Effective Non-reflective Boundary for Lamb Waves: Theory, Finite Element Implementation, and Applications

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ABSTRACT

This article presents a new approach to designing non-reflective boundary (NRB) for inhibiting Lamb wave reflections at structural boundaries. Our NRB approach can be effectively and conveniently implemented in commercial finite element (FE) codes. The paper starts with a review of the state of the art: (a) the absorbing layers by increasing damping (ALID) approach; and (b) the Lysmer-Kuhlemeyer absorbing boundary conditions (LK ABC) approach is briefly presented and its inadequacy for Lamb wave applications is explained. Hence, we propose a modified Lysmer-Kuhlemeyer approach to be used in the NRB design for Lamb wave problems; we call our approach MLK NRB. The implementation of this MLK NRB was realized using the spring damper elements which are available in most commercial FE codes. Optimized implementation parameters are developed in order to achieve the best performance for Lamb-wave absorption. Our MLK NRB approach is compared with the state of the art ALID and LK ABC methods. Our MLK NRB shows better performance than ALID and LK ABC for all Lamb modes in the thin-plate structures considered in our examples. Our MLK NRB approach is also advantageous at low frequencies and at cut-off frequencies, where extremely long wavelengths exist. A comprehensive study with various design parameters and plate thicknesses which illustrates the advantages and limitations of our MLK NRB approach is presented. MLK NRB applications for both transient analysis in time domain and harmonic analysis in frequency domain are illustrated. The article finishes with conclusions and suggestions for future work.

1 INTRODUCTION

Finite element method (FEM) has been widely investigated as a convenient easy-to-use tool for the study of ultrasonic wave propagation and its interaction with structural flaws and damage [1]. However, FEM is computationally intensive: to ensure computational accuracy, strict rules of spatial and temporal discretization need to be adhered to, i.e., the element size must be smaller than one twentieth of the smallest wavelength and the time step must be smaller than one twentieth of the smallest period [2]. Thus, the propagation of high-frequency short-wavelength waves over long distances may become computationally prohibitive to model because of the very fine mesh and very small time step required to ensure validity of the simulated wave signals, especially in interaction with structural flaws and damage [3].

In order to make the computational burden manageable, one notices that the FEM model is mainly necessary in the study of the scattering interaction between waves and structural

damage because the FEM approach allows the detail modeling of the damage geometry that is not possible analytically. However, outside the damage area, the wave propagation can be modeled analytically and the use of large plate models has only been justified by the need to avoid boundary reflections contaminating the scatter signal. Hence, researchers have developed the concept of non-reflective boundaries (NRB), which would eliminate the unwanted boundary reflections and allow the use of a finite size FEM model to simulate infinite medium conditions. The benefit of an NRB approach is that computational resources can be focused on the region of interest without modeling the redundant outside domain merely for the purpose of avoiding boundary reflections.

Current techniques for removing boundary reflections fall into three main categories: (i) infinite element methods; (ii) non-reflective boundary conditions [4]; and (iii) absorbing layer methods [5]. The infinite element methods use only one layer of specialty elements around the boundary to absorb the incident waves. This technique is available in commercial software such as ABAQUS. But discrepancies have been reported in elastic wave scattering problems when both the pressure and shear wave modes are present [6, 7]. Lysmer and Kuhlemeyer [4] introduced the absorbing boundary condition (LK ABC) which imposes matching reaction forces at the boundaries to simulate time domain wave propagation into infinite medium. This technique is only available in special commercial software package such as ANSYS LS-DYNA solver in time domain simulations. LK ABC was found to work well with bulk waves, but its application to Lamb waves experiences noticeable reflections and poor performance for antisymmetric Lamb modes. Other successful non-reflective boundary conditions have been developed by Givoli and Keller in the frequency domain [8, 9]. This approach was extended by Moreau et al. [10] to solve scattered waves from irregular defects using a frequency domain small-size FEM. However, these non-reflective boundary conditions require the modification of standard FEM solving procedure and the development of specialist codes. The absorbing layer techniques can be classified into two main subcategories: (iiia) the Perfectly Matched Layers (PMLs); and (iiib) Absorbing Layers by Increasing Damping (ALID). Both techniques extend the boundary using several layers of absorbing elements with gradually varying properties. The damping mechanism contributes to the imaginary part of the wavenumbers, which results in the attenuation of the wave amplitudes along the propagation path. The PML technique optimizes the material properties to match the acoustic impedance of successive layers, so that no reflections occur between adjacent layers [11, 5, 12]. PML works well in both the frequency and time domain analyses but it is only available in a very limited number of commercial FE packages such as the COMSOL RF Module, which is an optional COMSOL add-on package. On the other hand, the ALID method adopts increasing damping along wave propagation to absorb incident waves, accompanied by impedance mismatches between successive layers [13, 14, 15, 16, 17]. The impedance-mismatch reflections may be minimized by optimizing the damping properties. It has been reported that ALID is much easier to be implemented in commercial FE software packages, because the users only need to define the increasing damping properties of the layer materials, which can be achieved with the standard techniques. A more recent contribution to ALID-type method was given by Pettit et al.

[18], who develop a stiffness reduction method (SRM) to further optimizing its performance. The SRM showed improved results compared with traditional ALID.

It is apparent that the state of the art in removing boundary reflections has generally focused on bulk waves, while only a few results have so-far been reported on removing boundary reflections for Lamb waves and other guided waves [10, 12, 19, 20, 21]. Current solutions for Lamb wave absorption mainly stem from the absorbing-layer family of methods, while other techniques are also desired to develop more effective solutions. In this article, we propose a different and new approach which employs an absorbing mechanism that is custom-built for the suppression of Lamb-wave reflections. In developing this method, we started with the conventional LK ABC method and endeavored to modify it to address the issues stemming from the multi-modal dispersive character of Lamb waves propagating in thin-plate structures. We call our method “modified LK non-reflective boundaries (MLK NRB)”. We have also noticed that most of the existing studies used the time domain simulations, while frequency domain simulations are only a few and sometimes limited to specialist FE codes. Our MLK NRB approach, which is especially designed for Lamb waves, is effective for both time-domain and frequency-domain simulations and is easy to implement in commercial FE packages. In this article, we will present and discuss three aspects of our MLK NRB approach:

- (1) The theoretical background for designing NRB conditions specific to Lamb-wave applications: we will first briefly review the LK ABC theory and identify the reason behind its inadequacy for Lamb wave applications. Then, we will develop the theory of our MLK NRB method that underpins its suitability for Lamb wave absorption. Parametric studies are used to develop guidelines for the choice of effective MLK NRB design variables.
- (2) The MLK NRB implementation in commercial FE packages and performance tests: we will demonstrate the MLK NRB implementation in both 2-D and 3-D FE models. Performance tests and comparison with conventional LK ABC and ALID methods will be given for multiple Lamb modes at various frequencies. The advantages and limitations of our proposed MLK NRB method will be evaluated through a parametric study to assess plate thickness and implementation length effects.
- (3) The capability of the MLK NRB method in both time domain and frequency domain will be demonstrated using two application case studies: (a) time domain simulation of Lamb wave generation, propagation, and interaction with a structural feature; (b) frequency domain simulation of Lamb wave scattering from damage and determination of optimum interrogation frequency.

2 MODIFICATION OF THE LYSMER-KUHLEMEYER METHOD TO OVERCOME LAMB-WAVE RELATED DIFFICULTIES

This section discusses the difficulties encountered when trying to apply the conventional LK ABC method to Lamb-wave problems and then develops our proposed MLK NRB method. Guidelines for the proper choice of MLK NRB parameters are also developed.

2.1 THEORY OF CONVENTIONAL LYSMER-KUHLEMEYER ABSORBING BOUNDARY CONDITION

Lysmer and Kuhlemeyer [4] developed a dynamic model for the absorption of P-wave and S-wave reflections at the boundary of an semi-infinite half plane using two coefficients, a and b , first proportional with the P-wave speed $c_p = \sqrt{(\lambda + 2\mu)/\rho}$, the other proportional with the S-wave speed $c_s = \sqrt{\mu/\rho}$, where λ and μ are Lamé's constants and ρ is the material density. They demonstrated that the proper choice of these a and b coefficients will result in the maximum energy absorption capability of a viscous boundary impinged upon by P and S waves. They also extended their method to Rayleigh wave absorption.

2.2 INADEQUACY OF CONVENTIONAL LK ABC FOR LAMB WAVES ABSORPTION

However, the Lysmer and Kuhlemeyer paper [4] did not consider the case of Lamb-wave reflections at a plate boundary. To understand the intricacies of such a problem, recall that Lamb waves, by their nature, are the constructive and destructive interferences and superposition of pressure wave (P-wave) and shear vertical wave (S-wave) undergoing multiple reflections between the top and bottom plate surfaces. As illustrated in the LK theory [4], it is apparent that the absorbing capability of a conventional LK viscous boundary placed at the edge of the plate is sensitive to the incident wave angle, which makes it improper for Lamb wave absorption. Figure 1 shows the multiple reflections of P-wave and S-wave in the formation of Lamb waves. The reflection angles of P-wave and S-wave are denoted by α and β , respectively. Their incident angle to the plate boundary are denoted by θ and γ .

Figure 1: Lamb wave formation: multiple reflections of P-wave and S-wave.

Hence, when analyzing the Lamb-wave reflection at a plate boundary, one should actually analyze the reflections of the P-wave and S-wave components as shown in Figure 2.

It is apparent from Figure 2 that the incident P and S waves interacting at a plate boundary get reflected and mode converted both at the free-end surface which is normal to the midplane and at the upper and lower free surfaces of the plate which are adjacent to the free end. If these reflected and mode-converted waves are to be suppressed by an absorbing viscous boundary, then this absorbing boundary has to extend around the complete contour at the end of the plate and not just be placed on the free-end surface as implied by the LK ABC method.

Recall the potential formulation of z-invariant Lamb waves [22]. The displacements are expressed as

$$u_x = \frac{\partial \Phi}{\partial x} + \frac{\partial H_z}{\partial y}; \quad u_y = \frac{\partial \Phi}{\partial y} - \frac{\partial H_z}{\partial x} \quad (1)$$

where Φ and H_z are the displacement potentials of the P and S waves, respectively.

Assuming a unit-amplitude harmonic incident P-wave, we can write the total-wave potentials as a superposition of incident and reflected waves, i.e.,

$$\begin{aligned} \Phi^P &= 1e^{i(\xi_P \cos \theta x + \xi_P \sin \theta y - \omega t)} + A_P e^{i(-\xi_P \cos \theta x + \xi_P \sin \theta y - \omega t)} \\ H_z^P &= B_P e^{i(-\xi_S \cos \gamma x + \xi_S \sin \gamma y - \omega t)} \end{aligned} \quad (2)$$

where θ , ξ_P and γ , ξ_S are the reflection angles and the wavenumbers of the P-wave and S-wave, respectively, whereas ω and t are the angular frequency and time. Note that A_P and B_P are the amplitudes of the reflected P and S waves generated by a unit-amplitude incident P wave.

Figure 2: Schematic of P-wave and S-wave incidence at the plate boundary: (a) incident P-wave; (b) incident S-wave.

Similarly, for a unit-amplitude incident S-wave case, we can express the total-wave potentials as

$$\begin{aligned} \Phi^S &= A_S e^{i(-\xi_P \cos \theta x + \xi_P \sin \theta y - \omega t)} \\ H_z^S &= 1e^{i(\xi_S \cos \gamma x + \xi_S \sin \gamma y - \omega t)} + B_S e^{i(-\xi_S \cos \gamma x + \xi_S \sin \gamma y - \omega t)} \end{aligned} \quad (3)$$

where A_S and B_S are the amplitudes of the reflected P and S waves generated by a

unit-amplitude incident S-wave. Note that in Eqs. (2) and (3), the superscripts on the wave potentials indicate the incident wave type, i.e., Φ^P represents the P-wave potential for a P-type incident wave, Φ^S represents the P-wave potential for an S-type incident wave, etc.

Next, we use the wave potentials to write the stress components [23, 22] and impose the viscous boundary conditions as

$$\begin{aligned}\sigma_{xx} &= (\lambda + 2\mu) \left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} \right) - 2\mu \left(\frac{\partial^2 \Phi}{\partial y^2} - \frac{\partial^2 H_z}{\partial x \partial y} \right) = -a\rho c_P \left(\frac{\partial \dot{\Phi}}{\partial x} + \frac{\partial \dot{H}_z}{\partial y} \right) \\ \tau_{xy} &= \mu \left(2 \frac{\partial^2 \Phi}{\partial x \partial y} - \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} \right) = -b\rho c_S \left(\frac{\partial \dot{\Phi}}{\partial y} - \frac{\partial \dot{H}_z}{\partial x} \right)\end{aligned}\quad (4)$$

where a and b are the previously discussed viscous coefficients for direct stress and shear stress as introduced by Lysmer and Kuhlemeyer [4]. Impose Snell's law and recall the wave speed ratio k to write

$$\frac{\sin \theta}{\sin \gamma} = \frac{\xi_S}{\xi_P} = \frac{c_P}{c_S} = k \quad (5)$$

For incident P-wave (Figure 2a), substitution of Eqs. (2) and (5) into Eq. (4) yields a set of linear algebraic equations in the reflected wave amplitudes A_p and B_p , i.e.,

$$\begin{bmatrix} 2\sin^2 \theta - k^2 - ak^2 \cos \theta & k^2 \sin 2\gamma + ak^3 \sin \gamma \\ \sin 2\theta + bk \sin \theta & k^2 \cos 2\gamma + bk^2 \cos \gamma \end{bmatrix} \begin{bmatrix} A_p \\ B_p \end{bmatrix} = \begin{bmatrix} k^2 - 2\sin^2 \theta - ak^2 \cos \theta \\ \sin 2\theta - bk \sin \theta \end{bmatrix} \quad (6)$$

Similarly, for incident S-wave (Figure 2b), substitution of Eqs. (3) and (5) into Eq. (4) yields a system of linear algebraic equations in A_s and B_s , i.e.,

$$\begin{bmatrix} 2\sin^2 \theta - k^2 - ak^2 \cos \theta & k^2 \sin 2\gamma + ak^3 \sin \gamma \\ \sin 2\theta + bk \sin \theta & k^2 \cos 2\gamma + bk^2 \cos \gamma \end{bmatrix} \begin{bmatrix} A_s \\ B_s \end{bmatrix} = \begin{bmatrix} k^2 \sin 2\gamma - ak^3 \sin \gamma \\ bk^2 \cos \gamma - k^2 \cos 2\gamma \end{bmatrix} \quad (7)$$

Lysmer and Kuhlemeyer [4] reported that when $a=b=1$, the viscous boundary will achieve the best energy absorption capability. This may be so for the case of bulk waves at a half plane boundary [4], but not necessarily so for Lamb waves reflecting at the end of a plate as shown in Figure 2. In this latter case, the incident angles θ and γ also play an important role in this process besides a and b . Figure 3 shows the reflected wave amplitudes versus the incident wave angle in an aluminum plate. It can be observed that for both incident P-wave and incident S-wave cases, the reflected wave amplitudes depend strongly on the incident wave angle. When the incident wave angle is very small, the reflected wave amplitudes approaches zero, demonstrating that the LK approach is effective in absorbing the wave energy. However, as the incident wave angle increases, the conventional LK viscous boundary begins to lose its

absorbing capability.

Figure 3: Reflected wave amplitudes versus incident wave angle calculated with the conventional LK ABC method (viscous boundary $a=b=1$) in an aluminum plate: (a) P-wave incidence; (b) S-wave incidence.

In order to calculate the incident angles θ and γ of Figure 2, recall the following notations used in the derivation of the Rayleigh-Lamb equation [23, 22, 24]

$$\eta_p^2 = \omega^2/c_p^2 - \xi^2; \quad \eta_s^2 = \omega^2/c_s^2 - \xi^2 \quad (8)$$

where ξ is the wavenumber of Lamb wave, and η_p and η_s may be interpreted as the vertical wavenumbers of the P and S waves that make up the Lamb waves (see page 315 of Ref [23]). Thus, for the benefit of Figure 2, the incident angles θ and γ of the P and S-waves that make up the Lamb waves can be calculated as

$$\theta = \arctan\left(\frac{\eta_p}{\xi}\right); \quad \gamma = \arctan\left(\frac{\eta_s}{\xi}\right) \quad (9)$$

Note: when solving Eq. (9), proper choice of real and imaginary solution should be made on η_p and η_s according to various regions the Rayleigh-Lamb equation (see page 447 in Ref [23]).

Figure 4: Incident angles of P-wave (θ) and S-wave (γ) at the plate edge in aluminum plates: (a) S0 Lamb wave case; (b) A0 Lamb wave case.

Figure 4 shows the incident angles of P-wave and S-wave at the plate edge for both fundamental S0 Lamb mode and A0 Lamb mode in aluminum plates. The unit fd is the combination of frequency and half plate thickness. It can be noticed that both S0 and A0 Lamb waves contain bulk wave components beyond the effective absorption angles shown in Figure 3. This illustrates the inadequacy of the conventional LK ABC approach for avoiding Lamb wave reflections at a plate boundary.

2.3 DEVELOPMENT OF THE MODIFIED LYSMER-KUHLEMEYER (MLK) APPROACH FOR LAMB-WAVE NRB DESIGN

Noticing that the conventional LK ABC approach is inadequate for effective absorption of Lamb waves at a plate boundary, we developed a MLK NRB approach that would effectively absorb the Lamb waves at plate free edges. Our concept takes into account that Lamb waves result from the superposition of P and S waves that undergo multiple reflections at the top and bottom surfaces of the plate. Hence, we want to inhibit these top and bottom reflections near the plate boundary. In order to achieve this, we added viscous boundaries on the top and bottom surfaces near the plate edge and smoothed them out by adopting a gradually increasing viscosity parameter from the inner region towards the plate edge (Figure 5). Thus, one part of our contribution is to extend the absorbing medium from the plate edge over the top and bottom surfaces of the plate in order to absorb the P and S wave reflection on these top and bottom surfaces. The other part of our contribution is to design a smooth-out law by which the top and bottom absorbing layers gradually participate in the absorption towards the plate edge; this second part is needed in order to prevent reflections from the Lamb waves interacting with the top and bottom viscous boundaries. Our MLK NRB design has the absorbing medium placed both at the plate edge and on its top and bottom surfaces as shown in Figure 5.

Figure 5: Extended viscous boundary on top and bottom surfaces for effective absorption of Lamb waves. The variation of coefficients a and b is described by the filled profile.

In this MLK NRB design, the P and S wave components of the Lamb wave interact with the viscous boundary multiple times, both at the plate surfaces and at the plate edge. This design of the absorbing boundary takes advantage of the multiple reflection character of the bulk waves forming the Lamb waves; thus the effective absorption of the Lamb waves is achieved through the multiple absorptions of the bulk wave components. Since the reflection angles at the plate surfaces and the plate boundary are complementary, the effective absorbing capability are also complementary at these two locations. This will ensure that the MLK NRB is effective for all the bulk wave angles of all the Lamb modes. Figure 5 shows the extended viscous boundary on the top and bottom surfaces, as well as the variation of coefficients a and b around the plate edge. It should be noted that the plate edges have the full value of the absorption coefficients ($a = b = 1$) as suggested by Lysmer and Kuhlemeyer [4] for the best absorption capability at the plate vertical edge. However, at the top and bottom surfaces, the value of these a, b absorption coefficients is not constant along the plate but it is smoothed out such as to avoid Lamb wave reflections due to the impedance mismatch that would be introduced by an abrupt change in the boundary viscosity at the plate top and bottom surfaces. To achieve this smooth transition, we used a transition function $f(x)$, which introduces the viscosity gradually over a length $n\lambda$ consisting of several wavelength.

3 GUIDELINES FOR THE PROPER CHOICE OF MLK NRB PARAMETERS

The proper choice of the MLK NRB parameters directly influences its performance. There are three major aspects to consider when constructing the MLK NRB, namely, the absorbing profile shape $f(x)$, the maximum damping parameter of the profile δ , and the minimum coverage length $n\lambda$ where λ is the longest wavelength under consideration.

Figure 6: Dispersion curves and wavelengths in an 8-mm thick aluminum plate: (a) phase velocity plot; (b) wavelength plot.

3.1 THE CHOICE OF THE ABSORBING FUNCTION $f(x)$

In our study, we considered three possible transitional profile functions $f(x)$: a linear function, a half Hanning window function, and a cubic function, i.e.,

$$f(x) = \delta \frac{x}{n\lambda}, x \in (0, n\lambda) \quad (\text{Linear function}) \quad (9)$$

$$f(x) = \frac{\delta}{2} \left[1 - \cos\left(\frac{\pi x}{n\lambda}\right) \right], x \in (0, n\lambda) \quad (\text{Half Hanning window function}) \quad (10)$$

$$f(x) = \delta \left(\frac{x}{n\lambda} \right)^3, x \in (0, n\lambda) \quad (\text{Cubic function}) \quad (11)$$

These three function profiles have different orders of smoothness, which can be evaluated using the Taylor series expansion: the order of smoothness for the linear function Eq. (9) is one; the order of smoothness for the half Hanning window Eq. (10) is two; the order of smoothness for the cubic function Eq. (11) is three.

Figure 7: Schematic of the FEM model used in the parametric study: (a) S0 mode excitation; A0 mode excitation.

To test the performance of the profile functions of Eqs. (9), (10), and (11), we conducted a parametric study on an 8-mm thick aluminum plate that has the phase velocity and wavelength curves shown in Figure 6. The parametric study was conducted with the FEM model shown in Figure 7 using ANSYS 14.0. The coverage length L was set to 50 mm for all three profile functions. The maximum damping parameter δ was increased from zero to 0.5 with a step of 0.05. The S0 and A0 modes were selectively excited by the application of a pair of symmetric and antisymmetric point forces at the top and bottom plate surfaces. A 100 kHz 10-cycle smoothed tone burst excitation was used to generate narrow-band Lamb waves. The analysis was performed in the time domain. The excited and reflected waves were recorded at the bottom node. The amplitude reflection coefficient was used as a performance and effectiveness metric.

Figure 8: Performance of various profile functions for a 100 kHz narrow-band signal: (a) S0 mode excitation; (b) A0 mode excitation

The results of this parametric study are shown in Figure 8. In general, all three profile functions

seemed to achieve better performance with higher δ values. However, the linear and half Hanning window profiles achieve better performance than the cubic profile. The performance also depends on the excitation mode, i.e., a clear difference can be noticed between S0 excitation (Figure 8a) and A0 excitation (Figure 8b).

For S0 excitation, the linear profile function has the best result quality at low δ values ($0 < \delta < 0.2$). Beyond $\delta = 0.2$, the half Hanning function seems to achieve a marginally better performance than the linear function. The cubic function is consistently worse because its reflection coefficients stay higher than the other two functions for all tested values of δ . In addition, the cubic function plateaus out beyond $\delta = 0.3$.

For A0 excitation, the half Hanning function seems to offer the best performance although the linear function performance is almost as good except at high δ where it plateaus out while the half Hanning function continues to improve.

The case of the conventional LK ABC is recovered at $\delta = 0$; as indicated in Figure 8, the LK ABC method achieves some effectiveness for S0 excitation (only 2% reflection) but worse results for A0 excitation (10% reflection).

It should be noted that in this case study the coverage length was $L = 50$ mm, which is approximately one wavelength of the S0 mode at 100 kHz (see Figure 6b). We expect that a longer coverage length would diminish the difference between the half Hanning window function and the linear function at low values of δ .

3.2 THE CHOICE OF δ VALUE AND THE COVERAGE LENGTH L

After the discussion for the proper choice of the smoothing function $f(x)$, the next important aspect to discuss is the proper choice of δ , i.e., maximum value of the damping parameter in the top and bottom plate surfaces. Recall that in the LK ABC approach, the damping was applied only to the vertical end of the plate and that its value was $a = b = 1$. In the MLK NRB approach, we apply additional damping of value $f(x)$ to the top and bottom surfaces of the plate. In our study, we took various δ values between 0 and 0.5 and studied their effect on reflection suppression. (Please note that this damping applied to the plate top and bottom surfaces does not exceed 50% of the LK ABC damping applied to the vertical end of the plate.)

According to Figure 8, a higher δ leads to a better performance for a 100-kHz narrow-band wave signal. The question that was investigated next was this: What happens at lower frequencies where the wavelengths are much longer?

To study this effect, we used the same FEM model of an 8-mm thick aluminum plate but with a signal of 100 kHz 2-cycle smoothed tone burst excitation which has a much broader frequency band. The MLK NRB coverage length was taken $L = 100$ mm. After signal processing, we were able to identify the wave components belonging to various frequencies in the broad-band signal.

Figure 9: High values of δ show poor performance at low-frequency long-wavelength.

The results shown in Figure 9 indicate that the absorbing effectiveness becomes worse at low frequency where long wavelength Lamb waves exist. It was also noticed that, in these low-frequency long-wavelength cases, the higher damping values (e.g., $\delta = 0.5$ in Figure 9) gave worse results than the lower damping (e.g., $\delta = 0.1$). It was also noticed that the cubic smoothing function had better performance in these low-frequency long-wavelength cases, but worse performance otherwise.

3.3 OPTIMAL MLK NRB DESIGN PARAMETERS

In many practical applications, multimodal Lamb waves at various frequency ranges with different wavelength contents (S0, A0, S1, A1, etc.) propagate simultaneously. The choice of optimal MLK NRB design parameters should be made taking all these factors into consideration.

Based on the results of Figure 8 and Figure 9 and on the authors' experience, the choice of half

Hanning window profile $f(x) = \frac{\delta}{2} \left[1 - \cos\left(\frac{\pi x}{n\lambda}\right) \right]$ with δ value between 0.15 and 0.3 will,

in general, ensure satisfactory results. The third important choice is a proper coverage length of the MLK NRB. The top and bottom viscous layer covers a length $L = n\lambda$, where λ is the longest wavelength of the Lamb mode under consideration. In general, an effective coverage requires $n \geq 2$.

Note that the lower δ values would apply to long wavelength components in which case a longer MLK NRB coverage length will also be necessary.

When broad band Lamb waves containing extremely long wavelength components participate in the interaction with MLK NRB, a high value of δ usually results in poor performance in the absorption of such components. This aspect will be further illustrated with parametric studies on coverage length L and plate thickness H presented later in Section 5.2.

4 MLK NRB IMPLEMENTATION IN COMMERCIAL FE CODES

The implementation of the MLK NRB method in commercial FE codes can be readily realized using spring-damper elements or dashpot elements, which are usually available in most commercial FE codes. (This approach does not require specialized FE codes or the modification of standard solving procedure.) Lysmer and Kuhlemeyer [4] have shown the successful implementation of the conventional LK ABC in a 2-D FE model with dashpot elements. Liu et al. [25] implemented a time domain viscoelastic artificial boundary for bulk waves in a 3-D FE model. In this section, the details and guidelines of MLK NRB implementation with spring-damper elements in both 2-D and 3-D FE ANSYS models will be presented.

4.1 SELECTION OF SPRING-DASHPOT PARAMETERS

According to Eq.(4), the viscous boundary reaction stresses should satisfy the following conditions:

$$\sigma_{xx} = -a\rho c_p \frac{\partial u_x}{\partial t}; \quad \tau_{xy} = -b\rho c_s \frac{\partial u_y}{\partial t} \quad (12)$$

where the reaction stresses depends on the normal and tangential particle velocities at the boundary. In this study, COMBIN14 spring damper elements in ANSYS were used to construct the MLK NRB. Similar element options such as dashpot elements can be found in ABAQUS. Figure 10 shows the schematic of COMBIN14 element in ANSYS, the 2-D FEM implementation, and the 3-D FEM implementation. Since the viscous reaction forces must be proportional to the boundary particle velocity, we only keep the damping coefficient and set the spring coefficient to zero.

Figure 10: Implementation of viscous boundary using COMBIN14 spring damper element: (a) COMBIN14 element [26]; (b) 2-D FEM implementation; (c) 3-D FEM implementation.

For 2-D FEM implementation, a pair of COMBIN14 elements are used, one in the normal direction, and the other in the tangential direction. One side of the elements are attached to the structural boundary node, while the degrees of freedom (DOFs) of other side are fixed. Thus, the motion of the structural boundary node will cause reaction forces in both normal and shear directions from the spring damper elements, which are proportional to the nodal velocities. The spring-damper coefficients K_N, K_T, C_N, C_T , which will generate reaction forces that correspond to the equivalent boundary stresses presented in Eq. (12), are as follows:

$$\begin{aligned} K_N &= 0; \quad C_N = \frac{a}{2}(L_1 + L_2)\rho c_P \\ K_T &= 0; \quad C_T = \frac{b}{2}(L_1 + L_2)\rho c_S \end{aligned} \quad (13)$$

where L_1 and L_2 are the element sizes in the neighborhood of the boundary node, and subscripts N and T represent normal and tangential directions. The spring coefficients are set to zero, while the damping coefficients depend on the bulk wave speeds c_P , c_S , the material density ρ , and the average element size $(L_1 + L_2)/2$. Coefficients a and b are the damping parameters; they are equal to one at the vertical boundary and follow the $f(x)$ function along the top and bottom surfaces of the plate. It should be noted that all the nodes along the defined MLK NRB should be connected to COMBIN14 element pairs; the drawing in Figure 10b demonstrates the implementation for only one boundary node.

For 3-D implementation, three COMBIN14 elements should be used, with one in the normal direction, and the other two along two mutually orthogonal tangential directions. One side of the three elements are attached to the structural boundary node, and the DOFs of other side are

fixed. The corresponding spring-damper coefficients are taken as

$$\begin{aligned} K_N &= 0; \quad C_N = \frac{a}{4}(A_1 + A_2 + A_3 + A_4)\rho c_p \\ K_T &= 0; \quad C_T = \frac{b}{4}(A_1 + A_2 + A_3 + A_4)\rho c_s \end{aligned} \quad (14)$$

where A_1, A_2, A_3, A_4 are the neighboring element facet areas surrounding the boundary node. Again, it should be noted that all the nodes on the NRB should be restricted by COMBIN14 elements, while Figure 10c only demonstrated the implementation for one of the boundary nodes.

4.2 SELECTION OF MESH SIZE

Figure 6b has shown that large wavelengths may be encountered for the S0 mode at low frequencies and for the higher modes near the cutoff frequencies. To effectively absorb Lamb modes with large wavelength, longer MLK NRB coverage is required. However, the longer wavelengths also allows us to achieve good accuracy with a coarser mesh. Hence, the total computational burden (degrees of freedom) should not change if one keeps the ratio between the wavelength and the mesh size constant and equal to the value that ensures computational accuracy. An illustration of this concept is given in Figure 11 which shows a finer MLK NRB mesh for shorter wavelengths and a coarser one for longer wavelengths.

Figure 11: Different FE meshes with the same wavelength to element size ratio. It should be noted that the total computational burden does not change.

Another concept depicted in Figure 11 is that the MLK NRB region may have a much coarser mesh than the FEM region modeling the phenomenon of interest. In addition, this NRB region mesh can become even coarser as we approach the plate boundary. The rationale for this is that the function of the NRB region is only to eliminate boundary reflections and thus the accuracy of the wave phenomena inside the MLK NRB coverage region is not of interest. Hence, we can relax the accuracy criteria in the MLK NRB coverage region by increasing the mesh size gradually towards the plate edge. This leads to a varying-mesh strategy that can further minimize the computational burden within the MLK NRB region.

When multiple Lamb modes exist simultaneously, the largest mesh size within the modeling region (as shown in Figure 11) should be smaller than one twentieth of the shortest wavelength. But the mesh density in the MLK NRB extended region toward the plate end does not need to satisfy this accuracy requirement and hence may have a coarse mesh. However, the coverage length in this MLK extended region should be larger than twice the longest wavelength.

4.3 CONSIDERATIONS FOR FREQUENCY DOMAIN ANALYSIS

Mesh-size considerations discussed in the previous section may become even more important for frequency-domain analysis, where a wide frequency range needs to be explored. In our experience, it is useful to identify several separate frequency regions within the whole frequency range and, for each region, choose different mesh sizes according to the wavelength to element size ratio criteria. Thus, the problem can be solved in the most efficient way using an adaptive FE mesh: the frequency ranges containing very long wavelengths can be treated separately using a large extended region with fairly coarse mesh, while the frequency ranges containing short wavelengths will use a small extended region with a dense mesh.

5 PERFORMANCE AND EFFECTIVENESS TESTS

To demonstrate the performance and effectiveness of the proposed MLK NRB, we conducted a case study comparing it with two existing methods: ALID and conventional LK ABC. We also carried out a parametric study on the coverage length and plate thickness with an asymptotic case, followed by the discussion on the advantages and limitations of the MLK NRB.

Figure 12: Comparative case study with existing ALID method and conventional LK ABC.

5.1 COMPARISON WITH EXISTING TECHNIQUES

Figure 12 shows the setup for the comparative study between our MLK NRB method and the existing ALID and LK ABC methods. We used the same plate thickness, material properties, and parameters as used by Drozd et al. [19] for the ALID method. An 8-mm thick aluminum plate was used, with 135-mm extended absorbing region. The loss factor η was determined by

the cubic function shown in Figure 12c. Drozd et al. [19] also suggested that, when $K = 5$, the ALID method would work well for both symmetric and antisymmetric modes. For our MLK NRB, the coverage length L was set to be the same length as the ALID extended region. For this comparative study, we chose MLK profile to be the half Hanning window function of Eq. (10) with δ equals 0.15. Similar to Figure 7, symmetric and antisymmetric excitations were used to selectively generate symmetric and antisymmetric Lamb modes. The left side of the model extended to a very long distance to avoid reflections from the left boundary. The transmitted and reflected waves passing the same location were recorded. A large model was first used to obtain the excited wave signal. Then, the subtraction method was used to extract the waves reflected from the boundary.

Figure 13: Time domain excitation signal and its frequency spectrum.

Figure 14: Symmetric mode waves reflected from different boundary conditions.

A broad-band 2-cycle 300 kHz smoothed tone burst was used for Lamb wave excitation. Figure 13 shows the time trace and frequency spectrum of the excitation signal. The dispersion curves of Figure 6a indicates that such a broad-band excitation will produce higher Lamb modes. Thus,

this case study allows us to examine the MLK NRB performance in the presence of multimodal Lamb waves near the cut-off frequencies.

Figure 15: Frequency spectra of the reflected symmetric waves.

As in previous sections, two situations were examined: (a) symmetric modes; (b) antisymmetric modes. The symmetric mode results are shown in Figure 14 (time traces) and Figure 15 (frequency spectra). The signal amplitudes were normalized to the full reflection condition, where no absorbing mechanism was applied. Three boundary absorption methods were compared: (i) our MLK NRB; (ii) conventional LK ABC; and (iii) ALID. Examination of the time traces presented in Figure 14 indicates that our MLK NRB achieves much better performance than both ALID method and the conventional LK ABC, because our MLK NRB method yields almost insignificant reflections amplitudes whereas the other two method have quite significant reflection signals. Examination of the frequency spectra of Figure 15 shows the frequency components of the reflections. It shows that the conventional LK ABC works generally well for symmetric modes at low frequency ranges, but its performance deteriorates when higher modes appear. ALID method achieves better performance than the conventional LK ABC at high frequency range and cut-off frequencies, but poor performance can be noticed for low frequency components. Compared with ALID method, the MLK NRB achieves much better performance, in general, for all the frequency components. The full-reflection energy has the maximum magnitude just after the cut-off frequencies of the S1 and S2 modes where these modes begin to participate in the wave propagation process. At these frequencies, our MLK NRB method yields very low reflection amplitudes, much smaller than both LK and ALID. It's

thus clear that our MLK NRB is better than the existing methods both at low frequencies for the fundamental modes and near the cut-off frequencies for the higher order modes.

Figure 16: Antisymmetric mode waves reflected from different boundary conditions.

The antisymmetric mode results are shown in Figure 16 (time traces) and Figure 17 (frequency spectra). Examination of the time traces presented in Figure 16 indicates that both our MLK NRB method and the existing ALID method perform much better than the LK ABC. The frequency spectra of the reflected waves are shown in Figure 17. At low frequencies, where the fundamental A0 mode has a long wavelength, the conventional LK ABC and our MLK NRB give excellent results, much better than ALID. At the A1 cut-off frequency, where the ‘newcomer’ A1 mode has a very long wavelength, our MLK NRB method behaves again much better than ALID and LK ABC methods. Thus, it can be said that our MLK NRB method is better than the existing ALID and LK ABC methods at all frequencies.

This comparative study demonstrates the effectiveness of the proposed MLK method and its advantage over conventional/existing techniques for both symmetric and antisymmetric Lamb wave modes, both fundamental and higher order.

Figure 17: Frequency spectra of the reflected antisymmetric waves.

5.2 PARAMETRIC STUDY ON COVERAGE LENGTH AND PLATE THICKNESS

Section 3 provided some guidelines for the proper choice of MLK NRB parameters, with the coverage length L identified as very important for good performance. In this section, we present a parametric study on the coverage length L and plate thickness H in order to substantiate these guidelines and to further demonstrate the effectiveness of our MLK NRB method.

We used the FE model shown in Figure 7. The coverage length L was varied from 20 mm to 100 mm in steps of 20 mm. The plate thickness was changed from 2 mm to 10 mm in steps of 2 mm. The other parameters were consistent with those presented in Section 5.1. We used a 100 kHz 10-cycle tone burst to generate narrow-band Lamb waves in the plates. The transmitted and reflected waves were recorded and the reflection coefficients were calculated to serve as the evaluating metric. Figure 18 shows the results of this parametric study for both symmetric and antisymmetric modes: our MLK NRB is compared with the ALID method. Each bar represents a numerical case with a specific combination of the L and H parameters. The bar height and color are indicative of the reflection coefficient amplitude.

Figure 18: Parametric study results for MLK coverage length and plate thickness.

It can be noticed that when the coverage length goes beyond $L = 2\lambda$, the MLK NRB method achieves good performance for all the plate thicknesses with the reflection coefficients staying below 0.01, which implies that only 0.01% of the wave energy is reflected. This observation applies to both symmetric (S0) and antisymmetric (A0) Lamb wave modes. In contrast, the ALID method displays worse performance for S0 Lamb wave mode in thin plates (2 mm to 6 mm), although it works well for the A0 Lamb wave mode in thick plates. Based on this parametric study, we can conclude that the MLK NRB has advantages over the ALID method in thin plates, which are extensively used in aerospace and many mechanical engineering structures. For thick plates, which are found in naval and nuclear engineering structures, ALID method remains a better choice. This parametric study also confirms the guideline of $L = 2\lambda$ coverage for the effective absorption of Lamb waves with MLK NRB. As an illustration, the 2λ curves corresponding to the S0 and A0 modes were plotted on the charts to serve as a reference for when the MLK NRB method starts to become strongly effective.

5.3 AN ASYMPTOTIC CASE WITH DISCUSSION ON MLK NRB LIMITATIONS

Although the focus of this article is on Lamb wave absorption, the case of Rayleigh waves was also considered because the Rayleigh waves can be viewed as the asymptotic behavior of Lamb waves at high-frequency in thick plates. We considered the wave propagation in a semi-infinite medium with an end boundary. In such a situation, excitations will introduce both bulk waves and surface guided Rayleigh waves (Figure 19). The finite element model of this situation consisted of MLK NRB absorbing layers on the right and upper free surface and ALID boundary conditions at the bottom of the model to simulate the semi-infinite extension of the medium as shown in Figure 19. We used two excitation points: one placed on the surface (F_1)

and the other placed at a certain depth (F_2). The surface excitation F_1 would produce Rayleigh waves whereas the subsurface excitation F_2 would produce mostly bulk waves. A 500-kHz 5-cycle smoothed tone burst was used. The response was recorded at three depth locations, R1, R2, and R3 (Figure 19).

Figure 19: Finite element model for wave absorption in a semi-infinite medium.

The simulation results are presented in Figure 20 in the form of wave propagation snapshots. When surface excitation F_1 was applied, P-wave and strong Rayleigh wave were generated. The Rayleigh waves showed dominant surface motion with rapid amplitude decrease along the depth. The MLK NRB displayed good absorbing capability for the surface Rayleigh waves (Figure 20a).

Figure 20: MLK NRB effectiveness in a semi-infinite medium: (a) surface excitation generating predominantly Rayleigh waves; (b) sub-surface excitation simultaneously generating P-waves, S-waves, and Rayleigh waves.

When sub-surface excitation F_2 was applied, three wave types (P-wave, strong S-wave, and Rayleigh wave) were generated. When Rayleigh waves entered the MLK NRB region, they were again effectively absorbed. However, it was noticed that the S-wave reflection was not completely eliminated. When the S-wave entered the MLK NRB region, it did not decay much along the coverage path due to the fact that its dominant motion was far away from the absorbing top surface. Then, the S-wave interacted with LK ABC layer at the right hand end of the domain where it was damped significantly but not completely eliminated.

Time traces of the receiver signals at three locations R1, R2, and R3 are shown in Figure 21. When surface excitation was applied, the signal recorded close to the surface at R1 had the highest amplitude as expected from the surface-dominant motion of the Rayleigh wave. Since

the Rayleigh wave decays rapidly with the depth, the signals recorded at R2 and R3 are progressively smaller. A zoom-in was applied to the signal tail to identify reflections. As shown in the top of Figure 21, these reflections are indeed very small which indicates that the MLK NRB method has a good performance for the absorption of Rayleigh waves. When sub-surface excitation was applied (bottom of Figure 21), the reflections were found to be much stronger. We attribute this behavior to the fact that, in this case, S-waves also exist in addition to Rayleigh waves. As already shown in Figure 20b, these S-waves had large motion deep into the medium. This was especially clear at the R3 receiver, where strong reflected S-waves were picked up.

From this asymptotic example, we learned that our MLK NRB is also effective in absorbing surface guided waves (Rayleigh waves) in addition to absorbing plate guided waves (Lamb waves). However, its performance in absorbing bulk waves is not better than that of the conventional LK ABC method.

Figure 21: Receiver signals (u_y) at three locations R1, R2, and R3 for surface excitation (top) and sub-surface excitation (bottom).

6 MLK NRB APPLICATION TO TIME DOMAIN AND FREQUENCY DOMAIN ANALYSES

In this section, we will demonstrate the application of our MLK NRB method to two 3-D FEM Lamb-wave damage interaction examples in which its effectiveness was found particularly useful. The first example will be the time-domain transient analysis of Lamb wave propagation and interaction with a structural feature. The second example will be the frequency-domain harmonic analysis of Lamb wave scattering from a structural damage.

6.1 TIME DOMAIN TRANSIENT ANALYSIS OF LAMB-WAVE INTERACTION WITH A STRUCTURAL FEATURE

We wanted to analyze the interaction between Lamb waves and a structural feature such as a 5-mm diameter rivet hole in a 2.032-mm thick aluminum plate. The Lamb waves were generated by a transmitter piezoelectric wafer active sensor (T-PWAS) surface-mounted at a location 100 mm away from the hole. The measured signal was picked up at a sensing location placed onto the 100-mm wave propagation path between the transmitter and the hole. The pickup point was 10 mm away from the transmitter. The Lamb waves were generated by a 200-kHz 3-cycle smoothed tone burst excitation applied on the T-PWAS, which, because of its surface mounted location, generates both S₀ and A₀ Lamb waves. The measured signal was the in-plane strain along the wave propagation direction. The analysis was 3D using SOLID45 ANSYS elements with a 1-mm mesh size along in-plane directions and an approximately 0.5-mm mesh size in the thickness direction.

Figure 22: Transient analysis of the interaction between Lamb waves and a rivet hold in a 3D FEM model using the MLK NRB method: (a) wave propagation in a pristine plate; (b) wave propagation in a plate with a rivet hole.

In a conventional FEM analysis, one would have to consider a plate at least 0.5-m long and wide in order to avoid the boundary reflections from contaminating the signal scattered from the hole. This would likely lead to very large problem of approximately 3×10^6 DOFs which would take considerable time and much computer resources. However, with our implementation of the MLK NRB method, we were able to use only a small model of 0.28-m by 0.18-m. Our MLK NRB boundary absorbing layer extended 40 mm inward away from the

plate edges all the way around the plate.

Figure 23: Distinctive scatter effect identified by comparing with baseline pristine signal.

Figure 22 presents snapshots of the 3-D transient simulation showing Lamb wave generation, propagation, and interaction with the rivet hole. Two wave propagation cases were simulated: (a) a clean plate; (b) a plate with a rivet hole. In the clean plate (Figure 22a), the absorption of each of the S_0 and A_0 wave modes at the boundary can be clearly noticed; no boundary reflections are present. In the plate with a rivet hole (Figure 22b), the interaction between the incoming Lamb waves and the rivet hole resulted in scattered S_0 , SH_0 , and A_0 waves. These waves are clearly identifiable in Figure 22b. Again, no reflections from the boundaries were present, hence the scattered waves are easy to pick up. A time-trace of the signal picked up at the sensing location is given in Figure 23; the reflected S_0 and A_0 wave packets are clearly shown. The SH_0 reflection is not shown in this signal because the scattered SH_0 wave reaches its minimum amplitude along the sensing direction (as shown in Figure 22b). This example has demonstrated that important wave interaction phenomena can be efficiently modeled with a small FEM model by using the MLK NRB method to avoid boundary reflection contaminations to the scatter signal.

6.2 FREQUENCY-DOMAIN HARMONIC ANALYSIS OF LAMB-WAVE SCATTERING FROM A STRUCTURAL DAMAGE

Another important use of our MLK NRB method can be found in the frequency domain wave scatter analysis. Such frequency domain analysis would be used to compute the damage scatter coefficients to be used in a normal modes expansion (NME) representation of the wave-damage interaction phenomenon.

For example, consider the interaction of Lamb waves with the butterfly cracks emanating from a rivet hole as shown in Figure 24 and Figure 25. The difficulty of the problem resides in the fact that the rivet hole, even if pristine, is a scatterer. Hence, when crack damage occurs around the hole, the nature of this scatterer will change, but not very much. The challenge of the problem is to separate from the scattered field the effect of the crack from the effect of the pristine hole.

For this problem, the analysis is conducted in the frequency domain. Using our MLK NRB

technique, we constructed a small-size local FEM using the frequency domain harmonic analysis module of a commercial FE software. The schematic of this small-size local FEM model for the analysis of a rivet hole with butterfly cracks is shown in Figure 24. The MLK NRB treatment was applied around this local FEM model such that the incident wave and the scattered waves will be fully absorbed at the model boundaries. The loading nodes were used to selectively generate the desired Lamb modes into the local FEM. A circular sensing boundary was used to pick up the scattered waves propagating in all the directions. Harmonic analysis was carried out to get the frequency domain response of the structure. It should be noted that the harmonic wave field will not support standing waves between the scatterer and the boundaries because the boundaries are fully absorbent. Instead, a propagating harmonic wave field will be created since the MLK NRB treatment simulates an infinite region outside the small FEM model. Due to its small size, the local FEM with MLK NRB treatment is very fast and efficient. The benefit of a harmonic analysis lies in the fact that it can provide the structural response under all the frequencies of interest with only one run of the simulation, i.e., in commercial FE software packages, one only needs to define the specific frequency range and step size, and the software will calculate the results for all the specified frequencies. This is an important benefit and advantage compared with time-domain transient analysis in which one has to conduct the simulation separately again and again to cover all the frequencies of interest. Of course, Fourier transform based post-processing of the transient analysis results may be applied to cover a wider frequency band, but this would require additional effort.

Figure 24: Schematic of the small-size local FEM for a 2.032-mm thick aluminum plate and the rivet hole with butterfly cracks.

The scatter field was described in terms of wave-damage interaction coefficients (WDICs), as discussed in Ref [27, 28]. In our study, two situations were computed: (1) a plate with undamaged rivet hole generating the $WDIC_{undamaged}$ scatter field; and (2) a plate with a damaged rivet hole containing butterfly cracks generating the $WDIC_{damaged}$ scatter field. The

scatter field of the butterfly crack $WDIC_{crack}$ was extracted from the total scatter field $WDIC_{damaged}$ by subtraction of the undamaged scatter field $WDIC_{undamaged}$, i.e.,

$$WDIC_{crack} = WDIC_{damaged} - WDIC_{undamaged} \quad (15)$$

For illustration, Figure 25 present a few results from our analysis for the case of an S0 Lamb wave incident.

Figure 25: (a) WDIC directivity plots under various frequencies; (b) WDIC in the forward direction as a function of frequency.

Figure 25a shows the scattered S0 wave amplitude $C_{ss}(\omega, \theta)$ from an incident S0 mode impinging upon the damage. It can be observed that the scattered wave amplitude is a function of frequency ω and the azimuthal angle θ . Directivity plots are presented at various frequencies; one notices that the azimuthal scatter pattern of the directivity plots changes dramatically with frequency. It can be noticed that, when the butterfly cracks are directly facing the transmitter direction, the best sensing location seems to be along the wave propagation direction as indicated by the red arrows in Figure 25. Similar results can be obtained for the A0

incidence [27, 28], but they will not be presented here for sake of brevity. For the same reason, mode conversion effects are not presented here either.

Since, the additional scatter field due to the butterfly crack damage is frequency dependent, the possibility of an optimal interrogation frequency arises. Figure 25b presents a frequency plot of scatter field amplitude measured along the wave propagation direction. This frequency plot indicates that the best interrogation frequency seems to be around 482 kHz when the damage scatter field amplitude reaches a peak. Also noticed in Figure 25b is the fact that certain frequencies might be bad choices because the scatter field would be very small, e.g., the 326 kHz frequency when the scatter amplitude is very small. It should be noted that the result and discussion was specific for this example. For different damage type, crack orientation, and interrogating wave modes, the results and best detection strategy may vary from case to case.

7 SUMMARY AND CONCLUSIONS

This article presented a new method for constructing non-reflective boundaries (NRB) when using finite element method (FEM) analysis to simulate Lamb wave interaction with structural defects and damage. Our method is an extension of the LK ABC method that was developed for absorbing P and S waves at the edge of a semi-infinite half space. Our new contribution consists in extending the LK concept to the case of a plate. In our modification of the LK concept, we wrapped the LK absorbing layer to the top and bottom surfaces of the plate. (In the LK model, the absorbing layer was only applied to the vertical end of the semi-infinite half space.) We call our approach the modified Lysmer-Kuhlemeyer non-reflective boundaries (MLK NRB).

The main idea in our MLK NRB method is to take into account the fact that Lamb waves, by their nature, are the superposition of P and S bulk waves that undergo multiple reflections at the top and bottom surfaces of the plate. Hence, we extended the LK absorbing layer over the top and bottom surfaces of the plate in order to attenuate these multiple P and S reflections in addition to attenuating their reflections at the vertical edge of the plate.

The paper started with a review of the existing non-reflective boundary literature and identified three major directions (i) infinite element methods; (ii) non-reflective boundary conditions, e.g., the conventional LK ABC method; and (iii) absorbing layer methods, e.g., the ALID family of methods. Current Lamb wave absorption techniques were found to stem mainly from the ALID family. The paper continued to discuss why the LK ABC approach is inadequate for Lamb waves and how it could be modified to compensate for this inadequacy. Subsequently, the paper presented details of our MLK NRB method. Several parametric studies were conducted to develop guidelines for the proper choice of MLK NRB parameters. Details of the implementation method and strategy of MLK NRB was provided in a systematic manner.

Comparative studies between our MLK NRB method and the existing ALID NRB and LK ABC methods were conducted. The studies showed that our MLK NRB method has better performance for all Lamb modes. The case of long wavelength Lamb waves that may appear at

low frequencies for the fundamental S0, A0 modes and near the cut-off frequencies for the higher modes was also studied. It was again found that MLK NRB gives better performance.

Parametric study on the coverage length and plate thickness were also conducted. It was found that our MLK NRB method is better than the ALID method when applied to thin plate structures. However, in thicker plates, the ALID method seems to work better than our MLK NRB method.

Two application examples of our MLK NRB method were given. One example was performed in the time-domain and showed how a Lamb wave package scatters from a structural feature such as a rivet hole. The other example was performed in the frequency domain and illustrated how additional scatter due to butterfly crack damage in a rivet hole can be extracted from the scatter field of the pristine hole. Interesting effects related to the variation of the damage scatter with frequency were noticed and desirable interrogation frequencies (as well as undesirable ones) were identified. The application of our MLK NRB method to reflection suppression in a semi-infinite half space was also studied. It was found that our MLK NRB method can effectively suppress Rayleigh wave reflections, but is not very effective in suppressing S-wave reflections.

Since our MLK NRB method is based on a totally different mechanism than the ALID family of methods, the question arises as to whether these two methods could be combined to get an even better performance for suppressing Lamb wave reflections at the plate boundaries. We believe that the mechanisms behind MLK NRB and ALID methods are complimentary to each other; hence, the investigation of a hybrid technique combining these two mechanisms is fully justified and should be pursued in future work.

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9 REFERENCES

- [1] D. Alleyne and P. Cawley, "The interaction of Lamb waves with defects," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 39, pp. 381-397, 1992.
- [2] F. Moser, L. Jacobs and J. Qu, "Modeling elastic wave propagation in waveguides with the finite element method," *NDT&E International*, vol. 32, pp. 225-234, 1999.
- [3] M. Gresil, Y. Shen and V. Giurgiutiu, "Benchmark problems for predictive FEM simulation of 1-D and 2-D guided waves for structural health monitoring," *Review of Progress in*

Quantitative Nondestructive Evaluation, vol. 31, pp. 1835-1842, 2012.

- [4] J. Lysmer and R. Kuhlemeyer, "Finite dynamic model for infinite media," *Journal of the Engineering Mechanics Division Proceedings of the American Society of Civil Engineers*, vol. EM4, pp. 859-877, 1969.
- [5] F. Hastings, J. Schneider and S. Broschat, "Application of the perfectly matched layer (PML) absorbing boundary conditions to elastic wave propagation," *Journal of Acoustic Society of America*, vol. 100, pp. 3061-3096, 1996.
- [6] G. Liu and S. Jerry, "A non-reflecting boundary for analysing wave propagation using the finite element method," *Finite Elements In Analysis and Design*, vol. 39, pp. 403-417, 2003.
- [7] P. Rajagopal, M. Drozd, E. Skelton, M. Lowe and R. Craster, "On the use of absorbing layers to simulate the propagation of elastic waves in unbounded isotropic media using commercially available finite element packages," *NDT&E International*, vol. 51, pp. 30-40, 2012.
- [8] D. Givoli and J. Keller, "Non-reflecting boundary conditions for elastic waves," *Wave Motion*, vol. 12, pp. 261-279, 1990.
- [9] D. Givoli, "High-order local non-reflecting boundary conditions: a review," *Wave Motion*, vol. 39, pp. 319-326, 2004.
- [10] L. Moreau, A. Velichko and P. Wilcox, "Accurate finite element modelling of guided wave scattering from irregular defects," *NDT&E International*, vol. 45, pp. 46-54, 2012.
- [11] J. Berenger, "A perfectly matched layer for the absorption of electromagnetic waves," *J. Comp. Phys.*, vol. 114, pp. 185-200, 1994.
- [12] E. Skelton, S. Adams and R. Craster, "Guided elastic waves and perfectly matched layers," *Wave Motion*, vol. 44, pp. 573-592, 2007.
- [13] M. Israeli and S. Orszag, "Approximation of radiation boundary conditions," *Journal of Computational Physics*, vol. 41, pp. 115-135, 1981.
- [14] M. Castaings, C. Bacon, B. Hosten and M. Predoi, "Finite element predictions for the dynamic response of thermo-viscoelastic material structures," *Journal of Acoustical Society of America*, vol. 115, pp. 1125-1133, 2004.
- [15] M. Castaings, C. Bacon, B. Hosten and M. Predoi, "Finite element modeling of torsional wave modes along pipes with absorbing materials," *Journal of Acoustical Society of America*, vol. 119, pp. 3741-3751, 2006.
- [16] W. Ke, M. Castaings and C. Bacon, "3D finite element simulations of an air-coupled

- ultrasonic NDT system," *NDT&E International*, vol. 42, pp. 524-533, 2009.
- [17] J. Semblat, L. Lenti and A. Gandomzadeh, "A simple multi-directional absorbing layer method to simulate elastic wave propagation in unbounded domains," *International Journal of Numerical Methods in Engineering*, vol. 85, pp. 1543-1563, 2011.
- [18] J. Pettit, A. Walker, P. Cawley and M. Lowe, "A stiffness reduction method for efficient absorption of waves at boundaries for use in commercial finite element codes," *Ultrasonics*, vol. 54, pp. 1868-1879, 2014.
- [19] M. Drozd, L. Morreau, M. Castaings, M. Lowe and P. Cawley, "Efficient numerical modelling of absorbing regions for boundaries of guided waves problems," *AIP Conference Proceedings*, vol. 820, pp. 126-133, 2006.
- [20] Z. Fan and M. Lowe, "Interaction of weld-guided waves with defects," *NDT&E International*, vol. 47, pp. 124-133, 2012.
- [21] S. Hosseini, S. Duczek and U. Gabbert, "Non-reflecting boundary condition for Lamb wave propagation problems in honeycomb and CFRP plates using dashpot elements," *Composites: Part B*, vol. 54, pp. 1-10, 2013.
- [22] J. Rose, *Ultrasonic Waves in Solid Media*, Cambridge University Press, 2004.
- [23] K. Graff, *Wave Motion in Elastic Solids*, New York: Dover Publications, INC., 1975.
- [24] V. Giurgiutiu, *Structural health monitoring with Piezoelectric Wafer Active Sensors (Second Edition)*, Elsevier Academic Press, ISBN 978-0-12-418691-0, 2014.
- [25] J. Liu, Y. Du, X. Du, Z. Wang and J. Wu, "3D viscous-spring artificial boundary in time domain," *Earthquake Engineering and Engineering Vibration*, vol. 5, no. 1, pp. 93-102, 2006.
- [26] "ANSYS 14.0 Help Manual".
- [27] Y. Shen and V. Giurgiutiu, "WFR-2D: an analytical model for PWAS-generated 2D ultrasonic guided wave propagation," in *SPIE Smart Materials and Structures/NDE*, San Diego, 2014.
- [28] Y. Shen, *Structural health monitoring using linear and nonlinear ultrasonic guided waves*, Doctoral dissertation, University of South Carolina, Columbia, SC, August 2014, <http://scholarcommons.sc.edu/etd/2883/>, 2014.