Robust Economic Dispatch Considering Automatic Generation Control with Affine Recourse Process

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Abstract

A robust economic dispatch (ED) considering automatic generation control (AGC) with affine recourse process is proposed in this paper. The approach co-optimizes the base points and participation factors of the AGC units using preemptive goal programming and robust optimization while considering the uncertain nodal power injections and the network constraints. The proposed approach is realized by two steps. The aim of the first step is to maximize the system effective acceptable disturbance range (EADR) while minimize the generation costs and reserve costs with respect to the obtained EADR in the second step. The novelty of the approach is as follows: (a) The security of the power system is optimized by maximizing the system EADR. The approach can obtain a solution which can cover the disturbance as much as possible even when the system does not have enough adjustable capacity to cover it all. The obtained nodal EADR can quantitatively represent the anti-disturbance capability of a node. (b) The economics of the system is significantly improved by minimizing the generation costs and reserve costs while the constraint of EADR requirement is respected. (c) The conservative level of the solution can be tuned according to the user's requirements. A simplified one-step linear model is also deduced. The effectiveness and validity of the proposed approach are demonstrated by a 6-bus system, the IEEE 118-bus system, and a real 445-bus system.

Keywords: affine recourse process, automatic generation control, preemptive goal programming, robust economic dispatch.

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1 Nomenclature

Descriptions: In this paper, means x corresponds to the upward adjustment. Similarly, $\check{x}$ means x corresponds to the downward adjustment. $\tilde{x}$ means x is a random variable.

1.1 Indices

i	Index of the online units, $i = 1, 2, \dots, N_G$.
j	Index of the load nodes, $j = 1, 2, \dots, N_D$.
l	Index of the transmission branches, $l = 1, 2, \dots, N_L$.

1.2 Random variables

$\tilde{p}_i$	Generation output of the i th unit.
$\Delta\tilde{d}_j$	Net load disturbance at the j th node.
$\Delta\tilde{p}_i$	Power adjustment of the i th unit.

1.3 Deterministic variables

α_i	Participation factor of the i th unit. It is assumed that all the participation factors are nonnegative. α_i is 0 for the non-AGC units.
p_i	Base point of the i th unit.
$\hat{r}_i, \check{r}_i$	Upward and downward AGC reserve requirements for the i th unit.
$\hat{y}_j, \check{y}_j$	Upper limits of the effective acceptable upward/downward disturbance of the j th node.
z_E, z_S	Optimal values of the economic and security objective functions.
$\underline{\gamma}_{jl}, \bar{\gamma}_{jl}$	Introduced control variables which correspond to the worst cases of the left-hand side of (18) and (19).
$\Delta\hat{d}_j^{acpt}$	Upper limit of the acceptable upward disturbance of the j th node.
$\Delta\check{d}_j^{acpt}$	Upper limit of the acceptable downward disturbance of the j th node.

1.4 Parameters

c_i	Generation cost of the i th unit.
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$\hat{c}_i, \tilde{c}_i$	Prices of the upward/downward reserve of the i th unit.
D_j	Expected demand on the j th node.
M_{il}	Sensitivity coefficients for the power flow in the l th transmission branch with respect to the output power of the i th unit. It is commonly referred to as the generation shift factor.
M_{jl}	Sensitivity coefficients for the power flow in the l th transmission branch with respect to the demand of the j th node. It is commonly referred to as the load shift factor.
N_D	Number of the load nodes.
N_G	Number of the online units.
N_L	Number of the transmission branches.
p_i^0	Initial generation of the i th unit.
$p_i^{\max}, p_i^{\min}$	Upper and lower generation limits of the i th unit.
R_i^D, R_i^U	Allowed upward/downward ramp rate of the i th unit.
$\hat{R}_i^{\max}$	Limit of the upward AGC reserve provided by the i th unit.
$\tilde{R}_i^{\max}$	Limit of the downward AGC reserve provided by the i th unit.
$\underline{T}_l, \bar{T}_l$	Bidirectional transmission capacity limits of the l th branch.
$\Delta \hat{d}_j^{load}$	Maximum upward load disturbance of the j th node.
$\Delta \tilde{d}_j^{load}$	Maximum downward load disturbance of the j th node.
β	An introduced parameter which is used to control the conservative level of the decision. $\beta \in [0, 1]$.

2 Introduction

Economic dispatch (ED) [1] and automatic generation control (AGC) [2] are two important components of the intraday dispatch. ED determines the base points (BPs) of the online units with respect to the short-term load forecasting results, while AGC maintains the system frequency and/or scheduled tie-line interchanges using the area control error (ACE) signals. Generally, the time scales for ED and AGC are 5~15 minutes and 1~2 minutes respectively. By using the AGC regulation, the load changes within each ED decision interval are allocated to the online units according to the participation factors (PFs) [3].

In spite of the close relationship between ED and AGC, the control process of AGC is barely considered in the ED models. BPs and PFs are usually optimized sequentially [3, 4]. Although the sequential optimization approach is easy to implement, the optimality of the solution is weakened. Reference [5] applied AGC reserve requirement constraints to coordinate ED and AGC, and the PFs were calculated in proportion to the AGC reserve capacity of the units. However the approach cannot quantitatively reflect the relationship between the AGC reserve and system operating security. And the transmission limits are not enforced for the AGC regulation in the model, which may be unsafe in case of large power injection fluctuations.

Robust optimization (RO) is one of the most effective approaches to make decision with uncertain parameters [6]. Different from the stochastic programming (SP) [7] which has been widely applied on the unit commitment (UC) and ED problems [8-12], RO does not depend on the probability distribution functions (PDFs) of the uncertain parameters. Instead, RO can capture robust optimization results with respect to a given disturbance range by satisfying the worst case within the range. With the integration of the large scale intermittent generation resources, RO attracts more and more attentions, and it has been used in the coordination of UC and ED with uncertain injections [13-17].

Mathematically, AGC can be seen as an affine recourse [18-20] for ED since the unbalance power is allocated to the online units according to the affine function [3, 4]:

$$\tilde{p}_i = p_i + \Delta\tilde{p}_i = p_i + \alpha_i \cdot \sum_{j=1}^{N_D} \Delta\tilde{d}_j \quad (1)$$

When the BPs and PFs are determined, this affine recourse mechanism can be used to estimate the generation and transmission status of the system with respect to any realization of the random injection. This feature distinguishes the coordination of ED and AGC from that of UC and ED [13, 14].

References [15] and [20] analyze the co-optimization of ED and AGC within the RO framework. In [15], a two-stage robust ED combined with AGC is proposed. The model aims to minimize the sum of the dispatch cost, regulation capacity cost and the worst-case regulation performance cost. In [20], an affine adjustable robust OPF

with renewable energy sources is proposed. The BPs of AGC units are calculated to match the forecast load and the PFs are calculated to ensure a feasible solution for the realizations of the uncertain injections within a prescribed box uncertainty set. However, the system security requirements are treated as operating constraints rather than being optimized in [15, 20]. The models will have a feasible solution only if the system has enough adjustable capacity. Otherwise, the models will be infeasible and cannot give any operating suggestion to the system operators.

This paper proposes a novel robust ED approach considering AGC with affine recourse process. The system operating security and economics are optimized simultaneously considering the uncertain nodal power injections and network constraints. The approach bases on RO and preemptive goal programming (PGP) [21] which is suitable for multi-objective optimization with a strict priority order of the objectives. A simplified linear model is also deduced to facilitate the calculation. The tests of the paper are performed on the General Algebraic Modelling System (GAMS) [22]. Solvers CONOPT [23] and CPLEX [24] are selected to solve the proposed BLP and LP problems, respectively. The convergence criterion is set as 1×10^{-6} . The tests run on an Intel Core i5 personal computer with 3.2 GHz processor and 4GB RAM. The novelty of the proposed approach is as follows;

1. The security of power system is optimized by maximizing the system effective acceptable disturbance range (EADR) in the first level of PGP. The proposed approach can obtain a solution which can cover the disturbance as much as possible even when the system does not have enough adjustable capacity. The nodal EADR can be obtained to quantitatively represent the anti-disturbance capability of the node.
2. The operating costs, i.e., the sum of the generation costs and AGC reserve costs are minimized in the second level of PGP with respect to the obtained EADR. The system operating economics can be improved significantly on the premise that the system anti-disturbance capability is not degraded.
3. A parameter referred to as guarantee ratio (GR) is introduced into the model to control the conservative level of the results. This provides a measure to tune the tradeoff between the operating security and economics.

The rest of the paper is organized as follows. In Section 3, the detailed formulation of ED considering AGC with affine recourse is described. In Section 4, the proposed model is transformed to a two-level deterministic bilinear programming (BLP) problem basing on the PGP and RO. Section 5 gives case studies and Section 6 draws the conclusions.

3 Formulations of ED with AGC affine recourse

3.1 Uncertain set and EADR

For simplicity, the sum of existing load and generation from uncertain power resources is represented as net load. The box uncertain set is selected to describe the deviation of the uncertain variables [20]. Comparing with the PDF used in SP, the box uncertainty set is preferable since the bounds of the box uncertain set are much easier to be determined. The bounds of the uncertain nodal net load can be obtained by truncating the convolution of the probabilities of the uncertain resources and loads on the node. They can also be obtained by combining the disturbance ranges of the uncertain resources and loads using the Arithmetic Operation [25]. The type and size of the uncertain set will influence the conservatism of the solution. They can be determined by analyzing the stochastic nature of the uncertain injections.

The EADR of a node is defined as the overlap of the acceptable disturbance range and actual net load disturbance range on the node [26]. The nodal acceptable disturbance range corresponds to the allocated AGC reserve which acts on the disturbance of the node. It is constrained by the total AGC reserve amount and the line flow constraints. In this range, the disturbance can be accepted safely. The system EADR is defined as the sum of the nodal EADRs. Fig. 1 shows an example on how the EADR is determined. EADR can quantitatively express the anti-disturbance capability of the system.

Fig. 1. The determination of EADR.

The maximal EADR of a system will be no more than the sum of the net load disturbance ranges since it is not necessary and providing EADR is not free. Through maximizing the EADR, the real time dispatch can always make the system at the most robust status whether the system has enough adjustable capacity or not. The calculated

acceptable disturbance range on each node is valuable information for the operators to rearrange centralized and/or distributed resources to regulate the disturbances.

3.2 Objective functions

ED with AGC affine recourse can be modeled as a bi-objective optimization problem. The first level aims to maximize the system EADR.

$$z_S = \max_{\substack{\alpha_i, p_i, \bar{r}_i, \bar{r}_i, \bar{y}_j, \bar{y}_j, \\ \underline{\gamma}_{ji}, \bar{\gamma}_{ji}, \Delta \hat{d}_j^{acpt}, \Delta \hat{d}_j^{acpt}}} \left\{ \sum_{j=1}^{N_D} EADR_j \right\} = \max_{\substack{\alpha_i, p_i, \bar{r}_i, \bar{r}_i, \bar{y}_j, \bar{y}_j, \\ \underline{\gamma}_{ji}, \bar{\gamma}_{ji}, \Delta \check{d}_j^{acpt}, \Delta \check{d}_j^{acpt}}} \left\{ \sum_{j=1}^{N_D} \left(\min \left(\Delta \hat{d}_j^{acpt}, \Delta \hat{d}_j^{load} \right) + \min \left(\Delta \check{d}_j^{acpt}, \Delta \check{d}_j^{load} \right) \right) \right\} \quad (2)$$

where $EADR_j$ is the EADR on node j .

Equation (2) can be rewritten as

$$z_S = \max_{\substack{\alpha_i, p_i, \bar{r}_i, \bar{r}_i, \bar{y}_j, \bar{y}_j, \\ \underline{\gamma}_{ji}, \bar{\gamma}_{ji}, \Delta \hat{d}_j^{acpt}, \Delta \hat{d}_j^{acpt}}} \left\{ \sum_{j=1}^{N_D} (\hat{y}_j + \check{y}_j) \right\} \quad (3)$$

Where

$$\begin{aligned} \hat{y}_j &= \min \left(\Delta \hat{d}_j^{acpt}, \Delta \hat{d}_j^{load} \right), & j &= 1, 2, \dots, N_D \\ \check{y}_j &= \min \left(\Delta \check{d}_j^{acpt}, \Delta \check{d}_j^{load} \right), & j &= 1, 2, \dots, N_D \end{aligned} \quad (4)$$

To avoid the logical operation, $\hat{y}_j$ and $\check{y}_j$ can be equally linearized as below [27]:

$$\begin{cases} \hat{y}_j \leq \Delta \hat{d}_j^{acpt}, & j = 1, 2, \dots, N_D \\ \hat{y}_j \leq \Delta \hat{d}_j^{load}, & j = 1, 2, \dots, N_D \end{cases} \quad (5)$$

$$\begin{cases} \check{y}_j \leq \Delta \check{d}_j^{acpt}, & j = 1, 2, \dots, N_D \\ \check{y}_j \leq \Delta \check{d}_j^{load}, & j = 1, 2, \dots, N_D \end{cases} \quad (6)$$

The aim of the second level is to minimize the operating costs which are equal to the generation costs plus reserve costs.

$$z_E = \min_{\substack{\alpha_i, p_i, \bar{r}_i, \bar{r}_i, \bar{y}_j, \bar{y}_j, \\ \underline{\gamma}_{ji}, \bar{\gamma}_{ji}, \Delta \hat{d}_j^{acpt}, \Delta \hat{d}_j^{acpt}}} \left\{ \sum_{i=1}^{N_G} c_i \cdot p_i + \sum_{i=1}^{N_G} \bar{c}_i \cdot \bar{r}_i + \sum_{i=1}^{N_G} \check{c}_i \cdot \check{r}_i \right\} \quad (7)$$

3.3 Constraints

The operating constraints are shown below:

1. Power balance constraint.

$$\sum_{i=1}^{N_G} p_i = \sum_{j=1}^{N_D} D_j \quad (8)$$

The above equation means that the BPs of the online units should match the forecasted net load.

2. AGC reserve requirement constraints.

The AGC reserve requirement for each unit is determined by the system net load disturbance and PF of the unit.

They can be computed according to the affine recourse mechanism:

$$\begin{cases} \hat{r}_i \geq \alpha_i \cdot \sum_{j=1}^{N_D} \Delta \hat{d}_j^{acpt}, & i = 1, 2, \dots, N_G \\ \check{r}_i \geq \alpha_i \cdot \sum_{j=1}^{N_D} \Delta \check{d}_j^{acpt}, & i = 1, 2, \dots, N_G \end{cases} \quad (9)$$

And the PFs satisfy

$$\sum_{i=1}^{N_G} \alpha_i = 1$$

(10)

3. Limits of the AGC reserve

$$\begin{cases} 0 \leq \hat{r}_i \leq \hat{R}_i^{\max}, & i = 1, 2, \dots, N_G \\ 0 \leq \check{r}_i \leq \check{R}_i^{\max}, & i = 1, 2, \dots, N_G \end{cases}$$

(11)

4. Capacity limits of the units.

$$\begin{cases} p_i + \hat{r}_i \leq p_i^{\max}, & i = 1, 2, \dots, N_G \\ p_i - \check{r}_i \geq p_i^{\min}, & i = 1, 2, \dots, N_G \end{cases}$$

(12)

5. Ramp rate constraints.

$$-R_i^D \leq p_i - p_i^0 \leq R_i^U, \quad i=1,2,\dots,N_G$$

(13)

6. Transmission capacity constraints.

The power flow on a branch is expressed using the generation shift factors and load shift factors [28] and it is constrained by the transmission capacity limits.

$$-T_l \leq \sum_{i=1}^{N_G} M_{il} \cdot (p_i + \Delta \tilde{p}_i) + \sum_{j=1}^{N_D} M_{jl} \cdot (D_j + \Delta \tilde{d}_j) \leq \bar{T}, l=1,2,\dots,N_L$$

(14)

Considering the relationship between $\Delta \tilde{p}_i$ and $\Delta \tilde{d}_j$ mentioned in (1), constraints (14) can be rewritten as

$$-T_l' \leq \sum_{j=1}^{N_D} \left(M_{jl} + \sum_{i=1}^{N_G} M_{il} \alpha_i \right) \Delta \tilde{d}_j \leq \bar{T}_l', \quad l=1,2,\dots,N_L$$

(15)

where

$$\begin{cases} T_l' = T_l + \sum_{i=1}^{N_G} M_{il} p_i + \sum_{j=1}^{N_D} M_{jl} D_j, & l=1,2,\dots,N_L \\ \bar{T}_l' = \bar{T}_l - \sum_{i=1}^{N_G} M_{il} p_i - \sum_{j=1}^{N_D} M_{jl} D_j, & l=1,2,\dots,N_L \end{cases}$$

(16) The derivation process is explained in Appendix B.

7. Acceptable disturbance range constraints.

Because of the limited transmission capacity, the constraints in (15) may be violated when the disturbance $\Delta \tilde{d}_j$ is extremely large. However, the transmission constraints should be satisfied when $\Delta \tilde{d}_j$ varies within the acceptable disturbance range.

$$-\Delta \tilde{d}_j^{acpt} \leq \Delta \tilde{d}_j \leq \Delta \tilde{d}_j^{acpt}, \quad j=1,2,\dots,N_D$$

(17)

The proposed model is composed of the objective functions (3), (7) and constraints (5~6), (8~13), (15). The complete formulation of the model is shown in Appendix A. The model is a bi-objective optimization problem including uncertain variables. The solution methodology will be explained in the next section.

4 Solution methodology based on RO and PGP

4.1 Eliminating the uncertain variables using the linear RO

When $\Delta\tilde{d}_j$ satisfies (17), the transmission constraints (15) should always be satisfied. This means that the following two constraints should be satisfied simultaneously.

$$\max_{\Delta\tilde{d}_j \in [-\Delta\tilde{d}_j^{acpt}, \Delta\tilde{d}_j^{acpt}]} \left\{ \sum_{j=1}^{N_D} \left(\left(M_{jl} + \sum_{i=1}^{N_G} M_{il} \alpha_i \right) \Delta\tilde{d}_j \right) \right\} \leq \bar{T}_l', l=1,2,\dots,N_L$$

(18)

$$\min_{\Delta\tilde{d}_j \in [-\Delta\tilde{d}_j^{acpt}, \Delta\tilde{d}_j^{acpt}]} \left\{ \sum_{j=1}^{N_D} \left(\left(M_{jl} + \sum_{i=1}^{N_G} M_{il} \alpha_i \right) \Delta\tilde{d}_j \right) \right\} \geq -\underline{T}_l', l=1,2,\dots,N_L$$

(19)

The above two constraints are satisfied if and only if the extreme values of the left hand sides of (18) and (19) satisfy the constraints. Obviously, the expressions in the curly braces are all linear with respect to $\Delta\tilde{d}_j$ when the terms in the inner parenthesis are seen as a parameter. Therefore, the random variable $\Delta\tilde{d}_j$ should take $\Delta\tilde{d}_j^{acpt}$ in (18) and $-\Delta\tilde{d}_j^{acpt}$ in (19) if the parameter $\left(M_{jl} + \sum_{i=1}^{N_G} M_{il} \alpha_i \right)$ is positive to achieve their extreme values, and vice versa. Unfortunately, the sign of this parameter is not known beforehand because the parameter contains the decision variable α_i .

Therefore, in order to eliminate the uncertain variable in (18) and (19), Soyster's method [29, 30] is employed. Constraints (18) and (19) can then be rewritten as

$$\begin{cases} \sum_{j=1}^{N_D} \left(\left(M_{jl} + \sum_{i=1}^{N_G} M_{il} \cdot \alpha_i \right) \left(-\Delta \check{d}_j^{acpt} \right) + \bar{\gamma}_{jl} \right) \leq \bar{T}'_l, l = 1, 2, \dots, N_L \\ \bar{\gamma}_{jl} \geq \left(M_{jl} + \sum_{i=1}^{N_G} M_{il} \cdot \alpha_i \right) \left(\Delta \check{d}_j^{acpt} + \Delta \hat{d}_j^{acpt} \right), j = 1, 2, \dots, N_D, l = 1, 2, \dots, N_L \\ \bar{\gamma}_{jl} \geq 0, \quad j = 1, 2, \dots, N_D, l = 1, 2, \dots, N_L \end{cases}$$

(20)

$$\begin{cases} \sum_{j=1}^{N_D} \left(\left(M_{jl} + \sum_{i=1}^{N_G} M_{il} \cdot \alpha_i \right) \Delta \hat{d}_j^{acpt} + \underline{\gamma}_{jl} \right) \geq -\underline{T}'_l, l = 1, 2, \dots, N_L \\ \underline{\gamma}_{jl} \leq - \left(M_{jl} + \sum_{i=1}^{N_G} M_{il} \cdot \alpha_i \right) \left(\Delta \check{d}_j^{acpt} + \Delta \hat{d}_j^{acpt} \right), j = 1, 2, \dots, N_D, l = 1, 2, \dots, N_L \\ \underline{\gamma}_{jl} \leq 0, \quad j = 1, 2, \dots, N_D, l = 1, 2, \dots, N_L \end{cases}$$

(21)

Constraints (20) and (21) are deduced using the linear strong duality theory. The transformation process is explained in Appendix C.

4.2 Bi-objective optimization using PGP

PGP is a classical method for multi-objective optimization [21], and it is useful when the objectives have a strict priority order. PGP optimizes the objective functions successively. The optimal value of the higher order objective function is used as a constraint in the subsequent optimizations, by which means the optimality of the higher order optimizations can be guaranteed.

Operators of power systems usually concern security over economics. When the PGP is applied, objective function (3) was given a higher priority. The proposed model can be explicitly formulated as follows:

Optimization level one:

$$z_s = \max_{\alpha_i, p_i, \bar{r}_i, \bar{y}_j, \bar{\gamma}_{jl}, \underline{\gamma}_{jl}, \Delta \check{d}_j^{acpt}, \Delta \hat{d}_j^{acpt}} \left\{ \sum_{j=1}^{N_D} (\hat{y}_j + \check{y}_j) \right\}$$

s.t.

Constraints (5~6), (8~13), and (20~21)

(22)

Optimization level two:

$$\begin{aligned}
 z_E = & \min_{\substack{\alpha_i, p_i, \hat{r}_i, \tilde{r}_i, \hat{y}_j, \tilde{y}_j, \\ \underline{I}_{jl}, \tilde{I}_{jl}, \Delta d_j^{apd}, \tilde{\Delta d}_j^{apd}}} \left\{ \sum_{i=1}^{N_G} c_i \cdot p_i + \sum_{i=1}^{N_G} \hat{c}_i \cdot \hat{r}_i + \sum_{i=1}^{N_G} \tilde{c}_i \cdot \tilde{r}_i \right\} \\
 s.t. & \left\{ \begin{aligned} & \sum_{j=1}^{N_D} (\hat{y}_j + \tilde{y}_j) \geq \beta \cdot z_S \\ & \text{Constraints (5~6), (8~13), and (20~21)} \end{aligned} \right.
 \end{aligned}$$

(23)

The objective function in (22) is reproduced from (3). The aim of level one is to maximize the system EADR. Z_S is the optimal value of the objective function. The decision variables are the base point p_i and participation factor α_i . Other variables in the model can be derived from them. Similarly, the objective function in (23) is reproduced from (7). The goal of level two is to minimize the operating cost. Z_E is the optimal value of the objective function. The decision variables of level two are the same as those of level one, and they will be updated in level two. The parameter Z_S in level two is carried forward from the solution of level one.

Different from the common PGP, a GR parameter β is introduced in the model to control the conservative level of the results. $\beta \in [0, 1]$. A larger β indicates the system prefers security. When β takes the value of 1, the maximum EADR obtained in the first level of the optimization is entirely guaranteed in the second level. This condition is in accordance with the classical PGP. A smaller β indicates that the system prefers economics. When β takes the value of 0, the problem will degrade to a deterministic ED problem [31].

The models expressed by (22) and (23) belong to the bilinear programming (BLP), whose objective functions and/or constraints are bilinear. BLP is a special class of quadratic constrained quadratic programming problem, and it can be solved by many commercial solvers effectively [32].

4.3 A simplified linear model

If the system has enough adjustable resources to deal with the disturbances and the decision makers are willing to do so, the proposed model will degrade to a linear model where the optimization of the first level is not considered and the acceptable disturbance range is fixed to the load disturbance range. This special case of the proposed model can be formulated as below.

$$\begin{aligned}
z_E = \min_{\substack{\alpha_i, p_i, \hat{r}_i, \\ \tilde{r}_i, \underline{\gamma}_{jl}, \bar{\gamma}_{jl}}} & \left\{ \sum_{i=1}^{N_G} c_i \cdot p_i + \sum_{i=1}^{N_G} \hat{c}_i \cdot \hat{r}_i + \sum_{i=1}^{N_G} \tilde{c}_i \cdot \tilde{r}_i \right\} \\
s.t. & \left\{ \begin{aligned} & \hat{r}_i = \alpha_i \cdot \sum_{j=1}^{N_D} \Delta \hat{d}_j^{load}, & i = 1, 2, \dots, N_G \\ & \tilde{r}_i = \alpha_i \cdot \sum_{j=1}^{N_D} \Delta \tilde{d}_j^{load}, & i = 1, 2, \dots, N_G \\ & \sum_{j=1}^{N_D} \left(\left(M_{jl} + \sum_{i=1}^{N_G} M_{il} \cdot \alpha_i \right) (-\Delta \tilde{d}_j^{load}) + \bar{\gamma}_{jl} \right) \leq \bar{T}_l', & l = 1, 2, \dots, N_L \\ & \bar{\gamma}_{jl} \geq \left(M_{jl} + \sum_{i=1}^{N_G} M_{il} \cdot \alpha_i \right) (\Delta \tilde{d}_j^{load} + \Delta \hat{d}_j^{load}), & j = 1, 2, \dots, N_D, l = 1, 2, \dots, N_L \\ & \bar{\gamma}_{jl} \geq 0, & j = 1, 2, \dots, N_D, l = 1, 2, \dots, N_L \\ & \sum_{j=1}^{N_D} \left(M_{jl} + \sum_{i=1}^{N_G} M_{il} \cdot \alpha_i \right) \Delta \hat{d}_j^{load} + \underline{\gamma}_{jl} \geq -\underline{T}_l', & l = 1, 2, \dots, N_L \\ & \underline{\gamma}_{jl} \leq - \left(M_{jl} + \sum_{i=1}^{N_G} M_{il} \cdot \alpha_i \right) (\Delta \tilde{d}_j^{load} + \Delta \hat{d}_j^{load}), & j = 1, 2, \dots, N_D, l = 1, 2, \dots, N_L \\ & \underline{\gamma}_{jl} \leq 0, & j = 1, 2, \dots, N_D, l = 1, 2, \dots, N_L \end{aligned} \right. \\
& \text{Constraints (5~6), (8), and (10~13)}
\end{aligned}$$

(24)

In model (24), the variables $\Delta \hat{d}_j^{acpt}$ and $\Delta \tilde{d}_j^{acpt}$ in constraints (9), (20) and (21) are replaced by the parameters $\Delta \hat{d}_j^{load}$ and $\Delta \tilde{d}_j^{load}$, respectively. Therefore, the proposed model is simplified from a two-level BLP problem to a one-level linear programming (LP) problem.

5 Case studies

5.1 Tests on a 6-bus system

The structure of the 6-bus system is shown in Fig. 2.

Fig. 2. Structure of the 6-bus system.

All units in the system are seen as AGC units. Parameters of the units and transmission lines are listed in Table 1 and Table 2 respectively. The generation/load shift factors computed based on DC power flow model are given in Table 3 [3]. The expected demands of L1, L2 and L3 are 70 MW, 170 MW and 70 MW respectively. The disturbance of each load bus varies within $\pm 5\%$ of the expected demand.

Table 1: Parameters of AGC Units of the 6-Bus System.

Table 2: Parameters of Transmission Lines of the 6-Bus System.

Table 3: Generation/Load Shift Factors of the 6-Bus System.

Based on the proposed two-level optimization approach, the calculated EADRs and costs with different values of β are summarized in Table 4. The corresponding BPs and PFs are listed in Table 5. The following observations can be obtained.

Table 4: Summary of the Optimization Results.

Table 5: BPs and PFs of the Units.

1. After the first level of the optimization, the maximum EADR of the system is 31 MW which is equal to the sum of the load disturbance ranges. This means that the system has capability to deal with all load disturbances.
2. The EADR used in the second level of the optimization is tuned by the value of β . In Table 4, it can be appreciated that with the increase of β , the EADR increases linearly at the expense of a larger operating costs and the dispatch results become more conservative. The operator can control the conservatism of the results by tuning the value of β .
3. When $\beta = 0$, the disturbances are completely ignored from the ED module, and the PFs will not be given by the model. Under this condition, it has been checked that the dispatch results are exactly the same as those of the deterministic economic dispatch [31].
4. When $\beta = 1$, the demand disturbances are totally covered by the system. Under this condition, it has been checked that the dispatch results are exactly the same as those of the simplified linear model of (24).

5. It can be found in Table 5 that when $\beta = 0.2$ the AGC reserve is all provided by G2 whose reserve price is relatively low. With the increase of β , more and more units will take part in the AGC regulation to satisfy the EADR requirement.

In the second level of the optimization, the EADR is allocated to the load buses economically with respect to the operating constraints. Fig. 3 shows the acceptable disturbance ranges and real load disturbance ranges for each load bus with respect to different β .

Fig. 3. Acceptable disturbance ranges and load disturbance ranges for each load bus.

In the 6-bus system the prices of the upward and downward AGC reserves are the same. However, from Fig. 3 it is found that when β is relatively small the system prefers to deal with the downward load disturbances. This is because the deployment of the upward AGC reserve of the economical units is restricted by the transmission constraints. Take $\beta = 0.2$ as an example. Under this condition, the computed EADR is 6.2 MW (see Table 6). When the load disturbances are not considered, the power on Line2 flows from Bus 2 to Bus 3 and this power flow has already reached the transmission limit. In order to maintain the EADR requirement with less cost, G2 should be adjusted in response to a possible disturbance since it has the lowest reserve price. However, due to the transmission limit of Line2, G2 can only address downward load disturbances.

In order to check the performance of the proposed approach on a larger load disturbance, the disturbances are set as $\pm 8\%$ of the expected demands. The corresponding test results are shown in Table 6 and Table 7.

Table 6: Summary of the Optimization Results with Severe Disturbance.

Table 7: BPs and PFs of the Units with Severe Disturbance.

In this case, the maximum load disturbance reaches up to 49.6 MW. The disturbance is so large that the approaches based on the common RO cannot get a feasible solution [19, 20]. Similarly, the simplified linear model of (24) cannot be used either. However, the proposed two-level optimization approach still can maximize the EADR and optimally allocate the EADR to the load buses.

After the first level of the optimization, the maximum EADR obtained is 41.3 MW. This value is mainly limited by the transmission capacity of Line2 and Line5. Fig. 4 shows the acceptable disturbance range and real load disturbance range for each load bus when $\beta = 1$. This information is quite useful for the system operator to arrange other centralized or distributed resources to deal with the unacceptable disturbances. Similarly, the system has a stronger capability to cover the downward load disturbances due to the transmission constraints.

Fig. 4. Acceptable disturbance ranges and load disturbance ranges with severe load disturbances ($\beta = 1$).

The second level of the optimization is to minimizing operation cost. As demand disturbance rate varies from 2% to 8%, the operation cost reduces on different degrees after considering economic target, as shown in Fig. 5. When load has slight disturbance, power system has enough regulating capacity to deal with it, thus there are multiple optimal solutions. However the costs of different groups of solutions vary considerably. They decrease obviously after considering the economy. As the load fluctuation rate increases, the solution is mainly determined by the EADR, while the economic goal is not obviously affected.

Fig.5 Variation curves of cost considering different objectives

5.2 Tests on the IEEE 118-bus system

The IEEE 118-bus system has 54 generation units, 91 load buses and 186 transmission lines. The total capacity of the online units is 8270 MW, and the demand is 3708 MW. The capacity of the units ranges from 20 MW to 650 MW. During the test, fifteen units with the capacity of 100 MW are selected as the AGC units. Five uncertain loads are selected randomly from the system. And these loads are assumed to vary within $\pm 20\%$ of their forecasted value. For the topology and parameters of this system, please refer to [33].

The two-level optimization approach is used to solve the problem, and the test results are summarized in Table 8.

Table 8: Summary of the Optimization Results of the IEEE 118-Bus System.

Because the 118-bus system has enough AGC reserve, all load disturbances can be covered. When $\beta = 1$, the operating costs increase 0.79% comparing to that of the deterministic ED. The acceptable disturbance ranges and real load disturbance ranges are shown in Fig. 6.

Fig. 6. Acceptable disturbance ranges and load disturbance ranges of the IEEE 118-bus system.

In order to show the impact of β on the computation efficiency, Fig. 7 shows the run time of the two-level optimization approach with different values of β . It can be observed the calculation speed of the model does not have a close relationship to β .

Fig. 7. Calculation time with different values of β .

Fig. 8 and Fig. 9 show the run time of the two proposed approaches when the number of the uncertain loads increases from 5 to 30 ($\beta = 1$). In order to make sure that the simplified model can get a feasible solution, the disturbances are decreased from $\pm 20\%$ to $\pm 5\%$ of the forecasted demands. In Fig. 8 and Fig. 9, the run time of the two approaches increases continuously as the number of uncertainty load buses increases. However, the rate of increase of the complete model is higher. If thirty uncertain load buses are selected, the run time of the complete model is 14.132 seconds, while for the simplified model the time is 0.658 seconds.

Fig. 8. Calculation time with different number of uncertain loads.

Fig. 9. Calculation time of the simplified model with different number of uncertain loads.

5.3 Tests on a real 445-bus system

The 445-bus system of Shandong Province, China is used here to demonstrate the calculation performance of the proposed approach on a relatively large system. The system has 48 generation units, 194 load buses, and 693 transmission lines. The total capacity of the units is 19618 MW, and the demand is 17103 MW. In the system, fifteen units are selected as the AGC units and their capacity ranges from 100 MW to 250 MW. Five to thirty uncertain loads are selected randomly to test calculation efficiency, and these loads are assumed to vary within $\pm 5\%$

of their forecasted value. The calculation time of the complete and simplified models are shown in Fig. 10 and Fig. 11.

Fig. 10. Calculation time with different number of uncertain loads of the 445-bus system..

Fig. 11. Calculation time of the simplified model with different number of uncertain loads of the 445-bus system.

In Fig. 10, the calculation time of the two-level optimization approach varies from 5.4 seconds to 128.7 seconds. Although the calculation time is longer than that of the 118-bus system, it is still acceptable for on-line application (typically the algorithm runs every 5 ~ 15 minutes). Furthermore, the bus aggregation technique can be applied in practice to simplify the network and improve computation efficiency [34]. On the other hand, the linear simplified model can always converge rapidly, and the calculation time of the approach varies from 0.4 seconds to 3.7 seconds, which is very desirable.

6 Conclusions

Based on PGP and RO, a novel robust ED considering AGC affine recourse process is proposed in this paper. The approach considers the coordination between ED and AGC by co-optimizing the BPs and PFs of the AGC units in a single model. In the proposed approach, the maximum EADR which reveals the anti-disturbance capability of the system is calculated in the first level of the optimization. The BPs and PFs are determined in the second level by minimizing the operating costs with respect to the EADR constraint and other operating constraints. The conservatism of the results can be controlled by the introduced GR. Furthermore, a simplified linear model with high computation efficiency is deduced. The effectiveness and validity of the proposed approaches are illustrated by a 6-bus system, the IEEE 118-bus system, and a real 445-bus system.

7 Acknowledgments

The research work described in this paper is supported by National Science Foundation of China (51007047, 51477091), National Basic Research Program of China (973 Program) (2013CB228205) and the Natural Science Foundation of Shandong Province, China (ZR2014EEM022).

8 Appendix A

The complete formulation of the proposed model can be expressed by (25), where $\Delta\tilde{d}_j$ is a random variable varying within $[-\Delta\tilde{d}_j^{acpt}, \Delta\tilde{d}_j^{acpt}]$.

$$\begin{aligned}
 \text{Obj_1: } z_S &= \max_{\substack{\alpha_i, p_i, \tilde{r}_i, \tilde{r}_i, \tilde{y}_j, \tilde{y}_j, \\ \underline{\gamma}_{jl}, \tilde{\gamma}_{jl}, \Delta\tilde{d}_j^{acpt}, \Delta\tilde{d}_j^{acpt}}} \left\{ \sum_{j=1}^{N_D} (\hat{y}_j + \tilde{y}_j) \right\} \\
 \text{Obj_2: } z_E &= \min_{\substack{\alpha_i, p_i, \tilde{r}_i, \tilde{r}_i, \tilde{y}_j, \tilde{y}_j, \\ \underline{\gamma}_{jl}, \tilde{\gamma}_{jl}, \Delta\tilde{d}_j^{acpt}, \Delta\tilde{d}_j^{acpt}}} \left\{ \sum_{i=1}^{N_G} c_i \cdot p_i + \sum_{i=1}^{N_G} \hat{c}_i \cdot \hat{r}_i + \sum_{i=1}^{N_G} \check{c}_i \cdot \check{r}_i \right\} \\
 \text{s.t.} \\
 &\left\{ \begin{aligned} &\left\{ \begin{aligned} \hat{y}_j &\leq \Delta\tilde{d}_j^{acpt} \\ \tilde{y}_j &\leq \Delta\tilde{d}_j^{load} \end{aligned} \right. , & \left\{ \begin{aligned} \hat{r}_i &\geq \alpha_i \cdot \sum_{j=1}^{N_D} \Delta\tilde{d}_j^{acpt} \\ \check{r}_i &\geq \alpha_i \cdot \sum_{j=1}^{N_D} \Delta\tilde{d}_j^{acpt} \end{aligned} \right. , & \sum_{i=1}^{N_G} p_i = \sum_{j=1}^{N_D} D_j , \\ &\left\{ \begin{aligned} \tilde{y}_j &\leq \Delta\tilde{d}_j^{acpt} \\ \tilde{y}_j &\leq \Delta\tilde{d}_j^{load} \end{aligned} \right. , & \left\{ \begin{aligned} 0 &\leq \hat{r}_i \leq \hat{R}_i^{\max} \\ 0 &\leq \check{r}_i \leq \check{R}_i^{\max} \end{aligned} \right. , & \sum_{i=1}^{N_G} \alpha_i = 1 , \\ &-R_i^D \leq p_i - p_i^0 \leq R_i^U , & \left\{ \begin{aligned} p_i + \hat{r}_i &\leq p_i^{\max} \\ p_i - \check{r}_i &\geq p_i^{\min} \end{aligned} \right. , \\ &\underline{T}_l + \sum_{i=1}^{N_G} M_{il} p_i + \sum_{j=1}^{N_D} M_{jl} D_j \leq \sum_{j=1}^{N_D} \left(M_{jl} + \sum_{i=1}^{N_G} M_{il} \alpha_i \right) \Delta\tilde{d}_j \leq \bar{T}_l - \sum_{i=1}^{N_G} M_{il} p_i - \sum_{j=1}^{N_D} M_{jl} D_j \\ &\quad i = 1, 2, \dots, N_G, \quad j = 1, 2, \dots, N_D, \quad l = 1, 2, \dots, N_L \end{aligned} \right.
 \end{aligned}$$

(25)

9 Appendix B

Considering the relationship between $\Delta\tilde{p}_i$ and $\Delta\tilde{d}_j$ mentioned in equation (1), transmission capacity limits (14) can

be rewritten as

$$-\underline{T}_l \leq \sum_{i=1}^{N_G} M_{il} \left(p_i + \alpha_i \cdot \sum_{j=1}^{N_D} \Delta\tilde{d}_j \right) + \sum_{j=1}^{N_D} M_{jl} (D_j + \Delta\tilde{d}_j) \leq \bar{T}_l, l = 1, 2, \dots, N_L \quad (26)$$

After expansion and combining like terms, equation (26) can be rewritten as.

$$-\underline{T}_l \leq \sum_{j=1}^{N_D} \left(M_{jl} + \sum_{i=1}^{N_G} M_{il} \alpha_i \right) \Delta\tilde{d}_j + \sum_{i=1}^{N_G} M_{il} p_i + \sum_{j=1}^{N_D} M_{jl} D_j \leq \bar{T}_l, l = 1, 2, \dots, N_L \quad (27)$$

Equations (15) and (16) can be obtained after transposing.

10 Appendix C

By introducing a new random variable w , random variable $\Delta \tilde{d}_j$ can be replaced by $\Delta \hat{d}_j^{acpt}$, $\Delta \check{d}_j^{acpt}$ and w as

$$\Delta \tilde{d}_j = -\Delta \check{d}_j^{acpt} + w(\Delta \hat{d}_j^{acpt} + \Delta \check{d}_j^{acpt}), \quad 0 \leq w \leq 1$$

(28)

Equation (18) can be rewritten as:

$$\max_{0 \leq w \leq 1} \left\{ \sum_{j=1}^{N_D} \left(\left(M_{jl} + \sum_{i=1}^{N_G} M_{il} \alpha_i \right) \cdot \left(-\Delta \check{d}_j^{acpt} + w(\Delta \hat{d}_j^{acpt} + \Delta \check{d}_j^{acpt}) \right) \right) \right\} \leq \bar{T}_l'$$

(29)

The dual problem of the left hand side of (29) is

$$\begin{aligned} \min_{\bar{\gamma}_{jl}} & \sum_{j=1}^{N_D} \left(\left(M_{jl} + \sum_{i=1}^{N_G} M_{il} \cdot \alpha_i \right) \left(-\Delta \check{d}_j^{acpt} \right) + \bar{\gamma}_{jl} \right) \\ \text{s.t.} & \\ & \bar{\gamma}_{jl} \geq \left(M_{jl} + \sum_{i=1}^{N_G} M_{il} \cdot \alpha_i \right) \left(\Delta \check{d}_j^{acpt} + \Delta \hat{d}_j^{acpt} \right) \\ & \bar{\gamma}_{jl} \geq 0 \end{aligned}$$

(30)

According to the duality theory, the objective value of (30) is always larger than or equal to the objective value of left hand side of (29), i.e.,

$$\sum_{j=1}^{N_D} \left(\left(M_{jl} + \sum_{i=1}^{N_G} M_{il} \alpha_i \right) \Delta \tilde{d}_j \right) \leq \sum_{j=1}^{N_D} \left(\left(M_{jl} + \sum_{i=1}^{N_G} M_{il} \cdot \alpha_i \right) \left(-\Delta \check{d}_j^{acpt} \right) + \bar{\gamma}_{jl} \right)$$

(31)

Then, constraint (18) can be transformed into constraints (20). During the optimization, (31) will be minimized to enlarge the feasible region of the main problem. Therefore, according to the strong duality theory the maximum value of the left hand side of (29) is obtained. The constraint (19) can be transformed into constraints (21) similarly.

11 References

- [1] B. H. Chowdhury and S. Rahman, "A review of recent advances in economic dispatch," *IEEE Trans. on Power Syst.*, vol. 5, no. 4, pp. 1248-1259, Nov. 1990.
- [2] N. Jaleeli, L. S. VanSlyck, D. N. Ewart, L. H. Fink, and A. G. Hoffmann, "Understanding automatic generation control," *IEEE Trans. on Power Syst.*, vol. 7, no. 3, pp. 1106-1122, Aug. 1992.

- [3] A. J. Wood and B. F. Wollenberg, *Power Generation, Operation, and Control*, New York: Wiley, 1984.
- [4] P. Kundur, *Power system stability and control*, New York: Tata McGraw-Hill Education, 1994.
- [5] F. D. Galiana, F. Bouffard, J. M. Arroyo, and J. F. Restrepo, "Scheduling and pricing of coupled energy and primary, secondary, and tertiary reserves," *Proc. IEEE*, vol. 93, no. 11, pp. 1970-1983, Nov. 2005.
- [6] A. Ben-Tal, L. E. Ghaoui, and A. Nemirovski, *Robust Optimization*. Princeton: Princeton University Press, 2009.
- [7] J. R. Birge and F. Louveaux, *Introduction to Stochastic Programming*. New York: Springer, 1997.
- [8] K. Methaprayoon, C. Yingvivatanapong, W. J. Lee, and R. Liao, "An integration of ANN wind power estimation into unit commitment considering the forecasting uncertainty," *IEEE Trans. Ind. Appl.*, vol. 43, no. 6, pp. 1441-1448, Nov./Dec. 2007
- [9] F. Bouffard and F. D. Galiana, "Stochastic security for operations planning with significant wind power generation," *IEEE Trans. on Power Syst.*, vol. 23, no.2, pp. 306–316, May 2008.
- [10] L. Wu, M. Shahidehpour, and T. Li, "Stochastic security-constrained unit commitment," *IEEE Trans. on Power Syst.*, vol. 22, no. 2, pp. 800-811, May 2007.
- [11] X. Liu and W. Xu, "Economic load dispatch constrained by wind power availability: a here-and-now approach," *IEEE Trans. on Power Syst.*, vol. 1, no. 1, pp. 2–9, Apr. 2010.
- [12] H. Zhang and P. Li. "Probabilistic analysis for optimal power flow under uncertainty," *IET Gen., Transm., Distrib.*, vol. 4, no. 5, pp. 553-561, May 2010.
- [13] R.W. Jiang, J. H. Wang, and Y. P. Guan, "Robust unit commitment with wind power and pumped storage hydro," *IEEE Trans. on Power Syst.*, vol. 27, no. 2, pp. 800-810, Feb. 2012.
- [14] D. Bertsimas, E. Litvinov, X. A. Sun, J. Y. Zhao, and T. X. Zheng, "Adaptive robust optimization for the security constrained unit commitment problem," *IEEE Trans. on Power Syst.*, vol. 28, no.1, pp. 52–63, Feb. 2013.
- [15] T. Zheng, J. Zhao, E. Litvinov and F. Zhao, "Robust optimization and its application to power system operation", presented at 2012 CIGRE, Paris, France, 2012.
- [16] M. Zhang, Y. Guan, Two-stage robust unit commitment problem, [online]. Available: http://www.optimization-online.org/DB_FILE/2009/10/2427.pdf.

- [17] R. Jiang, M. Zhang, G. Li and Y. Guan, Two-stage robust power grid optimization problem, [online]. Available: http://www.optimization-online.org/DB_FILE/2010/10/2769.pdf.
- [18] A. Ben-Tal, A. Goryashko, E. Guslitzer, and A. Nemirovski, "Adjustable robust solutions of uncertain linear programs," *Math. Program.*, vol. 99, no. 2, pp. 351-376, Mar. 2004.
- [19] J. Warrington, P. Goulart, S. Mariethoz, and M. Morari, "Policy-based reserves for power systems," *IEEE Trans. on Power Syst.*, vol. 28, no.4, pp. 4427-4437, Nov. 2013.
- [20] R. A. Jabr, "Adjustable robust OPF with renewable energy sources", *IEEE Trans. on Power Syst.*, vol. 28, no. 4, pp. 4742-4751, Nov. 2013.
- [21] H. D. Sherali, A. L. Soyster, and P. L. Yu, "Preemptive and non-preemptive multi-objective programming: relationships and counter examples," *J. Optim. Theory Appl.*, vol. 39, no.2, pp. 173-186, Feb. 1983.
- [22] A. Brooke, D. Kendrick, A. Meeraus, and R. Raman, *GAMS: A User's Guide*. Washington, D.C., GAMS Development Corp., 1998.
- [23] A. S. Drud, *CONOPT: A System for Large Scale Nonlinear Optimization*, Bagsvaerd: ARKI Consulting and Development A/S, 1995.
- [24] ILOG CPLEX. (2007, Sep.). 11.0 user's manual. [Online]. Available: <http://www-eio.upc.es/lceio/manuals/cplex-11/pdf/usrcplex.pdf>
- [25] L. V. Barboza, G. P. Dimuro, and R. H.S. Reiser. "Towards interval analysis of the load uncertainty in power electric systems", In 2004 8th *International Conference on Probabilistic Methods Applied to Power Systems*, pp. 538-544.
- [26] J. Z. Zhu, R. Q. Fan, G. Y. Xu and C. S. Chang, "Construction of maximal steady-state security regions of power systems using optimization method," *Electr. Power Syst. Res.*, vol. 44, no.2, pp. 101-105, Feb. 1998.
- [27] C. A. Floudas, *Nonlinear and Mixed-Integer Optimization: Fundamentals and Applications*, New York: Oxford University Press, 1995.
- [28] S. J. Wang, S. M. Shahidehpour, D. S. Kirschen, S. Mokhtari, and G. D. Irisarri. "Short-term generation scheduling with transmission and environmental constraints using an augmented Lagrangian relaxation," *IEEE Trans. on Power Syst.*, vol. 10, no.3, pp. 1294-1301, Aug. 1995.
- [29] A. L. Soyster, "Convex programming with set-inclusive constraints and applications to inexact linear programming," *Oper. Res.*, vol. 21, no.5, pp. 1154-1157, Sep.-Oct. 1973.

- [30] D. Bertsimas and M. Sim, “The price of robustness,” *Oper. Res.*, vol. 52, no.1, pp. 35-53, Jan.-Feb. 2004.
- [31] R. D. Zimmerman, C. E. Murillo-Sanchez, R. J. Thomas, “MATPOWER: steady-state operations, planning, and analysis tools for power systems research and education,” *IEEE Trans. on Power Syst.*, vol. 26, no.1, pp. 12-19, Feb. 2011.
- [32] S. Kolodziej, P. M. Castro, and I. E. Grossmann. “Global optimization of bilinear programs with a multiparametric disaggregation technique”, *Journal of Global Optimization*, vol.57, no. 4, pp. 1039-1063, Jan. 2013.
- [33] R. Christie. (1993, May). Power system test case archive. [Online]. Available: http://www.ee.washington.edu/research/pstca/pf118/pg_tca118bus.htm.
- [34] H. Oh, “Aggregation of buses for a network reduction,” *IEEE Trans. on Power Syst.*, vol. 27, no. 2, pp. 705-712, May, 2012.

Tables and Figures:

Fig. 1. The determination of EADR.

Fig. 2. Structure of the 6-bus system.

Fig. 3. Acceptable disturbance ranges and load disturbance ranges for each load bus.

Fig. 4. Acceptable disturbance ranges and load disturbance ranges with severe load disturbances ($\beta = 1$).

Fig.5 Variation curves of cost considering different objectives

Fig. 6. Acceptable disturbance ranges and load disturbance ranges of the IEEE 118-bus system.

Fig. 7. Calculation time with different values of β .

Fig. 8. Calculation time with different number of uncertain loads.

Fig. 9. Calculation time of the simplified model with different number of uncertain loads.

Fig.10. Calculation time with different number of uncertain loads of the 445-bus system.

Fig.11. Calculation time of the simplified model with different number of uncertain loads of the 445-bus system.

Table 1: Parameters of AGC Units of the 6-Bus System.

Unit ID	G1	G2	G3
P_i^{max} (MW)	200	150	100
P_i^{min} (MW)	100	50	20
c_i (\$/MWh)	70	65	135
$\hat{c}_i, \tilde{c}_i$ (\$/MWh)	75	70	140
R_i^U / R_i^D (MW)	15	11.25	7.5
P_i^0 (MW)	150	100	60

Table 2: Parameters of Transmission Lines of the 6-Bus System.

Branch ID	Line1	Line2	Line3	Line4	Line5	Line6	Line7
Reactance (pu)	0.17	0.037	0.258	0.197	0.037	0.14	0.018
Transmission Limit (MW)	200	130	200	80	124	100	100

Table 3: Generation/Load Shift Factors of the 6-Bus System.

Injection Line	G1	G2	G3	L1	L2	L3
Line1	0.482	-0.199	-0.152	0.168	0.032	-0.152
Line2	0.222	0.367	-0.482	0.531	0.101	-0.482
Line3	0.517	0.199	0.152	-0.168	-0.032	0.152
Line4	0.261	0.433	0.330	-0.364	-0.069	0.330
Line5	-0.222	-0.368	-0.518	0.469	0.899	-0.518
Line6	-0.222	-0.368	-0.518	0.469	-0.101	-0.518
Line7	0.222	0.368	-0.482	-0.469	0.101	-0.482

Table 4: Summary of the Optimization Results.

β	EADR (MW)	Operating Costs (\$)	Generation Costs (\$)	Reserve Costs (\$)
0	0	2077.45	2077.45	0.00
0.2	6.2	2113.61	2077.45	36.17
0.4	12.4	2150.26	2077.45	72.81
0.6	18.6	2200.69	2090.06	110.63
0.8	24.8	2267.14	2119.64	147.50
1	31	2361.63	2133.93	227.71

Table 5: BPs and PFs of the Units.

β	Participation Factors			Base Points (MW)		
	a_1	a_2	a_3	p_1	p_2	p_3
0	NA	NA	NA	164.7	88.7	56.5
0.2	0	1	0	164.7	88.7	56.5
0.4	0.093	0.907	0	164.7	88.7	56.5
0.6	0.274	0.726	0	158.8	92.1	59.1
0.8	0.274	0.726	0	142.9	101.8	65.3
1	0.016	0.726	0.258	146.4	96.1	67.5

Table 6: Summary of the Optimization Results with Severe Disturbance.

β	EADR (MW)	Operating Costs (\$)	Generation Costs (\$)	Reserve Costs (\$)
0	0	2077.45	2077.45	0.00
0.2	8.3	2125.61	2077.45	48.17
0.4	16.5	2175.97	2077.45	98.53
0.6	24.8	2227.58	2077.45	150.13
0.8	33	2313.06	2112.88	200.19
1	41.3	2447.03	2129.17	317.86

Table 7: BPs and PFs of the Units with Severe Disturbance.

β	Participation Factors			Base Points (MW)		
	a_1	a_2	a_3	p_1	p_2	p_3
0	NA	NA	NA	164.7	88.7	56.5
0.2	0	1	0	164.7	88.7	56.5
0.4	0.319	0.681	0	164.7	88.7	56.5
0.6	0.546	0.454	0	164.7	88.7	56.5
0.8	0.546	0.454	0	141.7	104.1	64.2
1	0.244	0.454	0.302	135	107.5	67.5

Table 8: Summary of the Optimization Results of the IEEE 118-Bus System.

β	EADR (MW)	Operating Costs (\$)	Generation Costs (\$)	Reserve Costs (\$)
0	0	3664.42	3664.42	0.00
0.2	10.83	3669.84	3664.42	5.42
0.4	21.66	3675.25	3664.42	10.83
0.6	32.50	3680.67	3664.42	16.25
0.8	43.33	3686.08	3664.42	21.66
1	54.16	3693.29	3665.17	28.12