A Partially-Averaged Navier-Stokes Model for the Simulation of Turbulent Swirling Flow with Vortex Breakdown

Hosein Foroutan^{*}

Department of Mechanical and Nuclear Engineering, The Pennsylvania State University
325 Reber Building, The Pennsylvania State University, University Park, PA 16802, USA
Email: hosein@psu.edu
Tel: +1-814-441-4060

* Corresponding author.

Savas Yavuzkurt

Department of Mechanical and Nuclear Engineering, The Pennsylvania State University
327 Reber Building, The Pennsylvania State University, University Park, PA 16802, USA
Email: sqy@psu.edu

Abstract

This paper presents the development and validation of a new Partially-Averaged Navier-Stokes (PANS) model which can successfully predict turbulent swirling flow with vortex breakdown. The proposed PANS model uses an extended low Reynolds number k - ε model as the baseline model. Furthermore, a new formulation for the unresolved-to-total turbulent kinetic energy ratio f_k is developed using partial integration of the complete turbulence energy spectrum. Therefore, the present formulation of f_k is believed to be superior to the previously used constant or computed values. The newly developed PANS model is used in unsteady numerical simulations of two turbulent swirling flows containing vortex breakdown, namely swirling flow through an abrupt expansion and flow in a draft tube of a hydraulic turbine operating under partial load. The present PANS model accurately predicts time-averaged and root-mean-square (rms) velocities in the case of the abrupt expansion, while it is shown to be superior to the Delayed Detached Eddy Simulation (DDES) and Shear Stress Transport (SST) k - ω models. Predictions of the reattachment length using the present model shows at least 14% and 23% improvements compared to the DDES and the SST k - ω models respectively. Also, transient features of the flow, e.g. vortex rope formation and precession, is well captured in the case of the complex draft tube flow. The frequency of the vortex rope precession, which causes severe fluctuations and vibrations, is well predicted by only 7% deviations from the experimental data.

Keywords

PANS, Turbulence model, Swirling flow, Vortex breakdown, Draft tube

1. Introduction

Turbulent swirling flows are present in several industrial applications, such as gas turbine combustors, furnaces, cyclone separators, and draft tube of hydraulic turbines. Therefore, there have been extensive theoretical, experimental, and computational studies aiming to understand and characterize this type of flows (Gupta et al., 1984). The early analytical and experimental investigations were focused on the understanding of the development of swirling flow in circular tubes with constant diameter. The swirl number, defined as the ratio between the flux of circumferential momentum and the flux of axial momentum, has been shown to have large effects on the flow field. Increasing the swirl level, an internal stagnation point on the vortex axis followed by a recirculating region acting as a blockage is induced, which is called the vortex breakdown. Harvey (1962) was the first to visualize this

phenomenon experimentally with an air swirling flow in a straight pipe. Depending on different combinations of Reynolds number and swirl number, different types of vortex breakdown may appear as observed by Sarpkaya (1971). Cassidy and Falvey (1970) focused on the helical form of the vortex breakdown. They set up an experimental apparatus to study its occurrence and to measure wall pressure fluctuations related to the vortex core precession. Afterwards many experimental investigations on the vortex breakdown phenomenon have been carried out (see Lucca-Negro and O'doherty (2001) for a review). Moreover many theories and explanations of the vortex breakdown have been brought up by different researchers. Some of them (Benjamin, 1962) assume that this phenomenon is based on the concept of critical state related to the wave phenomena, analogous to the hydraulic jump in open-channel flow, whereas others (Hall, 1967) believe in an analogy between vortex breakdown and boundary layer separation.

Nevertheless, the vortex breakdown phenomenon is quite complex and experiments, although being necessary for validating numerical results, cannot provide all the details to fully understand it and comprehend the mechanisms involved. The low-cost and availability of detailed set of data make computational fluid dynamics (CFD) an important tool for obtaining additional information concerning the structure of the breakdown, as well as identifying the various parameters affecting its occurrence and development. The simulation of a turbulent swirling flow, however, is a challenging task, and accurate numerical calculations of the flow parameters require a careful choice of turbulence closure method. Several previous turbulent swirling flow predictions have been carried out employing the steady Reynolds-Averaged Navier-Stokes (RANS) equations with various turbulence closure models (Yaras and Grosvenor, 2003; Shamami and Birouk, 2008; Foroutan and Yavuzkurt, 2014a). A review of the literature reveals, however, that RANS turbulence models show inadequate performance in simulating highly swirling flows. This has been investigated and shown to be true for a wide range of RANS models including the standard $k-\epsilon$ (Leschziner and Hogg, 1989), RNG $k-\epsilon$ (Xia et al., 1998), SST $k-\omega$ (Yaras and Grosvenor, 2003), algebraic Reynolds stress model (ASM) (Fu et al., 1988), and Reynolds stress model (RSM) (Lu and Semiao, 2003). Furthermore, there is a great challenge in accurately predicting the unsteady features of the flow, coherent structures, and turbulence statistics for a swirling flow with vortex breakdown using unsteady version of RANS (URANS) models since these models tend to damp out the unsteadiness of the flow (Ruprecht et al., 2002). The time scale of unsteadiness of the vortex breakdown is close to that of turbulent fluctuations of energy carrying eddies, and therefore, this small scale unsteadiness is averaged off by the URANS approach.

In the unsteady simulation framework, large eddy simulation (LES) is a candidate for studying turbulent swirling flows (Schlüter et al., 2004). In the LES approach, the large, energy carrying, dynamically important, and flow-dependent eddies are solved directly, leaving only smallest (subgrid) scale of turbulence with very low energy and supposedly universal behavior (assumed isotropic) to be modeled. However, one of the major obstacles to the use of LES in complex confined industrial flows, such as swirling flow in cyclones and draft tubes, is the modeling of the near-wall region (Sagaut, 2002). The dynamics of the flow near the wall are strongly anisotropic even at the small scales, and turbulence production in this region is associated with an up-scale energy cascade that is largely dominant over the commonly assumed down-scale energy cascade, presenting elsewhere. Also, the large eddies that must be captured on the computational grid to perform an accurate LES shrink in size and are not isotropic as one approaches the wall, leading to excessive computational costs. In fact the small eddies are not isotropic near a wall either. Current LES modeling approaches require that either the near-wall region be adequately resolved (using a DNS-like grid near the wall which makes it inapplicable for industrial flows), or that an LES wall-model be used, which to date has not provided accurate results in relatively complex flows.

Hybrid URANS/LES methods aim at combining the best of RANS and LES turbulence models. In this approach, the URANS models are mainly employed in the near-wall region, while LES treatment is applied to regions away from the wall. A wide variety of hybrid URANS/LES models, e.g. detached eddy simulation (DES) (Spalart, 2009), very large eddy simulation (VLES) (Speziale, 1998), extra-large eddy simulation (XLES) (Kok et al., 2004), partially-integrated transport model (PITM) (Chaouat and Schiestel, 2005), partially-averaged Navier-Stokes (PANS) (Girimaji, 2006), and scale-adaptive simulation (SAS) (Menter and Egorov, 2006), have been developed and several of them have been used for unsteady simulation of turbulent swirling flow (Ruprecht et al., 2002; Widenhorn et al., 2009; Paik and Sotiropoulos, 2010).

The partially-averaged Navier-Stokes (PANS) model was developed by Girimaji (2006) as a continuous approach for hybrid URANS/LES simulations with seamless coupling between the URANS and LES regions. It is a bridging closure model that can be used with any level of grid resolution between RANS and direct numerical simulation (DNS) (Girimaji, 2006). The PANS model has been successfully used in several turbulent separated flow problems, including flow past a square (Jeong and Girimaji, 2010) and circular (Lakshminpathy and Girimaji, 2010) cylinder, flow over a backward facing step (Freund et al., 2006), flow around a simplified vehicle model (Han et al., 2013), and flow around a rudimentary landing gear (Krajnović et al., 2012). Nevertheless, it has been seldom

applied in simulation of turbulent confined flows. Specifically, no previous report is found in the literature on the development and application of the PANS model in simulation of turbulent confined swirling flows.

The objective of the present paper is to develop a new PANS model, and to apply it in simulation of turbulent swirling flow with vortex breakdown, where complex, unsteady, coherent structures develop in the flow. The paper is organized as follows. The formulation of the new PANS model is presented first. Then, it is applied to simulation of two cases of turbulent swirling flow, namely 1) swirling flow through an abrupt expansion, and 2) flow in a draft tube of a Francis turbine operating under partial load. For each test case, a description of the problem, numerical methods, and results are presented and discussed in details. Finally, major findings and conclusions of the paper is summarized.

2. Model Formulation

The partially-averaged Navier-Stokes (PANS) (Girimaji, 2006) method is a bridging model between RANS and DNS. Bridging models (Speziale, 1997) aim to provide the best possible closure at any given level of computational grid size, and improve accuracy with increasing the resolution. They can be seen as an LES model with a cut-off wavenumber that can go continuously to zero, to become a RANS model. Therefore, bridging models including PANS are of particular interest in moderate grids where the cut-off is in the wavenumber range between RANS and LES. In these conditions PANS can provide better results than the RANS, since it resolves part of the flow, while RANS models the whole flow. The PANS is advantageous in comparison with the LES, since the grid is not fine enough to fulfill the requirement of LES, therefore less number of grid points and less computational resources would be needed. The seamless, continuous nature of the PANS model is a results of the so-called “partial averaging” concept, which corresponds to averaging (filtering) a portion of the fluctuating scales. The corresponding arbitrary explicit or implicit filter is linear and constant preserving, and commutes with temporal and spatial differentiation (Girimaji, 2006).

The partially-averaged Navier-Stokes equations for a turbulent incompressible flow are (Girimaji, 2006):

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \quad (1)$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial (\bar{u}_i \bar{u}_j)}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\nu \frac{\partial \bar{u}_i}{\partial x_j} - \tau_{ij} \right) \quad (2)$$

where $\bar{u}_i$ and $\bar{p}$ are the partially-averaged velocity and pressure respectively. The sub-filter scale (SFS) stress tensor τ_{ij} results from the partial-averaging of non-linear terms and represents the effects of the unresolved motions on the resolved field. It is similar to the Reynolds stress tensor resulting from the Reynolds-averaging in RANS approach, or to the subgrid scale (SGS) stress tensor after the filtering in LES method. To close the system of PANS equations, as in RANS and LES, a model is needed for the SFS stress tensor τ_{ij} relating the unresolved field stress and the resolved flow field. Girimaji (2006) proposed a generalized Boussinesq approximation for SFS stress tensor:

$$\tau_{ij} = -2\nu_u \bar{S}_{ij} + \frac{2}{3} k_u \delta_{ij} \quad (3)$$

where $\bar{S}_{ij}$ is the resolved rate of strain tensor:

$$\bar{S}_{ij} = \frac{1}{2} \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \quad (4)$$

ν_u denotes the PANS eddy viscosity (eddy viscosity of unresolved scales) and can be obtained, as in RANS, by various turbulence closure models. In the original PANS model (Girimaji, 2006), the eddy viscosity of unresolved scales ν_u is formulated based on the turbulent kinetic energy (k_u) and turbulent dissipation rate (ϵ_u) of unresolved scales as follows:

$$\nu_u = C_\mu \frac{k_u^2}{\epsilon_u} \quad (5)$$

Therefore, in order to close the system of PANS equations, two transport equations for the unresolved turbulent kinetic energy k_u and its dissipation rate ε_u was derived by Girimaji (2006). He derived the evolution equation for k_u and ε_u by asking the following question: “If the RANS two-equation model represents the closure for the fully averaged kinetic energy and dissipation, what are the implied model equations for their partially averaged counterparts?” In order to answer this question, Girimaji (2006) introduced two parameters f_k and f_ε , being the unresolved-to-total ratios of turbulent kinetic energy and turbulent dissipation rate respectively:

$$f_k = \frac{k_u}{k} \quad , \quad f_\varepsilon = \frac{\varepsilon_u}{\varepsilon} \quad (6)$$

These parameters control the extent of averaging in PANS model. With $f_k = 1$, for example, all the turbulent kinetic energy of the flow field is unresolved (modeled) and the PANS model would render a RANS solution. On the other hand, setting $f_k = 0$ on a sufficiently fine grid results in resolving all the turbulent kinetic energy and giving a DNS solution asymptotically. With a value of f_k between 0 and 1, the PANS model resolves the turbulent structures partially and leaves the unresolved motions to be modeled.

In the PANS approach, the unresolved (partially-averaged) kinetic energy k_u and dissipation rate ε_u transport equations are obtained by substituting the total kinetic energy k and dissipation rate ε in a parent RANS model using Eq. (6) as shown in details by Girimaji (2006) and Lakshmipathy and Girimaji (2006). Therefore, a PANS model can be derived from any parent RANS model based on the ratio between unresolved and total turbulence quantities. The original PANS model (Girimaji, 2006) was derived based on the standard k - ε model, while Lakshmipathy and Girimaji (2006) suggested PANS equations based on the k - ω model. Later, a near-wall formulation of PANS was proposed by Basara et al. (2011) in the form of the PANS k - ε - ζ - f model, and a low Reynolds number variant of PANS was developed by Ma et al. (2011) using the low-Reynolds number k - ε model of Abe et al. (1994).

In the present paper a new version of PANS model based on the extended k - ε turbulence model of Chen and Kim (1987) is developed. Chen and Kim (1987) proposed a modified k - ε model in which two different time scales are employed for calculating the rate of generation of ε . In their model, the rate of generation of ε is given by:

$$C_{1\varepsilon} \frac{P\varepsilon}{k} + C_{3\varepsilon} \frac{P^2}{k} \quad (7)$$

The two time scales involved are the dissipation rate time scale k/ε and the production time scale k/P . Since P is based on the mean strain rates, this modification enables the ε equation to respond more strongly to changes in the mean strain. The new term (the one including $C_{3\varepsilon}$) may be viewed as the energy transfer rate from large to small scales controlled by the production time scale. The net effect of these changes is to increase ε when the mean strain is strong and to decrease it when the mean strain is weak. Chen and Kim (1987) showed improved predictions of the flow field for several turbulent flows including plane and axisymmetric jets, turbulent boundary layer, flow over a backward facing step, and confined swirling flow.

Later, Monson et al. (1990) developed a low-Reynolds version of the extended k - ε model of Chen and Kim (1987) by combining this model with the low-Reynolds number k - ε model of Lam and Bremhorst (1981). The final form of the low-Reynolds number extended k - ε model of Chen and Kim (1987) is as follows (Monson et al., 1990):

$$\frac{\partial k}{\partial t} + \bar{u}_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + P - \varepsilon \quad (8)$$

$$\frac{\partial \varepsilon}{\partial t} + \bar{u}_j \frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + C_{1\varepsilon} \tilde{f}_1 \frac{P\varepsilon}{k} - C_{2\varepsilon} \tilde{f}_2 \frac{\varepsilon^2}{k} + C_{3\varepsilon} \tilde{f}_3 \frac{P^2}{k} \quad (9)$$

$$\nu_t = C_\mu \tilde{f}_\mu \frac{k^2}{\varepsilon} \quad (10)$$

where

$$\begin{aligned} \sigma_k &= 0.75 & \sigma_\varepsilon &= 1.15 & C_{1\varepsilon} &= 1.15 & C_{2\varepsilon} &= 1.9 & C_{3\varepsilon} &= 0.25 \\ \tilde{f}_1 &= \tilde{f}_3 = 1 + (0.05 / \tilde{f}_\mu)^3 & \tilde{f}_2 &= 1 - \exp(-\text{Re}_T^2) & \tilde{f}_\mu &= (1 - \exp(-0.0165 \text{Re}_y))^2 \cdot (1 + 20.5 / \text{Re}_T) \\ \text{Re}_T &= \frac{k^2}{\nu \varepsilon} & \text{Re}_y &= \frac{\sqrt{k} y}{\nu} \end{aligned}$$

Equations (8)-(10) are used as the parent RANS equations for development of the present PANS model. Derivation of the k_u and ε_u transport equations for the present PANS model is performed by multiplying the RANS equation for k (Eq. (8)) by f_k and for ε (Eq. (9)) by f_ε , as shown in detail in Girimaji (2006). Without repeating the details of the procedure, which can be found in Girimaji (2006), Lakshmipathy and Girimaji (2006), and Ma et al., (2011), the final form of the present PANS model is given by:

$$\frac{\partial k_u}{\partial t} + \bar{u}_j \frac{\partial k_u}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_u}{\sigma_{ku}} \right) \frac{\partial k_u}{\partial x_j} \right] + P_u - \varepsilon_u \quad (11)$$

$$\frac{\partial \varepsilon_u}{\partial t} + \bar{u}_j \frac{\partial \varepsilon_u}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_u}{\sigma_{\varepsilon u}} \right) \frac{\partial \varepsilon_u}{\partial x_j} \right] + C_1^* \frac{P_u \varepsilon_u}{k_u} - C_2^* \frac{\varepsilon_u^2}{k_u} + C_3^* \frac{P_u^2}{k_u} \quad (12)$$

where P_u is the production of unresolved turbulent kinetic energy, and can be found by:

$$P_u = \nu_u \frac{\partial \bar{u}_i}{\partial x_j} \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) = f_k \left(P - \frac{\varepsilon_u}{f_\varepsilon} \right) + \varepsilon_u \quad (13)$$

All model coefficients in Eq. (11) and (12) are functions of the original RANS model coefficients (Chen and Kim, 1987), and f_k and f_ε parameters as follows:

$$C_1^* = (C_{1\varepsilon} \tilde{f}_1 + 2C_{3\varepsilon} \tilde{f}_3) - 2C_{3\varepsilon} \tilde{f}_3 \frac{f_\varepsilon}{f_k}$$

$$C_2^* = (C_{1\varepsilon} \tilde{f}_1 + 2C_{3\varepsilon} \tilde{f}_3) + (C_{2\varepsilon} \tilde{f}_2 - C_{1\varepsilon} \tilde{f}_1 - C_{3\varepsilon} \tilde{f}_3) \frac{f_k}{f_\varepsilon} - C_{3\varepsilon} \tilde{f}_3 \frac{f_\varepsilon}{f_k}$$

$$C_3^* = C_{3\varepsilon} \tilde{f}_3 \frac{f_\varepsilon}{f_k}$$

$$\sigma_{ku} = \sigma_k \frac{f_k^2}{f_\varepsilon}$$

$$\sigma_{\varepsilon u} = \sigma_{\varepsilon} \frac{f_k^2}{f_{\varepsilon}}$$

The PANS eddy viscosity, which is used in Eq. (3), is defined by:

$$\nu_u = C_{\mu} \tilde{f}_{\mu} \frac{k_u^2}{\varepsilon_u} \quad (14)$$

In order to close the present PANS system of equations, the unresolved-to-total ratios of turbulent kinetic energy f_k and turbulent dissipation rate f_{ε} are needed to be defined. The parameter f_{ε} plays an important role when the dissipative scales are resolved, which is most likely in a low Reynolds number flow. In a high Reynolds number flow, where there is a clear separation between energy-carrying and dissipative scales, the dissipative small scales are unlikely to be resolved, hence, $\varepsilon_u = \varepsilon$ and $f_{\varepsilon} = 1$ (Basara et al., 2011; Girimaji and Abdol-Hamid, 2005).

In the early stages of PANS application, the unresolved-to-total ratio of turbulent kinetic energy f_k was prescribed as a constant (Girimaji, 2006; Jeong and Girimaji, 2010; Lakshmipathy and Girimaji, 2010). As the assumption of constant f_k is not reasonable, Girimaji and Abdol-Hamid (2005) proposed a formula for f_k to become a variable parameter that depends upon the grid size and turbulence length scale. They assumed that the smallest resolved length scale can be determined by the local dissipation and local eddy viscosity similar to the Kolmogorov scale. Requiring the grid size to be of the order of this “resolved-field Kolmogorov scale”, Girimaji and Abdol-Hamid (2005) derived the following formula for f_k :

$$f_k = \frac{1}{\sqrt{C_{\mu}}} \left(\frac{\Delta}{\Lambda} \right)^{2/3} \quad (15)$$

where Δ is the smallest grid dimension, and $\Lambda = k^{3/2} / \varepsilon$ is the turbulence length scale (Girimaji and Abdol-Hamid, 2005). Equation (15) has been widely used in PANS simulations by Basara et al. (2008), Basara et al. (2011), and Han et al. (2013), however, they considered Δ to be the geometric-average grid cell dimension, i.e. $\Delta = (\Delta_x \Delta_y \Delta_z)^{1/3}$. This parameter f_k is implemented in the computational procedure as a dynamic parameter,

changing at each point at the end of every time step, and then it is used as a fixed value at the same location during the next time step (Basara et al., 2008). The formula in Eq. (15), however, does not guarantee that the f_k parameter remains bounded between 0 and 1 as required by Eq. (6). Specifically, for situations where $\Delta \approx \Lambda$, this equation, in this form, may give f_k values larger than 1. Furthermore, studying the turbulent channel and hump flow, Davidson (2014) found very recently that the actual ratio of modeled to total turbulent kinetic energy k_u/k is much smaller than f_k from Eq. (15). The f_k obtained from Eq. (15) gives much too large a turbulent viscosity which kills all resolved turbulence (Davidson, 2014).

In the present study, a new formulation for f_k is developed which overcomes these problems. Consider the turbulence energy spectrum $E(\kappa)$ and a cut-off wave number κ_c , as shown in Fig. 1. The cut-off wave number is the spectral filter size usually related to the grid size by $\kappa_c = \pi / \Delta$ (Fadai-Ghotbi et al., 2010), and it is assumed constant or slowly variable in the case of variable step size of the grid. The total turbulent kinetic energy k can be obtained by:

$$k = \int_0^{\infty} E(\kappa) d\kappa \quad (16)$$

while the turbulent kinetic energy associated with the unresolved (modeled) scales (the shaded area in Fig. 1) is:

$$k_u = \int_{\kappa_c}^{\infty} E(\kappa) d\kappa \quad (17)$$

Consequently, the unresolved-to-total ratio of turbulent kinetic energy, i.e., f_k can be written as:

$$f_k = \frac{k_u}{k} = 1 - \frac{k_r}{k} = 1 - \frac{\int_0^{\kappa_c} E(\kappa) d\kappa}{\int_0^{\infty} E(\kappa) d\kappa} \quad (18)$$

Therefore, f_k can be analytically calculated using an energy spectrum equation. In this study, an accurate energy spectrum $E(\kappa)$ equation inspired from a Von Kármán like spectrum (Schiestel and Dejoan, 2005) is used:

$$E(\kappa) = C_k \varepsilon^{2/3} \kappa^s \left[\left(\frac{C_k \varepsilon^{2/3}}{C_s} \right)^{\frac{2}{5+3s}} + \kappa^{2/3} \right]^{\frac{5+3s}{2}} \quad (19)$$

where C_k , C_s and s are constants. The use of this spectrum, Eq. (19), valid in the entire range of wavenumbers evolving from large to small eddies allows to obtain a more accurate result for the f_k than the one obtained by considering the Kolmogorov spectrum ($E(\kappa) = C_k \varepsilon^{2/3} \kappa^{-5/3}$) which is only valid in the inertial range. It is shown (see the Appendix) that Eq. (15) for f_k implies the use of the Kolmogorov spectrum, which is not valid for the entire range of wavenumbers. This is specifically important when performing PANS simulations on coarse grids implying that the cut-off wavenumber may happen to be located before the inertial zone.

Using Eq. (18) and (19) (see the Appendix for details), $f_k = k_u/k$ is calculated analytically:

$$f_k = 1 - \left[\frac{\left(\frac{\Lambda}{\Delta} \right)^{2/3}}{\left(\frac{C_k}{(1+s)\pi^{2/3}} \right) + \left(\frac{\Lambda}{\Delta} \right)^{2/3}} \right]^{\frac{3}{2}(1+s)} \quad (20)$$

where $\Delta = (\Delta_x \Delta_y \Delta_z)^{1/3}$ is the grid length scale, and $\Lambda = k^{3/2} / \varepsilon$ is the turbulence integral length scale. The total turbulent kinetic energy required to calculate the turbulence length scale is obtained in three steps. First, the total TKE field is obtained from a preliminary steady RANS simulation. This RANS simulation requires a considerably smaller computational time, and is needed anyway to initialize transient simulations, since starting an unsteady simulation without any initialization often results in severe numerical instabilities. Second, the PANS simulations are performed for sufficiently long time in order to obtain preliminary statistics of k_u and k_r using the “frozen” RANS field for k . Finally, the last part of PANS simulations are performed in which the total turbulent kinetic energy is calculated ($k = k_u + k_r$) and updated on a regular basis.

With Kolmogorov constant $C_k=1.5$ and $s=2$ (Pope, 2000) (see the Appendix) f_k , in its final form, reads:

$$f_k = 1 - \left[\frac{\left(\frac{\Lambda}{\Delta}\right)^{2/3}}{0.23 + \left(\frac{\Lambda}{\Delta}\right)^{2/3}} \right]^{4.5} \quad (21)$$

For a very coarse grid, where $\Lambda \ll \Delta$, $f_k \rightarrow 1$ according to Eq. (21) and the model is identical to a RANS approach with all scales being modeled. When the grid size is much smaller than the turbulence length scale, f_k decreases, and more and more of the turbulent kinetic energy can be resolved. Furthermore, Eq. (21) guarantees that the f_k value remains between 0 and 1, a condition required by the definition of the f_k , Eq. (6).

As a summary, the present partially-averaged Navier-Stokes (PANS) model includes Eq. (1) and (2), where the PANS eddy viscosity is obtained by Eq. (14), solving two transport equations for unresolved turbulent kinetic energy Eq. (11), and unresolved dissipation rate Eq. (12). Two important parameters in Eq. (11) and (12), i.e., f_k and f_ϵ are obtained by Eq. (21) and $f_\epsilon = 1$, respectively.

3. Applications

The present PANS model is used in simulations of two flow problems, namely swirling flow through an abrupt expansion, and flow in a draft tube of a Francis turbine operating under partial load. In both cases, the swirl number is high enough for the vortex breakdown to happen and the flow is very unsteady and includes several complex phenomena.

Although the pure central differencing scheme is the ideal choice for LES and hybrid URANS/LES simulations due to the low level of numerical diffusion, it results in numerical instabilities and some unphysical oscillations added to the solution. Therefore, simulations are performed using a bounded central-differencing scheme (Jasak et al., 1999) for the momentum equation, while a second-order upwind scheme is used for turbulence quantities. The bounded central-differencing scheme, which blends the central differencing and second-order upwind schemes as well as forces the convection boundedness criterion (Gaskell and Lau, 1988), is shown to be competitive in terms of minimizing the numerical errors (Park et al., 2004 and Adedoyin et al., 2006). The temporal differencing is carried out using the second-order implicit scheme. The SIMPLE algorithm is used to couple the velocity and pressure fields.

3.1. Swirling flow through an abrupt expansion

3.1.1. Test case description and numerical methodology

Turbulent swirling flow through an abrupt expansion is simulated using the present model. It is a complex flow possessing various dynamic phenomena including vortex breakdown, recirculation, detachment and reattachment, and enhanced mixing. Therefore, correctly predicting the flow behavior is quite challenging and special considerations should be paid to choosing the turbulence closure model. Furthermore, the swirling flow through a sudden expansion is of high industrial interest since it resembles the flow in several technical applications such as the gas turbine combustor.

The considered test case corresponds to the experimental study of Dellenback et al. (1988), for which several numerical investigations, using different turbulence models, have been reported in the literature (Schlüter et al., 2004; Gyllenram and Nilsson, 2008; Paik and Sotiropoulos, 2010). The experimental configuration consists of water flow at an axisymmetric expansion (with the expansion ratio D_2/D equal to 1.94) as shown in Fig. 2. The axial and circumferential components of time-averaged and root-mean-square (rms) velocity were measured by Dellenback et al. (1988) at several cross-sections downstream of the sudden expansion. In addition, measurements were performed for two sections upstream of the expansion which can be used as the inflow boundary condition for the numerical simulations. Here, the inlet section of the computational domain is put at two diameters upstream of the expansion ($z/D = -2.0$) and the outlet boundary is placed at $z/D = 10$. This computational domain is shown to be sufficient for this flow problem (Paik and Sotiropoulos, 2010). The swirl number defined as (Dellenback et al., 1988):

$$S = \frac{\int_0^R U V r^2 dr}{R \int_0^R U^2 r dr} \quad (22)$$

is approximately 0.6, with R being the inlet radius, $R = D/2$, and V and U denoting the time-averaged circumferential and axial velocities respectively. The Reynolds number based on the inlet diameter D and the bulk velocity is 30,000.

At the inlet section, the radial profiles of axial and circumferential velocity components are obtained from the measured data of Dellenback et al. (1988) by interpolation. Figure 3 shows the velocity profiles applied at the inlet

of the computational domain. The low axial velocity region near the center ($r/R=0$) is formed due to the high circumferential velocity (swirl) transferring momentum away from the center. In this case, with a high level of swirl, the turbulence is mainly generated after the expansion, and regions of high turbulence production and shear layers created by the recirculating flow are basically independent of the inflow conditions. Therefore, the flow is almost independent of the initial turbulence conditions as shown in detail by Schlüter et al. (2004). Hence, no unsteadiness is added to the steady inlet conditions shown in Fig. 3. A constant uniform inlet turbulence intensity of 10% is estimated from measurements of Dellenback et al. (1988). No-slip conditions are applied at wall boundaries.

The main computational grid (with which all simulations are performed) consists of 1,660,384 hexahedral cells. The grid is refined in areas of large variable gradients, i.e., near-wall and centerline region, and near the expansion. The size of the grid is comparable to those used in previous LES and hybrid URANS/LES simulations of this test case (Schlüter et al., 2004; Gyllenram and Nilsson, 2008; Paik and Sotiropoulos, 2010). Furthermore, attention is paid to make sure that this grid is within the range of the guidelines for DES grids (Spalart, 2001). To examine the grid sensitivity of the present PANS model, simulations are also performed for a finer grid with 3,428,160 cells (results will be discussed later). The computational time on the finer grid is about 2.3 times the computational time of the main grid. The first cell center normal to the wall is placed at $y^+ \approx 0.5$ everywhere in both grids.

Unsteady simulations are carried out using the presently developed PANS model, as well as the delayed DES (DDES) (Spalart et al., 2006), and the URANS (SST $k-\omega$) (Menter, 1994) turbulence models. All unsteady simulations are initialized by a RANS steady solution. The dimensionless time-step size (normalized by the inlet diameter and the bulk velocity) is taken as 0.0044. The convergence criterion of the residuals for each time-step is set to 3 orders of magnitude drop or maximum 30 sub-iterations. Unsteady simulations are carried out for about 12 through-flow time (the time required by the mean flow to pass through the domain once), corresponding to about 32,000 time-steps.

3.1.2. Results and discussion

Figure 4 shows the instantaneous axial velocity contours on the meridian plane, obtained using various turbulence closure approaches, namely URANS, DDES and present PANS models. The development of the reverse flow region at the center of the pipe, and the strong shear layer between this region and the main flow is clearly seen in all

figures. However, as shown in Fig. 4a, the SST $k-\omega$ URANS model cannot capture the self-induced unsteadiness of the vortex breakdown and gives steady symmetric results (similar to RANS) due to steady symmetric boundary conditions. A detailed discussion on this issue can be found in Foroutan and Yavuzkurt (2014). Applying hybrid URANS/LES models when the vortex breakdown unsteadiness is resolved in an LES manner, detailed unsteady features of the flow can be captured sufficiently resulting in non-symmetric unsteady results as shown in Fig. 4b for the DDES and in Fig. 4c for the PANS model.

A snapshot of the flow is presented in Fig 5. The three-dimensional vortical structures are visualized by the non-dimensional iso-surfaces of the Δ -criterion. The Δ -criterion was developed by Chong et al. (1990) to identify the vortex. They proposed that a vortex core is a region with complex eigenvalues of velocity gradient tensor $\partial u_i / \partial x_j$. Complex eigenvalues imply that the local streamline pattern is closed or spiral in a reference frame moving with the point. This physically can happen only if a vortex exists in the flow where fluid particles are rotating around the vortex axis. Complex eigenvalues will occur when

$$\Delta = \left(\frac{Q}{3}\right)^3 + \left(\frac{R}{2}\right)^2 > 0 \quad (23)$$

where $Q \equiv -\frac{1}{2} \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i}$ and $R \equiv \text{Det} \left(\frac{\partial u_i}{\partial x_j} \right)$ are the invariants of $\frac{\partial u_i}{\partial x_j}$ (Chong et al., 1990).

It is seen in Fig. 5 that the flow is dominated by the precessing vortex core that rotates around the geometrical axis of symmetry and its subsequent breakdown to small coherent structures downstream of the expansion. Both the present PANS model and the DDES model simulate the vortex core and its breakdown, however, the present model is less dissipative and captures more detailed structures. This implies that the switch between RANS and LES is more efficient in the present model leading to resolving of more turbulent structures on the same grid.

Another important physical phenomenon in this flow is the separation and reattachment of the flow with respect to the wall of the expansion section. As the flow enters the wider section, it separates from the wall and a recirculating region is formed near the wall of the wider pipe (see Fig. 4). The flow reattaches to the wall at some distance downstream of the sudden expansion. This distance is called the reattachment length. Dellenback et al. (1988) measured the reattachment length of the flow and obtained a value of $z_r/h=2.5$, where h is $0.5(D_2-D)$. In this

study, the reattachment length is obtained by calculating the skin friction coefficient. The reattachment length obtained from using the present PANS model is $z_r/h=2.54$ which is in very good agreement with the experimental result (less than 2% deviation), while a reattachment length of $z_r/h=2.90$ is obtained by using the DDES model results in 16% deviation. The SST $k-\omega$ model underpredicts the reattachment length by 25%, giving a value of $z_r/h=1.87$.

Figure 6 shows radial distributions of the mean (time-averaged) and rms axial and circumferential velocities on nine planes after the expansion, corresponding to the z/D values of 0.25 to 8.0. All time-averaged results are computed by temporal averaging of the results of 24,000 time-steps (with normalized time-step size of 0.0044) after setting the simulations to run for an initial 8000 time-steps (initial transient). Results obtained using the present model is compared to the experimental data of Dellenback et al. (1988) as well as those obtained using the DDES and URANS (SST $k-\omega$) models. It can be seen that the agreement between the predictions of the PANS model and experimental data is very good. The DDES model also gives quite good results; however, some deviations are seen around the reattachment point ($0.75 \leq z/D \leq 2.0$). Specifically, the axial velocity is overpredicted by DDES just upstream of the reattachment point ($z/D=1.0$), resulting in a 16% overprediction of the reattachment length as discussed above. Using the present PANS model, an overall improvement is seen in results comparing to the DDES predictions with as much as 51% and 28% improvements in predictions of the rms axial and circumferential velocities respectively, near the centerline at $z/D=1.0$. The SST $k-\omega$ URANS model gives poor results and the shear layer between the central reverse flow region and outer flow cannot be captured (see the mean axial velocity curves). Furthermore, the level of swirl cannot be predicted reasonably, resulting in underpredictions of the circumferential velocity as reported also by Gyllenram and Nilsson (2008). As discussed above, results of the SST $k-\omega$ model converge to a steady solution, hence, no rms velocity associated with this model is plotted in Fig. 6.

To investigate the sensitivity of the numerical solutions to the grid size, in addition to the main grid (with 1,660,384 cells) simulations are also carried out for a finer grid (with 3,428,160 cells), and results are shown in Fig.7. Profiles of the mean and rms velocities obtained using PANS and DDES models on both grids are shown in Fig. 7 on four planes with z/D values of 0.5, 1.0, 3.0, and 6.0. Both PANS and DDES results can be slightly improved using a finer mesh, however, it can be seen that the grid dependence is not a major issue.

The evolution of the f_k parameter in PANS simulations is shown in Fig. 8 for three axial cross sections, one, two, and four diameters downstream of the expansion. At each section, three curves are compared, namely, the f_k

parameter calculated using Eq. (21) (the present formula), the f_k parameter calculated using Eq. (15) (the previous formula), and the computed f_k parameter based on the calculated k_u and k using Eq. (6). It is seen that the present f_k parameter approaches 1 near the wall, resulting in a more RANS-like simulation for this region. Near the centerline f_k decreases, and the present PANS model resolves more turbulent motions in an LES manner. Also f_k has lower values at the section $z/D=1.0$ where the vortex breaks down and the flow is very unsteady, therefore it needs to be resolved. Further downstream ($z/D=4.0$) where the flow reattaches to the wall, f_k has higher values, although the model is still in LES mode in most of the pipe, according to the Pope's criterion (Pope, 2000). Furthermore, there is a good agreement between the present prescribed value of f_k (Eq. (21)) and the calculated value. It should be noted that the equality between the prescribed and computed values of f_k is exactly reached only in strict equilibrium flows; therefore, it is not surprising to see deviations between these two values in this complex flow. The previously used formula for f_k (Eq. (15)), however, shows considerable overprediction (Also see Fig. A1). Higher values of f_k calculated by Eq. (15) result in predicting larger turbulent viscosities, damping more resolved turbulence, and showing slower change from RANS to LES, as also observed by Kubacki et al. (2013) and Davidson (2014). Here, this effect is shown in Fig. 9 where predictions of PANS simulations using two different formulas for f_k , i.e., Eq. (15) and Eq. (21), are compared. All other parameters are kept unchanged. It is seen that predictions are improved using the present model, while the previous model tends to return to the RANS solution. Specifically at downstream sections ($z/D = 3.0$ and 6.0), the previous formulation considerably overpredicts the value of f_k resulting in underprediction of the rms velocities and returning the mean velocity values to the RANS predictions.

3.2. Flow in a draft tube of a Francis turbine operating under partial load

3.2.1. Test case description and numerical methodology

Francis turbines operating at partial load have a high level of residual swirl at the draft tube inlet as a result of the mismatch between the swirl generated by the wicket gates and the angular momentum extracted by the turbine runner. The decelerated swirling flow in the draft tube cone may lead to flow instabilities resulting in the helical vortex breakdown called the “vortex rope”. The flow instability with formation of the vortex rope is the main cause of efficiency reduction, pressure fluctuations, and vibrations experienced by a Francis turbine operating at partial load (Dörfler et al., 2013). Given the strong effects that vortex rope can have, accurate numerical simulations that

can capture its physical behavior are necessary for improving hydropower plant efficiency and preventing structural vibrations.

For the present PANS simulations, the Francis turbine draft tube investigated in the FLINDT project (Avellan, 2000) is considered. The FLINDT project experiments were carried out on a scaled model of a Francis turbine (See Fig. 10) with specific speed of 0.56. The turbine model has a spiral case, stay ring of 10 stay vanes, a distributor made of 20 wicket gates, a 17-blade runner of 0.4 m outlet diameter, and an elbow draft tube with one pier (Avellan, 2000). Many experimental and numerical studies have been carried out under the FLINDT project (Mauri, 2002; Ciocan et al., 2007; Iliescu et al., 2008), however, the exact details of the draft tube geometry is not available in open literature. Therefore, the simplified FLINDT draft tube was investigated in the previous studies by authors (Foroutan and Yavuzkurt, 2014a,b). For this study, a comprehensive investigation of the previously published articles within the FLINDT project is performed, in order to build a complete database on details of the draft tube geometry. Using this database the three-dimensional FLINDT draft tube geometry is rebuilt as shown in Fig. 11. It includes the runner crown cone, the short draft tube cone of 17° angle followed by a 90° curved elbow, and a rectangular section diffuser with a pier. The inlet section (section S0 in Fig. 11) is located just downstream of the trailing edge of runner blades.

A no-cavitation, partial load operating condition with the head coefficient of $\psi = 2gH/(\omega^2 R^2) = 1.18$ and flow rate coefficient (discharge coefficient) of $\phi = Q/(\pi\omega R^3) = 0.26$ is considered. In this operating condition, the wicket gates are 7° more closed comparing to the best efficiency point (BEP), resulting in reduction of the flow rate to 70% of the BEP flow rate. Experimental studies (Ciocan et al., 2007) showed that a large precessing vortex rope forms inside the draft tube in this condition, causing severe pressure fluctuations. The swirl number (Eq. (22)) at inlet is 0.64, and the Reynolds number based on the runner's diameter and angular velocity is 6.3×10^6 (Ciocan et al., 2007).

The inlet to the computational domain is chosen to be the surface swept by the trailing edge of the runner (section S0 in Fig. 11), where the circumferentially-averaged mean velocity components and turbulent quantities are available from (Stein et al., 2006). Figure 12 illustrates the axial, radial, and circumferential velocity profiles at the inlet of the draft tube. The increase in velocity components near the band (see Fig. 12) is related to the secondary flows in the blade channel at partial load with formation of inter-blade vortices (Avellan, 2004). In order to overcome back-flows at the outlet, which could introduce numerical instabilities, the outflow boundary is located further downstream after the draft tube discharge. Nevertheless, it has been shown (Mauri, 2002) that the outlet

boundary plays an unimportant role in this case. No-slip conditions are applied on all walls, while the crown cone is defined as a rotating wall with an angular velocity equal to that of the runner. Similar considerations, as in the previous case of the abrupt expansion, are applied in generating the computational grid including the grid sensitivity analysis. In this case, the computational grid includes 3,361,198 cells, with the first cell center adjacent to the wall being placed at $y^+ \approx 2$.

Unsteady simulations are initialized by a RANS steady solution. The time-step size is taken corresponding to one degree rotation of the runner (2.2×10^{-4} second) which is known to be sufficient for hydroturbine applications (Ciocan et al., 2007). In order to make sure that a quasi-steady condition is reached, unsteady simulations are continued for a very long time. Specifically, simulations are performed for 73,000 iterations corresponding to about 200 rotations of the runner, and about 37 through-flow time.

3.2.2. Results and discussion

Overall evaluation of the present PANS simulations is performed considering two global parameters: 1) the pressure recovery coefficient defined as (Mauri, 2002):

$$\chi \equiv \frac{p_2 - p_1}{\frac{1}{2} \rho \left(\frac{Q}{A_1} \right)^2} \quad (24)$$

and calculated between section S1 and S2 in Fig. 10, and 2) the portion of the flow exiting the “left” channel (see Fig. 11 for the definition of “left” and “right”). The pressure recovery coefficient obtained from the PANS simulations is 0.123 which is in very good agreement with experimental value (Susan-Resiga et al., 2006) of 0.116 (only 6% difference). Also, the present PANS model predict 81% of flow to be exiting through the left channel which is in excellent agreement with 81% value numerically obtained by Mauri (2002).

Figure 13 shows contours of instantaneous pressure and axial velocity for three arbitrary instances in time, as well as contours of the mean axial velocity in the draft tube. Contours are plotted in the symmetry plane, showing the draft tube cone and elbow up to the pier. In each figure, the average location of the vortex core in the draft tube cone obtained from linear curve fitting of PIV data of Ciocan and Iliescu (2007) is shown using two black lines. In fact, these lines represent the 2D cross section of the conical surface that vortex rope wraps around. Figure 13a

shows low pressure discoid regions in the draft tube representing cross sections of the helical vortex rope. In a vortex, pressure tends to have a local minimum on the axis of a circulating flow when the centripetal force is balanced with the radial pressure gradient ($\partial p / \partial r = \rho V_\theta^2 / r$). It can be seen that, there is an overall good agreement between the locations of the vortex core predicted by the PANS simulations and those obtained from PIV measurements (Ciocan and Iliescu, 2007). This also can be seen in Fig. 13b where instantaneous axial velocity contours are plotted at the same three time instants. Furthermore, Fig. 13b shows the strong shear layer which is the cause of the formation of the vortex rope. The direction of rotation of the vortex can be easily found out using Fig. 13b and considering the direction of the axial velocity at each shear layer. Note that in Fig. 13, downward is positive and upward (reverse flow) is negative. As discussed by Foroutan and Yavuzkurt (2014a), the vortex rope forms due to the roll-up of the shear layer at the interface between the low-velocity inner region created by the wake of the crown cone and highly swirling outer flow. This low-velocity inner region (stagnant region) is clearly shown in Fig. 13c by contours of the mean axial velocity. Again, it is interesting to note the agreement between the average locations of the vortex cores from PIV data (black lines) and the extent of the stagnant region predicted by the PANS simulations.

As shown in Fig. 14, the present PANS model can capture the strong precessing vortex rope in the draft tube. The helical vortex is visualized in Fig. 14 at three instants by the iso-pressure surfaces corresponding to $p = -16,000$ Pa. The vortex rope has a very unsteady nature, and its shape can be dramatically changed over time. The tail of the vortex rope may impact the inner part of the elbow wall as seen in Fig. 14 at $t=6.6$ s. This impact, called the “shock phenomenon”, induces strong acoustic noise, pressure fluctuations, and even structural vibrations (Dörfler, 1994).

To demonstrate fluctuations in the draft tube due to the formation of the vortex rope, the wall pressure is monitored on draft tube wall. Figure 15a shows the evolution of wall pressure with time in the draft tube for point “pp” in Fig. 11, 0.24 m downstream of the draft tube inlet section, during unsteady PANS simulations. Results are shown for 6 seconds (about 27,000 iterations and 75 rotations of the runner) after letting the simulations to run for an initial 4 seconds to make sure that a periodic unsteady (quasi-steady) state is reached. Pressure fluctuations due to the vortex rope have large amplitude ($p_{rms} = 2423$ pa) and low frequency (~ 4 Hz) as shown in Fig. 15a. The dominant frequency of the pressure fluctuations can be obtained performing Fast Fourier Transform on the results. Figure 15b shows the normalized frequency spectrum obtained from present PANS simulations. The vortex rope frequency is found to be about 0.321 of the runner rotation frequency. This is in very good agreement with the value

of 0.3 seen in experimental studies (Ciocan et al., 2007) (only 7% difference). Furthermore, results of the present simulations give better predictions in comparison with the numerical study of Ciocan et al. (2007) who used the $k-\varepsilon$ URANS model and reported 13% overestimation of the vortex rope frequency in comparison with experimental data.

4. Conclusions

A new partially-averaged Navier-Stokes (PANS) turbulence model for predicting unsteady turbulent swirling flow with vortex breakdown is developed. The present model is formulated based on the extended $k-\varepsilon$ turbulence model of Chen and Kim (1987) by employing the PANS methodology. The main distinctive feature of the present model is to incorporate a newly developed relation for the unresolved-to-total turbulent kinetic energy ratio f_k , using partial integration of the complete turbulence energy spectrum. The new expression overcomes the problem of overestimated f_k and damped turbulent motions where the grid cut-off wave number is below the inertial range. It is clearly shown that the new formulation improves the predictions compared to the previously used expression for f_k . Numerical simulations are carried out using the present PANS model for two flow problems, namely swirling flow through an abrupt expansion and flow in a draft tube of a hydraulic turbine operating under partial load. In both cases, the flow is very unsteady and includes high level of swirl and vortex breakdown. Furthermore, both problems are of high industrial interest. Results for the case of the abrupt expansion obtained using the present model are in very good agreement with experimental data, while improvements are seen comparing to the results of DDES and URANS (SST $k-\omega$) models. Specifically, mean and rms axial and circumferential velocity profiles, as well as the reattachment length are accurately predicted. Although the DDES model also performs quite well, it overpredicts the velocity values around the reattachment point resulting in a 16% overprediction of the reattachment length. The present model predicts the reattachment length by only 1.6% error. The new PANS model is also applied to simulations of complex unsteady flow in a draft tube of a Francis turbine operating at partial load where a precessing vortex rope forms inside the draft tube. It is shown that the pressure recovery factor is quite precisely (only 6% deviation from experimental data) predicted by the PANS simulations. The precessing vortex rope is captured showing highly unsteady behavior. The vortex rope frequency is obtained by monitoring wall pressure fluctuations, and it is found to be about 0.321 of the runner rotation frequency which shows only 7% difference compared to the

experimental data. Overall, the present PANS model shows very promising results in both cases. Further investigation of the model and its application to several other flow problems are the future steps of this study.

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Nomenclature

$C_{1\varepsilon}, C_{2\varepsilon}, C_{3\varepsilon}, C_{\mu}$	Turbulence model constants
D	Diameter
$\tilde{f}_1, \tilde{f}_2, \tilde{f}_3, \tilde{f}_{\mu}$	Damping functions
f_k	Unresolved-to-total ratio of turbulent kinetic energy
f_{ε}	Unresolved-to-total ratio of turbulence dissipation rate
H	Turbine head
k	Turbulent kinetic energy
P	turbulence production
p	Pressure
Q	Volume flow rate
R	Radius
r	Radial coordinate
S	Swirl number
$\bar{u}_i$	Partially-averaged velocity
V_r, V_{θ}, V_z	Velocity components in cylindrical coordinates
x, y, z	Coordinates
y^+	Dimensionless wall distance

Greek symbols

ε	Dissipation rate of turbulent kinetic energy
Δ	Grid length scale
φ	Flow rate (discharge) coefficient ($= Q/(\pi\omega R^3)$)
ψ	Head (energy) coefficient ($= 2gH/(\omega^2 R^2)$)
Λ	Turbulence length scale
τ_{ij}	Sub-filter stress tensor
κ	Wave number
ν_u	PANS eddy viscosity
ω	Angular velocity of the runner

Subscripts

u	Unresolved
r	Resolved

Abbreviations

BEP	Best Efficiency Point
DDES	Delayed Detached Eddy Simulation
LES	Large Eddy Simulation
PANS	Partially-Averaged Navier-Stokes
RANS	Reynolds-Averaged Navier-Stokes
SST	Shear Stress Transport
TKE	Turbulent Kinetic Energy
URANS	Unsteady RANS

Appendix

Details of the derivation of Eq. (21) for f_k are presented in this appendix. The energy spectrum at very low wavenumbers ($\kappa \rightarrow 0$) behaves as (Lesieur, 2008):

$$E(\kappa) \approx C_s \kappa^s \quad (\text{A1})$$

With C_s being a constant (the hypothesis of permanence of big eddies, Lesieur (2008)). In the inertial range, on the other hand, the Kolmogorov spectrum has to be recovered:

$$E(\kappa) \approx C_k \epsilon^{2/3} \kappa^{-5/3} \quad (\text{A2})$$

A smooth approximation that has the correct behavior both at small wavenumbers and in the inertial range is proposed by Schiestel and Dejoan (2005):

$$E(\kappa) = C_k \epsilon^{2/3} \kappa^s \left[\left(\frac{C_k \epsilon^{2/3}}{C_s} \right)^{\frac{2}{5+3s}} + \kappa^{2/3} \right]^{\frac{5+3s}{2}} \quad (\text{A3})$$

Integration of the spectrum over all wavenumbers gives the total turbulent kinetic energy:

$$k = \int_0^\infty E(\kappa) d\kappa = \frac{1}{1+s} \left[C_s^{\frac{2}{3(1+s)}} C_k \mathcal{E}^{2/3} \right]^{\frac{3+3s}{5+3s}} \quad (\text{A4})$$

Using Eq. (A4), the constant C_s can be written as:

$$C_s = \left[\frac{\left(k(1+s)^{\frac{5+3s}{3+3s}} \right)^{\frac{3+3s}{2}}}{C_k \mathcal{E}^{2/3}} \right] \quad (\text{A5})$$

Now, the parameter f_k can be calculated as follows:

$$f_k = \frac{k_u}{k} = 1 - \frac{k_r}{k} = 1 - \frac{\int_0^{\kappa_c} E(\kappa) d\kappa}{k} = 1 - \frac{\int_0^{\kappa_c} C_k \mathcal{E}^{2/3} \kappa^s \left[\left(\frac{C_k \mathcal{E}^{2/3}}{C_s} \right)^{\frac{2}{5+3s}} + \kappa^{2/3} \right]^{\frac{5+3s}{2}} d\kappa}{\frac{1}{1+s} \left[C_s^{\frac{2}{3(1+s)}} C_k \mathcal{E}^{2/3} \right]^{\frac{3+3s}{5+3s}}} \quad (\text{A6})$$

Analytically calculating the integral in the numerator of Eq. (A6), then substituting for C_s from Eq. (A5) and simplifying yields:

$$f_k = 1 - \left[\frac{\kappa_c^{2/3}}{\left(\frac{C_k \mathcal{E}^{2/3}}{k(1+s)} \right) + \kappa_c^{2/3}} \right]^{\frac{3}{2}(1+s)} \quad (\text{A7})$$

Defining $\Lambda = k^{3/2} / \mathcal{E}$ and $\Delta = \pi / \kappa_c$ as the turbulence and the grid length scale, the f_k parameter can be written as:

$$f_k = 1 - \left[\frac{\left(\frac{\Lambda}{\Delta}\right)^{2/3}}{\left(\frac{C_k}{(1+s)\pi^{2/3}}\right) + \left(\frac{\Lambda}{\Delta}\right)^{2/3}} \right]^{\frac{3}{2}(1+s)} \quad (\text{A8})$$

which was presented by Eq. (20).

Two constants, namely C_k and s need to be defined. For C_k , the Kolmogorov constant, a value of 1.5 is widely accepted (Pope, 2000). The question of permissible values for s , however, is a controversial one. A complete discussion about the choice of s can be found in Lesieur (2008). The two most widely used values for s are 2 and 4 (Pope, 2000; Lesieur, 2008), giving the following relations for f_k :

$$f_k = 1 - \left[\frac{\left(\frac{\Lambda}{\Delta}\right)^{2/3}}{0.23 + \left(\frac{\Lambda}{\Delta}\right)^{2/3}} \right]^{4.5} \quad (s = 2) \quad (\text{A9})$$

$$f_k = 1 - \left[\frac{\left(\frac{\Lambda}{\Delta}\right)^{2/3}}{0.14 + \left(\frac{\Lambda}{\Delta}\right)^{2/3}} \right]^{7.5} \quad (s = 4) \quad (\text{A10})$$

In order to investigate the sensitivity of f_k to the choice of s , both relations are plotted against (Λ/Δ) in Fig. A1. It can be seen that the curves are almost coincide, and the deviation between Eq. (A9) and (A10) for f_k is less than 3%. Therefore, Eq. (A9) is chosen and presented (Eq. (21)) as the final form for f_k . Furthermore, Fig. A1 compares the variations of f_k obtained from Eq. (A9), Eq. (A10), and Eq. (15) with respect to the ratio of Λ/Δ . For the present formulation, in situations where $\Lambda/\Delta \rightarrow 0$, i.e. for coarse grids or very near the wall, $f_k \rightarrow 1$ and RANS equations retrieved, while $f_k \rightarrow 0$ asymptotically for $\Lambda/\Delta \rightarrow \infty$. The f_k obtained from Eq. (15) is considerably higher specially at lower Λ/Δ values. It reaches the value of one around $\Lambda/\Delta \approx 6$, and gives even higher values for lower values of Λ/Δ .

It is worth mentioning that if only the Kolmogorov energy spectrum (Eq. (A2)), would be used to obtain the f_k parameter, the result reads as:

$$f_k = \frac{k_u}{k} = \frac{\int_{\kappa_c}^{\infty} C_k \varepsilon^{2/3} \kappa^{-5/3} d\kappa}{k} = \frac{3}{2} C_k \frac{\varepsilon^{2/3}}{k} \kappa_c^{-2/3} \quad (\text{A11})$$

Using the above-mentioned definitions for the turbulence and the grid length scale, Δ and Λ , f_k can be written as:

$$f_k = \frac{3}{2} C_k \left(\frac{1}{\pi} \right)^{2/3} \left(\frac{\Delta}{\Lambda} \right)^{2/3} \approx 1.05 \left(\frac{\Delta}{\Lambda} \right)^{2/3} \quad (\text{A12})$$

which is similar (but with a different constant coefficient) to the formula proposed by Girimaji and Abdol-Hamid (2005) and Basara et al. (2008) (Eq. (15)).

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Figure9

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