Application of crack tip plasticity based fatigue model on high temperature alloy HAYNES® 282®

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[ABSTRACT] Confined crack tip plasticity model is employed to predict time independent fatigue crack growth rate (FCGR) behavior of HAYNES® 282® alloy at temperatures 1200F and 1400F. Crack growth tests were done in lab air, vacuum and steam environments at load ratios $R=K_{min}/K_{max}$ ranging from 0.05 to 0.5. Calibrated model predicts average cyclic crack growth rate behavior of the material reasonably well. Predictions do not capture the accelerated fatigue crack growth rates observed in the data at low load levels. Such effects are believed to be caused by environmentally driven factors, which are not expected to be predicted by plasticity based models.

Keywords: crack growth, high temperature, crack tip plasticity, crack closure, HAYNES282

1. Introduction and background

Fatigue crack growth modeling in metals has gone through several stages of development over the past few decades. Most of the developed models are based on the assumption that small scale yielding (SSY) conditions hold, in which plastic deformation region is confined to the crack tip vicinity. This assumption allows for applications of linear elastic fracture mechanics (LEFM) relationships such as stress intensity factor or energy release rate as similitude crack tip driving forces in damage tolerant design. Majority of the damage models used for macro cracks are similar to the Paris-Erdogan equation, where fatigue crack growth rate relates to the crack tip driving force in a power law [1]. The simplest and original form of this equation is given by [2]

$$\frac{da}{dN} = C \Delta K^n \quad (1)$$

where da/dN is the fatigue crack growth rate, $\Delta K = K_{max} - K_{min}$ is the applied stress intensity factor range, C and n are material parameters at the given temperature and

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environment. Besides being dependent on the material type, the parameter C also changes with the applied load ratios $R = K_{min}/K_{max}$.

Discovery of crack closure phenomenon introduced additional insights into crack growth modeling. It was suggested that crack tip damaging effects occur only when the crack is fully open [3]. Instead of using the entire far field load range (ΔK), the effective part of the range corresponding to the open crack $\Delta K_{eff} = K_{max} - K_{op}$ (K_{op} = crack opening SIF) could be used in a power law relationship:

$$\frac{da}{dN} = C_o (\Delta K_{eff})^n = C_o U^n \Delta K^n \quad (2)$$

where $U = \Delta K_{eff}/\Delta K$ is the normalized effective SIF range and C_o is the value of C for the load case with no closure (i.e. when $U=1$), which typically occurs at high R levels. Because U depends on load ratio, R effect (or equivalently mean stress effect) is already incorporated in Eq.(2). Crack closure levels can be measured [4], or alternatively estimated using analytical [5-7] and numerical models [8-13]. Closure has also been known to explain load history effects such as crack growth retardation due to tensile overloads [14-17].

The equations above represent the Paris (log-log linear) regime in da/dN vs dK plots. Modified forms of these equations have been suggested in sigmoidal forms producing smooth transitions to threshold and fracture regions (see for example [6]).

It must be noted that the damage laws discussed hitherto are cycle based. Thus, non-cyclic missions have to be converted to a series of constant amplitude cyclic loads using some type of counting techniques such as Rainflow method before these damage laws can be employed for life predictions. Reconstruction of load missions, however, would lead to loss of load history effects such as crack growth retardation due to overloads. Moreover, if time dependent damages are to be coupled with the equations above, the cyclic form needs to be transformed to the time-based form (or vice versa) to make the mathematical forms of different damage modes compatible with one another. But if both non-cyclic and time-dependent damaging mechanisms are present, then it could be challenging to couple cyclic-based and time-based portions of the damage model. Therefore, it is advantageous to start with the time-based fatigue model from the beginning, so that the damage law could be applied to the load mission as is. A number of researchers have lately been using the crack tip plasticity levels to characterize crack growth behavior [18, 19]. This approach is consistent with the well-known phenomenon first observed by Laird and Smith [20]

$$da/dN \approx CTOD \quad (3)$$

which states that the crack tip opening displacement ($CTOD$) is proportional to the fatigue crack growth rate (FCGR). In the present work, one of such models, referred as confined crack tip plasticity (CCTP) approach, is evaluated to capture the time-

independent crack growth behavior of high temperature HAYNES® 282® alloy. The model is time based (i.e. da/dt), which makes it conveniently extendable to incorporate time-dependent damage due to creep deformation and corrosive environment. Remainder of the paper is organized as follows.

Section II starts with brief observations related to micro- and macro-structural aspects of the material in consideration. A material constitutive model is calibrated to represent the bulk plasticity of the material at the temperatures of interest.

Section III describes the FE modeling used to transition from bulk material plasticity behavior to near crack tip plasticity behavior. Elastic-plastic FE simulations are run to determine plasticity occurring near the crack tip and crack closure levels. The evolution equations of the CCTP model are calibrated based on the FE results. A single damage law parameter is estimated from the available crack growth rate data as a last step of model calibration.

Section IV presents the model predictions of constant amplitude FCGR data for different load levels and load ratios at 1200F and 1400F.

Finally in Section V, summary and conclusions drawn from this study as well as potential areas for future work are outlined.

2. Material properties

Round bar and compact tension (CT) specimens were used to collect bulk stress-strain and fatigue crack growth rate data. These specimens were machined from a rolled HAYNES® 282® alloy billet. Before the specimens were extracted, the billet was solution heat treated at 1010°C for a period of 2 hours in vacuum followed by fast quenching in Argon to room temperature. The billet was then vacuum aged at 788°C for 8 hours and Argon cooled to room temperature.

Table I lists nominal chemical composition of the material [21]. Figure 1 shows a typical microstructure of the material after solution hardening. Consistent with other Ni-base superalloys, HAYNES® 282® alloy has a network of twin boundaries, as indicated in red in Figure 1. Figure 2 shows grain size histogram obtained from grain areas detected by Electron Back Scatter Diffraction (EBSD) measurement technique. The histogram appears to be left-skewed, and the mean effective grain diameter is $\mu_d=13.6$ mils. The longitudinal yield strength (0.02% offset) and ultimate tensile strength (UTS) of the material are $\sigma_y=92$ ksi and $\sigma_u=166$ ksi, respectively, at room temperature. These strengths in transverse direction are $\sigma_y=91$ ksi and $\sigma_y=148$ ksi, indicating some level of anisotropy in UTS.

A material constitutive model representing the cyclic stress-strain behavior is needed for CCTP model calibration [22]. Figure 3 presents an example stress-plastic strain curve for HAYNES® 282® alloy at 1200F in lab air. The data in Fig.3a corresponds to the

applied total strain ratio $R_\epsilon = \epsilon_{min}/\epsilon_{max} = -1$, whereas Fig.3b is for $R_\epsilon = 0$. ANSYS [23] built-in Chaboche type constitutive model used with both isotropic and kinematic hardening behaviors to capture this bulk material response. In Chaboche non-linear kinematic hardening model employed in ANSYS [23], the yield function is defined as

$$F = \sqrt{\frac{3}{2}(S - \beta):(S - \beta)} - R = 0$$

Where S is the deviatoric stress tensor, R is the radius of the yield locus, and β is the back-stress tensor that translates the yield surface during plastic deformation. The back-stress tensor is defined by the superposition of several kinematic hardening models

$$\beta = \sum_{i=1}^n \beta_i$$

Here, n is the number of kinematic models to be superimposed. The evolution of the back-stress for each model is expressed as

$$\Delta\beta_i = \frac{2}{3}C_i\Delta\epsilon^{pl} - \gamma_i\beta_i\Delta\hat{\epsilon}^{pl}$$

Where constants C_i , γ_i ($i=1, 2, \dots, n$) are the material parameters for kinematic hardening behavior, tensor $\Delta\epsilon^{pl}$ is the plastic strain increment and scalar $\Delta\hat{\epsilon}^{pl}$ is the equivalent plastic strain increment. Isotropic hardening model is governed by the following equation that expands the yield surface during plastic deformation

$$R = \sigma_0 + R_0\hat{\epsilon}^{pl} + R_\infty(1 - e^{-b\hat{\epsilon}^{pl}})$$

Where constants σ_0 =initial yield strength, R_0 , R_∞ , b =material parameters characterizing isotropic hardening behavior and $\hat{\epsilon}^{pl}$ = accumulated equivalent plastic strain.

Material parameters listed in Table II are calibrated for test temperatures 1200F and 1400F using in-house optimization software PEZ (PEZ, Perl Version of Easy_OPT, is an internal design tool of GE used for computer simulation design studies related to probabilistic design and optimization). In calibrating the material parameters, nonlinear optimization technique amoeba built-into PEZ, was employed. Predictions based on 1200F optimized parameters are plotted in Fig.3 by solid lines. The optimized material model captures the $R_\epsilon = -1$ test data reasonably well (Fig.3a). The plastic strain ranges are somewhat under-predicted when $R_\epsilon = 0$ (Fig.3b).

3. CCTP model calibration

Crack tip blunting

The essence of the CCTP model is the crack tip blunting, a parameter that characterizes the extent of plasticity occurring near the crack tip. As shall be seen in the upcoming Section 4, the crack tip blunting, denoted by ρ , directly relates to the crack propagation rate. Hence the main objective here is to use FE simulations of a crack to determine the evolution of ρ with respect to the applied remote load (in terms of K_I) and the crack growth (da).

Following the development in [24], the crack tip blunting is obtained by decomposing the total crack face displacement profile into elastic and plastic components:

$$u_{ep}(r, t) = \tilde{K}_I u_e(r) + \rho(t) \quad (4)$$

where $u_{ep}(r, t)$ is the elastic-plastic crack face displacement profile for the applied K_I , and $u_e(r)$ is the elastic crack face displacement field corresponding to unit stress intensity factor ($K_I = 1 \text{ ksi}\sqrt{\text{in.}}$). The term $\tilde{K}_I$ is the plastic stress intensity, which reduces to the elastic stress intensity factor in the absence of plasticity. Once u_{ep} and u_e are computed using FE simulations, the parameter ρ can be determined by the least squares method [24].

To keep consistency with the test data, the CT specimen is modeled to obtain the crack face displacement solutions. Figure 4 shows the model sizes and the 2D FE mesh with prescribed boundary conditions. Although plane strain conditions were used in [24], authors choose to use plane stress elements in this work since the crack growth rate predictions better correlates with such an assumption. One half of the specimen is modeled using the symmetry condition along the crack plane. The crack tip region is refined to adequately capture plastic deformation occurring locally. Preliminary mesh study indicates that the elements of size 2e-4in. in the vicinity of the crack tip produce sufficiently converged crack tip displacement results.

Fig.5 presents the applied SIF against crack tip blunting solutions obtained from the FE model. The circular symbols represent the monotonic plasticity curve, and "x" symbols depict cyclic plasticity. These curves represent the evolution of ρ with respect to monotonic and cyclic loads. Since this type of near crack tip plastic deformation behavior is difficult to measure experimentally, FE model is employed along with the bulk material plasticity laws to produce such curves. Next, transfer functions of the CCTP model are fitted to these FE solutions.

Comprehensive details of the model formulations are described in [25], but we would like to review a few key aspects in this paper. In particular, the equations used to capture the monotonic and cyclic loading behavior observed in Fig.5 are modified slightly from the original formulation suggested in [25]. The monotonic plasticity curve used is given by

$$K_I = a_m \rho^{1/m} + b_m \quad (5)$$

Similarly the cyclic plasticity equation is taken as

$$K_I = a_c \rho^{1/c} + b_c \quad (6)$$

where a_m, b_m, m, a_c, b_c , and c are material constants. In Ref [25], both m and c were enforced to be 2, whereas they are generalized to be part of calibration parameters to better capture the deformation behavior obtained from the FE simulations. It is shown later that the forms given by Eqs. (5-6) above improve the model predictions appreciably at lower ΔK_I levels. In the CCTP model, the equations are implemented in incremental forms to determine the amount of crack tip blunting per incremental applied K_I . For monotonic loading case, the incremental change in ρ can be expressed as follows:

$$d\rho = \rho_i - \rho_{i-1} = \left(\frac{K_i - b_m}{a_m} \right)^m - \left(\frac{K_{i-1} - b_m}{a_m} \right)^m \quad (7)$$

and for the cyclic loading it is

$$d\rho = \rho_i - \rho_{i-1} = \left(\frac{K_i - b_c}{a_c} \right)^c - \left(\frac{K_{i-1} - b_c}{a_c} \right)^c \quad (8)$$

where subscripts i and $i-1$ correspond to current and previous load steps, respectively.

Figure 6 also shows the CCTP model predictions of crack tip blunting compared with the FE model solutions. The solid line is the prediction through Eqs. (7-8) above (generalized formulation), and the dashed line is when quadratic formulations is used. The plots indicate improvement in capturing the FE response with the generalized formulation as opposed to the quadratic form to characterize both monotonic and cyclic behaviors.

Overload and crack closure effects

Evolution equations for the size of the monotonic plastic zone K_m and the contact point K_{xm} with respect to the crack growth da govern the overload and crack closure effects. To capture the behaviors of K_m and K_{xm} obtained by FE simulations, the authors in [25] suggested following forms for these equations

$$\frac{\partial K_m}{\partial a} = \frac{1}{2} p_a K_m \quad (9)$$

$$\frac{\partial K_{xm}}{\partial a} = 2k_a K_m + 2k_b (K_m K_{xm})^{1/2} \quad (10)$$

where, p_a, k_a , and k_b are material parameters yet to be calibrated. The incremental form of the equations above can be expressed as:

$$K_{m_i} = K_{m_{i-1}} e^{p_a(a_i - a_{i-1})/2} \quad (11)$$

$$K_{xm_i} = \left[\left(K_{xm_{i-1}}^2 - K_{xm_{i-1}}^2 \frac{k_b}{k_b - p_a} \right) e^{k_a(a_i - a_{i-1})} - K_{m_i}^2 \frac{k_b}{k_b - p_a} \right]^{1/2} \quad (12)$$

We followed the same procedure suggested in [25] to estimate the parameters p_a, k_a , and k_b . The parameter p_a controls the extent of crack growth retardation after the onset of

an overload event; higher p_a leads to shorter lasting overload effect and vice versa. Its magnitude is estimated from the slope of the average equivalent plastic strain near the crack tip when plotted against the distance from the crack tip in a *log-log* scale. Figure 6a gives an example of such a plot for HAYNES® 282® alloy at 1200F obtained from the same FE model described above after it is subjected to a single load cycle. It must be noted that although [25] makes qualitative connection between p_a and the slope, there is no direct conceptual correlation of these two quantities. In fact, the unit of p_a comes out to be $1/\text{length}$, whereas the slope as defined has no units. In [25] the slope was used as an estimation for p_a , which was defined in *SI* units (i.e. $1/\text{mm}$). To be consistent with their procedure, the slope -1.485 given in Fig.6 was also assumed to be in $1/\text{mm}$. Hence, the parameter p_a was estimated to be $p_a = -1.485/\text{mm} = -37.3/\text{in}$. Going forward, more deterministic approach needs to be established for p_a estimation.

The parameters k_a and k_b for the evolution equation of the closure level K_{xm} given in Eq.(10) are estimated based on the FE simulations of crack closure. Crack growth was simulated by node release using the same FE model shown in Fig.4, but with rigid contact elements added along the crack line as part of symmetry. The closure level is defined as a load point corresponding to the onset of contact between any crack surface node and the rigid contact surface during unloading. The symbols given in Fig.6b are the FE solutions of crack closure levels during crack propagation. The parameters k_a and k_b listed in Table III are calibrated to fit this FE solution for temperatures 1200F and 1400F. The CCTP model response of K_m and K_{xm} are plotted by dashed and solid lines in Fig.6b, respectively.

Note that for the case of constant amplitude cyclic loading, the Eq. (10) can be set to zero to obtain the following expression for the saturated closure level normalized by the applied K_{max} :

$$\frac{K_{xm}}{K_{max}} = \sqrt{\frac{k_b}{k_a - k_a}} \quad (13)$$

In deriving the equation above, the relation $K_{max}^2 = K_m^2 + K_{xm}^2$ is used, which holds for the constant amplitude cyclic loads in the CCTP model. Figure 7 shows the normalized closure predictions obtained from Eq. (13) plotted against the applied load ratio $R=K_{min}/K_{max}$. The Newman's strip yield model predictions [6] corresponding to plane stress and plane strain conditions also included in the plot. Dashed line in the figure represents the absence of plasticity induced crack closure. It is observed that the closure levels predicted by the current model are constant up to $R=0.27$, beyond which there's no closure. This response looks to be similar to the Newman's closure predictions for plain strain conditions, but simplified to a straight horizontal line.

It is worth noting that all the model parameters up to this point are estimated using push-pull test data along with a few sets of FE simulations. One last parameter left to be calibrated makes use of a single FCGR dataset, as is discussed next.

Damage law

Time based damage law proposed in [18] suggests that the crack growth rate is proportional to the crack tip blunting rate

$$\frac{da}{dt} = \alpha \left| \frac{d\rho}{dt} \right| \quad (14)$$

where α is a proportionality constant that can be estimated from the crack growth rate data at a single load level in the Paris regime. Note that, because the material model is rate independent, the crack tip blunting is also rate independent. For example, at a constant applied load, ρ does not change with time (i.e. no visco-plasticity or creep incorporated). Hence the damage law given by Eq. (14) accounts for the time-independent cyclic and non-cyclic damage, but the damage law is derived on time basis to keep track of time. Once this rate-independent part is set up, additional time dependent terms can be directly coupled with the cyclic term to incorporate creep or environment effects. These additional effects are currently being implemented and will be published in a follow up paper. The reader is advised to see [26] for a possible formulation of oxidation related damage term.

For constant amplitude cyclic loading, the damage law above can be readily converted to a cyclic form

$$\frac{da}{dN} = \alpha |d\rho_{cycle}| \quad (15)$$

where $d\rho_{cycle}$ is the change in crack tip blunting per cycle. This form of damage law allows us to calibrate the parameter α against cyclic FCGR data. Figure 8 shows such data where da/dN is plotted against ΔK_I for HAYNES® 282® alloy tested in-house at 1200F and 20 cycles per minute (CPM). As Eq. (15) suggests, α vertically translates the model predictions in a log-log scale plot, but the slope of the predicted line remains the same. Solid line in the figure shows the CCTP predictions with $\alpha=1$, which compares reasonably well with the data. Also included in the plot by a dashed line is the model prediction when the parameters m and c in the Eq (7-8) are set to 2. It is observed that the quadratic forms of transfer functions for monotonic and cyclic plastic loading result in significantly lower da/dN predictions at the lower tail. At load levels above $\Delta K_I = 13ksi\sqrt{in.}$, the two predictions coincide.

Table III summarizes the CCTP model parameters calibrated for HAYNES® 282® alloy at test temperatures of 1200F and 1400F.

4. Results and Discussion

Figure 9a presents the CCTP model predictions of cyclic crack growth rates for 1200F test temperature at three different load ratios: $R=0.05, 0.25$ and 0.5 for which experimental data have been collected. The slope of the lines representing the linear Paris regime of the da/dN curve is predicted by the model. Also, a typical trend of upward shift

in crack growth rates due to higher load ratios can be observed in predictions. This behavior is controlled by the crack closure formulations included in the model.

In Fig.9b, results from specimens tested at $R=0.05$ and 20CPM are plotted along with the model predictions. The dashed lines also included in the figure represent 2x bands, a typical scatter observed with crack growth data. The 2x bands are obtained using the predicted curve (solid line) in such a way so that the upper boundary is twice the lower boundary, while the prediction is at the midpoint of the two. Most of the experimental data fall within this band except for the low ΔK regions in specimens CT2 and CT3. These two data sets exhibit an abrupt change in slope near $20ksi\sqrt{in}$. Such a behavior is rather frequently observed in our test data for HAYNES® 282® alloy at both 1200F and 1400F in steam and air. The exact cause of this behavior is not fully understood yet, but one possible explanation could be the presence of an environment effect at reduced crack growth rates. The phenomenon was observed during both K-decreasing and K-increasing experiments.

Figures 9c and 9d present similar types of comparisons at two higher load ratios, $R=0.25$ and 0.5. These plots also include a test in vacuum, indicated by “x” symbols. Figures 9c and 9d indicate an absence of the abrupt slope change observed in air at $20ksi\sqrt{in}$, again indicating that the source of the phenomenon is an environmental-fatigue interaction.

Within the Paris regime (i.e. region II), majority of the air and vacuum data at $R=0.25$ and 0.5 lie within the 2x band predictions. Hence, the calibrated CCTP model captures average load ratio effects for the range $R=0.05$ to 0.5 reasonably well. It must be noted, however, that the model does not predict further upward shifts in da/dN curves beyond $R=0.5$. This is because load ratio sensitivity in the model is coming from predicted crack closure levels, and the model predicts no closure for $R>0.3$ as is shown in Fig.7.

Figure 10 presents da/dN data tested at 1400F along with corresponding predictions. Similar to previous plots, 2x bands estimated from the predicted line are also included by dashed lines in the figure. Figure 10a shows the data tested in air, steam and vacuum at the load ratios of $R=0.05$ and 0.1. The model calibrated at 1400F captures the average crack growth rate behavior. Majority of the experimental scatter are bounded by the 2x band with a couple of exceptions. The specimen CT11 tested in air at $R=0.05$ exhibits change in slope near $20ksi\sqrt{in}$, in a similar way to the 1200F dataset discussed above. This data falls outside the upper boundary of 2x band at load levels $\Delta K < 15ksi\sqrt{in}$. Also, CT12 violates the 2x band for $\Delta K < 26ksi\sqrt{in}$. The dataset labeled CT16 was tested with a 100second hold at max load, in steam. This data indicates that, for a 100second hold time, the presence of additional damage due to the steam environment is negligible.

The FCGR measurements along with model predictions for $R=0.25$ and 0.5 at 1400F are plotted in Figs.10b and c. Only a single specimen is evaluated per load ratio. Thus, the experimental scatter is not captured at these two load ratios. However, the model

predicts the slope of the data reasonably well at higher stress intensity factor ranges. As before, a change of slope is observed in the data for $\Delta K < 20ksi\sqrt{in}$.

5. Summary and future work

The CCTP model is calibrated for HAYNES® 282® alloy separately at 1200F and 1400F using the bulk plasticity material model and a series of FE crack simulations. Chaboche type material constitutive model is set up using the stress-strain data obtained from round bar cyclic tests at high temperature. The FE model of a CT specimen is used to generate local plasticity solutions in terms of crack tip blunting. CCTP model parameters are established based on crack tip blunting solutions. The calibrated crack growth model predicts average cyclic crack growth rate behaviors and the load ratio effects in the Paris regime when $0.05 < R < 0.5$. Predictions do not capture the accelerated fatigue crack growth rates frequently observed at lower ΔK levels. Existence of such experimental trends could be due to additional environmentally driven micro-mechanisms. These additional damaging effects are not expected to be captured by bulk material plasticity, on which the model is based.

Going forward, the model needs to be validated against experimental data at higher load ratios (i.e. $R > 0.5$) as well as negative load ratios. Note that the CT specimens used in current study are typically not used for negative load ratio tests, but MT (Middle Crack in Tension) specimens could be possible alternative for such tests. Additionally, load history capability of the model should be validated by running overload and non-cyclic load mission tests.

Next step after comprehensive model validation is to bring in environment effect by coupling the time-independent damage law given by Eq.(14) with the time dependent crack growth models.

As part of future developmental work, the model capability could be enhanced to capture near threshold crack growth rate behaviors since most of the structural component life is expended in the threshold region. Furthermore, the current model is restricted to Mode I propagation. Theoretical background on mixed mode propagation laws has been proposed in the literature [27,28]; however, its application and extensive validation against the experimental data are not publicized yet. In particular, model validations of both crack growth life and path predictions against the test data from controlled mixed mode load tests would put the methodology on solid ground for practical implementations.

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Table Captions:

Table I: Nominal Chemical Composition of HAYNES® 282® alloy, Weight % [21]

Table II: ANSYS built-in Chaboche material model parameters for HAYNES® 282® alloy

Table III: CCTP model parameters for HAYNES® 282® alloy

Table I

Ni	Cr	Co	Mo	Ti	Al	Fe	Mn	Si	C	B
57	20	10	8.5	2.1	1.5	1.5	0.3	0.15	0.06	0.005

Table II

T	E , ksi	ν	σ_y , ksi	C_1 , ksi	γ_1	C_2 , ksi	γ_2	R_0 , ksi	R_∞ , ksi	b
1200F	2.5E4	0.3	72	8.3E4	7.6E3	1.7E4	360	2.6	0.036	5.9
1400F	2.3E4	0.3	63	8.6E4	4.8E3	1.0E4	600	5.0E-5	5.2E-4	7E-3

Table III

T	1200F	1400F
α	1	0.9
a_c , ksi.in ^(1/2-1/c)	1740	1479
b_c , ksi.in ^{1/2}	1	1.6
c	2.7	2.65
a_{mv} ksi.in ^(1/2-1/m)	1134	955
b_{mv} ksi.in ^{1/2}	1.11	0.85
m	2.65	2.65
a_{xmv} ksi.in ^{-1/2}	-52100	-52600
p_a , in ⁻¹	-37	-34.5
k_a , in ⁻¹	-850	-250
k_b , in ⁻¹	160	70

Figure Captions

Fig.1: Micrograph of HAYNES® 282® alloy. $\Sigma 3$ boundaries marked with red.

Fig.2: Grain diameter distribution

Fig.3: Stress-plastic strain data and material model prediction for HAYNES® 282® alloy at 1200F test temperature: (a) total strain ratio $R_\epsilon = -1$, (b) total strain ratio $R_\epsilon = 0$

Fig.4: FE model geometry and mesh of a CT test specimen

Fig.5: FE model solutions of crack tip blunting and CCTP model predictions

Fig.6: Calibrating the CCTP model parameters p_a , k_a and k_b

Fig.7: Normalized crack closure predictions of CCTP model compared to Newman's strip yield model [6]

Fig.8: CCTP model calibration of cyclic FCGR at $R=0.05$

Fig.9: CCTP model predictions of cyclic FCGR for 1200F test temperature

Fig.10: CCTP model predictions of cyclic FCGR for 1400F test temperature

Figure2

Figure3

Figure4

Figure5

Figure6

Figure7

Figure8

Figure9

(a)

(c)

(b)

(d)

Figure10