An Evaluation Method to Predict Progressive Collapse

Resistance of Steel Frame Structures

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Abstract:

In the current work, a probabilistic assessment method of a steel framed building under abrupt removal of a column due to catastrophic events is developed. A multi-storey steel framed model taking into account the influence of catenary effect has been analyzed. Uncertainties in the structural variables are incorporated in the probabilistic simulation approach. Based on the changes of component internal energy, the progressive collapse sensitivity to abrupt removal of a column has been investigated. Besides, a simplified beam damage model is proposed to analyze the energies absorbed and dissipated by structural beams under large deflections. In addition, the correlation **incorporating catenary action** between bending moment and axial force in a beam during the whole deformation development process is studied. With the methodologies adopted for progressive collapse assessment under removal of a column, a deterministic method has been developed, framed within the Advanced First Order Reliability Method (AFORM). A robustness index (RI) is proposed to evaluate the structural robustness performance based on the acceptable probability of global failure and structural collapse probability.

Keywords: Progressive collapse, Probability, Energy, Structural Reliability, Advanced First Order Reliability Method, Robustness

1. Introduction

Progressive collapse is a catastrophic partial or total structural failure that **is caused by** local structural damage, which cannot be absorbed by the inherent continuity and ductility of the structural system. The local damage or failure initiates a chain reaction of failures that propagates vertically or horizontally through the structural system, leading to an extensive partial or total collapse. Virtually all the structural collapses initiate from local as opposed to system-wide damage (earthquakes being a possible exception), **the key feature distinguishing progressive collapse is that the**

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resulting damage is disproportionate to the local damage caused by the initiating event. Such collapses can be initiated by abnormal loads (e.g., gas explosions, vehicular collisions, and sabotage), fires, extreme environmental effects, human errors in design and construction. [1].

The capability of a structure to withstand damage can be developed based on structural reliability analysis and probability assessment [2]. Bassam et al. [3] proposed the probabilistic risk assessment framework to multi-story steel buildings subject to extreme loading. A brief outline of the probabilistic framework [4-6] was applied in their work, including the main requirements of describing uncertainty in the hazards, the associated local damage, as well as the consequences of global failure.

The probabilistic analysis of buildings against progressive collapse has recently been emphasized [5-6]. The probabilistic analysis will allow design engineer to decide whether further protective measures would be required to against tolerable collapse risk. The collapse risk of a building can be calculated as [5-6]:

$$P(C) = P(C|DH) \cdot P(D|H) \cdot P(H) \quad (1)$$

where: P(C) represents the probability of progressive collapse, P(H) is the probability of the occurrence of a hazard H, P(D|H) is the probability of local damage D as a result of a hazard H, P(C|DH) stands for the probability of progressive collapse of the structure as a result of local damage caused by hazard.

Starossek and Haberland [7] assigned appropriate terms for the formula as shown in Fig. 1. Based on Eq. (1) and Fig.1, the probability of progressive collapse can be minimized in three ways: controlling abnormal events, local element behavior and/or controlling global system behavior. Predicting abnormal events is difficult, however, the local and global system behavior i.e. P (D|H) and P (C|DH) can be partially controlled.

$$P(C) = P(C|DH) \cdot P(D|H) \cdot P(H)$$

$\left. \begin{array}{c} \text{collapse resistance} \\ \text{robustness} \quad \text{element behaviour} \quad \text{event control} \end{array} \right\} \text{maximise}$
 $\left. \begin{array}{c} \text{vulnerability} \quad \text{hazard} \end{array} \right\} \text{minimise}$

Fig.1. Terms in context of progressive collapse

According to the Eurocode [8] and US guidelines [9], robustness shown in Fig.1 is defined as the ability of a structure to withstand events like fire, explosions, impact or the consequences of human error, without being damaged to an extent disproportionate to the original cause. In terms of those codes, a localized failure due to accidental actions **could** be acceptable, as long as it will not endanger the stability of the whole structure, the overall load-bearing capacity of structure is maintained and allows necessary emergency measures to be taken.

Fewer studies have focused on the evaluation of $P(C|DH)$. Recently Bassam [3] investigated the failure probability of a structure, a probabilistic risk assessment framework to multi-storey buildings subject to extreme loading is developed based on the First Order Reliability Method (FORM). However, according to SSC-351 [10], the FORM method has the following shortcomings: Firstly, if the performance function $g(.)$ is non-linear and the linearization takes place at the mean values of x_i , errors may be introduced by neglecting higher-order terms. Secondly, the method is not consistent with different equivalent formations for the same problem, which means that the reliability index β depends on the formulation of limit state equation. Thirdly, in the FORM method the reliability index β represents failure probability when the variables x_i are normally distributed and the performance function $g(.)$ is linear in x_i . It is noted that many variables follow other distribution types, such as Weibull or exponential distribution, not just normal distribution. Therefore, an advanced First Order Reliability Method (AFORM) is adopted in the current study to establish the conditional probability of failure. Structural reliability theory allows some uncertainties to be quantified and included explicitly in the analysis. Considering these uncertainties, structural reliability analysis yields a quantifiable measure of structural safety, the failure probability, as shown in Fig.2 [11-12]. Structural reliability methods can be employed to determine the characteristic values.

Fig.2 Reliability analysis of a structure with random parameters

Assuming that the ultimate limit-state criterion can be expressed in terms of the resistance R (capacity) and the effect of the loading S (demand). The values of R and S are assumed to be uncertain and can be described by probability density functions $f_R(r)$ and $f_S(s)$. The safety limit-state will be violated if:

$$Z = g(R, S) = R - S \leq 0 \quad (2)$$

As shown in Fig.3, the probability of failure is given by:

$$P_f = P(R - S \leq 0) = P(g(R, S) \leq 0) < P_d \quad (3)$$

where: $g(.)$ is 'limit-state function' with negative values defining the failure scenario, P_d is the predefined probability of failure level equaling to acceptable probability of failure.

Fig.3 Probabilistic safety concept

The main objective of the current study is to develop an evaluation method to predict progressive collapse resistance of steel frame structure based on probability and energy principle.

2. The Probabilistic Model of Progressive Collapse

2.1 The sensitivity structural analysis

The component's internal energy (elastic deformation work) is defined as:

$$E_{i,int} = \iiint_{V_i} u_i dV \quad (4)$$

Where: i is the number of the undamaged component; u_i and V_i are elastic strain energy density per unit volume and the volume of a member, respectively. If a component is formed by single material and has uniform section,

$$u_i = \frac{E_{i,int}}{V_i} \quad (5)$$

The change in the elastic strain energy density per unit volume is given by:

$$\Delta u_i = \frac{|\Delta E_{i,int}^d|}{V_i} = \frac{|E_{i,int}^d - E_{i,int}^0|}{V_i} \quad (6)$$

Where: $E_{i,int}^0$ is the internal energy (elastic deformation work) of a member before being damaged, $E_{i,int}^d$ is the internal energy (elastic deformation work) of a member after being damaged, and $\Delta E_{i,int}^d$ is the variation of internal energy caused by abrupt removal of a column.

Furthermore, the relative variation of the elastic strain energy density per unit volume is:

$$\Delta u_{i,rel} = \frac{\Delta u_i}{\max_i \{\Delta u_i\}} \quad (7)$$

The sensitivity index of all structural components is defined as:

$$SI_{i,j} = \Delta u_{i,rel} \quad (8)$$

where: i is the number of a undamaged component, j is the number of a removed component.

According to the sensitivity index of a structural component, a sub-structure resisting progressive collapse can be obtained. Thus, the proposed sub-structure becomes the analysis target, instead of a single beam or column in the traditional analysis. The whole structural performance resisting progressive collapse is characterized by analyzing the behavior of the sub-structure in a column removal scenario. Thus, the analysis process proposed in the current study is closer to the realistic collapse scenario.

2.2 A simplified beam damage model

A steel frame beam under normal load is mainly subjected to the combined effect of moment and shear force, and the axial force in steel beam is usually ignored. With the increase of load, bending moments at midspan and ends of the steel frame beam increase accordingly, which develop plastic hinges at these locations and large deflection at the beam midspan. In this case, the steel beam endures considerable axial force, which cannot be neglected. As the total cross section yields, the axial force in the steel beam can further increase to resist the additional external loading, which results in decreasing of the bending moment at plastic hinge section. Finally, the steel beam mainly depends on the axial force to resist external loading and the steel beam fails at midspan or ends or connections. This process is commonly referred as tensile catenary action [13-17].

Fig.4. The simplified mechanical damage model of a beam.

A simplified beam damage model is proposed in this paper to analyze beam failure energy incorporating catenary action, meeting the inelastic axial load–bending moment interaction, as shown in Fig.4. Four moment plastic hinges and

two axial force **springs** are identified, the rest of the beams are assumed as rigid rod.

Beam motion can be decoupled into rotation (producing bending in the moment hinges) and extending (producing extending in the force **springs**) of beams. The key variable for quantitative analysis is the total vertical displacement (δ) by rotation and extending:

$$\delta = v_\theta + v_\Delta = (L_b^0 + \Delta) \cdot \sin(\theta + \varphi) \quad (9)$$

$$\theta + \varphi = \cos^{-1} \left(\frac{L_b^0 \cos \theta}{L_b^0 + \Delta} \right) \quad (10)$$

where: v_θ and v_Δ are vertical displacement due to rotation and extending, respectively. Δ and φ are the extending deflection and the corresponding rotation [14]. L_b^0 is the distance between the two moment plastic hinges, as shown in Fig.4. $L_b^0 = (1-d) L_b$, and d is a length factor of a moment plastic hinge, **equal** to 0.1 in the current study.

Furthermore, considering catenary action in beams, a simplified load-displacement relationship for the hinges and springs are developed based on the Federal Emergency Management Agency [18], as shown in Fig.5. The hardening phase (BC) of the developed moment hinge model is ignored in the current work, as shown in Fig.5 (b). Firstly, the beam moment begins to decrease gradually after reaching flexural strength, and then the beam axial force increases until rotational angle of the beam achieves the maximal value, meanwhile, the beam fails.

Fig5. Load-displacement relationship for the hinges and springs (a) Axial force deformation relation for axial springs.

(b) Modified moment rotation relation for a beam plastic moment hinges.

The plastic deformations ratio (b_M) and the residual strength ratio (c_M), are determined based on the FEMA 356

[18], as shown in Fig. 5 (b). The flexural strength and axial yield force of the member are calculated by:

$$M_y = Zf_{ye} \quad (11)$$

$$P_y = (2A_f + A_w) \cdot f_{ye} \quad (12)$$

And the rotation at yield is:

$$\theta_y = Zf_{ye}L_b / 6EI_b \quad (13)$$

where f_{ye} is the expected material yield strength; L_b is beam length; Z is plastic section modulus; I_b is moment of inertia.

Catenary action is considered in the developed model. When the total cross-section yields, the axial force and moment correlation equation is given by Yin et al. [15]. If the plastic neutral axis is in the web,

$$\frac{M}{M_y} + \frac{(1+\alpha)^2}{\alpha[2(1+\beta)+\alpha]} \left(\frac{P}{P_y}\right)^2 = 1 \quad (14)$$

where $\alpha = A_w / (2A_f)$, A_w is web plate area and A_f is flange area; $\beta = t / h_0$, t is flange thickness and h_0 is web plate height. M and P are bending moment and axial force, respectively; M_y and P_y are expected flexural strength and axial yield force of the member, respectively. If the plastic neutral axis is in the flange:

$$\frac{1-\gamma}{1 - \frac{(1+\alpha)^2\gamma^2}{\alpha[2(1+\beta)+\alpha]}} \frac{M}{M_y} + \frac{P}{P_y} = 1 \quad (15)$$

where $\gamma = A_w / (2A_f + A_w)$.

However, flange thickness t is much smaller than web plate height h_0 in engineering practice, the ratio of flange thickness to web height is approximately equal to zero, namely, $\beta \approx 0$. For simplicity, the nonlinear curve from Eq. (14) can be simplified by a straight line [13]. Thus the Eq. (14) and Eq. (15) can be rewritten as Eq. (16) and Eq. (17) as:

$$\frac{M}{M_y} + \zeta \frac{P}{P_y} = 1 \quad (16)$$

$$\lambda \frac{M}{M_y} + \frac{P}{P_y} = 1 \quad (17)$$

where $\zeta = (1+\alpha)^2 / [\alpha(2+\alpha)]$; $\lambda = (1-\gamma) / (1-\zeta\gamma^2)$.

Based on the experimental results [19-24], the axial force of a beam is less than the axial yield force (P_y) before

the beam fails. Thus, it is assumed that if the plastic neutral axis reaches the critical point of flange and web in section, meanwhile the moment and rotational deformation of the beam are $c_M \cdot M_y$ and $\theta_{\max}$, respectively, the beam fails.

Hence, combining with the proposed model shown in Fig.5 (b), Eq. (16) can be modified as Eq. (18), and the modified axial force and moment correlation curve is shown in Fig.6.

$$\frac{M}{M_y} + \frac{1-\gamma}{1-\lambda c_M} \frac{P}{P_y} = 1 \quad (18)$$

Fig.6. The modified relationship between bending moment and axial force

Different failure phases in a structural beam are presented in Fig.7. The relationship of beam axial force and vertical displacement is shown in Fig.7 (a). The failure of a beam has two different failure mechanisms, as shown in Fig.7 (b). The two primary mechanisms are beam mechanism (Phase ①-②) and catenary mechanism (Phase ②-④), respectively.

Fig.7. The damaged process of a beam incorporating catenary action. (a) Beam axial force versus vertical displacement of center column. (b) Different phases in process of failure.

According to Fig.6 and Fig.7, the behavior was dominated by flexure in the early phases, and the beam axial force is zero. With increased vertical displacement, axial tension develops in the beams, and the catenary action will occur. The axial force in the beams increases with increased downward displacement, until the beam can no longer bear the combined axial and flexural stresses and fails. In other words, the beam will fail if moment and rotational deformation of the beam reaches $c_M \cdot M_y$ and θ_{max} , and the corresponding axial force of the beam and tensile deformation reaches $(1-\lambda c_M)P_y$ and Δ_3 , respectively.

Hence, the plastic deformation work of a beam in the limit state is given by:

$$E_{i,beam} = 2 \int_0^{\theta_{max}} M(\theta) d\theta + \int_0^{\Delta_3} P(\Delta) d\Delta \quad (19)$$

where $E_{i,beam}$ is the flexural failure energy considering catenary action of a beam; Δ_3 and θ_{max} are the axial tensile length and the maximum rotation, respectively.

Combining Eqs. (9) – (19), the energy dissipation capacities of a beam is given by:

$$R_i = E_{i,beam} = [c_M(k-1) + k] \frac{Z^2 f_{ye}^2 L_b}{6EI_b} + \frac{AL_b^0}{2E} f_{ye}^2 (1-\lambda c_M)^2 \quad (20)$$

where:

$$Z = \frac{1}{4}b_f(2t_f + h_w)^2 - \frac{1}{4}(b_f - t_w)h_w^2 \quad (21)$$

$$I_b = \frac{1}{12}b_f(h_w + 2t_f)^3 - \frac{1}{12}(b_f - t_w)h_w^3 \quad (22)$$

$$A = 2A_f + A_w = 2t_f b_f + t_w h_w \quad (23)$$

$$k = \frac{\theta_{\max}}{\theta_y} \quad (24)$$

Meanwhile, the maximum vertical displacement at the beam midspan **in the limit state** is calculated by:

$$\delta_{\max} = v_{\theta} + v_{\Delta} = (L_b^0 + \Delta_3) \cdot \sin(\theta_{\max} + \varphi) \quad (25)$$

where:

$$\theta_{\max} + \varphi = \cos^{-1} \left(\frac{L_b^0 \cos \theta_{\max}}{L_b^0 + \Delta_3} \right) \quad (26)$$

And the external force work due to deformation of a beam **in the limit state** is:

$$S_i = E_{i,potential,beam} = (DL + LL) \cdot \left[\frac{1}{2} \delta_{\max} \left(\frac{d}{2} L_b + L_b^0 \cdot \cos \theta_{\max} \right) \right] \quad (27)$$

where: DL is the applied dead load , which is generated based on element volume and material density. LL is the live load.

2.3 Model Validation

Fahim et al. [19-21] conducted an experimental assessment **to** the performance of steel beam-column assemblies with two types of moment-resisting connections under a middle-column-removal scenario, which are Bolted Web Connection (WUF-B) and Reduced Beam-Section Connection (RBS), respectively. The test set-up is shown in Fig.8.

A validation analysis is performed to a beam connected to columns by RBS connections. The RBS connection is created by cutting away a portion of the top and bottom flanges of the beam at a distance from the beam-column interface, such that yielding is concentrated in this reduced area. The reduced section acts as a fuse to protect the connection against premature fracture. The radius-cut section was found to yield the most reliable performance [25]. The RBS connection has been commonly used for seismic design since the **1994 Northridge earthquake in California**. FEMA 355D (FEMA 2000b) [25] provides extensive information on the testing and performance of the RBS connections under seismic loading. The RBS connection details and the failure modes are given in Fig.9 and Fig.10,

respectively (adapted from Fahim et al. [19-21]).

Fig.8. Fahim's test set-up

Fig.9. RBS connection details

Fig.10. The representative failure mode of steel beam

The experimental results of the beam axial force versus the vertical displacement are compared with predictions of the proposed model, as given in Fig.11. It can be seen that in the early phases of the response, the behavior was dominated by flexure with less axial forces in the beams. With the increase of vertical displacement, axial forces developed in the beams and the behavior was dominated by catenary action. The beam axial force increased with larger downward displacement, until the beam could no longer carry the combined axial and flexural stresses, resulting in the

failure of the beam and the **fracture propagated from bottom flange to web**. At the time of failure, the axial force in the beams was about 2450 kN and the vertical displacement was approximately 850 mm. The failure was characterized by fracture of the bottom flange in the middle of the reduced connection section near the center column. In general, the modified model (considering RBS connections by reducing the flange width) proposed in the current study shows good agreement compared with the experiment results. The proposed model considering RBS connections can be applied for analysis of complete structural systems connected by RBS connections to evaluate their vulnerability to disproportionate collapse.

Fig.11. Beam axial force versus vertical displacement of center column

2.4 The structural performance function

In the current work, the effects of secondary elements are ignored because response of the major load carrying members should be clarified. Among all the energies absorbed and dissipated by a structure, the total failure energy of a column, E_{column} , and the total failure energy of a beams considering catenary action, E_{beam} , are crucial.

$$R = E_{EC} = \sum_i SI_{i,j} \cdot R_i = \sum_i SI_{i,j} \cdot E_{i,beam} + \sum_k SI_{k,j} \cdot E_{k,column} \quad (28)$$

where R is the structural resistance and E_{EC} is the structural energy dissipating capability after damaged.

The inherent potential energy of the structure across over the removed column is:

$$S = E_{ED} = \sum_i SI_{i,j} \cdot S_i = \sum_i SI_{i,j} \cdot E_{i,potential,beam} + \sum_k SI_{k,j} \cdot E_{k,potential,column} \quad (29)$$

where S is the structural response. E_{ED} is the energy demand to support upper structure after removal of a column.

$E_{i,beam}$ is the flexural failure energy considering catenary action of a beam and $E_{k,column}$ is the failure energy of a column. $E_{i,potential,beam}$ and $E_{k,potential,column}$ are the external work by applied forces due to deformation of a beam and a

column, respectively. Thus, we assume that the column and connecting joint are strong enough and cannot be destroyed under progressive collapse, namely, $E_{k,column} = 0$ and $E_{k,potential,column} = 0$. Thus, Eq. (28) and Eq. (29) can be simplified as:

$$R = \sum_i SI_{i,j} \cdot E_{i,beam} \quad (30)$$

$$S = \sum_i SI_{i,j} \cdot E_{i,potential,beam} \quad (31)$$

Hence, combining Eq.(2), Eq.(30) and Eq.(31), the structural limit-state (or performance) function $g(.)$ is given by :

$$g(R,S) = (\sum SI_{i,j}) \cdot \left\{ [c_M (k-1) + k] \frac{Z^2 f_{ye}^2 L_b}{6EI_b} + \frac{AL_b^0}{2E} f_{ye}^2 (1 - \lambda_{c_M})^2 \right\} - (\sum SI_{i,j}) \cdot \frac{(DL + LL)}{2} \left[\delta_{\max} \left(\frac{d}{2} L_b + L_b^0 \cdot \cos \theta_{\max} \right) \right] \quad (32)$$

The structural limit-state (or performance) function $g(.)$ consists of seven random variables containing Modulus of elasticity of steel (E), Expected yield strength of the material (f_{ye}), Sectional area (A), Plastic section modulus (Z), Moment of inertia (I), Dead load (DL) and Live load (LL). Table 1 summarizes the statistical information for the basic random variables.

Table 1 Probabilistic models for the steel structure

Variable	Distribution	Mean Nominal	Coefficient of Variation (COV, v)	Ref.	Variable Symbol
Modulus of elasticity of steel (E)	Lognormal	1.0	0.03	[8,26,27]	X_1
Expected yield strength of the material (f_{ye})	Lognormal	1.0	0.07	[8,26,27]	X_2
Sectional area (A)	Normal	1.0	0.05	[8,26,27]	X_3
Plastic section modulus (Z)	Normal	1.0	0.02	[8,26,27]	X_4
Moment of inertia (I)	Normal	1.0	0.05	[8,26,27]	X_5
Dead load (DL)	Normal	1.05	0.1	[8,26,27]	X_6
Live load (LL)	Lognormal	0.24	0.6	[8,26,27]	X_7

3. Structural Reliability Analysis

3.1 Advanced first order reliability method (AFORM)

The basic principle of the First Order Reliability Method (FORM) is illustrated in Fig.12 [28-29]. The curved

failure surface is approximated by a hyperplane (i.e. a straight line for the two-dimensional case) at the critical point in the region, which gives the highest contribution to the failure probability. A specific numerical search algorithm, such as the Rackwitz–Fiessler algorithm, is generally required to obtain this point, which is typically referred as the “design point”. Subject to normalization of the original distributions of the different variables, the distance of the design point from the origin β (reliability index) as shown in Fig.12 [28], can be used to approximate the conditional probability of failure $P(C|DH)$:

$$P_f = P(C|DH) = \Phi(-\beta) \quad (33)$$

where Φ is the standard normal cumulative distribution function. The parameter β is given as:

$$\beta = \frac{\mu_Z}{\sigma_Z} \quad (34)$$

Meanwhile, the corresponding reliability of the structure is given by:

$$P_r = 1 - P_f \quad (35)$$

The proposed analytical methods have apparent physical interpretations and do not require excessive computation time.

Fig.12 First-order approximation in standard normal space

However, given the inherent drawbacks of FORM, as mentioned in the introduction, the Advanced First Order Reliability Method (AFORM) is utilized in the current study to establish the conditional probability of failure. The first two shortcomings discussed previously are avoided by using a procedure proposed by Hasofer and Lind (1974) [30]. Instead of expanding Taylor’s series about the mean value point, which cause the invariance problem, the linearization point is taken on the failure surface. On the failure surface, the limit-state (or performance) function $g(\cdot)$ containing the load and strength variables determines the performance or the structure state. In the Hasofer-Lind procedure, the load and resistance variables, x_i , are transformed to standardized (reduced) variables with zero mean and unit variance $N(0, 1)$ by:

$$y_i = \frac{x_i - \mu_i}{\sigma_{x_i}} \quad (36)$$

The Hasofer-Lind reliability index is defined as the shortest distance from the origin to the failure surface in the reduced (standardized) space. This point is found by solving the following set of equations [10]:

$$G(y_1^*, y_2^*, \dots, y_n^*) = 0 \quad (37)$$

$$y_i^* = -\alpha_i^* \beta \quad (38)$$

$$\alpha_i^* = \frac{\left(\frac{\partial G}{\partial y_i} \right)_{y_i^*}}{\sqrt{\sum_i \left(\frac{\partial G}{\partial y_i} \right)_{y_i^*}^2}} \quad (39)$$

where $G(\cdot)$ is the failure surface in the reduced space, also known as the limit state function, and y_i^* is the coordinates of point closet to the origin in the reduced space (i.e., design point). The α -vector is the outward directed normal vector to the failure surface of the design point y_i^* . All partial derivatives are evaluated at the checking point. It is noted that this procedure is equivalent to linearizing the limit state function in the reduced variables space at the checking point and determining corresponding β value.

In the original space, the checking point is obtained from [10]:

$$x_i^* = \mu_i + \sigma_{x_i} y_i^* = \mu_i - \alpha_i \sigma_{x_i} \beta \quad (40)$$

In general, for a linear limit state function, the Hasofer-Lind method will yield the same β values as the FORM method. For the nonlinear limit-state functions, this method yields a safety index β which is invariant to formulation of the performance function.

In the case of non-normal variables, a transformation of these variables into equivalent normal variables is necessary prior to each iteration. As the limit-state function is non-linear, the design point can be found iteratively [28]:

$$\beta = \min_{y \in \{g(y)=0\}} \sqrt{\sum_{i=1}^n y_i^2} = \min(y^T \cdot y)^{1/2} \quad (41)$$

where: $y = (y_1, y_2, \dots, y_n)^T$, n is the number of variables.

Assuming the cumulative distributions and the probability density functions of the actual and normal distribution are equal to each other at the linearization point, one can determine the mean μ_x' and standard deviation σ_x' of the equivalent normal variable (as shown in Fig.13):

$$F_x(x^*) = \Phi\left(\frac{x^* - \mu'_x}{\sigma'_x}\right) \quad (42)$$

$$f_x(x^*) = \frac{1}{\sqrt{2\pi}\sigma'_x} \exp\left[-0.5\left(\frac{x^* - \mu'_x}{\sigma'_x}\right)^2\right] = \frac{1}{\sigma'_x} \varphi\left\{\Phi^{-1}\left[F_x(x^*)\right]\right\} \quad (43)$$

where $\varphi(\cdot)$ is the standard normal probability density.

Fig.13 The illustration of conditions for equivalent normalization

The parameters of the equivalent normal distribution can be determined as follows:

$$\sigma'_x = \frac{\varphi\left\{\Phi^{-1}\left[F_x(x^*)\right]\right\}}{f_x(x^*)} \quad (44)$$

$$\mu'_x = x^* - \Phi^{-1}\left[F_x(x^*)\right]\sigma'_x \quad (45)$$

Since the linearization point x_i^* changes with each iteration, σ'_{x_i} and μ'_{x_i} should be calculated for each iteration.

These values are then inputted into Eq.37 to Eq.39. **It should be noted** that if the iteration is performed in the reduced (or standard) space, distribution transformation into reduced space has to be performed in each step [10]. Following are the main steps to calculate structure reliabilities by the proposed analytical method:

- a) Transform the physical basic variables into the space of standard normal variables;
- b) Calculate design points;
- c) Determine failure reliability.

So far, the random variable X_i has been assumed to be uncorrelated. If the variables are normal and correlated through a correlation matrix $[C]$, a transformation to a set of uncorrelated variables Z_i is possible and essential. The safety index β is determined by the new uncorrelated set [10].

The uncorrelated variables Z_i can be determined from the reduced variables X'_i using the orthogonal transformation:

$$Z = T^t X' \quad (46)$$

Where T is an orthogonal transformation matrix and the superscript t indicates the transpose. The transformation matrix T is:

$$T' [C'] T = [\lambda] \quad (47)$$

where $[C']$ is the covariance matrix of X' and $[\lambda]$ is a diagonal matrix of the eigen values of $[C']$. The covariance matrix of $[C']$ is to be related to the covariances (cov) of the original variables X through

$$[C'] = \begin{bmatrix} 1 & \rho_{12} & \rho_{13} & \cdots & \rho_{1n} \\ \rho_{21} & 1 & \rho_{23} & \cdots & \rho_{2n} \\ \rho_{31} & \rho_{32} & 1 & \cdots & \rho_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_{n1} & \rho_{n2} & \rho_{n3} & \cdots & 1 \end{bmatrix} \quad (48)$$

where:

$$\rho_{ij} = \frac{\text{cov}(X_i, X_j)}{\sigma_{X_i} \sigma_{X_j}} \quad (49)$$

$$\text{cov}(X_i, X_j) = E[X_i \cdot X_j] - E[X_i] \cdot E[X_j] \quad (50)$$

where $E[.]$ is expected value of random variables.

Since T is orthogonal, ($T^{-1} = T^t$) and from Eq.(46):

$$X' = TZ \quad (51)$$

and

$$X = [\sigma_X] X' + \mu_X \quad (52)$$

where:

$$[\sigma_X] = \begin{bmatrix} \sigma_{X_1} & & & 0 \\ & \sigma_{X_1} & & \\ & & \ddots & \\ 0 & & & \sigma_{X_n} \end{bmatrix} \quad (53)$$

and

$$\mu_X = \begin{bmatrix} \mu_{x_1} \\ \mu_{x_2} \\ \vdots \\ \mu_{x_n} \end{bmatrix} \quad (54)$$

It is shown that the eigen values $[\lambda]$ are also function of the variables Z_i . The uncorrelated reduced variables Y_i and the original random variables X_i can be expressed by:

$$Y = [\sqrt{\lambda}]^{-1} Z \quad (55)$$

$$X = [\sigma][T][\sqrt{\lambda}]Y + \mu_x \quad (56)$$

where: $[\sqrt{\lambda}]$ is given by

$$[\sqrt{\lambda}] = \begin{bmatrix} \sqrt{\lambda_1} & & & \\ & \sqrt{\lambda_2} & & \\ & & \ddots & \\ & & & \sqrt{\lambda_n} \end{bmatrix} \quad (57)$$

An iterative solution scheme to calculate the reliability index and design point is provided in Fig.14:

Fig.14 The iterative solution scheme

3.2 Robustness Index (RI)

Approaches to quantify robustness via a robustness index can be classified into three levels [5]: 1) A risk-based robustness index based on a complete risk analysis where the consequences are divided into direct and indirect risks; 2)

A probabilistic robustness index based on probabilities of the structural system failure for an undamaged and a damaged structure; 3) A deterministic robustness index based on structural measures, e.g. pushover load bearing capacity of an undamaged and a damaged structure.

The current study takes the probabilistic robustness index based on probabilities as an assessment method to evaluate the damaged structural performance of progressive collapse. Frangopol & Curley (1987) and Fu & Frangopol (1990) proposed probabilistic measures related to structural redundancy, which also indicates the level of robustness [4]. It is defined as a “redundancy index” (RI):

$$RI = \frac{P_{f(\text{damaged})} - P_{f(\text{intact})}}{P_{f(\text{intact})}} \quad (58)$$

where $P_{f(\text{damaged})}$ is the probability of failure for a damaged structural system and $P_{f(\text{intact})}$ is the probability of failure of an intact structural system. This redundancy index provides a measure to the redundancy of a structural system. The value ranges between zero and infinity, smaller values indicates larger robustness.

The failure probability of an intact structural system $P_{f(\text{intact})}$ is ignored in the current study. Therefore, a new robustness index (RI) is proposed based on the acceptable probability of global failure $P_{f(\text{acceptable})}$ and structural collapse probability $P_{f(\text{collapse})}$ ($P_{f(\text{collapse})} = P(C)$) as in Eq. (1), defined by:

$$RI = \frac{P_{f(\text{acceptable})} - P_{f(\text{collapse})}}{P_{f(\text{acceptable})}} \quad (59)$$

Based on the COST Action TU0601 [4], acceptable failure probability for global failure of buildings in earthquake regions in the U.S.A., $P_A = 2 \times 10^{-5}$ per year, is utilized as an acceptable failure probability of structural progressive collapse in the current study.

Other than the complete collapse of a building, the global failure could be a partial collapse that greater than a considered limit. Based on Eq. (59), a higher value of RI indicates better structural robustness, and then the structural robustness index can be classified as four performance levels as shown in Table 2.

Table 2 Performance level accounting for robustness levels

Robustness Index	RI 1 ($RI \leq 0$)	RI 2 ($0 < RI \leq 1/3$)	RI 3 ($1/3 < RI \leq 2/3$)	RI 4 ($2/3 < RI \leq 1$)
Performance Level	Collapse	Low Robustness	General Robustness	High Robustness

3.3 The framework of calculating program

The flow chart of the probability and energy principle proposed in the current study is shown in Fig.15.

Fig.15. General flow chart of simplified analysis based on probability and energy principle

4. The Analysis Model

4.1 The analysis model

The current work focuses on hazard types (fire, explosion, human error and vehicular collisions) that could lead to the removal of a single column in multi-story buildings. The representative building in our analysis, **earlier studied by**

Marjanishvili and Agnew (2006) [31] using the GSA (2003) approach, is a nine-story steel frame structure, as shown in Fig. 16, with six bays in the longitudinal direction and three in the transverse direction in accordance with the AISC Load Resistance Factor Design code [32]. The longitudinal direction has a uniform column spacing of 8.25 m (27 ft), the three-bay side columns are spaced every 9.75m (32 ft). Main girders are W21×57. Floor-to-floor height for every story is 4.3 m (14 ft). W14×159 columns span from the ground to the fifth floor and W14×90 columns span from the sixth floor to the roof. Grade A992Fy50 steel with a modulus of elasticity of 200 GPa, a yield stress (F_y) of 345 MPa (50 ksi), and effective yield stress (F_{ye}) of 379.2 MPa (55 ksi) is adopted in the example. A three-dimensional finite-element model of the structure is developed, as shown in Fig.16. The two-dimensional frame enclosed in the dotted rectangle is taken out as the analysis objects. Live load is assumed to be 3.92N/mm distributed uniformly across the entire beam span including roof. To estimate the dead load, we assumed a uniform concrete slab thickness of 90 mm (3.5 in), with normal weight concrete density of 23.6 kN/m³ (150 pcf), and the equivalent linear load is 4.28N/mm. Additionally, the dead load includes perimeter wall weight of 19.7 N/mm (1,350 plf) at every floor, except roof level.

Fig.16 Three-dimensional model and structural plan of example building

The assumed probabilities $P(H)$ for the occurrence [2, 33-35] of damage events, regardless of intensity, over a 50 year period at any location within the building are given in Table 3. It is worth noting that extreme events arising from planned human action, such as vandalism and terrorism, are excluded, as the associated probabilities cannot be rationally established from statistical data [2]. These are presented simply for illustrative purposes to provide a context for the subsequent discussion.

Table 3 Estimated probabilities for the column removal case

Hazard	$P(H)$ (50 years)	$P(D H)$	Ref.
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Explosion	2×10^{-3}	0.1	[2]
Fire	20×10^{-3}	0.1	[2]
Human error	2×10^{-3}	0.1	[2]
Vehicular collisions	3×10^{-2}	0.1	[2,33,34,35]

4.2 Structural model analysis

Based on Eq. (8), the sensitivity index of structural components to abrupt removal of the **long-side central** column shown in Fig.17 (a) can be determined. Analysis results are showed in Fig.17 (b). Then, a sub-structure enclosed in the dotted rectangle in Fig.17 (b) to resisting progressive collapse is proposed based on the sensitivity of structural components.

Fig.17 Results of the structural sensitivity analysis. (a) Element labels; (b) Sensitivity coefficients of the elements

Based on the modified correlation of axial force and moment incorporating catenary action, the basic parameters with regard to the section W21×57 to calculate structural performance are calculated and listed in Table 4.

Table 4 Basic parameters to calculate the structural performance function

θ_y (rad)	$\theta_{\max}$ (rad)	M_y (kN·m)	P_y (kN)	Δ_y (mm)	α	ζ	γ	λ	c_M	Δ_3 (mm)
0.0113	0.1356	801.65	4085.7	15.65	0.940	1.362	0.485	0.758	0.6	8.53

Furthermore, in accordance with the values in the Fig.17 (b), it is noted that energies absorbed and dissipated by structure mainly are contributed by the failure of beams in the sub-structure, so the failure state of structural progressive collapse is shown in Fig.18. Hence, in accordance with Eq.(32), the structural limit-state (or performance) function $g(\cdot)$ is given by Eq.(60).

Fig.18 The limit failure state of structural progressive collapse

$$g(R, S) = 370790 \frac{x_4^2 x_2^2}{x_1 x_5} + 15998.5 \frac{x_3 x_2^2}{x_1} - 59981467.8(x_6 + x_7) \quad (60)$$

Variation of structural plastic deformation work and potential energy (external force work) in the different phases is shown in Fig.19. The relationship of beam axial force and vertical displacement is shown in Fig.20.

Fig.19 Energy versus vertical displacement

Fig.20 Beam axial force versus vertical displacement

It is observed that the energies absorbed and dissipated by the catenary mechanism are greater than the beam mechanism based on Fig.19. Hence, based on Fig.11 and Fig.20, the catenary mechanism is the last defense against progressive collapse. In addition, the corresponding vertical displacement (defined as δ_{cr}) is approximately 500mm after the critical point, the plastic deformation work is always greater than the potential energy until the vertical displacement reaches the maximum value ($\delta_{max} = 1359.8mm$).

Structural Safety Reserve (SSR) is defined as an index for a comparative analysis between the energy method proposed in the current work and the GSA guideline. SSR_BE is obtained by the energy method, as given by $SSR_BE = \delta_{max} / \delta_{cr}$. Where: δ_{cr} is the critical displacement value when internal energy and potential energy is equal as shown in Fig.19. δ_{max} obtained by Eq.(25) is the maximum vertical displacement at the beam midspan in the limit state. Thus, the structural safety reserve (SSR) of the analyzed example building for progressive collapse can be defined as $SSR_BE = \delta_{max} / \delta_{cr} = 2.72$.

Based on the GSA guideline, Nonlinear Static (NS) and Nonlinear Dynamic (ND) analysis are performed by SAP2000 software as shown in Fig.21 (a) and Fig.21 (b). The loading combination Load = 2(DL + 0.25LL) is applied in NS analysis, and Load = DL + 0.25LL is applied in ND analysis [9]. Also, recommended limited values of ductility ratio ($\mu = \delta_{max,1} / \delta_e$) and rotation angle ($\theta = \delta_{max,2} / L$) for a steel beam are 0.21 and 20, respectively. Hence, the limited vertical displacement δ_{limit} is equal to $\min\{20\delta_e, 0.21L_b\}$, namely, damage of structural beams occurs when the vertical displacement of the point connected to the removed column reaches minimum value between $\delta_{max,1}$ and $\delta_{max,2}$. The maximum vertical displacement is 161.5 mm and 444.5 mm by ND and NS analysis, respectively. As shown

in Fig.21, they are lower than the limited value $\delta_{\text{limit}} = \min\{20\delta_e, 0.21L_b\} = 1732.5\text{mm}$, where $\delta_e = 131\text{mm}$ is obtained by NS analysis. The SSR_NS by NS analysis and the SSR_ND by ND analysis are defined as $\delta_{\text{limit}} / \delta_{\text{max}}$. As a result, SSR_NS is 3.9 and SSR_ND is 10.7. It is noted that SSR_NS and SSR_ND are greater than SSR_BE by the proposed based-energy method, namely, $\text{SSR}_{\text{BE}} < \text{SSR}_{\text{NS}} < \text{SSR}_{\text{ND}}$. Thus, analysis results obtained by the proposed based-energy method are reasonable accurate.

Fig.21 Structural progressive collapse analysis based on traditional method recommended in GSA2003. (a) Nonlinear Static (NS) analysis. (b) Nonlinear Dynamic (ND) analysis.

4.3 Structural reliability analysis

From section 3.1, the structural reliability index computed by the MATLAB program is 2.61. If the correlations of random variables are not considered, the structural reliability index is equal to 2.49, as shown in Fig.22. Different results could be obtained if the correlations of random variables by Eq.(21) to Eq.(23) are taken into account in the calculating procedure. Therefore, the reliability index considered the correlations of random variables is more accurate. The failure probability of structural progressive collapse is given in Table 5 based on Eq. (1). The structural robustness index on account of vehicular collisions is 0.986, so the model analyzed in the current study has high robustness, as shown in Table 5.

Fig.22 The reliability indexes in the process of iterative calculation

Table 5 The failure probability of the structural collapse

Hazard	$P(H)$ (50 years)	$P(D H)$	$P(C/DH)$	$P(C)$
Explosion	2×10^{-3}	0.1	4.5×10^{-3}	9×10^{-7}
Fire	20×10^{-3}	0.1	4.5×10^{-3}	9×10^{-6}
Human error	2×10^{-3}	0.1	4.5×10^{-3}	9×10^{-7}
Vehicular collisions	3×10^{-2}	0.1	4.5×10^{-3}	1.4×10^{-5}

5 Conclusion

The current work focuses on evaluating the conditional failure probability for specific local damage scenarios, consisting of single column loss in multi-story frame structure, and developed the probabilistic assessment framework.

1) Based on the changes of component internal energy, sensitivity of structural members to abrupt removal of a column is proposed to determine a sub-structure resisting progressive collapse. A simplified beam damage model is proposed to analyze beam failure energy incorporating catenary action at large deflections, which shows reasonable accuracy compared with experimental results.

2) A general overview on the methodologies used for evaluating the structural collapse probability has been proposed. The adopted probabilistic assessment approach relies on the availability of the probability of hazard occurrence, which can be obtained from statistical data.

3) The analyses show that the correlations of random variables should be taken into account when calculating the reliability index. Based on the acceptable probability of global failure and structural collapse probability, a robustness index (RI) is proposed in the current study to evaluate the structural robustness performance level.

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