Uniaxial fatigue behavior of Cu-Nb micro-composite conductor, Part II: modeling and simulation

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Abstract

A damage-coupled constitutive model has been built to predict the ratcheting effect and fatigue life of Cu-Nb micro-composite conductor under different loading conditions. The model is rate-independent and has no isotropic hardening effect according to the experiments reported in Part I. The Abdel-Karim-Ohno model [Abdel-Karim M, Ohno N. Kinematic hardening model suitable for ratcheting with steady-state. *Int J Plast* 2000;16:225-240] is improved for a better description of the ratcheting effect. Two damage factors D_t and D_a are respectively defined to be related to the ratcheting strain and accumulative plastic strain. The damage factor D_t is coupled to the model to reflect the stress reduction. On the contrary, the damage factor D_a is not coupled to the model but still calculated to decide the failure of the conductor. Accordingly, there are three failure criteria for different peak stress levels without considering the ratcheting strain above 5%. When the peak stress is above 758.5 MPa, the failure is decided by the peak stress being equal to the tensile stress corresponding to the ratcheting strain. When the peak stress is between 650 MPa and 758.5 MPa, the failure is caused by the competition between the ratcheting strain and D_a whose critical values are respectively 5% and 0.065. When the peak stress is below 650 MPa, failure happens due to $D_a > 0.065$. As a result, the simulated ratcheting effect is more or less overestimated for some loading conditions, which leads to certain deviation between the predicted and experimental fatigue life. Nevertheless, the precision is still reasonable with the fatigue life ranging from 10^2 to 10^5 which is large enough for a further analysis of the fatigue behavior of pulsed magnets where the conductor is applied.

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1 Introduction

Ratcheting is the unidirectional progress of deformation with loading cycles in stress control. It has long been a crucial factor to influence the fatigue property of materials and structures. The ratcheting-fatigue problem is usually dealt with in two steps.

The first step is to accurately describe the ratcheting effect using a constitutive model. For this purpose, extensive efforts have been made in past two decades to improve the A-F kinematic hardening model (A-F model) proposed by Armstrong and Frederick [1]. The improvement mainly includes the following four aspects:

- 1) Decomposition of the A-F model into multiple components by Chaboche [2,3] who also introduced a “threshold” into dynamic recovery to include both stable and unstable ratcheting.
- 2) Introduction of the transient radial recovery by Burlet and Cailletaud [4] as well as a non-proportionality parameter by Tanaka [5] to reflect additional hardening effect on multiaxial ratcheting.
- 3) Piecewise linearization of the Chaboche multi-component model by Ohno and Wang [6,7] to largely simplify the process of parameter calibration.
- 4) Introduction of the weighing coefficient by Abdel-Karim and Ohno [8] to combine different models.

Other improvements in the A-F model are based on some of the above aspects such as those proposed by McDowell [9], Jiang and Sehitoglu [10], Bari and Hassan [11], Kang [12], Chen and Jiao [13] and so on. More comprehensive comments on the advanced constitutive models can be referred to the recent reviews by Chaboche [14], Kang [15] and Chen [16].

The second step is to predict fatigue life with ratcheting especially under multiaxial loading conditions. It is usually done using an engineering fatigue model where the influence of ratcheting is reflected either by mean stress or ratcheting strain. Fatemi and Shamsaei [17] summarized some important issues of a fatigue model and several new models were recently proposed by Liu [18], Ince and Glinka [19], Varvani-Farahani [20], Chaboche [21] and Chen [22] using classical approaches based on equivalent stress/strain theory or critical-plane theory. These fatigue models, however, are actually empirical and independent of the constitutive model. Therefore, it is difficult for them to give an accurate life prediction especially when the loading condition is complicated or varied. In this case, another approach is proposed by Lemaitre and Chaboche [23] where a damage-coupled constitutive model is combined with a group of failure criteria to predict fatigue life. Examples can be found in the studies by Kang [24], Zhang [25], Szusta and Seweryn [26] and Martinez [27] where the damage factor can be a function of stress variables, accumulative plastic strain, fracture energy and number of loading cycles. On the other hand, the failure criteria can be classified into four types: numerical non-convergence, critical damage, critical ratcheting strain and critical stress. The first two are equivalent if the critical damage is 1, which can be automatically checked by the program of constitutive model. Other

criteria, however, should be added to an extra program to decide the occurrence of failure. As is pointed out by Kang [24], this method features in the capability to reflect the effect of fatigue damage back on the ratcheting effect as well as the versatility to incorporate classical fatigue models.

In this paper, a damage-coupled constitutive model is built to predict the uniaxial fatigue behavior Cu-Nb micro-composite conductor. According to the experimental results in Part I, the conductor can be seen as rate-independent and has no significant cyclic hardening effect. In this case, the Abdel-Karim-Ohno model (AKO model) proposed in [8] is improved for the ratcheting effect of the conductor. After that, the evolution of damage factor is discussed according to the experimental results in Part I and failure criteria are set according to the discussion. In the end, the ratcheting effect and the fatigue life ranging from 10^2 to 10^5 are predicted using the proposed model and discussions are given for the simulation results.

2 AKO model

2.1 Basic properties and improvement

The AKO model is originally proposed in the following form:

$$f = \sqrt{\frac{3}{2}(\boldsymbol{\sigma} - \boldsymbol{\alpha}) : (\boldsymbol{\sigma} - \boldsymbol{\alpha})} - y_0 \quad (1)$$

$$\boldsymbol{\alpha} = \sum_{i=1}^n \boldsymbol{\alpha}_i \quad (2)$$

$$d\boldsymbol{\alpha}_i = b_i \left(\frac{2}{3} r_i d\boldsymbol{\epsilon}_p - \boldsymbol{\alpha}_i dp_i \right) \quad (3)$$

$$dp_i = \left[\mu_i - H(g_i) \left\langle \frac{\partial f}{\partial \boldsymbol{\sigma}} : \frac{\boldsymbol{\alpha}_i}{r_i} - \mu_i \right\rangle \right] dp \quad (4)$$

$$g_i = \|\boldsymbol{\alpha}_i\| - r_i = \sqrt{\frac{3}{2} \boldsymbol{\alpha}_i : \boldsymbol{\alpha}_i} - r_i \quad (5)$$

$$dp = \sqrt{\frac{2}{3} d\boldsymbol{\epsilon}_p : d\boldsymbol{\epsilon}_p} \quad (6)$$

where n is the total number of kinematic hardening components $\boldsymbol{\alpha}_i$ and y_0 is the linear limit which usually increases with the accumulative plastic strain p to reflect the cyclic hardening effect. As the Cu-Nb conductor does not have such effect, y_0 is supposed to be constant in this study. b_i , r_i and μ_i are the parameters to be obtained for the ratcheting effect and μ_i is the weighing coefficient ranging from 0 to 1, which reflects the influence of the A-F model on the behavior of $\boldsymbol{\alpha}_i$. H is the Heaviside function and $\langle \rangle$ is the Macaulay bracket. If the material is loaded proportionally, equation (4) can be simplified as:

$$dp_i = [\mu_i - H(g_i)\langle 1 - \mu_i \rangle]dp \quad (7)$$

where $\mu_i = 1$ corresponds to the A-F model while $\mu_i = 0$ is for the Ohno-Wang I model proposed in [6]. Furthermore, in the uniaxial tensile test, the evolution of α_i can be derived as:

$$\alpha_i = \begin{cases} r_i \left[1 - \left\langle 1 - \frac{1 - \exp(-\mu_i b_i \varepsilon_p)}{\mu_i} \right\rangle \right] & 0 < \mu_i \leq 1 \\ r_i [1 - \langle 1 - b_i \varepsilon_p \rangle] & \mu_i = 0 \end{cases} \quad (8)$$

According to the above equations, the tensile $\alpha_i - \varepsilon_p$ curve of single kinematic hardening component with different μ_i is shown in **Fig. 1(a)** supposing that $r_i = 1$ MPa and $b_i = 100$. It is found that α_i stops increasing if its norm $\|\alpha_i\| = r_i$. Furthermore, the nonlinear curve when $\|\alpha_i\| < r_i$ gradually becomes linear as μ_i decreases from 1 to 0. **Fig. 1(b)** shows the $\varepsilon_{pm} - N$ curves of different μ_i under the same cyclic loading condition where the peak and valley values of α_i are respectively 0.8 MPa and zero. ε_{pm} is the maximum plastic strain of each loading cycle while N is the cycle number. It is found that every $\varepsilon_{pm} - N$ curve has an almost constant slope $d\varepsilon_{pm}/dN$ that increases with μ_i . If $\mu_i = 0$, ε_{pm} does not increase with loading cycles, indicating that the hysteresis loop of $\alpha_i - \varepsilon_p$ curve is closed.

Fig. 1. Basic properties of a kinematic hardening component α_i with $r_i = 1$ MPa and $b_i = 100$ (a) Tensile $\alpha_i - \varepsilon_p$ curves with different μ_i . (b) $\varepsilon_{pm} - N$ curves of different μ_i under the same cyclic loading condition with the peak and valley values of α_i being respectively 0.8 MPa and zero

The above basic properties enables the AKO model to describe the ratcheting effect of a material. However, a constant μ_i cannot reflect the transiting process of ratcheting which requires that $d\varepsilon_{pm}/dN$ should gradually decrease to be constant. In this case, μ_i is improved to vary with the accumulative plastic strain p which is expressed as:

$$d\mu_i = (\mu_{si} - \mu_{oi})c_i dp \quad (9)$$

$$\mu_i = \mu_{si} - (\mu_{si} - \mu_{oi}) \exp(-c_i p) \quad (10)$$

where μ_{oi} and μ_{si} are respectively the original and saturated value of μ_i while c_i decides the varying speed of μ_i between these two values. **Fig. 2** has shown the ε_{pm} - N curves of different c_i with $r_i=1$ MPa and $b_i=100$, which are also compared with that of $\mu_i=\mu_{oi}$ and $\mu_i=\mu_{si}$. It is found that the original and final values of $d\varepsilon_{pm}/dN$ are respectively decided by μ_{oi} and μ_{si} while a larger c_i leads to a faster transition between the two values. This improvement is actually similar to that of Kang [12] without considering the effect of temperature.

Fig. 2. Influence of c_i on the ε_{pm} - N curve of with $r_i=1$ MPa and $b_i=100$

2.2 Calibration of r_i and b_i

According to equation (8), **Fig. 3** shows the general way to obtain r_i and b_i from an experimental α - ε_p curve when $\mu_i = 0$ and $n=3$. It is actually a multi-linear fit for the experimental curve that satisfies the relationship expressed by the following equations (10) and (11) with $\varepsilon_{p0} = 0$ and $A_0 = 0$. α remains to be the maximum value $\alpha_{max} = A_n$ if the plastic strain is larger than ε_{pn} . It has to be noted that this way to obtain r_i and b_i can also be applied to the situations where $\mu_i \neq 0$ but small enough such that the fitting error of the tensile α - ε_p curve is acceptable. One can increase n to reduce the error but should also consider the larger time cost in numerical calculation.

$$b_i = \frac{1}{\varepsilon_{pi}} \quad 1 \leq i \leq n \quad (10)$$

$$r_i = \begin{cases} \left(\frac{A_i - A_{i-1}}{\varepsilon_{pi} - \varepsilon_{p(i-1)}} - \frac{A_{i+1} - A_i}{\varepsilon_{p(i+1)} - \varepsilon_{pi}} \right) \varepsilon_{pi} & 1 \leq i < n \\ \frac{A_i - A_{i-1}}{\varepsilon_{pi} - \varepsilon_{p(i-1)}} \varepsilon_{pi} & i = n \end{cases} \quad (11)$$

Fig. 3. The general way to obtain r_i and b_i from an experimental $\alpha - \epsilon_p$ curve with $\mu_i = 0$ and $n=3$.

3 Damage evolution

3.1 Different definitions of damage factor

According to Lemaitre and Chaboche [23], there are actually two ways to define the damage factor D which are expressed as:

$$D = \begin{cases} 1 - \frac{\sigma_T}{\sigma_0} \\ 1 - \frac{E_T}{E_0} \end{cases} \quad (12)$$

where σ_T is the measured stress, σ_0 is the equivalent stress without damage, E_T is the measured elastic modulus for every loading cycle and E_0 is the initial elastic modulus. Note that both definitions are based on uniaxial tests and thus the stress and elastic modulus are scalar variables. These two equations are used in different situations.

The first equation is usually used in a monotonic tensile test where σ_0 is set to be the tensile strength σ_{max} and $D = D_t$ is defined as a function of the tensile strain ϵ or plastic strain ϵ_p . D_t starts to increase only after the tensile strength is reached.

The second equation is often used under cyclic loading conditions and the evolution of $D = D_a$ is a function of accumulative plastic strain p . It is suggested in [23] that this function should have the following form:

$$dD_a = kY^m dp \quad (13)$$

$$Y = \frac{1}{2} \sigma_0 : C_0^{-1} : \sigma_0 = \frac{1}{2} \frac{\sigma_T}{1 - D_a} : C_0^{-1} : \frac{\sigma_T}{1 - D_a} \quad (14)$$

where k and m are constants to be obtained and C_0 is the initial elastic tensor without damage. In uniaxial tests, the above equations can be simplified as:

$$dD_a = kY^m dp = k \left(\frac{\sigma_0^2}{2E_0} \right)^m dp = k \left[\frac{1}{2E_0} \left(\frac{\sigma_T}{1-D_a} \right)^2 \right]^m dp \quad (15)$$

It is interesting that this equation can also be applied to describe the damage evolution in monotonic tensile test where $dp = d\varepsilon_p$ and $\sigma_0 = \sigma_{max}$, indicating a linear relationship between D_a and ε_p . However, this relationship is not proper for cyclic loading conditions where $p \gg \varepsilon_p$ and D_a evolves much slower than expected. That is an important reason why many researchers still use different equations or at least different parameters to express the damage evolution for different loading conditions.

Nevertheless, it can still be concluded that the damage evolution can be simultaneously related with ε and p which respectively reflect the state and history of deformation. Considering the results in Part I, Table 1 summarizes the fatigue failure mechanism of Cu-Nb conductor and its related damage factors that will be further discussed below.

Table 1. Fatigue failure mechanism of Cu-Nb conductor

Fracture mode	Fracture morphology	Range of peak stress	Failure strain	Damage factors
Necking	Axial dimples	> 720 MPa	> 5%	$D_t - \varepsilon$
Shear fracture	Tortuous dimples	650 MPa ~ 720 MPa	2% ~ 5%	$D_t - \varepsilon$ or $D_a - p$
Nominal fracture	Interfacial cracks	< 650 MPa	< 2%	$D_a - p$

3.2 Damage by tensile strain D_t

D_t is defined using the first equation of (12) with $\sigma_0 = \sigma_{max} = 765\text{MPa}$. **Fig. 4** shows the $\sigma_T - \varepsilon$, $\sigma_0 - \varepsilon$ and $D_t - \varepsilon$ curves of Cu-Nb conductor measured by different extensometers. It is seen that D_t starts to increase only if $\varepsilon > 2.4\%$ and the $D_t - \varepsilon$ curve depends on the gauge length (GL) of extensometer when $\varepsilon > 5\%$ due to the occurrence of apparent necking. To simplify the situation, only the curve below the strain of 5% is fitted in the following form:

$$D_t = a < \varepsilon - 0.024 >^t \quad (16)$$

Fig. 4. Definition of the damage factor D_t using different extensometers

3.3 Damage by accumulative plastic strain D_a

Given that the maximum strain of each loading cycle is ε_m , D_a can be obtained using the following equation:

$$D_a = 1 - \frac{E_T}{E_0} - D_t \quad (17)$$

where $E_0 = 100$ GPa and D_t is calculated using equation (16) with $\varepsilon = \varepsilon_m$. Note that ε_m is defined as the ratcheting strain of Cu-Nb conductor in Part I and this paper. **Fig. 5** shows the evolution of D_a under different loading conditions with $\varepsilon_m < 5\%$ where R is the stress ratio. As is seen in **Fig. 5(a)**, when $\sigma_P < 680$ MPa, the fatigue failure of Cu-Nb conductor is decided by D_a which lies between 0.05 and 0.07 with the average value of about 0.065. As for the remaining curves in **Fig. 5(a)** and all the curves in **Fig. 5(b)**, $\sigma_P > 680$ MPa and D_a is even lower than 0.05, implying that it is not the cause of failure under these loading conditions. On the other hand, all the curves in **Fig. 5** have a similar variation trend, indicating that the effect of peak stress σ_P is not apparent probably because of the subtraction of D_t . Therefore, the $D_a - p$ curve can be fitted using equation (15) with $m = 0$ and k being a power function of p , which turns out to be in the following form:

$$D_a = \beta p^\gamma \quad (18)$$

$$k = \beta \gamma p^{(\gamma-1)} \quad (19)$$

Fig. 5. The evolution of the damage factor D_a under different loading conditions. (a) Different peak stress levels with $R = 0$ (b) Different stress ratios with $\sigma_P = 760$ MPa

3.4 Failure criteria

According to the above discussions, there are basically two criteria $D_t = D_{tcr}$ and $D_a = D_{acr}$ for the failure of Cu-Nb conductor under cyclic loading conditions. D_{acr} is set to be constant as the average value of 0.065 while D_{tcr} varies with the peak stress σ_P . As is shown in **Fig. 6(a)**, $D_t = D_{tcr}$ is equivalent to $\sigma_P = \sigma_{Tcr}$ or $\varepsilon = \varepsilon_{cr}$ with the point $(\varepsilon_{cr}, \sigma_{Tcr})$ located on the $\sigma_T - \varepsilon$ curve. As the strain above 5% is not considered, there are three situations dominated by different failure criteria shown in **Fig. 6(b)**. When $\sigma_P > 758.5$ MPa, the failure does not happen until $\sigma_P = \sigma_{Tcr}$. When $650 \text{ MPa} < \sigma_P \leq 758.5 \text{ MPa}$, the failure is decided by either $\varepsilon = \varepsilon_{cr} = 5\%$ or $D_a = D_{acr}$. When $\sigma_P \leq 650$ MPa, the ratcheting effect is not significant and the failure is mainly caused by $D_{acr} = 0.065$. These three criteria can correspond to the failure modes of the conductor shown in Table 1.

Fig. 6. Definition of failure criteria. (a) The failure by $D_t = D_{tcr}$ is equivalent to $\sigma_P = \sigma_{Tcr}$ or $\varepsilon = \varepsilon_{cr}$ (b) Three situations dominated by different failure criteria.

4 Simulation results

4.1 Parameter calibration

The first step is to obtain r_i and b_i by fitting the tensile $\sigma_0 - \varepsilon$ curve of Cu-Nb conductor shown in **Fig. 4** with the AKO model including 11 components. The following two aspects are particularly considered:

- 1) All the fitted points are selected below the strain of 2.4% after which $\sigma_0 = \sigma_{max} = 765 \text{ MPa}$ and can be satisfied automatically by the AKO model. In the fitting process, $\mu_i = 0$ for all the components, which means that the slight nonlinearity caused by nonzero μ_i is not considered.
- 2) The linear limit $y_0 = 55 \text{ MPa}$ and the first four components, satisfying the following equation (20), are used to describe the hysteresis loop of the conductor in tension-tension loading cycles. **Fig. 7** shows how they make it using the real $\sigma - \varepsilon_p$ curve of Cu-Nb conductor with one loading cycle. It is seen that as y_0 is lower than a half of the tensile strength $\sigma_{max} = 765 \text{ MPa}$, the hysteresis loop can be expected in tension-tension loading cycles. Moreover, the slope of hysteresis loop changes only if one of these components is reversely saturated.

$$\sum_{i=1}^4 2r_i \leq \sigma_{max} - 2y_0 = 655 \text{ MPa} \quad (20)$$

Fig. 7. The way to describe the tension-tension hysteresis loop of Cu-Nb conductor using the first four kinematic hardening components of AKO model.

The second step is to obtain the weighing parameter μ_i . As the ratcheting rate $d\varepsilon_m / dN$ of Cu-Nb conductor is nonzero only when $\sigma_P > 650 \text{ MPa}$, some components of the AKO model have a zero μ_i while others have a varying μ_i expressed by equation (10). The related parameters μ_{oi} , μ_{si} and c_i are obtained by trial and error according to the fitness of the simulated ratcheting strain for the first 400 loading cycles to the corresponding experimental results at different peak stress levels with $R = 0$.

The final step is to obtain the parameters of D_t and D_a based on the following considerations:

- 1) D_t is coupled to the model mainly to reflect the decrease in σ_T in monotonic tension. In cyclic loading conditions, it is used as a failure criterion for $\sigma_P > 758.5$ MPa.
- 2) Coupling D_a to a constitutive model is mainly to reflect the rapid increase in the ratcheting strain before failure. However, as is observed in Part I, the ratcheting rate of the conductor is almost constant when $\varepsilon_m < 5\%$. As the strain above 5% is not considered, there is no need to couple D_a to the model which, however, still increases with p to indicate the failure.

With all the above considerations, the constitutive parameters of Cu-Nb conductor are finally summarized in Table 2.

Table 2. Parameters of the constitutive model of Cu-Nb conductor

Elastic parameters	Parameters of the AKO model $n = 11$	Parameters of the damage factors
Elastic modulus $E_0 = 100$ GPa	$\mu_{oi} = \mu_{si} = 0, c_i = 0 (i \neq 8, 9, 10)$	Damage factor D_t $a = 1.98$ $t = 1.47$
	μ_i $\mu_{o8} = 0.01, \mu_{s8} = 0.002, c_8 = 50$ $\mu_{o9} = 0.045, \mu_{s9} = 0.01, c_9 = 15$ $\mu_{o10} = 0.1, \mu_{s10} = 0.03, c_{10} = 5$	
Poisson's ratio $\nu = 0.33$	r_i $r_1 = 66.15, r_2 = 72.85, r_3 = 58.27,$ $r_4 = 57.91, r_5 = 70.89, r_6 = 61.07,$ $r_7 = 72.71, r_8 = 98.78, r_9 = 68.56,$ $r_{10} = 59.82, r_{11} = 23.02$	Damage factor D_a $\beta = 0.027$ $\gamma = 0.3627$
Linear limit $y_0 = 55$ MPa	b_i $b_1 = 95449, b_2 = 15149, b_3 = 3452.1,$ $b_4 = 1267.9, b_5 = 450.56, b_6 = 273.96,$ $b_7 = 196.81, b_8 = 132.60, b_9 = 108.61,$ $b_{10} = 84.30, b_{11} = 60.42$	

4.2 Discussion

The simulated ratcheting effect of Cu-Nb conductor at different cyclic loading conditions is shown in **Fig. 8**, which is found to be in a reasonable fit with the experimental results. However, there is still an overestimation in the ratcheting strain especially when $\sigma_P = 680$ MPa and $R = 0$. As a result, the predicted failure is due to $\varepsilon_m = 5\%$ rather than $D_a = 0.065$ which should have been the cause according to the experiments where $\varepsilon_m < 2.5\%$.

The predicted fatigue life of the Cu-Nb conductor at different peak stress levels and stress ratios are shown in **Fig. 9(a)**. It is found that there is certain deviation between the predicted and experimental fatigue life at $\sigma_P = 740$ MPa with $R = 0.4$. The $D_a - \varepsilon_m$ curve of this loading condition is shown in **Fig. 9(b)**, which indicates that the failure is decided by $\varepsilon_m = 5\%$ rather than $D_a = 0.065$. Considering that the predicted fatigue life is lower than the experimental one, this deviation must also result from the overestimated ratcheting effect by simulation.

According to the above discussions, the overestimated ratcheting effect under some loading conditions is the main problem for life prediction of Cu-Nb conductor. It can be solved by adopting a better optimization method than trial and error to obtain the weighing parameters μ_{oi} , μ_{si} and c_i . Nevertheless, the precision is still reasonable with the fatigue life ranging from 10^2 to 10^5 which is large enough for the pulsed magnets where the conductor is applied.

Fig. 8. Simulated ratcheting effect and the failure types at different loading conditions. (a) Different peak stress levels with $R = 0$. (b) Different stress ratios with $\sigma_p = 760$ MPa.

Fig. 9. Fatigue life prediction under different loading conditions. (a) Reasonable precision in general except the deviation when $\sigma_p = 740$ MPa and $R = 0.4$. (b) Probable overestimated ratcheting effect that results in the failure by $\varepsilon_m = 5\%$ when $\sigma_p = 740$ MPa and $R = 0.4$.

5 Conclusion

The ratcheting effect and fatigue life of Cu-Nb conductor under different loading conditions are predicted using a damage-coupled constitutive model with the following considerations:

- 1) The isotropic hardening and rate-dependency are not considered according to the experimental results in Part I. The elastic modulus $E_0 = 100$ GPa, the Poisson's ratio $\nu = 0.33$ and the linear limit $y_0 = 55$ MPa.
- 2) The tensile stress-strain curve is fitted using the improved AKO model with 11 components. Some of the weighing parameters μ_i are varying with the accumulative plastic strain p and μ_{oi} , μ_{si} and c_i are obtained by comparing the simulated and experimental ratcheting strain for the first 400 loading cycles under different loading conditions.
- 3) The damage factors D_t and D_a are respectively in a power function of ε and p , D_t is coupled to the model while D_a is not but still calculated to decide the occurrence of failure. The failure criteria are expressed by $\sigma_P = \sigma_{Tcr}$, $\varepsilon_{cr} = 5\%$ and $D_a = 0.065$. The first two are actually related to failure by D_t or the ratcheting strain while the last one represent the failure due to D_a or the accumulative plastic strain.

In spite of certain deviation by overestimated ratcheting effect for some loading conditions, the predicted fatigue life ranging from 10^2 to 10^5 is still in a reasonable precision with the experimental results in Part I. The next step is to use this constitutive model to analyze the fatigue behavior of pulsed magnets that incorporate the Cu-Nb conductor.

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