These five events offer very important and perhaps the most useful information on the pack ice driving force in the Beaufort Sea.

Table 2: Results of the multi-year ice interactions with the Molikpaq (after Wright et al. 1992)

Event	Date	Floe Size	Pack Ice thickness	Global Load	Line Load	Edge Loading normalized to 1 m thickness by $h^{1.25}$	Pack Ice Pressure	Pack Ice Failure
	1986	m	m	MN	MN/m	MN/m	kPa	
M1	March 6	240	1.5	200	0.84	0.51	560	ridging
M2	March 7	900	1.5	245	0.27	0.16	180	no
M3	March 8	3140	1.5	260	0.085	0.05	55	no
M4	April 12	1100	1.6	385	0.35	0.19	220	no
M5	April 12	470	1.6	210	0.45	0.25	280	no

Observations of extreme ice features

The third method to estimate pack ice driving forces does not make use of any directly measured forces. Instead the approach is to observe and measure large ice pile-up features and then use some method to calculate (or estimate) the forces necessary to create these large ice pile-ups. These features result from moving ice sheets breaking apart and piling-up when the ice sheet encounters an ice floe, a beach, shallow water or other obstacles. Observations of these pile-ups have shown that they can be quite massive with reported pileup heights up to 30 m for grounded features generated with thick ice (Barker and Timco 2017). Clearly high forces are required to generate these large ice features. An estimate of

the force required to build the feature could then be interpreted in terms of a pack ice pressure using the feature's geometry and the thickness of the pack ice. Recall that events from which the line loads and pack ice pressures determined in *Table 4* are from "extreme" features and therefore they should not be taken as representative of the natural population of pile-up heights in the Arctic.

In contrast to the first two approaches discussed above, this approach does not use any actual measured ice stress or force information. It is, in essence, a detective approach to try to solve a problem. In this section, there are four different types of extreme ice features that can be used to provide an approximate estimate of pack ice pressures:

1. Ice pile-ups on shorelines and offshore shoals or relic berms;
2. Grounded offshore shear features;
3. Multi-year ice floe splitting;
4. Floating first-year ridges with deep keels.

Ice Pile-ups

Ice Shoreline Pile-ups

Grounded ice pile-ups are common features along the shore lines and in the near-shore waters of the Beaufort Sea. Also there are a number of offshore pile-ups which form over natural shoals or on the relic berms of past drilling sites and these are discussed later in this paper. Barker and Timco (2017) have recently presented a comprehensive review of these pile-up features. Clearly the moving pack ice has sufficient force to create these large features. The following sections provide salient information on a large number of more extreme pile-up events. For these events to be useful here, information on the ice thickness, pile-up height and pile-up length was required. This criterion prevented the use of many of the reported pile-up features, including even more extreme events.

Mahoney *et al.* (2004) report on an ice shove along the Alaskan Chukchi Sea coast which occurred in June 2001. This ice shove affected the communities of Barrow and Wainwright which are about 150 km apart. Analysis of aerial photography before and after the event indicated that up to 395 m of ice motion was accommodated almost entirely in discrete ridges up to 5m high. The ice was greater than 1 m thick and contained some multi-year ice floes up to 3.2 m thick. The ice was well decayed so it had relatively low strength. The length of the interaction region is estimated to be about 2.5 km based on an analysis of the sketches and photos in Mahoney *et al.* (2004). They estimated the forces required to build these ridges to be in the range of 0.035 – 0.062 MN/m (Event P1, *Table 3*).

There have been many observations of pile-ups along the shores of the Beaufort Sea. Shapiro *et al.* (1984) reported on a number of ice pile-ups along the Alaskan northern shoreline. Two in particular have sufficient information for a pack ice force analysis (Events P2 and P3, *Table 3*). Kovacs (1983, 1984) reported on pile-ups along the northern coast of Alaska; many of these were onshore and often had limited height. But there were five observations that had a significant height and the information required for the present pack ice analysis (Events P4 to P8, *Table 3*). Coastal Frontiers (2010) provided detailed information on the freeze-up of the Alaskan Beaufort Sea and the Chukchi Sea in 2009-2010. As part of their study, they did some on-ice measurements and reported on seven pile-up events, two of which are suitable for a driving force analysis (Events P9 and P10, *Table 3*). It should be noted that one of their events at Narwhal Island only listed the maximum pileup height of 16 m along the 1.8 km pileup.

But observations of the feature on the cover of their report indicate that a more representative pileup height would be about 12 m.

Fairway Rock is an island situated in the Bering Strait about 24 km west-southwest of Cape Prince of Wales, Alaska. It is a 350 m diameter igneous rock that rises almost vertically out of a 50 m water depth to a height of about 165 m. At sea level it exhibits surfaces of near-vertical rock and steep talus slopes comprised of large boulders. Its location in the Bering Strait is in the path of large amounts of sea ice that pass through from the Chukchi Sea to the Bering Sea. In the early 1980s, Kovacs *et al.* (1981a, 1982) visited Fairway Rock and using stereo imagery, profiled the icefoot along the its edge. An analysis of their data yielded three pile-ups that could be useful here (Events P11 to P13, *Table 3*). The lengths ranged from 180 m to 420 m with heights ranging from 9.6 m to 14.7 m. Kovacs noted that the ice thickness ranged from 0.5 to 1.5 m in these pile-ups.

In March and April of 1980, Hudson and a number of other researchers (Hudson *et al.*, 1980) conducted a three week field study to investigate the ice dynamics and features along the northwest edge of the Canadian Arctic Archipelago. Fourteen ground sites were visited and more than 2300 line km of aerial photography were studied. Investigations at six sites provided Hudson *et al.* (1980) with information to back calculate the average driving stress in the pack ice. Similar to other pack ice calculations, several assumptions were made in the calculations. The observations included both pile-ups and pushing of large ice features onto the beach. All of these events are summarized here to illustrate the scale of these extreme events, but only the pile-up events with width information are used in the subsequent analysis. Their findings are summarized below:

- Site 3 – there was a very large pile-up event observed southwest of Cape Andreasen on Prince Patrick Island. This feature was 2.5 km long and varied in width from 150 m to 600 m. The maximum sail height was about 12 m and averaged about 6 m. The ice was very weathered and no information on the ice thickness was obtained. (Event P14, *Table 3*). Since there was no ice thickness information available, this event could not be included in the analysis.
- Site 5 – two large blocks of multi-year ice were pushed ashore about 200 m apart at the corner of Eglinton Island. The block at site 5a was 31 m long and 17 m thick and was 60 m from the coastline. The slope of the beach was about 13°. The block at Site 5b was 19 m long and 12 m thick and was found 120 m inland. Hudson *et al.* (1980) calculated the force necessary to push these blocks to their resting position and determined that the driving stresses were 0.53 MPa for Site 5a and 0.18 MPa for Site 5b (Events P15 and P16, *Table 3*). These events are not used in the present analysis.
- Site 6 – There were a number of large multi-year ice blocks pushed up and grounded close to shore south of Cape Mecham on Prince Patrick Island. They observed a very large ice feature which was grounded in about 22 m water depth. They estimated the force necessary to put the block to its observed position and determined that the driving stress in the ice was 0.92 MPa. No ice thickness information was given for this event (Event P17, *Table 3*). This event was not used in the present analysis.
- Site 7 – At Site 7 west of Cape Andreasen, there was significant buckling observed in 0.6 m thick first-year sea ice. The peak-to-peak displacement was 0.5 and the wavelength was about 15 m. Hudson *et al.* (1980) calculated the stress necessary to cause this buckling and determined it to be 0.88 MPa (Event P18, *Table 3*). This event was not used in the analysis.
- Site 8 – The most extreme ice ride-up event occurred north of Cape Malloch on Borden Island. There, an ice island fragment approximately 45 m thick grounded at the edge of the landfast ice. A large pile-up feature was observed where a 5 m thick multi-year floe had bent and slid up at

an angle of 26°. The height of the pile was 22 m. Hudson *et al.* (1980) determined the average driving stress for this feature was 0.28 MPa (Event P19, *Table 3*).

- Site 12 – Fragments of an ice island were observed ashore west of Bernard Island. Hudson *et al.* (1980) calculated the driving force necessary to push these large ice fragments to their final position and estimated it to be 0.33 MPa (Event P20, *Table 3*). This event was not used in the present analysis.
- Site 13 – There was an extreme pile-up event along a 3 km stretch of Banks Island. The feature was formed from first-year ice which was 1.3 m thick. The pile-up feature had a maximum height of 15 m and an average height of 10 m. Hudson *et al.* (1980) estimated the driving stress to create this feature as 0.54 MPa (Event P21, *Table 3*).

Pile-ups on Shoals and Relic Berms

There are a number of reports of very large ice rubble piles that have built up over natural shoals or relic submarine berms in the Beaufort Sea. A few contain the information required for a pack ice force analysis.

Katie's Floeberg is an extremely large ice rubble feature that forms in the Chukchi Sea as discussed in the section on *in situ* measurements in this paper. During the early 1980s, there were a number of site visits to Katie's Floeberg and it was studied in detail. Vaudrey and Thomas (1981) investigated several sites in and around the Floeberg in the spring of 1980. They found that it was a mixture of multi-year and first-year ice, and was highly ridged and extremely rough. Typical elevations were on the order of 3 to 5 m throughout the Floeberg with high values on the order of 12 to 15 m. Site 11, near the southern tip of the Floeberg, is especially interesting for this study since there was a rubble pile (140 m by 300 m) built from ice of 0.4 m thick. The maximum pile-up height was about 13 m (Event 22, *Table 3*). Thomas and McGonigal (1983) visited the Floeberg in March and April of 1981. They reported on a major rubble pile up to about 28 m high in a water depth of about 30 m. This is an extreme pile-up with a total height from base to peak of almost 60 m. The thickness of the ice in the rubble was not measured but the photographs indicated that a suitable range would be from 1 to 3 m. This pile-up was mostly linear with a total length of about 120 m (Event P23, *Table 3*). It was part of a larger rubble pile which was about 1400 m long (Event P24, *Table 3*). The highest peak along this length (neglecting the peak of 28 m) was about 17 m.

McGonigal *et al.* (1986) studied the properties of a very large rubble pile that formed over a natural shoal in the Beaufort Sea in 1978/79. The rubble pile was grounded and stable until late June 1979. It consisted of a nucleus that was 350 m by 150 m across with an average height of 5.4 m. There were more extreme pile-ups with heights 12 m above the water line. This nucleus was well grounded in water depths of 15 to 19 m. By mid-May 1979, this rubble field had grown to a maximum extent of 8.9 km by 6.9 km. The rubble pile was composed of first-year ice with thickness ranging from 0.2 m to 1.5 m. McGonigal *et al.* (1986) estimated the horizontal extent of these features could be on the order of 1 km long and 0.5 km wide. They are very rough with sail heights up to 10 to 14 m. There is usually a wide range of ice thickness that forms the rubble fields. But evidence from an analysis by Canatec (1994) showed that most of the analyzed rubble field forming events involved ice less than 0.3 m thick, with 90% of the events involving ice less than 0.8 m thick. Canatec (1994) felt that this indicated that most rubble fields formed before January 1. The average values of these features are used to estimate the pack ice force that would be required to form the rubble field (Events P27, P28, *Table 3*).

As part of the Barker and Timco (2016) research study, several dedicated field programs were carried out (along with Brian Wright) in the Beaufort Sea. During these field programs, there was a site visit to the Minuk site in the spring of 2010. Figure 3 and Figure 4 show an overview of the rubble field at that time. There was a long (about 400 m) rubble pile-up extending along the western edge of the rubble pile (Event P29, *Table 3*). The highest peak (which was named “Mount Minuk” by the Wildlife Observer) was 13 m high. It was in about 8 m water depth. The ice which created this pile-up was quite thin, about 0.3 m. The average height along this pile-up was about 6 to 9 m.

Figure 3: Photograph of the 400 m long rubble pile at the Minuk site in 2010. The arrow shows the location of the highest peak which was named “Mount Minuk” (photo A. Barker, NRC).

Figure 4: Photograph showing the rubble pile “Mount Minuk” from ice level inside the rubble field (photo G. Timco)

Pile-Up Analysis

Before discussing the other methods for estimating pack ice forces from extreme ice features, it is useful here to discuss how observations of pile-ups can be used to estimate the pack ice forces necessary to generate them.

Recently Barker and Timco (2017) have documented over 230 pile-up features reported in the literature from several regions around the world. Data were chosen if the maximum pile-up height and the block ice thickness were reported. In contrast to the present situation, information on the length of the feature was not a criterion for selection. Thus there are many more data points than analyzed for the present paper. A plot of the rubble sail height as a function of the ice thickness at the time of the rubble formation shows considerable scatter but a general increase in sail height with increasing ice thickness. An upper bound of the data can be represented by $H_{s,max} = 19 h_i^{0.33}$ where $H_{s,max}$ is the maximum pile-up height [m], and h_i is the ice thickness [m]. This equation covers the data range up to 2 m in ice thickness. The analysis of the field data also clearly showed that very high pile-ups (on the order of 10 to 12 m high) can be generated from ice less than 0.5 m thick.

Timco and Barker (2002) performed a parametric study using a numerical model and found that three conditions must be met to generate the highest rubble piles:

1. sufficient driving force,
2. a large fetch area of ice available for feeding the pile-up, and
3. sufficient time for the pile-up to occur.

Investigating the largest pile-ups should provide insight into the higher range of pack ice driving forces. Field observations have been recorded throughout the literature as discussed earlier. The most appropriate features for the present analysis are described below and summarized in *Table 3*. It should be noted that these events were selected since they all had the required information of ice thickness, maximum pile-up height, and pile-up length. But a view of these data points with respect to the Barker and Timco (2017) curve (see Figure 5) shows that these points are below the Barker and Timco (2017) upper bound curve for the maximum pile-up height. This suggests that the pile-up data points discussed in the present paper have not met all three of the criteria mentioned above. If so, they may not necessarily be representative of upper bound pack ice values. The data points closest to the upper bound values are those from Katie's Floeberg Event P22 ($h_i = 0.4$ m and $H_s = 13.1$ m).

Table 3: Summary of pile-up and ice shove events

Pileup Event #	Location	Region	Year	Ice Thickness	Maximum Pile-up Height	Pile-up Length	Source
				m	m	m	
P1	Alaskan coast	new Barrow	June 2002	>1 m	5	2500	Mahoney et al (2004)
P2	Tapkaluk Island	Northern Alaska	1978	0.9	10	900	Shapiro et al (1984)
P3	Barrow	Northern Alaska	1978	1.3	7	725	Shapiro et al (1984)
P4	Cape Halkett	Northern Alaska	1979-1980	0.25	3.5	300	Kovacs (1983)
P5	Point Drew	Northern Alaska	1980-1981	0.3	3	1000	Kovacs (1983)
P6	Lonely	Northern Alaska	1980-1981	0.5	4	500	Kovacs (1983)
P7	Collinson Pt	Northern Alaska	1980-1981	0.3	4	2000	Kovacs (1983)
P8	offshore Tulimanik Island	Northern Alaska	Aug 1983	0.5	5	500	Kovacs (1984)
P9	Arey Island	Northern Alaska	December 2009	0.6	6 to 9	930	Coastal Frontiers (2010)
P10	Narwhal Island	Northern Alaska	January 2010	0.75	16	1850	Coastal Frontiers (2010)
P11	Fairway Rock	North Side	March 1980	0.5 to 1.5	9.6	420	Kovacs et al (1982)
P12	Fairway Rock	South Side	March 1980	0.5 to 1.5	14.7	420	Kovacs et al (1982)
P13	Fairway Rock	East Side	March 1980	0.5 to 1.5	11.1	180	Kovacs et al (1982)
P14	Prince Patrick Island	Cape Andraesen	April 1980		6 to 12	2500	Hudson et al. (1980)
P15	Eglinton Island		April 1980	17	ice block push		Hudson et al. (1980)
P16	Eglinton Island		April 1980	12	ice block push		Hudson et al. (1980)
P17	Prince Patrick Island	Cape Mecham	April 1980		ice block push		Hudson et al. (1980)
P18	Prince Patrick Island	Cape Andraesen	April 1980	0.6	ice buckling		Hudson et al. (1980)
P19	Borden Island	Cape Malloch	April 1980	5	22		Hudson et al. (1980)
P20	Bernard Island		April 1980		ice block push		Hudson et al. (1980)
P21	Banks Island	Site 13	1980	1.3	15	3000	Hudson et al. (1980)
P22	Katie's Floeberg	Site 11	1980	0.4	13	300	Vaudrey & Thomas (1981)
P22	Katie's Floeberg	Site 11	1980	0.4	13	140	Vaudrey & Thomas (1981)
P23	Katie's Floeberg		1981	1 to 3	28	120	Thomas & McGonigal (1983)
P24	Katie's Floeberg		1981	1 to 3	28	1400	Thomas & McGonigal (1983)
P25	Offshore Shoal	Nucleus	1978/79	0.2 to 1.5	5.4	350	McGonigal et al. (1986)
P26	Offshore Shoal	Mid-winter width	1978/79	0.2 to 1.5	5 to 12	1100	McGonigal et al. (1986)
P27	Generic Rubble Field	Length		0.2 to 0.8	14	1000	Barker and Timco (2015)
P28	Generic Rubble Field	Width		0.2 to 0.8	14	500	Barker and Timco (2015)
P29	Minuk 2010	west edge	2010	0.3	6 to 13	400	Barker and Timco (2015)

Figure 5: Maximum pile-up sill height versus the ice thickness at formation including the upper bound equation developed by Barker and Timco (2017). The data points in Table 3 are highlighted with a solid symbol. The open symbols are pile-ups reported by Barker and Timco (2017) which do not contain information on the pile-up length. The majority of the pile-up data points used in this paper do not lay close to the upper bound limit values.

Table 3 provides a basis for a pack ice force analysis. But questions remain on the best approach to analyze this data: physical modelling, numerical modelling or analytical models. Each has advantages and disadvantages.

Physical modelling is one approach to analyze the field pile-up events (see e.g. Timco 1991; Hopkins 1997; Sodhi *et al.* 2003). However, dedicated physical model test programs are expensive and have limitations in terms of correctly simulating pile-up heights on grounded features, so this approach is not used in the present paper.

Numerical models provide another approach for analyzing pile-up events. For example, Barker and Timco (2007) modelled the rubble field development around the Molikpaq at the Isserk I-15 site. The results of the modelling showed good agreement with field observations of the rubble pile-up. However, the forces necessary to generate the rubble field were not presented since this particular numerical model essentially assumes an infinite driving force. A model would have to be specially developed or adapted to be useful in analysing the forces that might be generated by pack ice, and would require rigorous testing and validation.

Analytical models present the most convenient pack ice force analysis method but many assumptions are involved in describing the mechanics of a very complex interaction process. Further, analytical models, which are usually 2-D, are essentially based on static analysis and do not contain any

information on the dynamics of the process; that is, they do not include any temporal components or variation. This is a definite weakness since the complex mechanics do not occur simultaneously so a simple addition of the components may not accurately represent a true ice load. Nevertheless, analytical approaches are useful in the estimation of the pack ice forces necessary to have created large pile-ups. Analytical models employ, as a core, an assessment of the force required for the frictional force to overcome the potential energy of the rubble pile. The models usually include as input the height of the pile, coefficient of friction, ice thickness, weight of the ice, slope of the pileup and other factors depending upon the sophistication of the model. Some of these properties are difficult to measure or observe, and must be assumed. Various analytical theories have been presented to predict the force necessary to cause the pile-up. Earlier models were developed by Allen (1970) and Shapiro (1976) but Kovacs and Sodhi (Kovacs, 1983, 1984; Kovacs and Sodhi, 1980, 1981b) and Croasdale (Croasdale *et al.*, 1978, 1994; Croasdale, 1980, 2012a) did the pioneering work in this area. Christensen (1994) provided a considerable amount of information on ice pile-ups observed in Europe and based on these data, developed a model for ice ride-up and pile-up. Croasdale's papers outlined analytical equations describing the physics of ice moving up an inclined slope. The ISO Arctic Structures Standard (ISO 19906, 2010) basically follows the Croasdale (1980) and Croasdale *et al.* (1994) approach for ice actions on a sloped structure. The more recent Croasdale (2012a) approach is used in the present analysis. The main differences are a revised formulation for the bending failure load and the omission of a term for the overturning of ice blocks in Croasdale (2012a). It should be noted that these models are essentially two-dimensional to get a line load per unit width. This value is then multiplied by the width to get the total ice load. The application of the Croasdale (2012a) model and the assumptions used in its application is discussed in detail in Appendix A.

Table 4 provides a summary of the calculated total load, line load, and average pack ice pressure based on the input data from *Table 3* and the Croasdale (2012a) model. The table also includes the line load normalized to a thickness of 1 m with an exponent of 1.25. All values in the table were generated using the Croasdale (2012a) equations and the "base" values of the parameters discussed in Appendix A using the sail height, ice thickness and length of the pileup from *Table 3*. The pressure estimates made by various authors outlined in *Table 3* were not used in subsequent analysis since little or no information was supplied on their approach for estimating the pressure.

Table 4: Summary of the results for the ice pileup data

Pileup Event #	Ice Thickness for calculations	Pile-up Height for calculations	Pile-up Length	Total Load	Line Load	Edge Loading normalized to 1 m thickness by $h^{1.25}$	Pack Ice Pressure
	m	m	m	MN	MN/m	MN/m	kPa
P1	1	5	2500	347	0.14	0.14	139
P2	0.9	10	900	209	0.23	0.26	258
P3	1.3	7	725	178	0.25	0.18	189
P4	0.25	3.5	300	7	0.02	0.13	93
P5	0.3	3	1000	26	0.03	0.12	87
P6	0.5	4	500	28	0.06	0.13	112
P7	0.3	4	2000	65	0.03	0.15	108
P8	0.5	5	500	34	0.07	0.16	136
P9	0.6	7.5	930	109	0.12	0.22	195
P10	0.75	12	1850	423	0.23	0.33	305
P11	1	9.6	420	105	0.25	0.25	250
P12	1	14.7	420	156	0.37	0.37	371
P13	1	11.1	180	51	0.28	0.28	283
P21	1.3	12	3000	1210	0.40	0.29	310
P22	0.4	13	300	39	0.13	0.41	325
P22	0.4	13	140	18	0.13	0.40	321
P23	2	28	120	166	1.38	0.58	692
P24	2	17	1400	1210	0.86	0.36	432
P25	1	5.4	350	52	0.15	0.15	149
P26	1	8	1100	232	0.21	0.21	211
P27	0.8	10	1000	206	0.21	0.27	258
P28	0.8	10	500	103	0.21	0.27	258
P29	0.3	9	400	27	0.07	0.30	225

Grounded Offshore Shear Features

As part of the NRC field study program mentioned previously, a visit was made in April 2007 to the Minuk site. Figure 6 shows the southern “wall” of the rubble field. This shear wall was approximately 130 m long and 23 m high (from the seabed). Recently Timco and Barker (2015) analyzed this large-scale shear and the ride-up of the level ice in terms of the pack ice driving force necessary to have caused the feature. As in all efforts to refine the pack ice force, a number of assumptions had to be made in their analysis. Shear strength values for the rubble of 14 kPa and 30 kPa were used based on the assumption of partially consolidated rubble. Timco and Barker (2015) calculated six different estimates for the pack ice force depending upon the assumed length (90 m, 400 m and 3000 m) of the interaction and the shear strength of the ice rubble. *Table 5* presents a summary of their results. It should be noted that this was a frozen-in feature; that is, the ice was not mobile when the shear event began. It is also important to note that although this was one event, there were six estimates made for the force due to the uncertainty in the length of the interaction and the strength of the rubble. Finally it is necessary to note

that due to the high number of assumptions made in the analysis, the confidence level for the data from this shear wall event is very low.

Figure 6: Photograph of the shear wall at the Minuk site in the spring of 2007. The feature was approximately 130 m long and 23 m high from the seabed (photo A. Barker, NRC).

Table 5: Summary of the results of shear wall analysis

Event #	Source	Width of Interaction	Ice Thickness	Assumed Shear Strength	Load	Line Load	Edge Loading normalized to 1 m thickness by $h^{1.25}$	Pack Ice Pressure
		m	m	kPa	MN	MN/m	MN/m	kPa
S1	Minuk shear 07	90	1.5	14	42	0.47	0.28	310
S2	Minuk shear 07	90	1.5	30	90	1.00	0.60	660
S3	Minuk shear 07	400	1.5	14	42	0.10	0.06	70
S4	Minuk shear 07	400	1.5	30	90	0.22	0.14	150
S5	Minuk shear 07	3000	1.5	14	42	0.01	0.01	9
S6	Minuk shear 07	3000	1.5	30	90	0.03	0.02	20
S7	Tarsiut shear 07	80		30	54			

In April of 2007, there was also a site visit to the location of the former Tarsiut caissons. The submarine berm that remained created a very large rubble field as shown in Figure 7. The Tarsiut location is much closer to the edge of the landfast ice and consequently is in much more dynamic ice conditions for a longer period during the winter than the Minuk site. The aerial view of the rubble field shows that it has been sheared off by the moving pack ice. The rubble field shows clear signs of an upstream ice wedge front and a wake behind it. The shearing action created vertical walls as shown in Figure 7b. The top of the wall was 7.3 m from the sea level and in 15 m water depth so the total height of the shear wall was approximately 22.3 m from the seabed. It is estimated that the length of the shear wall was approximately 80 m. By using an estimate of rubble shear strength, it is possible to estimate the load necessary to create this shear wall. In large-scale field experiments Croasdale *et al.* (2005) measured keel strengths ranging from 4 to 23 kPa, with an apparent significant cohesive strength component.

Croasdale *et al.* (2005) suggest maximum shear strength of ridge keels of 30 kPa, with a sensitivity of 55 kPa. In the present analysis, a value of 30 kPa will be used. It should be noted that Croasdale *et al.* (2005) measured these strength values on ridge keels, which would be weaker than the partially consolidated rubble pile at the Tarsiut site. Nevertheless, with this gross assumption, a force of 54 MN is calculated for this event (Event S7, *Table 5*). Unfortunately there is no mechanism to convert this to a pack ice force. It is mentioned here to illustrate the extreme pile-ups and shear walls that can be created by higher pack ice driving forces.

Figure 7: Photographs of the ice rubble pile at the Tarsiut site in April 2007. Photo (a) shows the whole rubble field whereas Photo (b) shows the large vertical wall indicated by the arrow in the upper photo. (photos G. Timco)

Multi-year ice floe splitting at Katie's Floeberg

Vaudrey and Thomas (1981) made a site visit to Katie's Floeberg during late April 1980. There was a feature at Site 4 which was quite interesting and could be used to study pack ice forces. A large (1830 m by 2440 m) multi-year ice floe (3 m thick) was sandwiched between 1 m thick first-year ice at the Floeberg and the wind-driven moving multi-year ice pack. A ridge formed from the first-year ice between the Floeberg and the multi-year floe. The "trapped" multiyear ice floe was split with three different cracks along the lines of compression in the floe. Vaudrey and Thomas (1981) used a simple analysis assuming this was a typical "Brazil" test. They calculated a force of 0.950 MN/m over a length of 2130 m. The extremely high force level calculated for this event is quite anomalous when compared to other pack ice data. The reason is the incorrect use of the Brazil test analysis. Although these types of tests were very fashionable during the 1960s, it was later shown by CRREL that the mechanical properties of ice did not satisfy the assumptions for its use (see Hawkes and Mellor, 1970; Mellor and

Hawkes, 1971). The baseline data of this fracture process certainly appears to be quite relevant for a fracture mechanics analysis. The authors contacted Prof. John Dempsey who is an expert in the fracture mechanics of sea ice. He examined this information and noted that any solution will involve too many questionable assumptions, and felt that a reliable analysis could not be done using fracture mechanics, limit analysis, or a strength theory.

Floating ridges with deep keels

Reviews of the shape and size of sea ice ridges (Timco and Burden, 1997; Sudom *et al.*, 2011; Strub-Klein and Sudom, 2012) show that keel depths on the order of 28 m have been measured in the Beaufort Sea. This depth corresponds to the water depth in the Beaufort Sea in which seabed scours become much less prevalent (see e.g. Blasco *et al.*, 1998). This limiting depth could be a barometer of the available driving force to generate these deep keels, or it could reflect the properties of the broken ice rubble. If the assumption is made that these limiting keel depths are a result of the available driving force, there are various theories that can be used to predict the force. *Table 6* presents a summary of four different approaches.

Sayed and Frederking (1984) presented a plasticity theory to describe the free-floating ridge-building process. They determined a relationship between line force (F) and keel depth (D) as $F = C \gamma D^2$ where γ is the unit weight of the ice rubble in the sail or keel. The coefficient C depends on the shape of the rubble. The authors give a value of 1.08 for a 45° frontal slope of the ridge. The plasticity theory gives a line load of approximately 0.7 MN/m for a 1.5 m thick ice sheet generating the maximum keel depth discussed above.

Sayed and Frederking (1986, 1988) analytically examined the formation of ridges using a limit height approach. They determined a relationship between line force (F) and keel depth (D) as $F = 0.76 \gamma D^2$ where γ is the unit weight of the ice rubble in the sail or keel. They produced a plot of the total predicted ice force as a function of the keel depth (or sail height). This approach could be used to get an estimate of the force necessary to create deep ridges in the Beaufort Sea. The Sayed and Frederking (1986, 1988) approach suggests that the line force to generate this keel depth is approximately 0.49 MN/m, assuming an ice thickness of 1.5 m.

Kovacs and Sodhi (1980) presented a potential energy method. The force using this approach was given by $F = 0.5 \gamma_i h_i D$ where γ_i and h_i are the unit weight and thickness of ice respectively. The authors included a term for the friction of an ice sheet sliding over the slope as $F_f = \mu \gamma_i h_i D \cot \beta$ where μ is the coefficient of friction and β is the slope. This approach predicts a line force of about 0.030 MN/m assuming a 45° slope and a friction coefficient of 0.1 (and the same conditions discussed above).

Sayed and Frederking (1986, 1988) compared these three approaches to the results of a series of model tests (Timco and Sayed, 1986). They found that their analytical model and the plasticity model represented well the data (scaled to the appropriate conditions). On the other hand, the potential energy method largely underestimated the required force to build the ridge.

Hopkins (1998) used a two-dimensional particle model to investigate the mechanics of ice ridging. He discussed that there are four stages to ridging and derived the following expression for the average ridging force (in kN/m) as $F = 95.4 h^{1.5}$ where the ice thickness h is in meters. For a typical maximum ice thickness for first-year ice in the Beaufort Sea of 2 m, this gives a ridging force of 0.27 MN/m.

Table 6: Results from deep (28 m) ridge keel models

Event #	Approach	Ice Thickness	Line Load	Edge Loading normalized to 1 m thickness by $h^{1.25}$	Pack Ice Pressure	Source
		m	MN/m	MN/m	kPa	
T1	Plasticity theory	1.5	0.70	0.42	470	Sayed and Frederking (1984)
T2	Limit Height	1.5	0.49	0.30	330	Sayed and Frederking (1986, 1988)
T3	Potential Energy	1.5	0.03	0.02	20	Kovacs and Sodhi (1980)
T4	Particle Model	2	0.27	0.11	140	Hopkins (1998)

It should be noted that none of these approaches take into account in any explicit manner the width of the interaction process, so they cannot be used in the present pack ice analysis for determining R values. They are included here since they give some indication of the calculated pressures from deep ridge keel models. Note that Sayed and Frederking (1986) indicate that field observation of ridges shows that although ridges can be quite long in extent, they can have relatively uniform sail heights for distances on the order of 100 m or so.

Discussion and Summary Plots

The present paper has critically viewed past pack ice measurements and added several new events. All of the events have information on the width and the pack ice thickness, and the resulting (normalized) line loads and pack ice pressures have been calculated. The first question to address is: how relevant are these values to the pack ice question? Examining the data indicates that the ice thickness for the events outlined here are all representative of the range of pack ice thickness in the Arctic. With regard to the width, decisions must be made. Typically an offshore platform in the Arctic is on the order of 100 to 150 m wide. Floe sizes of these diameters or less would not be analyzed in terms of the pack ice process. Therefore events with interaction widths of less than 150 m are not included in the analysis for the remainder of this paper. Using this criterion reduces the number of events from 44 to 33.

The pack ice events noted in this paper involve interaction lengths up to about 3000 m. The question is: Is this a reasonable range of floe sizes in the Arctic? Johnston *et. al* (2015) analyzed the floe sizes of old ice floes in the Beaufort Sea using the Canadian Ice Service regional ice charts for the years 1980 to 2009. They showed that vast floes (2 – 10 km) are generally the most common, with only two years of giant floe (>10 km) prevalence. Canatec (2010) examined multi-year floe size in the Beaufort Sea from satellite imagery for 2008 and found that median floe diameter decreased from 6 km in mid-winter to 2 km in late winter, and down to 0.8 km by late summer. Thus the range of widths up to 3000 m from the present data is in the range of typical floe sizes that could be encountered in the Beaufort Sea.

It should also be noted that some of the events reported represent the situation where the pack ice is frozen-in before the interaction event. The five data points from the Molikpaq and the four events from the shear wall analysis are these types of events. These “frozen-in” events are plotted using solid symbols together with the “moving ice” data (open symbols) in Figure 9 and Figure 10. As pointed out earlier in this paper, ISO recommends that frozen-in events be treated differently as one might expect higher loads in this case.

It is possible to view the data outlined in this paper in a number of different ways. We start by considering the ISO 19906 (2010) approach discussed earlier (Equation 1). Figure 8 shows the normalized line load versus the interaction width for all data discussed in this paper included the *in situ* tests (Table 1), the Molikpaq data (Table 2) the ice pile-up data (Table 4) and the shear wall analysis (Table 5). The lines for $R=2$ and $R=10$ are also shown. These are the boundaries for R outlined in the ISO 19906 (2010) Standard. The highest line load values occur for the narrowest widths, but that there are only a few data points higher than a normalized line load of 0.3 MN/m. These few points along with the R curves give the appearance of an upward trend for smaller widths. And undoubtedly, this could be correct since line loads on a 100 m wide caisson have been shown to be 0.55 ± 0.15 MN/m for mixed mode ice failures (Timco 2007). However, if one neglects the data point above 0.5 MN/m (and ignores the two R curves superimposed on the graph) it could easily be argued that the normalized line load is not strongly dependent on the interaction width for widths greater than 150 m. This raises the question if, in fact, the observation of no width dependence is correct. Examining individual data sets may shed light on this question. First, there does appear to be a downward trend in the *in situ* stress data. But recall that the interaction width from these field experiments was essentially defined as the distance of the sensor to the floe edge. There is an open question if this is in fact a reasonable value for the width of ridging along the floe edge, so little confidence can be placed in the width dependence observation for the *in situ* stress data. The Molikpaq data does show a definite trend of width dependence. As noted previously, this data set appears to be reliable and well-documented for pack ice analysis. Further data and/or analysis are required to address this question. The pile-up data do not show any trend of width dependence. But these are based on a two-dimensional model that has been extrapolated to three dimensions by multiplying the line load by the width of the feature, and there is no mechanism to obtain a width effect. The shear wall data represent a single event with uncertainties in the geometry of the actual event so nothing can be said about width effect in this case. Unfortunately it is not possible to determine if there is a width effect using the available data presented here.

Figure 8: Graph of the normalized line load versus the interaction width. The data are grouped according to the approaches discussed in the paper. The lines for $R=2$ and $R=10$ are also shown. These lines set the bounds for the R value in the ISO 19906 (2010) Standard. Note that the solid symbols represent frozen-in conditions whereas the open symbols represent moving pack ice conditions.

It is possible to use Equation 1 with the data for each event to calculate the corresponding R value. Figure 9 shows the R values for all 34 events where an R value could be determined. This plot assumes that Equation 1 is correct and that the exponents for h and D are also correct. The majority of the R values are in the range of 2 to 10 as outlined in ISO 19906 (2010), but there are a few events with higher R values. It is interesting to note that the higher R values are associated with larger interaction widths.

Figure 9: Graph of the R values for each of the events. Note the arrows superimposed on the Molikpaq data. The upwards arrows indicate that these are not upper-bound values whereas the downward arrow indicates an upper bound value for this event. Note that the solid symbols represent frozen-in conditions whereas the open symbols represent moving pack ice conditions.

The question arises: “Does ISO 19906 (2010) give the best approach for dealing with pack ice driving forces?”. It is instructive to examine this approach based on the data presented in this paper:

- First, the ISO 19906 (2010) approach explicitly assumes that the pack ice will fail by ridging. While this is a likely failure mode, the pack ice could also fail by rafting, buckling, higher in-plane consolidation, etc. which would have different failure loads. Thus, this assumption of only a ridging failure mode may not be necessarily correct;
- Even if ridging is the failure mode, it is highly unlikely that the pack ice will fail simultaneously across the width of the multi-year ice feature;

- The form of the expression for the pack ice force was developed by Croasdale *et al.* (1992) using initially only the five data points from the *in situ* measurements. His analysis gave a value of $R = 2$;
- Subsequently with the addition of the Molikpaq data, the Wright *et al.* (1992) analysis of five data points gave indicated values of $R = 10$ (although 4 of the 5 data points were actually lower bound values). Thus based on these 11 data points, the ISO 19906 (2010) proposed a range for R of 2 to 10;
- The present paper adds many more data points and several do not lie in this range for R ;
- The ISO 19906 (2010) approach “normalizes” the ice to a 1 m thickness by raising the ice thickness to a power of 1.25. Croasdale (2012a, 2012b) recently questions this approach and suggests that the thickness should be normalized to a power of 1.1. He indicates that the power 1.25 arose because the bending term is proportional to this power. But Croasdale (2012a, 2012b) suggests that bending is less important than the frictional, gravity and buoyancy terms (which are approximately proportional to thickness to the power 1.0). Thus there is uncertainty in this approach;
- The exponent for taking into account the width effect (i.e. $D^{-0.54}$) was based only on the five original *in situ* data points and based on the unsubstantiated assumption that the interaction width was the same as the distance of the sensor to the floe edge. The new data presented here do not follow this trend.

The question is: Is there a better approach to treat pack ice driving forces? A simpler approach is to treat this situation by looking at the pressure estimated for each of the events. Figure 10 shows the pressure values for each event as a function of interaction width. Similar to the R -value plot (Figure 9), there is considerable scatter and no clear trend of pressure with interaction width. Note that although there are some higher pressures for narrower widths, the pressures are below 400 kPa for interaction widths of interest for pack ice pressures.

Figure 10: Pressure versus the interaction width for each of the four types of approaches discussed in this paper. Note that the deep ridge keel models predicted pressures in the range of 20 to 470 kPa which are in line with the data in this plot. Note that the solid symbols represent frozen-in conditions whereas the open symbols represent moving pack ice conditions. The upwards arrows indicate that these are not upper-bound values whereas the downward arrow indicates an upper bound value for this event.

Jordaan *et al.* (1993) and Frederking (2003) used an approach for analyzing measured values of local ice pressures on ships. It is possible to use the same approach here to estimate the probability of a higher value of a specific value of pressure (p). In this analysis, the pack ice pressure (p_p) values (in kPa) are arranged in descending order and plotted using the following expression for the probability of exceedance (P_e) as $P_e = i/(n+1)$ where i is the i^{th} ranked data point and n is the total number of data points. Figure 11 shows the ranked pressure data. The following 3-parameter Weibull function was found to fit the data well:

Equation 4
$$P_e = \exp (-[(p_p + 1.7174)/218.08]^{1.5967})$$

Figure 12 shows the log of the probability of exceedance versus the pack ice pressure. Note that the upper bound value from the Molikpaq data is the highest pressure (560 kPa) observed in all of the data discussed in this paper. It is interesting to note that the ridge-building forces predicted for deep keel ridges (discussed in the section Floating ridges) cover basically the same range as that shown in Figure 11. The analytical models predicted pressures of 20, 330 and 470 kPa and the numerical model predicted ridge-building pressures of 140 kPa (from Table 6).

Figure 11: Probability of exceedance versus the pack ice pressure. The data is well represented by a 3-parameter Weibull function (Equation 4).

Figure 12: Log graph of the probability of exceedance versus the pack ice pressure.

Engineering Implications

This paper has focused on obtaining new data that can be related to pack ice driving forces in Arctic regions. It has also critically examined the assumptions used in deriving pack ice driving forces. The analysis shows that the value of R covers a wider range and values for some events are higher than that those proposed in the ISO 19906 Arctic Standard (ISO 19906, 2010). This paper has also suggested that using a simple pressure approach to describe pack ice forces might be more advantageous since the assumptions of width dependence and ice thickness power-dependence are not required.

Figure 13 illustrates the basic scenario for consideration: a thicker ice feature (such as a multi-year floe) of width D and floe thickness H is being pushed by competent pack ice of thickness h against an offshore structure of width w . The load on the offshore structure will be dictated by the lower of the limit stress and limit force approaches.

Figure 13: Sketch of a multi-year ice floe pushed by pack ice against an offshore platform. The load on the platform will be the lesser of the limit force (from the pack ice) or the limit stress (from the failure of the multi-year floe).

As an example scenario, consider that the multi-year floe is 500 m wide and 4 m thick, and is being pushed by competent pack ice of 1.3 m thickness against a 100 m wide offshore platform. First, we estimate the global load from the larger ice feature crushing on the platform from the ISO 19906 (2010) Equations A8-20 and A8-21. The limit stress global load (F_G) on a wide, rigid structure from an ice feature of greater than 1 m thickness is given by

$$\text{Equation 5} \quad F_G = p_G w H = C_R (H/H_1)^{-0.3} (w/H)^{-0.16} w H$$

where the ice strength $C_R = 2.8$ MPa for the Beaufort Sea, H is the ice thickness of the multi-year ice feature (m), H_1 is a reference thickness of 1 m, and w is the structure width (m). ISO 19906 (2010) states that C_R can be assumed as 2.8 MPa for a deterministic analysis in an extreme level ice event (annual probability of exceedance not greater than 10^{-2}). ISO 19906 (2010) also notes that a probabilistic approach can be used by determining probability density functions for the parameter C_R and the ice thickness H . Using a simple deterministic approach for the example event given above, we calculate $F_G = (2.8 \text{ MPa}) (4 \text{ m} / 1 \text{ m})^{-0.3} (100 \text{ m} / 4 \text{ m})^{-0.16} (100 \text{ m}) (4 \text{ m}) = 441 \text{ MN}$.

Next, we calculate the limit force load (F_p) produced by the pack ice pressure (p_p) on the back side of the floe using the expression

$$\text{Equation 6} \quad F_p = p_p D h$$

In order to use the pressure method proposed in this paper, we must choose a pack ice pressure value with the same probability of exceedance (P_e) as that chosen for the limit stress approach. Using Equation 4 (or Figure 12), the pack ice pressure for a probability of exceedance of 10^{-2} for this example would be 565 kPa. The resulting pack ice driving force using Equation 6 is then $F_p = (0.565 \text{ MPa})(500 \text{ m})(1.3 \text{ m}) = 367 \text{ MN}$. Thus, in this case, F_p is less than F_G and the pack ice would not have sufficient force to drive the thicker multi-year ice feature against the offshore structure. The pack ice would fail behind the multi-year floe and an estimated force of 367 MN would be exerted on the platform.

For the same example scenario, we can compare the results from the ISO 19906 (2010) approach for the limit force. The expression for the ridge building line load p_D was given in Equation 1. The pack ice or ridge building force in ISO 19906 (2010) is then:

$$\text{Equation 7} \quad F_B = p_D D = (R (h^{1.25}) (D^{-0.54})) D$$

This approach requires choosing an appropriate value, or distribution, for R . ISO 19906 (2010) recommends using $R = 6$ (for an estimated 50% confidence level), or a uniform distribution of $R = 2$ to 10. In contrast to the previous case where the pressure value is expressed in terms of a probability of exceedance, the R value from the ISO 19906 (2010) is not given in this format. Therefore it is not possible to compare this situation with a $P_e = 0.01$ for both limit stress and limit force. But it is possible to determine an “upper bound” limit force from the ISO 19906 (2010) approach. First it is instructive to look at the range of values that would be given for the ISO 19906 (2010) approach and then estimate an upper bound value. For this simple example, using Equation 7 and assuming $R = 6$, we calculate the pack ice force to be $F_B = (6) (1.3 \text{ m})^{1.25} (500 \text{ m})^{-0.54} (500 \text{ m}) = 145 \text{ MN}$. If a higher R value is used, say $R = 10$, we would calculate $F_B = 242 \text{ MN}$ – still lower than the force calculated with the pressure method. But if the provision of a frozen-in condition is assumed, this value should be multiplied by 1.5, giving $F_B = 363 \text{ MN}$. This is in line with the value calculated using the pressure approach and would represent an upper bound value for the ISO 19906 (2010) approach for this example.

It is possible to do a similar probability of exceedance analysis for the R values. A two-parameter Weibull function gives the best representation as

$$\text{Equation 8} \quad P_e = \exp [(-R/7.6572)^{1.4395}]$$

In this case for $P_e = 0.01$, we calculate $R = 22$, giving a limit force value of $F_B = 532 \text{ MN}$ using Equation 7. This value is higher than the calculated limit stress so in contrast to the previous predictions, the limit stress would dictate the load on the platform. Note that there is a significant difference in the limit force of 367 MN calculated using the pack ice pressure approach and the value of 532 MN using the extrapolated R value approach. The reason for the large discrepancy can be found in the influence of the exponents for h and D in the ISO 19906 (2010) approach and that this analysis indicates values of R much higher than the original maximum value of 10 given in ISO 19906 (2010).

It is important to recall that the data from which pack ice pressures and R values were determined, and the probability of exceedance relations (Equation 4 and Equation 8) are derived largely from extreme pile-up heights. As noted previously, they are not representative of the natural populations of these features; rather they were selected to try to obtain pack ice forces which represent more extreme values (i.e. during high wind or storm events). These equations simply represent the best fit through the data outlined in this paper and as such they are not intended for design purposes.

Single representative values of ice thickness, floe size and pack ice pressure have been used in this example scenario for illustrative purposes. The comparative results of a calculation as given above would change for different values of floe size and pack ice thickness since the ISO 19906 (2010) approach is not linear with respect to these parameters. For design purposes, a more rigorous approach would be required, using the best available distributions (i.e. floe geometries, floe thickness, pack ice thickness, pile-up heights, limit stress values for crushing and/or mixed mode failure, pack ice pressure, R , etc.) in an appropriate probabilistic model. This paper provides information on the range of pack ice pressure values that could be used in a probabilistic model.

Final Comments

This analysis of the pack ice driving force approach has been humbling. It has been shown that in addition to the previously reported estimates of pack ice force, additional data can be obtained by observing and measuring an extreme ice feature and “back calculating” the force necessary to create the feature. However in all cases, a significant number of unsubstantiated assumptions must be made. Although some of the assumptions are based on known physical properties of sea ice, many of the parameters involved can have a wide range of values. Without actually witnessing the creation of the ice feature and taking measurements, it is not possible to narrow down this range. Even if one had direct observations of a ridge or rubble pile being built with ice of a known thickness and measurements of the resultant geometry of the ridge, there still would be a need for some sort of an 'assumed' analytical model and representative ice properties to estimate a pack ice pressure or get an R value. Also, many of the calculated values are based on analytical models which are sophisticated, but they cannot truly represent the complex nature of building a ridge or rubble pile since they do not take into account either the spatial or temporal components of the process. Nevertheless, this paper has been able to develop a much larger data set on pack ice force estimates than previously available. Further it showed that 19906 ISO Arctic Offshore Structures Standard (ISO 19906, 2010) in its present form is limited in how it describes pack ice driving forces based on all available data. A different approach using the calculated values of pressure that the pack ice can exert is proposed and was illustrated with a simple example.

The question is: Where does the ice mechanics community go from here? The paper has illustrated that getting new and reliable data on pack ice forces is extremely challenging and likely no new data will be obtained in the foreseeable future. Therefore it is necessary to build on the available data and insight gained from using it. The NRC is currently investigating the engineering implications of the uncertainty in the pack ice force. From this paper it is clear that some key questions to address include:

- Under what circumstances is pack ice pressure of importance for Arctic structures?
- What are the critical multi-year floe ice thicknesses and widths for the Arctic?
- Is there an interaction with dependence for the pack ice pressure? Past models of non-simultaneous failure (see e.g. Kry, 1978) suggest that there should be a reduction of the pack ice pressure with increasing width. Methods to investigate this would be desirable.

- Are there geographical differences for R and the pack ice pressure for different regions of the Arctic? That is, should the values chosen for an engineering analysis only be based on values obtained in the geographical location of interest for the offshore platform?
- Can the difference in the calculated values using the R approach or the pack ice pressure approach be reconciled? If not, which provides a more accurate approach?
- How sensitive is a probabilistic analysis of ice loads on an Arctic platform to the choice of R or pack ice pressure used for the analysis?
- How can the situations of moving pack ice and frozen-in conditions be quantitatively addressed?
- Is there an upper limit to pack ice pressures?
- Would a re-analysis of the pile-up data using a more sophisticated numerical model provide better estimates of pack ice forces from these features?

A rigorous engineering analysis should shed light on addressing these important questions.

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Appendix A

The Croasdale (2012a) analysis is quite sophisticated and uses a very detailed mechanics model of the pile-up process. The details will not be dealt with here and the reader is referred to the Croasdale (2012a, 2012b) papers for details. However, several of the key parameters used in the model, along with their range and the certainty of the values, are discussed below. For the present analysis, a representative value for each parameter is given and these are used as the “present base values” for the parametric analysis in *Table 7*. The base values used by Croasdale (2012b) in an analysis of ride-up behaviour are given in brackets.

- Flexural strength (500 kPa) - The flexural strength is a function of the porosity in the ice. Timco and O’Brien (1994) have shown that the flexural strength (σ_f) is related to the brine volume (v_b) by $\sigma_f = 1.76 e^{-5.88 \sqrt{v_b}}$. The range of flexural strength values can be quite large with values on the order of 750 kPa for cold sea ice and 200 kPa for warm sea ice. The value of 500 kPa represents a mid-range value and is also used as the present base value.
- Specific weight of ice (8.9 kN-m⁻³) – The specific weight is the density of ice times the gravitational constant (9.8 m-s⁻²). Ice density has measured values reported over a range from 0.72 to 0.94 Mg-m⁻³ with *in situ* values of 0.84 to 0.91 Mg-m⁻³ for ice above the waterline and 0.90 to 0.94 Mg-m⁻³ for sea ice below the water line (see Timco and Frederking 1996). The value for specific weight of 8.9 kN-m⁻³ chosen by Croasdale (2012b) appears to be a reasonable representative value based on the *in situ* density measurements.
- Specific weight of water (10.08 kN –m⁻³) – This value is related to the density of the sea water and although it varies with temperature and salinity, the base value is a reasonable choice for this analysis.

- Young's modulus (5 GPa) – Croasdale (2012b) has chosen a value of 5 GPa which is reasonable for Young's (strain) modulus for sea ice (see Timco and Weeks 2010).
- Sail angle of repose (25 degrees) – Timco and Burden (1997) have analyzed a large number of first year sea ice ridges from the Beaufort Sea. They found that the slope of the ridges approximately followed a log normal distribution with a mean of 33° with a standard deviation of 9.2° (for N=40). Thus the value of 25° used by Croasdale (2012b) is close to the mean value observed from first-year Beaufort Sea ridges. The mean observed slope of 33° is used as the present base value. It should be noted however that the data from Timco and Burden (1997) were obtained from free-floating ridges, not grounded pile-ups. Observations from the authors have shown that grounded features can have much steeper slopes in some cases (see also Kovacs *et al.* 1981a, 1982).
- Slope angle of interaction (35 degrees) – This parameter is largely unknown since much of the rubbing process can take place inside the rubble and is not visible. What happens inside the ridge sail is a guess; the slope angle of the penetration could be greater or less than the angle of repose. There is no reliable set of field observations. As a start, this is taken as 33°, the same value as the sail angle of repose.
- Sail height (m) – The value of the sail height or the ride-up height is input based on observations of a specific pile-up feature.
- Zone width of ride-up (m) – This value is input based on observations of a specific pile-up.
- Ice-Slope friction (0.15) – The friction value has a very large uncertainty associated with it. There are no large scale measurements and only a few medium-scale measurements. Pritchard *et al.* (2012) did experiments in the Beaufort Sea using blocks of sea ice that were approximately 1 m² in plan area and 0.25 m thick. They found that the dynamic friction coefficient was a function of the weight of the block such that it decreased from 0.6 to 0.4 as the ice block weight changed from 0.2 kN to 3 kN. The static friction coefficient changed from 0.9 to 0.6 over the same weight range. These values were obtained with no snow cover. They generally were 15-20% higher with a snow cover. Sukhorukov and Løset (2013) performed a very comprehensive set of experiments on ice-ice friction using samples with a weight range of 0.3 to 2 kN. They found a striking reduction in the friction coefficient with repeated runs with the same ice surfaces. The initial tests with un-used surfaces indicated dynamic friction coefficients in the range of 0.3 to 0.5. They did not find any dependence on velocity on un-used surfaces and sea water did not appreciably alter the coefficient. Based on these experiments, it would appear that a dynamic ice-ice friction coefficient in the range of 0.3 to 0.5 might be representative for sea ice. This range is higher than the value chosen by Croasdale (2012b) of 0.15. The base value used for the present analysis is 0.4.
- Ice thickness (m) - This value is input based on observations of a specific event.
- Rubble porosity (0.3) – Høyland (2002) has summarized values of the measured porosities of sea ice ridges and notes that most of the reported values range from 0.30 to 0.35. Thus the value of 0.3 chosen by Croasdale (2012b) is reasonable and also used for the present base value.
- Shear strength of rubble (5 kPa) – Croasdale (2012b) has used a value for the shear strength, or cohesion, of rubble as 5 kPa based on a number of large-scale measurements on shearing of sea ice ridges (Croasdale, 1998; 1999; Croasdale *et al.* 2001). This value is also used as the present base value. Rubble friction angle is assumed to be zero.

As seen from this discussion, there is considerable range and uncertainty in many of the parameters used in the model. Therefore it is important to understand the impact of the range of the parameters.

- operations. Cold Regions Science and Technology 121, pp 66-83.
- Barker, A., Timco, G.W. 2017. Maximum pile-up heights for grounded ice rubble. Cold Regions Science and Technology 135, pp 62-75.
- Blasco, S.M., Shearer, J.M., Myers, R. 1998. Seabed scouring by sea-ice: scouring process and impact rates: Canadian Beaufort Shelf. Proceedings Mombetsu Sea Ice Conference, Mombetsu, Japan.
- Canatec, 1994. Study of ice rubble formation in the Beaufort Sea. Canatec Report prepared for the National Energy Board of Canada, Mobil Research & Development Corporation, and Gulf Canada Resources Ltd, Calgary, AB, Canada.
- Canatec, 1994. Study of ice rubble formation in the Beaufort Sea. Canatec Report prepared for the National Energy Board of Canada, Mobil Research & Development Corporation, and Gulf Canada Resources Ltd, Calgary, Canada.
- Canatec, 2010. Beaufort and Chukchi seas ice design criteria extreme ice features and multi-year floe/ridge statistics. Canatec Report for Shell, ConocoPhillips, Chevron, ExxonMobil, BP, and the US Mineral Management Service, Calgary, Alberta, Canada.
- Christensen, FT. 1994. Ice ride-up and pile-up on shores and coastal structures. J. Coastal Research 10, No 3, pp 681-701.
- Coastal Frontiers, 2010. 2009 Freeze-up study of the Alaskan Beaufort and Chukchi Sea. Coastal Frontier Report CFC-800, Chatsworth, CA, USA.
- Comfort G., R. Ritch, 1990. Field measurements of pack ice stress. Proc 9th OMAE Conference, Volume 4, pp 177-182, Houston, TX, USA.
- Comfort G., Ritch, R., Frederking, R.M.W. 1992. Pack ice stress measurements. Proceedings 11th OMAE Conference, Volume 4, pp 245- 253, Calgary, AB, Canada.
- Croasdale, K.R. 1984. The limiting driving force approach to ice loads. Offshore Technology Conference, OTC Paper 4716, Houston, TX, USA.
- Croasdale, K.R. 1980. Ice forces on fixed rigid structures. in IAHR State-of-the-Art Report on Ice Forces on Structures, CRREL Special Report 80-26, Hanover, NH, USA.
- Croasdale, K.R. 1998. Measurement of large scale ice ridge strengths off the north east coast of Sakhalin Island, 1998. A proprietary study for Exxon Neftegas and partners, KR Croasdale & Assoc., Calgary, AB, Canada.
- Croasdale, K.R. 1999. Field study of ice characteristics off the West Coast of Newfoundland. K.R. Croasdale and Associates report to National Research Council, Ottawa, and Exxon Production Research Co. PERD/CHC Report 2-70 and 2-71 (Proprietary)
- Croasdale, K.R. 2009. Limit force ice loads – an update. Proceedings 20th POAC conference, Paper POAC09-030, Lulea, Sweden.
- Croasdale, K.R. 2012a. Ice rubbing and ice interaction with offshore facilities. Cold Regions Science and Technology 76-77, pp 37-43.
- Croasdale, K.R. 2012b. Ice ridging forces, ice rubble, and other neglected topics in ice-structure interaction. Proceedings ICETECH Conference, Paper No. ICETECH12-112-RF, Banff, AB, Canada.
- Croasdale, K.R., Bruneau, S., Christian, D., Crocker, G., English, J., Metge, M., Ritch, R. 2001. *In-Situ* measurements of the strength of first year ridge keels. Proceedings 16th POAC Conference, Vol 3, pp 1445-1454, Ottawa, ON, Canada.
- Croasdale, K.R., Cammaert, A.B., Metge, M. 1994. A method for the calculation of sheet ice loads on sloping structures. Proc. IAHR Ice symposium, Vol 2, pp 874-885, Trondheim, Norway.
- Croasdale, K.R., Comfort, G., Frederking, R., Graham, B.W. and Lewis, E.L. 1987. A pilot experiment to measure pack ice driving forces, POAC '87 Fairbanks, Alaska, 17-21 August 1987, Vol. 3, pp. 381-395
- Croasdale K.R., Comfort G., Graham, B.W. 1986. Arctic pack-ice forces research project, 1986. K.R. Croasdale and Associates report to Government of Canada, Dept. of Fisheries and Oceans, Institute for Ocean Sciences, Sidney, BC, Canada.
- Croasdale, K.R., Frederking, R., Wright, B., Comfort, G. 1996. Size effect on pack ice driving forces. Proceedings IAHR Ice Symposium, pp 1481-1496, Beijing, China.
- Croasdale, K.R., Frederking, R., Wright, B., Comfort, G. 1992. Size effect on pack ice driving forces. Proceedings 11th IAHR Ice Symposium, Vol. 3 pp 1468-1496, Banff, AB, Canada.
- Croasdale, K.R., Metge, M., Verity, P.H. 1978. Factors governing ice ride-up on sloping beaches. Proc. IAHR Ice Symposium, Vol 1, pp 405-420, Lulea, Sweden.

- Frederking, R. 2003. Determination of local ice pressures from ship transits in ice. Proceedings ISOPE'03, Paper No. 2003-JSC-393, Honolulu, Hawaii, USA.
- Frederking, R., Hewitt, K., Jordaan, I., Sudom, D., Bruce, J., Fuglem, M., Taylor, R. 2011. Overview of the Molikpaq multi-year ice load: Reanalysis 2007 JIP. Proceedings 21st POAC Conference, Paper POAC 11-165, Montreal, PQ, Canada.
- Hallam, S.D., Duckworth, R., Pickering, J. G., Westermann, P. H. 1985 Katie's Floeberg 1985: Pack-ice driving force measurements final report. BP Report prepared for the BP/SOHIO Programme, London, U.K.
- Hawkes, I., Mellor, M. 1970. Uniaxial testing in rock mechanics laboratories. Engineering Geology, Vol. 4, No.3, p. 177-285.
- Hopkins, M.A. 1997. Onshore ice pileup: a comparison between experiments and simulation. Cold Regions Science and Technology 26, pp 205-214.
- Hopkins, M.A. 1998. Four stages of pressure ridging. Journal of Geophysical Research 103, C10, pp 21883-21891.
- Høyland, K.V. 2002. Consolidation of first-year sea ice ridges. Journal of Geophysical Research Oceans 107, Issue C6, Pages 15-1–15-16.
- ISO 19906. 2010. Petroleum and natural gas industries- Arctic offshore structures. International Organization for Standardization, Geneva, Switzerland.
- Jefferies, M.G., Wright, W.H. 1988. Dynamic response of Molikpaq to ice-structure interaction. Proceedings OMAE'88, Vol. IV, pp 201-220, Houston, TX, USA.
- Johnston, M., Frederking, R., Tivy, A., 2015. Polar icebreaker alternatives analysis support: Ice trend analysis. NRC Report to Booz Allen Hamilton for US Coast Guard. Technical Report OCRE-TR-2015-003, Ottawa, ON, Canada.
- Johnston, M.E., Timco, G.W., Frederking, R. 1999. An overview of ice loads on slender structures. Proceedings 18th International Conference on Offshore Mechanics and Arctic Engineering, OMAE'99, Paper OMAE-99-1152, St. John's, NL, Canada.
- Jordaan, I.J., Maes, M.A., Brown, P.W., Hermans, I.P. 1993. Probabilistic analysis of local ice pressures. Proceedings, 11th International Conference on Offshore Mechanics and Arctic Engineering, Calgary, AB, Vol. II, pp. 7-13.
- Kovacs, A. 1983. Shore ice ride-up and pile-up features. Part 1: Alaska's Beaufort Sea coast. CRREL Report 83-9, Hanover, NH, USA.
- Kovacs, A. 1984. Shore ice ride-up and pile-up features. Part 11: Alaska's Beaufort Sea coast – 1983 and 1984. CRREL Report 84-26, Hanover, NH, USA.
- Kovacs, A., Sodhi, D.S. 1980. Shore pile-up and ride-up: field observations, models, theoretical analysis. Cold Regions Science and Technology 2, pp 209-288.
- Kovacs, A., Sodhi, D.S. 1981a. Sea ice piling at Fairway Rock, Bering Strait, Alaska: Observations and theoretical analysis. Proceedings POAC'81, Vol 2, pp 985-1000, Quebec City, PQ, Canada.
- Kovacs, A., Sodhi, D.S. 1981b. Ice pile-up and ride-up on Arctic and subarctic beaches. Coastal Engineering 5, pp 247-273.
- Kovacs, A., Sodhi, D.S., Cox, G.F.N. 1982. Bering Strait sea ice and the Fairway Rock icefoot. US Army CRREL Report 82-31, Hanover, NH, USA.
- Kry, P.R. 1978. A statistical prediction of effective ice crushing stresses on wide structures. Proc. Int. Assoc. for Hydraulic Research, Symposium on Ice Problems, Part 1, pp 33 – 47, Lulea, Sweden.
- Kubat, I., Sayed, M. Lamotagne, P. 2015. Analysis of vessel besetting over the Gulf of St. Lawrence and the Strait of Belle Isle, Winter 2013-2014. Proceedings POAC'15, Paper POAC15-177, Trondheim, Norway.
- Kubat, I., Sayed, M. Savage, S., Carrieres, T. 2010. Numerical simulations of ice thickness redistribution in the Gulf of St. Lawrence. Cold Regions Science and Technology 60, pp 15–28.
- Kubat, I. Watson, D. Sayed, M. 2016. Ice Compression Risks to Shipping over Canadian Arctic and Sub-Arctic Zones. Arctic Technology Paper OTC 27348, St. John's, NL, Canada.
- Mahoney, A., Eicken, H., Shapiro, L., Grenfell, T.C. 2004. Ice motion and driving forces during a spring ice shove on the Alaskan Chukchi coast. Journal of Glaciology 50, pp 195-207.
- McGonigal, D., Wright, B.D. Foo, P.M. 1986. Natural shoal rubble pile study: Beaufort Sea March-April 1979. Gulf Canada Resources Inc. Report, APOA 170, Calgary, AB, Canada.
- Mellor, M., Hawkes, I. 1971. Measurement of tensile strength by diametral compression of discs and annuli. Engineering Geology, Vol. 5, No. 3, p. 173-225.
- Pritchard, R.S., Knoke, G.S., Echert, D.C. 2012. Sliding friction of sea ice blocks. Cold Regions Science and

- Technology 76-77, 8-16.
- Sandwell and Canatec. 1994. Processing of pack ice stress data. Sandwell Report 113027, Calgary AB, submitted to National Research Council, Ottawa, ON, Canada.
- Sayed, M., Frederking, R. 1986. On modelling ice ridge formation. Proceedings IAHR Ice Symposium, Vol 1, pp 603-614, Iowa City, IO, USA.
- Sayed, M., Frederking, R. 1988. Model of ice rubble pileup. ASCE Journal of Engineering Mechanics 114, pp 149-160.
- Sayed, M., Frederking, R.M.W. 1984. Stresses in first- year pressure ridges. Proceedings, 3rd OMAE, Vol. 3, p. 173-177, New Orleans, USA.
- Shapiro, L.H. 1976. Mechanics of origin of pressure ridges, shear ridges and hummock fields in landfast ice. Environmental Assessment Program of the Alaskan Continental Shelf, Vol. 14, Ice, pp 117-153, Boulder, CO, USA.
- Shapiro, L.H., Metzner, R.C., Hanson, A., Johnson, J.B. 1984. Fast ice sheet deformation during ice-push and shore ice ride-up. *in* The Alaskan Beaufort Sea: Ecosystems and Environments. *Edited by* Barnes, P.W., Schell, D. and Reimnitz, E., pp 137-157, Academic Press, Toronto, ON, Canada.
- Sodhi, D.S., Hirayama, K. Haynes, F.D., Kato, K. 2003. Experiments on ice ride-up and pile-up. Annals of Glaciology 4, pp 266-270.
- Strub-Klein, L., Sudom, D. 2012. A comprehensive analysis of the morphology of first-year sea ice ridges. Cold Regions Science and Technology 82, pp 94-109.
- Sudom, D., Timco, G.W., Sand, B., Fransson, L. 2011. Analysis of first-year and old ice ridge characteristics. Proceedings of the 21st International Conference on Port and Ocean Engineering under Arctic Conditions, Paper POAC11-164, Montreal, QC, Canada.
- Sukhorukov, S., Løset, S. 2013. Friction of sea ice on sea ice. Cold Regions Science and Technology 94, pp 1-12.
- Thomas, B., McGonigal, D. 1983. Final Report – Katie’s Floeberg 1981. Gulf Research & Development Company, Houston, TX, USA. (CHC Report 5-24).
- Timco, G.W. 1991. The vertical pressure distribution on structures subjected to rubble-forming ice. Proceedings 11th Conference on Port and Ocean Engineering under Arctic Conditions, POAC'91, Vol. I, pp 185-197, St. John's, NL, Canada.
- Timco, G.W. 2007. Thickness dependence of the failure modes of ice from field observations on wide structures. Proceedings 19th POAC Conference, Vol. 1, pp 286-295, Dalian, China.
- Timco, G.W. 2012. Multi-year ice loads: Alignment of experts workshop. Arctic Technology Conference, OTC Paper 23759, Houston, TX, USA.
- Timco, G.W., Barker, A. 2002. What is the maximum pile-up height for ice? Proceedings of the 16th IAHR International Symposium on Ice, pp 69-77, Dunedin, New Zealand.
- Timco, G.W., Barker, A. 2015. Analysis of a large shear wall: implications for pack ice driving forces. Proceedings Arctic Technology Conference, Paper OTC-22535, Copenhagen, Denmark.
- Timco, G.W., Burden, R.P. 1997. An analysis of the shapes of sea ice ridges. Cold Regions Science and Technology 25, pp 65-77.
- Timco, G.W., Frederking, R. 2008. Overview of historical Beaufort Sea information. NRC Report CHC-TR-057, Ottawa, ON, Canada.
- Timco, G.W., Frederking, R. 2009. Historical perspectives on the 1970s and 1980s Beaufort Sea exploration. Proceedings 20th International Conference on Port and Ocean Engineering under Arctic Conditions, Paper POAC09-93, University of Luleå, Sweden.
- Timco, G.W., Frederking, R.M.W. 1996. A review of sea ice density. Cold Regions Science and Technology, Vol. 24, pp 1 - 6.
- Timco, G.W., Johnston, M.E. 2003. Ice loads on the Molikpaq in the Canadian Beaufort Sea. Cold Regions Science and Technology, Vol. 37, No. 1, pp 51-68.
- Timco, G.W., Johnston, M.E. 2004. Ice loads on the caisson structures in the Canadian Beaufort Sea. Cold Regions Science and Technology 38, pp 185– 209.
- Timco, G.W., O'Brien, S. 1994. Flexural strength equation for sea ice. Cold Regions Science and Technology, 22, pp 285-298.
- Timco, G.W., Sayed, M. 1986. Model tests of the ridge-building process in ice. Proceedings 8th International Association for Hydraulic Research Symposium on Ice, Vol. I, pp 591-602, Iowa City, IO, U.S.A.

- Timco, G.W., Weeks, W.F. 2010. A review of the engineering properties of sea ice. *Cold Regions Science and Technology* 60, No. 2, pp 107-129. doi:10.1016/j.coldregions.2009.10.003
- Vaudrey, K.D., Thomas, B. 1981. Final Report – Katie’s Floeberg 1980. Gulf Research & Development Company, Houston, TX, USA. (CHC Report 5-43).
- Wright, B.D., Frederking, R., Croasdale, K. 1992. Pack ice driving force inferred from Molikpaq ice loads. *Proc IAHR Ice Symposium*, pp 1468-1480, Banff, AB, Canada.
- Wright, B.D., Timco, G.W. 1994. A review of ice forces and failure modes on the Molikpaq. *Proceedings of the 12th IAHR Ice Symposium, IAHR'94*, Vol. 2, pp 816-825, Trondheim, Norway.
- Wright, B.D., Timco, G.W. 2000. First-year ridge interaction with the Molikpaq in the Beaufort Sea. *Cold Regions Science and Technology* 32, pp 27-44.