A novel variable step-size normalized subband adaptive filter based on mixed error cost function

Pengwei Wen and Jiashu Zhang*

Abstract—A novel variable step-size algorithm is proposed for normalized subband adaptive filter. The proposed algorithm is based on the mixed error cost function. By assuming the time-averaging estimate of the *priori* and *posteriori* errors equals the variance of subband noise, the step-size is obtained. Therefore, the proposed algorithm has more effective approach to the optimum solution. The power of noise-free subband *priori* error is obtained by using the shrinkage denoising method. Using the energy conservation method, the mean-square convergence performance analysis is presented. The analysis result shows this algorithm is stable and effective. The simulation results demonstrate the performance of proposed algorithm distinctly outperforms other conventional variable step-size algorithms in both steady-state error and abrupt tracking performance.

Index Terms—Normalized subband adaptive filter, mixed error cost function, variable step-size.

1. Introduction

As well known, the normalized least-mean-square (NLMS) algorithm is one of the popular algorithms because it is easy to implement and has low computational complexity. However, the performance of the NLMS algorithm will be degraded severely when the input signals are colored [1]. This problem is frequently encountered in a wide variety of situations, such as underwater acoustics, audio processing, broadband power line communications, robust multiuser detection, real time traffic prediction and low frequency atmospheric propagation [2].

To solve the problem, a normalized subband adaptive filtering (NSAF) algorithm has been developed and learned in [3]. The merit of NSAF is that it can decompose the signals into many almost mutually repulsive subband signals to whiten them [4]. Considering the property of the NSAF algorithm, a suitable choice of step-size is important because it controls the compromise between the convergence speed and the misalignment. The collide could be solved by using the variable step-size, and various ways have been presented [5–7]. By assuming that the l_2 -norm of the *a posteriori* error is equal to the measurement noise variance in [5], a called variable step-size matrix NSAF (VSSM-NSAF) has been designed and is capable of tracking unknown system, however, exhibits a large steady-state error. Two kind of variable step-size NSAF algorithms were derived based on minimizing the mean-square deviation (MSD), which are VSS-NSAF [6] and individual VSS-NSAF (VISS-NSAF) [7]. The VSS-NSAF algorithm achieves a low steady-state error and fast convergence speed under stationary systems, its re-adaptation ability degrades in changed environment. The performance of VISS-NSAF outperforms the former proposed variable step-size algorithms. However, it still has a large steady-state estimation error.

In this paper, a novel variable step-size NSAF based on mixed error cost function is proposed. The optimum individual step-size is derived by assuming that the cross-correlation function of *priori* and *posteriori* errors is equal to the power of subband noise. And the shrinkage denoising method is used to obtain the power of noise-free subband *priori* error, this method has been detailed described in [8–11]. Then, the convergence performance analysis of the proposed algorithm is presented in this paper, which demonstrates the process of weight update is stable. The performance of the proposed algorithm is evaluated for system identification and compared with that of the VSS- NSAF, VSSM-NSAF and VISS-NSAF algorithms. The simulation results show that the proposed algorithm achieves a fast convergence rate at the instant and a low steady-state error at the steady-state.

This paper is organized as follows. Section 2 introduces the NSAF algorithm. Section 3 describes the proposed algorithm. Section 4 illustrates the convergence analysis of the proposed algorithm. Section 5 illustrates the simulation results obtained using the proposed algorithm, and Section 6 presents some conclusions.

2. Conventional NSAF algorithm

Fig. 1 displays the construction of the subband adaptive filter [12]. Consider a desired signal

$$d(n) = \mathbf{u}^T(n) \mathbf{w}_o + v(n) \quad (1)$$

where $\mathbf{u}(n) = [u(n), u(n-1), \dots, u(n-M+1)]^T$ is the input signal, $\mathbf{w}_o$ and $v(n)$ denote the unknown weight vector with the length M and the background noise, respectively. The noise $v(n)$ and $\mathbf{u}(n)$ are statistical independence, and given to be stationary and zero mean. In Fig.1, the signals $\mathbf{u}(n)$ and $d(n)$ are decomposed into $\mathbf{u}_i(n)$ and $d_i(n)$ by the filters $H_i(z)$ ($i=0, 1, \dots, N-1$). And the subband desired output signals $y_i(n)$ are acquired from $\mathbf{u}_i(n)$ filtered by $\hat{\mathbf{w}}(k)$. N is the number of subband. Notice that we use the variable n to index the primitive signal sequences, and k to index the decomposed orders. The signals $d_i(n)$ and $y_i(n)$ are decimated to $d_{i,D}(k)$ and $y_{i,D}(k)$ respectively. We can know $d_{i,D}(k) = \mathbf{u}_{i,D}^T(k) \mathbf{w}_o + v_{i,D}(k)$. Then, the weight vector of the NSAF is expressed as

$$\hat{\mathbf{w}}(k+1) = \hat{\mathbf{w}}(k) + \mu \sum_{i=0}^{N-1} \frac{\mathbf{u}_i(k)}{\|\mathbf{u}_i(k)\|^2} e_{i,D}(k) \quad (2)$$

*Corresponding author.

E-mail addresses: 623101594@qq.com(P. Wen), jszhang@home.swjtu.edu.cn (J. Zhang).

the step size μ must meet $0 < \mu < 2$. where $\mathbf{u}_i(k)=[u_i(kN), u_i(kN-1), \dots, u_i(kN-M+1)]^T$ and $v_i(k)$ is a i th subband noise.

The i th subband output error is

$$e_{i,D}(k) = d_{i,D}(k) - y_{i,D}(k) = d_{i,D}(k) - \mathbf{u}_i^T(k) \hat{\mathbf{w}}(k) \quad (3)$$

Defining the *priori* and *posteriori* error signals respectively as

$$e_{a,i} = (\mathbf{w}_o - \hat{\mathbf{w}}(k))^T \mathbf{u}_i(k) + v_i(k) \quad (4)$$

$$e_{p,i} = (\mathbf{w}_o - \hat{\mathbf{w}}(k+1))^T \mathbf{u}_i(k) + v_i(k) \quad (5)$$

Fig. 1 Structure of subband adaptive filtering

3. The proposed algorithm

3.1) The proposed algorithm

To derive the proposed algorithm, (2) is rewritten as

$$\hat{\mathbf{w}}(k+1) = \hat{\mathbf{w}}(k) + \sum_{i=0}^{N-1} \mu_i(k) \frac{\mathbf{u}_i(k)}{\|\mathbf{u}_i(k)\|^2} e_{i,D}(k) \quad (6)$$

The nonparametric individual step size for (6) could be obtained according to the time-varying estimate of the cross-correlation of $e_{a,i}(k)$ and $e_{p,i}(k)$. Like the way in [13], the step-size would be proposed thus:

$$J_{ap,i}(k) = E(e_{a,i}(k) e_{p,i}(k)) = \sigma_{v_i}^2 \quad (7)$$

where $E\{\cdot\}$ denotes mathematical expectation and $\sigma_{v_i}^2$ is the variance of each subband system noise. Using the approximation in [14], $\mathbf{u}_i^T(k) \mathbf{u}_i(k) = M \sigma_{u_i}^2$ and $\sigma_{v_i}^2 = \sigma_v^2 / N$.

Through straightforward calculations, (7) could be written as

$$J_{ap,i}(k) = (1 - \mu_i(k)) \sigma_{e,i}^2(k) = \sigma_{v_i}^2 \quad (8)$$

Thus,

$$\mu_i(k) = 1 - \frac{\sigma_{v_i}^2}{\delta + \sigma_{e,i}^2(k)} \quad (9)$$

where δ is greater than zero used to avoid division by zero. $\sigma_{e,i}^2$ is the power of each subband error signal.

Remark: During the transient-state, we know $\sigma_{e,i}^2 \gg \sigma_{v_i}^2$. Thus, the proposed algorithm will resemble the NSAF scheme. And

when the algorithm starts to converge to the steady-state, i.e. $\sigma_{v_i}^2 \approx \sigma_{e,i}^2(k)$, which leads to $\mu_i(k) \approx 0$. As can be shown, the proposed algorithm achieves both fast convergence rate and low steady-state error.

Denoting the noise-free subband *priori error* $e_{f,i} = (\mathbf{w}_o - \hat{\mathbf{w}}(k))^T \mathbf{u}_i(k)$, thus

$$\sigma_{e,i}^2 = \sigma_{f,i}^2 + \sigma_{v_i}^2 \quad (10)$$

where $\sigma_{f,i}^2$ is the variance of $e_{f,i}$.

However, the noise-free subband *priori error* is unknown, the main difficulty in (9) is to calculate $\sigma_{f,i}^2$. The noise-free subband *priori error* $e_{f,i}$ can be acquired by using shrinkage denoising method in [8-11]. The cost function is

$$\min_{e_{f,i}(k)} \left(\frac{1}{2} (e_{i,D}(k) - e_{f,i}(k))^2 + t |e_{f,i}(k)| \right) \quad (11)$$

where the threshold parameter t is chosen as $\sqrt{Q\sigma_{v,i}^2}$. Its solution can be obtained as

$$e_{f,i}(k) = \text{sign}(e_{i,D}) \max(|e_{i,D}(k)| - t, 0) \quad (12)$$

Then, $\sigma_{f,i}^2$ can be estimated through

$$\sigma_{f,i}^2(k) = \lambda \sigma_{f,i}^2(k-1) + (1-\lambda) e_{f,i}^2(k) \quad (13)$$

where λ is a forgetting factor.

As a consequence, the proposed algorithm is summarized in Table I.

Table 1
Summary of the proposed algorithm

Initialization:	
$\hat{\mathbf{w}} = 0, e_{i,D} = 0 \text{ for } n \leq 0$	
$\delta = 0.001, \sigma_{v,i}^2, \sigma_e^2, \lambda, Q = 1$	Multiplication
For $k=1,2,3,\dots$ at $1/N$	
For $i=1,\dots,N$	
$e_{i,D}(k) = d_{i,D}(k) - \mathbf{u}_i^T(k) \hat{\mathbf{w}}(k)$	M
$e_{f,i}(k) = \text{sign}(e_{i,D}) \max(e_{i,D}(k) - t, 0)$	1
$\sigma_{f,i}^2(k) = \lambda \sigma_{f,i}^2(k-1) + (1-\lambda) e_{f,i}^2(k)$	3
$\mu_i(k) = 1 - \frac{\sigma_{v_i}^2}{\delta + \sigma_{f,i}^2 + \sigma_{v_i}^2}$	2
end	
$\hat{\mathbf{w}}(k+1) = \hat{\mathbf{w}}(k) + \sum_{i=0}^{N-1} \mu_i(k) \frac{\mathbf{u}_i(k)}{\ \mathbf{u}_i(k)\ ^2} e_{i,D}(k)$	$2M$
end	

3.2) Complexity analysis

With the length L of the analysis filters, the number of subbands N and the filter length M , the computational complexity of the proposed algorithm is compared with that of other cited algorithms in terms of the total number of additions and multiplications. The computational requirements for these adaptive filters with corresponding adaptive algorithm are summarized in Table 2. Compared with the NSAF algorithm, the additional calculation cost of the proposed algorithm stems from the computation of (9), (12) and (13) respectively. The proposed algorithm needs additional 6 multiplications and 5 additions for all subbands. The computational complexity of the proposed algorithm is in direct proportion to the number of subbands. In addition, the slight increase of the proposed algorithm in computational complexity could be compromised by its improved performance.

Table 2
Summary of the computational complexity

Algorithm	Multiplication	Addition
NSAF	$3M+3NL+1$	$3M+3N(L-1)$
VSSM-NSAF	$6M+3NL+8$	$5M+3N(L-1)+3$
VSS-NSAF	$3M+(2M+3)/N+3NL$	$2M+(M+2)/N+3NL-2N$
VISS-NSAF	$3M+3NL+N(M+2N+7)$	$3M+3N(L-1)+2MN$
proposed	$3M+3NL+7$	$3M+3N(L-1)+5$

4. Convergence analysis of proposed algorithm

In this section, the convergence of the proposed algorithm is analyzed based on the mean square deviation (MSD) defined by

$$c(k) = E[\|\mathbf{v}(k)\|^2] = E[\|\mathbf{w}_o(k) - \hat{\mathbf{w}}(k)\|^2] \quad (14)$$

where $\|\cdot\|$ denotes the Euclidean norm of a vector.

Subtracting (6) from $\mathbf{w}_o$, taking the squared Euclidean norm of both sides, and using the diagonal assumption [15], we have

$$\|\mathbf{v}(k+1)\|^2 = \|\mathbf{v}(k)\|^2 - 2 \sum_{i=0}^{N-1} \mu_i(k) \frac{(e_{i,a}(k) - v_i(k))e_{i,D}(k)}{\|\mathbf{u}_i(k)\|^2} + \sum_{i=0}^{N-1} \mu_i^2(k) \frac{e_{i,D}^2(k)}{\|\mathbf{u}_i(k)\|^2} \quad (15)$$

Taking the mathematical expectation of (15), we get

$$c(k+1) = c(k) - 2 \sum_{i=0}^{N-1} \mu_i(k) E\left[\frac{(e_{i,a}(k) - v_i(k))e_{i,D}(k)}{\|\mathbf{u}_i(k)\|^2}\right] + \sum_{i=0}^{N-1} \mu_i^2(k) E\left[\frac{e_{i,D}^2(k)}{\|\mathbf{u}_i(k)\|^2}\right] \quad (16)$$

Assuming $\|\mathbf{u}_i(k)\|^2$ is uncorrelated with $e_{i,D}(k)$ for sufficiently long filters [16], we have

$$c(k+1) - c(k) = -2 \sum_{i=0}^{N-1} \mu_i(k) \frac{E[e_{i,D}^2(k)] - \sigma_{v_i}^2}{E[\|\mathbf{u}_i(k)\|^2]} + \sum_{i=0}^{N-1} \mu_i^2(k) \frac{E[e_{i,D}^2(k)]}{E[\|\mathbf{u}_i(k)\|^2]} \quad (17)$$

Substituting (3)-(4) and (9) into (17) yields

$$c(k+1) - c(k) = - \sum_{i=0}^{N-1} \mu_i(k) \frac{3\sigma_{e,i}^2 - 2\sigma_{v_i}^2 - \frac{\sigma_{v_i}^2 \sigma_{e,i}^2}{\delta + \sigma_{e,i}^2}}{E[\|\mathbf{u}_i(k)\|^2]} \quad (18)$$

It is clear that $c(k+1) - c(k) \leq 0$ because of $\sigma_{e,i}^2 \geq \sigma_{v_i}^2$, which implies that the proposed algorithm is convergent as k increases [17]. The weight update process of the proposed algorithm is stable.

Fig. 2 Measured impulse response of the acoustic echo path

5. Simulation results

In this section, to evaluate the performance of the proposed algorithm, simulations through Monte Carlo (MC) are performed on the system identification problem. The cosine modulated filter bank is used in all the subband filter algorithms. We use the measured impulse response of the acoustic echo path as the unknown system, which is depicted in Fig. 4 [18]. In the exact-modeling, the measured impulse response of the acoustic echo path was applied as the unknown to be system identified and it was truncated to the first 512 weights. The mean square deviation (MSD) curves is used as performance evaluation, which is obtained by ensemble averaging the results of 100 independent trials. The input signal was colored, which was acquired by using a white Gaussian random sequence through an AR(1) with a pole at 0.9. The signal-to-noise ratio (SNR) at the system output is given by:

$$SNR = 10 \log_{10} \left(\frac{\sigma_d^2}{\sigma_v^2} \right) \quad (19)$$

where σ_d^2 is the variance of the uncorrupted output of the unknown system and σ_v^2 is the variance of observation noise $v(n)$.

Fig. 3 MSDs comparison of the proposed algorithm with different number of subbands, SNR=30dB.

Firstly, the performance of the proposed algorithm is compared for different number of subbands (i.e., $N=2, 4$, and 8) in Fig. 3. The adaptive filter tap-length is 512. The forgetting factor λ was set 0.9945. With the number of subbands increasing, the convergence speed of the proposed algorithm becomes fast and the steady-state MSD reduces. However, this situation will not apparent when the number of subbands is larger than a positive value (in this case is 4). Besides with the number of subbands increasing, the computational complexity also increases. For example, when N equals 4, the proposed algorithm needs 1927 multiplications and 1913 additions. And when N equals 8, the proposed algorithm needs 3079 multiplications and 3053 additions. Thus, a suitable number of subbands are determined as a trade-off between convergence rate and steady-state MSD. The number

of subbands is chosen as $N = 4$ and prototype filter length $L=32$ in the following simulations.

In the second experiment, we compare the proposed algorithm with the conventional NSAF, VSS-SSAF, VSSM-NSAF, VSS-NSAF and the VISS-NSAF algorithms with different SNRs. For fair comparisons, the parameters $\mu = 0.8$ and 0.4 , are chosen for the NSAF. $\delta=15\sigma_u^2$ and $\kappa = 6$ for the VSSM-NSAF, $\alpha = 0.997$ for the VSS-NSAF, $\text{Tr}\{P(0)\}$ is set to be 100 and $\beta = 1$ for the VISS-NSAF [4], the forgetting factor α is the same with that in the first experiment for the proposed algorithm. For more intuitive comparison, the numerical costs computational of the compared algorithms are displayed. In this experiment, the NSAF needs 1921 multiplications, VSS-NSAF needs 2177 multiplications, VSSM-NSAF needs 3464 multiplications and the VISS-NSAF algorithm needs 4028 multiplications. The simulation result is shown in Fig. 4.

As can be seen in Fig. 4, compared with the conventional NSAF, VSS-SSAF, VSSM-NSAF, VSS-NSAF and VISS-NSAF algorithms, the proposed algorithm achieves a faster convergence speed and an improved steady-state misalignment than the other cited algorithms under different SNRs environments. Especially for low SNR, the performance of the proposed algorithm is evidently better than other conventional algorithms. The proposed algorithm is more suitable for the environment with low SNR.

In the last experiment, the tracking performance of the proposed algorithm is compared with the conventional NSAF, VSSM-NSAF, VSS-NSAF and the VISS-NSAF algorithms in the scenario of sudden environment change, where the coefficients of the room impulse response change during the iterations. The entries of the impulse response vector were abruptly multiplied by -1 at iteration 25000. The simulation settings are the same as the former experiment.

As illustrated in Fig. 5, the VISS-NSAF algorithm fails to track the system change. This is because the subband *priori error* $e_{a,i}$ is not involved in the step-size updating scheme. Thus the step-size adaptation of VISS-NSAF can't reflect the changing of the system. Although the NSAF, VSSM-NSAF, and VSS-NSAF algorithms could track the system change, they have high steady-state MSDs. The proposed algorithm exhibits obviously good tracking ability and low steady-state estimate error superior to the comparative algorithms.

(a)

(b)

Fig.4 MSDs of NSAF, VSSM-NSAF, VSS-NSAF, VISS-NSAF and proposed algorithm for AR(1) input signal,(a) SNR=10dB. (b) SNR=20dB. (c) SNR=30dB.

Fig.5 Tracking performance comparison of the algorithms at SNR=30 dB.

6. Conclusion

In this paper, a novel variable step-size NSAF algorithm is proposed. The step-size is derived by assuming the time varying-averaged of the cross-correlation of the *priori* and *posteriori* errors is equal to the power of subband noise. And the shrinkage denoising method is used to obtain the variance of noise-free subband *priori* error. The complexity and convergence analysis of the proposed algorithm is carried out based on the energy conservation method. The analysis result shows the algorithm is stable and effective. By contrasting with several conventional variable step-size algorithms under different noise environments, simulations on system identification show that the proposed algorithm obtains 9dB attenuation in term of steady-state MSD than the conventional NSAF at the same convergence rate. Simultaneously, the proposed algorithm has the capacity of tracking unknown systems.

Acknowledgments

This work was partially supported by National Science foundation of P. R. China (Grant: 61671392).

Pengwei Wen and Jiashu. Zhang (*The Sichuan province Key Lab of Signal and Information Processing at Southwest Jiaotong University, Chengdu, 610031,P.R.China*)

References

- 1 B. Widrow and S. D. Sterns, Adaptive Signal Processing. Englewood Cliffs, NJ: Prentice-Hall, 1985.
- 2 A.H. Sayed, Fundamentals of Adaptive Filtering, Wiley–Interscience, New York, 2003.
- 3 K.A. Lee, and W.S. Gan,: ‘Improving convergence of the NLMS algorithm using constrained subband updates’, IEEE Signal Process. Lett., 2004, 11, (9), pp. 736–739.
- 4 K.A. Lee, and W.S. Gan, Inherent decorrelating and least perturbation properties of the normalized subband adaptive filter, IEEE Trans. Signal Process. 54 (11) (2006) 4475–4480.
- 5 J Ni, and F Li, ‘A variable step-size matrix normalized subband adaptive filter’, IEEE Trans. Audio Speech Lang. Process., 2010, 18, (6), pp. 1290–1299.
- 6 Jeong, J.J., Koo, K.H., Choi, G.T., and Kim, S.W.: ‘A variable step size for normalized subband adaptive filters’, IEEE Signal Process. Lett., 2012, 19, (12), pp. 906–909.
- 7 Seo J H, Park P G. Variable individual step-size subband adaptive filtering algorithm[J]. Electronics Letters, 2014, 50(3): 177-178.
- 8 S Zhang, J Zhang, H Han. Robust Shrinkage Normalized Sign Algorithm in Impulsive Noise Environment [J]. IEEE Transactions on Circuits and Systems II: Express Briefs (Accepted).
- 9 M.Z.A. Bhotto, M.O. Ahmad, M.N.S Swamy. Robust shrinkage affine-projection sign adaptive-filtering algorithms for impulsive noise environments[J]. IEEE Trans. on Signal Processing, 2014, 62(13): 3349-3359.
- 10 M.Z.A. Bhotto, A. Antoniou A family of shrinkage adaptive-filtering algorithms[J]. IEEE Trans. on Signal Processing, 2013, 61(7): 1689-1697.
- 11 W Xia, L Zhu, J Zhu, et al. A Shrinkage Variable Step Size for Normalized Subband Adaptive Filters[J]. Signal Processing, 2016.
- 12 P Wen, S Zhang, J Zhang. A novel subband adaptive filter algorithm against impulsive noise and it's performance analysis[J]. Signal Processing, 2016, 127: 282-287.
- 13 J Liu, X Yu, H Li. A nonparametric variable step-size NLMS algorithm for transversal filters[J]. Applied Mathematics and computation, 2011, 217(17): 7365-7371.
- 14 K.A. Lee, W.S. Gan, S.M. Kuo, Subband adaptive Filtering: Theory and Implementation, Wiley, Hoboken, NJ, 2009.
- 15 Jeong J J, Koo K, Koo G, et al. Steady-state mean-square deviation analysis of improved normalized subband adaptive filter[J]. Signal Processing, 2015, 106: 49-54.
- 16 T. Y. Al-Naffouri and A. H. Sayed, Transient analysis of adaptive filters with error nonlinearities, IEEE Trans. Signal Processing, vol. 51, pp. 653–663, Mar. 2003.
- 17 S. Zhang, J. Zhang, Transient analysis of zero attracting NLMS algorithms without Gaussian input signal, Signal Process. 97 (2013) 100–109.
- 18 J.H. Kim, J.H. Chang, S.W. Nam, Sign subband adaptive filter with l_1 - norm minimisation-based variable step-size, Electron. Lett. 49 (21) (2013) 1325–1326.
- 19 JW Shin, JW Yoo, PG Park. Variable step-size sign subband adaptive filter[J]. IEEE Signal Processing Letters, 2013, 20(2): 173-176.