A Multi-Agent Based Design of Bidding Mechanism for Transmission Loss Reduction

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Abstract

This paper investigates a new bidding mechanism to minimize the transmission losses in a day-ahead market based on generators' power utilization ratio. A reachable set based loss allocation method is developed to assign active losses to generating companies (GenCos). The line losses borne by GenCos are characterized and measured by a loss adjustment factor. Power is allocated to GenCos based on the virtual market clearing price calculated via an iterative power adjustment procedure, aiming at minimizing the overall transmission losses. Under the multi-agent game framework, we further analyze GenCos' responsive bidding strategy which has not been adequately addressed in literature. The performance of the proposed bidding mechanism is extensively tested on the IEEE 14-bus system. The results show that the proposed bidding mechanism is able to lower the overall transmission losses.

Keywords—Bayesian game theory, multi-agent system, power utilization ratio, support vector machine, virtual market clearing price.

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NOMENCLATURE

m	=	Number of GenCos
n	=	Number of nodes
$l_{\max}$	=	Maximum iterations
k	=	Index for GenCos, and $k=1, 2, \dots, m$
i, j	=	Index for nodes, and i or $j=1, 2, \dots, n$
l	=	Index for iterations, and $l=1, 2, \dots, l_{\max}$
D	=	Aggregate system demand
L	=	Aggregate system losses
$P(i, j)$	=	Active power flowing from nodes i to j (measured at node i)
$L(i, j)$	=	Line losses between adjacent nodes i and j
$L_k(i, j)$	=	Proportion of $L(i, j)$ incurred by GenCo k
L_k	=	Losses apportioned to GenCo k
$P(i)$	=	Active power injected into node i
$\phi_k(i)$	=	Percentage of the transmission losses injected into node i by GenCo k
$R_s(k)$	=	s -step reachable set of node k where a GenCo is located
r_s	=	s -step reachable nodes, and $r_s \in R_s(k)$
$\mathbf{Y}$	=	admittance matrix of the system
G_{ij}, B_{ij}	=	Conductance and susceptance in row i and column j in the admittance matrix
λ	=	Loss allotment coefficient
$\gamma_k^{(l)}$	=	Loss adjustment factor of GenCo k in iteration l
$L_k^{(l)}$	=	Losses assigned to GenCo k in iteration l
$L^{(l)}$	=	System losses in iteration l
$\omega^{(l)}$	=	Virtual market clearing price in iteration l
$PG_i^{(l)}, QG_i^{(l)}$	=	Active and reactive power supplied by GenCos for node i in iteration l
$V_i^{(l)}$	=	Voltage amplitude of node i in iteration l
$\theta_i^{(l)}$	=	Voltage phase angle of node i in iteration l
$\theta_{ij}^{(l)}$	=	Voltage angle between nodes i and j in iteration l
P_{Fi}, Q_{Fi}	=	Active and reactive load for node i
$P_k^{\min}, P_k^{\max}$	=	Active power capacity limit of GenCo k
$Q_k^{\min}, Q_k^{\max}$	=	Reactive power capacity limit of GenCo k
ρ_k	=	Bidding price of GenCo k
α_k, β_k	=	Bidding coefficient of GenCo k
P_k, L_k	=	Power and losses allocated to GenCo k
φ_k	=	Coefficient for high quotation for GenCo k
$q_k^{(b_k)}$	=	Probability for GenCo k to place a high ($b_k=1$) or a normal ($b_k=2$) bid
$\mathbf{b}$	=	Bidding strategy for all GenCos. $\mathbf{b}=[b_1, \dots, b_k, \dots, b_m]$
$E[\pi_k^{(b)}]$	=	expected profit for GenCo k when it chooses b_k , and its rivals choose $b_1, \dots, b_{k-1}, b_{k+1}, \dots, b_m$
$P_k(\mathbf{b}), L_k(\mathbf{b})$	=	Power and losses allocated to GenCo k under bidding strategy $\mathbf{b}$
c_0, c_1	=	Coefficients for the quadratic cost model
$\varepsilon_c, \varepsilon_L, \varepsilon_r$	=	Positive thresholds

1. INTRODUCTION

The hierarchical power generation system, though cost-effective, typically incurs 5-10% of energy

losses during the long-haul transmission and distribution process [1]. Driven by the requirements for high energy efficiency and environmental sustainability, reducing the transmission losses has become one of the priorities in power system. Current approaches to loss reduction mainly focus on installing energy-efficient equipment [2], reconfiguring the network topology [3], integrating distributed or onsite generators [4], [5] and optimizing dispatch scheduling [6]. Despite the technological variations, all these approaches aim to improve energy efficiency by mitigating the transmission losses through the enhancement of physical and operational performance.

As the vertically integrated power industry evolves into a deregulated and competitive paradigm, new methodologies for mitigating transmission losses have been proposed such that a higher level of energy delivery efficiency can be attained via price signals. Lu and Brammer [7] introduce a loss penalty factor to reflect the distance between the generating companies (GenCos) and the loading points. The underlying rationality is that the actual losses are highly correlated with GenCos' proximity to the loading points. An optimal economical dispatch can be achieved by penalizing the GenCos whose bids may lead to larger transmission losses. Later, the penalty factors are widely used to facilitate the locational marginal price calculation, or determine the nodal electricity rate in order to convey efficient generator dispatching and siting [8-11]. Recently, reducing transmission losses via market transaction mechanism also have sparked an active research. Zhang et al. [12] propose an adjustable bidding price to reflect the GenCos' contribution to the system losses. Wang et al. [13] develop a loss modification method through which each GenCo's bidding price can be adjusted in accordance with its generation losses, and the method is proved suitable in Northeast China area electricity market. In those studies, the resulting loss allocation is highly dependent on the selection of the marginal GenCo or the slack bus. As such, it might be possible that an unpredictable or even distorted dispatch decision is made due to the high nonlinearity between the aggregate transactions and the load flow equations.

A prerequisite for designing the loss-considered transaction mechanism requires the implementation of a fair and transparent loss allocation scheme, with which a precise assessment of the energy efficiency of individual Gencos can be made and an efficient dispatch and siting incentive could be

further deployed. Current loss allocation methods can be classified into four categories: the pro-rata method [14], the loss formula method [15-17], the incremental method [18-20], and the flow tracing method [21-24]. The pro-rata method is to assign the system losses to the generators and/or loads in proportion to their active power or demand profile. Since pro-rata neglects the effects of the network topology, it may lead to a substantially different result compared to other methods [25]. In the loss formula method [15-17], both the power and the losses are explicitly expressed in terms of system transactions. Quite often a complex procedure is required to solve the underlying mathematical models. The incremental allocation method [18-20] is designed to approximate the system losses through the first order sensitivity analysis with respect to the nodal real power and loads. Though the idea is widely used in most of the above-mentioned loss-considered transaction studies [7-13], this method may be volatile and subject to discriminatory judgments as its performance largely depends on the selection of the slack bus or the marginal GenCo. Accordingly, the loss-considered transactions that utilize the incremental method may also result in a volatile and uncertain pricing policy for losses reduction. The flow tracing method [21-24], also called proportional sharing method, states that “the power flow reaching a bus from any power line splits among the lines evacuating power from the bus proportionally to their corresponding power flows” [25]. Though the principle has not been proved yet, this method is shown to be quite accurate to allocate losses to GenCos who are geographically dispersed. In fact, Bialek [21] indicates that the flow tracing method is relatively independent to the selection of the marginal generator, and it always results in a positive monetary charge on the losses.

Under multi-agent system framework, this paper proposes a novel bidding mechanism to minimize the transmission losses in a day-ahead electricity market. GenCos place their bidding curve based on the net power for meeting the demand of loading points. A reachable set based power flow tracing method is devised to assign the losses to each GenCo after the bid is received by ISO. We propose a sequential adjustment procedure that enables the independent system operator (ISO) to make an energy-efficient power allocation decision based on GenCos’ power utilization ratio. The actual bidding curve viewed by the ISO is augmented by multiplying a loss adjustment coefficient which translates the GenCo’s generation

losses into a competitive effect. The actual traded power comprises the net power and the transmission losses, yet they are aggregated through the loss adjustment factor and not differentiable by the GenCos. The proposed multi-agent bidding mechanism not only minimizes the system losses, but also motivates strategically-behaving GenCos to maximize their profit by adopting responsive bidding policy. Hence a win-win outcome is expected by all the market participants.

The remainder of the paper is organized as follows: Section III presents the multi-agent bidding framework and describes the reachable set based power flow tracing method. Section IV designs and implements the loss-considered bidding mechanism. In Section V we analyze GenCo's strategic bidding policy to maximize its profit. In Section VI, we carry out a comprehensive test of the new bidding mechanism on IEEE 14-bus system. Section VII concludes the paper.

2. FRAMEWORK OF MULTI-AGENT BIDDING SYSTEM

2.1 System overview

The transmission network under study consists of m GenCos who generate the power to meet the total demand. Let L_k and P_k , respectively, be the transmission loss and the supplied power from GenCo k for $k=1, 2, \dots, m$. Then, the actual power utilization ratio for GenCo k can be expressed as $\eta_k=1-L_k/P_k$ and its value varies between $[0, 1]$. Obviously, if $\eta_k=1$, it means there are no transmission losses. Theoretically, the overall system losses could be reduced by increasing the power output of the GenCos with high η_k , or equivalently reducing the output of low utilization units. Due to the complex nonlinear relation between L_k and P_k , in reality η_k is rarely used as a key criterion in evaluating the competitiveness of individual GenCos.

In this paper, a loss-embedded bidding mechanism is proposed to reduce the system transmission losses by explicitly incorporating GenCo's power utilization ratio in a multi-agent system (MAS) framework. The proposed MAS bidding mechanism incorporates the common characteristics of day-ahead market, such as price bidding, bilateral transactions and market clearing price. The market is comprised of

three types of autonomous agents: ISO, GenCos, and customers. The functionality of these agents is briefly described below.

There is only one ISO agent in the MAS market and the role of the ISO is to design and implement the loss-considered bidding mechanism. To accommodate the power utilization ratio, a mathematical programming model is formulated to guide the ISO to optimize the allocation of the power and losses for each GenCo. In addition, there is a group of self-interested and competitive GenCos in the market pool. To gain the competitive edge, some of them behave strategically by adjusting their bidding curves to maximize the profit. The only customer agent that exists in the market is a group of energy consumers who are responsible for forecasting the electricity demand and reporting the consumption data to the ISO.

Depending on the role of the agents, there are two decision-support systems implemented in the MAS framework: 1) a day-ahead bidding mechanism that minimizes the overall system losses; and 2) a responsive bidding policy that maximizes GenCo's profit. The former is owned and operated by the ISO, and the latter pertains to those strategically behaving GenCos. To this end, a reachable set based (or RS-based) power flow tracing method is used to allocate the transmission losses to individual GenCos. Bayesian game theory combined with support vector machine is used to infer the posterior bidding strategy of rival GenCos.

2.1 RS-based power flow tracing method

Existing flow tracing methods [21-24], though effective and accurate, usually require complex mathematical operations, such as buses re-grouping or matrix inversions. These operations could be time-consuming, especially for large-scale systems. The RS-based power flow tracing method, can avoid these complex mathematical operation, yet without compromising the computational accuracy.

To illustrate the working principle of RS-based flow tracing method, consider a simple network consisting of seven nodes, one GenCo and two additional power sources shown in Fig. 1. Note that the GenCo at node 1 is denoted as GenCo 1 with output power $P(1)$.

Fig. 1: A Simple Power Flow Chart

Consider a line i - j where active power flows from node i into node j . Let $P(i, j)$ denote the active power flowing from i to j (measured in i). Then $|P(i, j)|$ is the non-negative value of the active power that is carried at the sending node i , and $|P(j, i)|$ is the non-negative value of the active power carried at the receiving node j . Let $L(i, j)$ represents the transmission losses across the line i - j . Then

$$L(i, j) = \{P(i, j) + P(j, i)\}^+, \quad (1)$$

where, $\{P(i, j) + P(j, i)\}^+$ denotes a non-negative value of $P(r_s, i) + P(i, r_s)$ when $P(r_s, i) + P(i, r_s) > 0$; or $L(i, j) = 0$ if $P(r_s, i) + P(i, r_s) \leq 0$. Let $L(i)$ represent the aggregate losses for node i which is cumulatively carried by the power flows leaving node i . For example, $L(1) = L(1, 4) + L(1, 5)$.

Assuming node k is a power generation node, and let $L_k(i)$ represent the proportion of $L(i)$ associated with GenCo k , and $\phi_k(i)$ denotes GenCo k 's share of the losses in percentage at node i . Since the power leaving from node 1 may not be fully provided by GenCo 1 (i.e. GenCo may only provide a certain percentage of power). Based on the share proportionality principle, we have

$$\phi_1(1) = \frac{P(1)}{P(1) + |P(1, 2)| + |P(1, 3)|}, \quad (2)$$

where $L_1(1)$ can be estimated as $L_1(1) = \phi_1(1)L(1)$. Similarly, $L_1(4)$ can be estimated as

$$L_1(4) = \phi_1(1)\phi_1(4)L(4), \quad (3)$$

with

$$\phi_1(4) = \frac{|P(4, 1)|}{|P(4, 1)| + |P(4, 6)| + P(4)}. \quad (4)$$

Let L_k denotes the losses apportioned to GenCo k . Then, the aggregate transmission losses incurred by GenCo 1 can be obtained as

$$L_1 = L_1(1) + L_1(4) + L_1(7) + L_1(5) \quad (5)$$

Obviously a GenCo needs to share the nodal losses for all the reachable nodes where injected power, more or less, is supplied by the same GenCo. If we treat the entire transmission network as a directed graph (see Fig. 1), any node whose injected power is directly or indirectly supplied by a generation node k can be characterized by a reachable node where the generation node k conveyances within limited steps. Let $R_s(k)$ denotes the set of nodes reachable by generation node k within s steps. Then

$$R_s(k) = \{i | iR^s k\}, \text{ for } i=1, 2, \dots, n \quad (6)$$

where $iR^s k$ denotes the power injected into node i is supplied by the generation node k via $(s-1)$ nodes' conveyance. Obviously, $1 \in R_0(1)$, $4 \in R_1(1)$, $5 \in R_1(1)$, and $7 \in R_2(1)$ for node 1. Given a network with n nodes, the RS-based loss allocation model for GenCo k now can be expressed as

$$L_k = \sum_{s=0}^{s_{\max}} L_k(r_s) = \sum_{s=0}^{s_{\max}} \sum_{r_s \in R_s(k)} \sum_{i=1}^n L(r_s, i) \phi_k(r_s) \quad (7)$$

with

$$L(r_s, i) = \{P(r_s, i) + P(i, r_s)\}^+, \text{ for } r_s \in R_s(k) \quad (8)$$

$$P(r_s, i) = \dot{V}_{r_s} Y_{r_s i} (\dot{V}_{r_s} - \dot{V}_i) \quad (9)$$

$$\phi_k(r_s) = \frac{\sum_{z=0}^{s-1} \sum_{r_z \in R_z(k)} |P(r_s, r_z)| \phi_k(r_z)}{P(r_s) + \sum_{i=1}^n |P(r_s, i)|}, \text{ for } 1 \leq s \leq s_{\max} \quad (10)$$

$$\phi_k(r_0) = \frac{P(r_0)}{P(r_0) + \sum_{i=1}^n |P(r_0, i)|}, \text{ for } r_0 \in R_0(k), \text{ and } r_0 = k \quad (11)$$

Eq. (7) computes the losses of the line that connects nodes i with r_s where $r_s \in R_s(k)$. Note that $s_{\max}$ is the maximum steps that the power of GenCo k can reach in the network. Eq. (8) simply calculates the losses incurred by line r_s - i . Eq. (9) computes the line power between nodes r_s and i . Here $\dot{V}_{r_s}$ and $\dot{V}_i$ are the

corresponding node voltage, and $\dot{Y}_{r,i}$ is the admittance between nodes r_s and i . Eqs. (10) and (11) compute the amount of losses at node r_s shared by GenCo k .

3. DESIGN OF LOSS MINIMIZATION BIDDING MECHANISM

3.1 General process of the bidding mechanism

From the perspective of ISO, the bidding mechanism that minimizes the total transmission losses consists of three core components: 1) a loss minimization model; 2) an iterative adjustment procedure; and 3) a RS-based loss allocation scheme. The loss minimization model aims to increase the power output of GenCos who possess higher utilization ratios. The iterative adjustment procedure is an optimization algorithm that sequentially searches for the optimal loss adjustment factors. The RS-based loss allocation scheme traces the line power and assigns the losses to each GenCos based on its power utilization ratio. For a given period t , the general procedure of the multi-agent bidding mechanism can be summarized as follows:

1). Initialization. Initial values are assigned to individual GenCos and the ISO. These values include bidding curves, GenCos' generating capacity, load predictions, loss allotment coefficient, and other technical constraints.

2). Submitting Bids. GenCos submit their bids based on the bidding curve that captures the linear relation between the generation capacity and the price.

3). Optimal Power Allocation. Based on power utilization ratio, the ISO determines the optimal power allocation policy among GenCos via the iterative adjustment procedure.

4). Engaging Transactions. The ISO engages the bilateral transactions with each GenCo at the price calculated based on the actual bidding curve.

5). Paying for power and losses: The ISO makes the payment to the GenCo, and the total compensations include the fees for purchasing the actual power and the losses.

This process is executed repeatedly as time evolves into the next period. Before we present the loss minimization model, several new terms including loss allotment coefficient, loss adjustment factor, and virtual market clearing price, are defined below.

3.2 Definitions of new terms

Let $\lambda \in [0, 1]$ be a loss allotment coefficient representing the percentage of the losses allocated between GenCos and ISO who represents energy consumers. In particular, $\lambda \times 100\%$ of transmission losses cost would be compensated by the ISO and the remaining part should be borne by the GenCos. Let ρ_k be the bidding price. The linear bidding curve of GenCo k is $\rho_k = \alpha_k + \beta_k P_k$ where α_k and β_k are bidding coefficients. Suppose the cost of transmission losses incurred by GenCo k is $\rho_k L_k$. Then, the ISO would pay $\lambda \rho_k L_k$ as the partial loss compensation to GenCo k .

Loss adjustment factor, denoted as γ_k , intends to incorporate the cost of transmission losses incurred by GenCo k into the bidding curve. When ISO purchases power P_k from GenCo k whose bidding curve is $\rho_k = \alpha_k + \beta_k P_k$, a regular payment $\rho_k P_k$ plus an additional fee $\lambda \rho_k L_k$ would be made to the GenCo. Hence the actual bidding curve perceived from the ISO becomes $\rho_k = \gamma_k (\alpha_k + \beta_k P_k)$ which consists of the actual power purchasing fee and the loss compensation fee. If L_k/P_k remains constant over time, then γ_k can be easily estimated as

$$\gamma_k = 1 + \lambda L_k / P_k, \text{ for } k=1, 2, \dots, m. \quad (12)$$

Since L_k is a nonlinear function of P_k , the actual value L_k/P_k varies over time. The following iterative equation is provided to find the approximated value of γ_k .

$$\gamma_k = \gamma_k^{(l)}, \quad \text{for } l=1, 2, 3, \dots \quad (13)$$

with

$$L_k^{(l-1)} / P_k^{(l-1)} = L_k^{(l)} / P_k^{(l)} \quad (14)$$

$$\gamma_k^{(l)} = \frac{(\alpha_k + \beta_k P_k^{(l-1)})(P_k^{(l-1)} + \lambda L_k^{(l-1)})}{(\alpha_k + \beta_k P_k^{(l-1)})P_k^{(l-1)}} \quad (15)$$

The detailed derivation of γ_k for each iteration as described in Eq. (15) is provided in the Appendix. Based on Eqs. (13)-(15), an iterative procedure that enables the ISO to search for the convergent value of γ_k will be presented in Section 3.4.

The virtual market clearing price, denoted as ω , is not the actual transaction price, rather it is a reference price with which the ISO can perform power re-allocation taking into account the transmission losses of each GenCo. The actual price transacted with a particular Genco is consistent with the GenCo's bidding curve. Once γ_k is obtained, ω in iteration l , denoted as $\omega^{(l)}$, can be derived based on the following equation

$$\omega^{(l)} = \gamma_k^{(l)} (\alpha_k + \beta_k P_k^{(l)}) \quad (16)$$

$$\text{Subject to:} \quad \sum_{k=1}^m P_k^{(l)} = D \quad (17)$$

Where, $\gamma_k^{(l)}$ can be estimated from Eq. (15), and D is the aggregate power demand of the system. In each iteration, different $\gamma_k^{(l)}$ may yield different values of $\omega^{(l)}$ which further generates different $P_k^{(l)}$ and $L_k^{(l)}$. If $L_k^{(l)} / P_k^{(l)} \cong L_k^{(l-1)} / P_k^{(l-1)}$, the value $\gamma_k^{(l)}$ can be treated as a good approximation to its true value γ_k . We use the following evolutionary process to illustrate the dynamic interactions among γ_k , L_k and P_k :

$$P_k^{(0)}, L_k^{(0)}, \alpha_k, \beta_k \rightarrow \gamma_k^{(1)} = (1 + \lambda L_k^{(0)} / P_k^{(0)}) \rightarrow P_k^{(1)} \rightarrow L_k^{(1)} \rightarrow \dots \rightarrow \gamma_k^{(l)} = (1 + \lambda L_k^{(l-1)} / P_k^{(l-1)}) \rightarrow P_k^{(l)} \rightarrow L_k^{(l)} \rightarrow \dots \quad (18)$$

Where l is the iteration number, and $P_k^{(l)}$ and $L_k^{(l)}$ are the power and the losses assigned to GenCo k in iteration l , respectively. If $L_k^{(l-1)} / P_k^{(l-1)} \cong L_k^{(l)} / P_k^{(l)}$, it is reasonable to assume that $\gamma_k^{(l)}$ is a deterministic value. The evolutionary process is illustrated in Eq. (18). Initially $P_k^{(0)}$ is calculated using Eqs. (16) and (17) with the value of $\gamma_k^{(0)} = 1$, and $L_k^{(0)}$ is computed using the Newton based power flow method. The process evolves to the next step, $\gamma_k^{(1)}$ is obtained using Eq. (15), and $P_k^{(1)}$ is calculated using Eqs. (16) and (17), and so is $L_k^{(1)}$. The dynamic interactions continue until the condition in Eq. (14) is satisfied.

3.3 Minimizing transmission losses

Based on the power utilization ratio, we present an optimization model, denoted as Problem P1, to minimize the total transmission losses through the power reallocation among m GenCos. That is

Problem P1:

$$\text{Min: } L = \sum_{k=1}^m L_k \quad (19)$$

Subject to:

$$(\alpha_k + \beta_k P_k) = \rho_k, \text{ for all } k \quad (20)$$

$$\gamma_k (\alpha_k + \beta_k P_k) = \omega, \text{ for all } k \quad (21)$$

$$(\alpha_k + \beta_k P_k)(P_k + \lambda L_k) = \gamma_k (\alpha_k + \beta_k P_k) P_k, \text{ for all } k \quad (22)$$

$$\sum_{k=1}^m P_k = D \quad (23)$$

The objective function (19) intends to minimize the total transmission losses among m GenCos. Constraint (20) calculates the transaction price for GenCo k for $k=1, 2, \dots, m$. Constraint (21) is the virtual market clearing price with which the ISO performs power reallocation based on the power utilization ratios of individual GenCos. Constraint (22) represents the payment for the power and the losses for GenCo k . Constraint (23) ensures that the system demand is always satisfied. The solution for Problem P1 depends on the value γ_k which shall be obtained by solving the following non-linear programming model.

Problem P2:

$$\gamma_k = \arg \min_{\gamma_k^{(l)}} \{L^{(l)}\} \quad (24)$$

Subject to:

$$(\alpha_k + \beta_k P_k^{(0)})(P_k^{(0)} + \lambda L_k^{(0)}) = \gamma_k^{(l)} (\alpha_k + \beta_k P_k^{(l-1)}) P_k^{(l-1)} \quad (25)$$

$$P_{Gi}^{(l)} - P_{Fi} = V_i^{(l)} \sum_{j=1}^n V_j^{(l)} (G_{ij} \cos \theta_{ij}^{(l)} + B_{ij} \sin \theta_{ij}^{(l)}) \quad (26)$$

$$Q_{Gi}^{(l)} - Q_{Fi} = V_i^{(l)} \sum_{j=1}^n V_j^{(l)} (G_{ij} \sin \theta_{ij}^{(l)} - B_{ij} \cos \theta_{ij}^{(l)}) \quad (27)$$

$$\gamma_k^{(l)}(\alpha_k + \beta_k P_k^{(l)}) = \omega^{(l)} \quad (28)$$

$$\sum_{k=1}^m P_k^{(l)} = \sum_{i=1}^n P_{Fi} = D \quad (29)$$

$$\sum_{k=1}^m P_{Gk}^{(l)} = \sum_{k=1}^m (P_k^{(l)} + L_k^{(l)}) \quad (30)$$

$$P_k^{\min} \leq P_k^{(l)} \leq P_k^{\max}, \quad \text{for all } k \quad (31)$$

$$Q_k^{\min} \leq Q_k^{(l)} \leq Q_k^{\max}, \quad \text{for all } k \quad (32)$$

$$\Delta L = L^{(l-1)} - L^{(l)} \leq \varepsilon_L \quad (33)$$

$$|\gamma_k^{(l)} - \gamma_k^{(l-1)}| \geq \varepsilon_r \quad (34)$$

$$1 \leq l \leq l_{\max} \quad (35)$$

$$\Delta C = \sum_{k=1}^m \left((\alpha_k + \beta_k P_k^{(l)})(P_k^{(l)} + \lambda L_k^{(l)}) - (\alpha_k + \beta_k P_k^{(0)})(P_k^{(0)} + \lambda L_k^{(0)}) \right) \leq \varepsilon_c \quad (36)$$

The objective of Problem P2 is to find the best approximation for γ_k using the iterative adjustment procedure. $L^{(l)}$ is the total system losses in iteration l . Constraint (25) prescribes the equivalent electricity purchase formula with losses included. Constraints (26) and (27) are the fundamental active and reactive power flow equations, respectively. $P_{Gi}^{(l)}$ and $Q_{Gi}^{(l)}$ are active and reactive power supplied by GenCos for node i in iteration l , respectively. P_{Fi} and Q_{Fi} are active and reactive load for node i , respectively. $V_i^{(l)}$ is voltage amplitude of node i in iteration l , and $\theta_{ij}^{(l)}$ is the voltage angle between nodes i and j . Constraints (28) and (29) compute ω and P_k in iteration l . Constraint (30) ensures both the power and the allocated losses are borne by the GenCos. Constraints (31) and (32) define the active and reactive generation capacity for each GenCo, respectively. $P_k^{\min}$ and $P_k^{\max}$ are the lower and the upper limit of active power capacity of GenCo k , and $Q_k^{\min}$ and $Q_k^{\max}$ are the lower and the upper limit of reactive power capacity, respectively. Constraints (33)-(36) are the termination conditions. In particular, constraint (36) states that a significant amount of cost growth is prohibited in order to reduce the system losses. Note that ε_L , ε_r , and ε_c are the positive thresholds, respectively.

Fig 2. Flow Chart for Iterative Adjustment Procedure

3.4 The iterative adjustment procedure

The purpose of the iterative adjustment procedure is to find an optimal power allocation scheme for all the GenCos so that the total system losses in Eq. (19) are minimized. The procedure consists of three steps including initialization, iterative search, and result output. The detailed steps are illustrated in the flow

chart in Fig. 2. Note that the swing bus is designed to be temporarily assigned all the generated losses in the Newton based power flow method when $L_k=0$ (for all k). During the loss allocation process, the deviated loss should be given to the GenCo with the minimum loss adjustment factor.

4. ANALYSIS OF STRATEGIC BEHAVIOR OF GENCOS

Under the loss-embedded bidding mechanism, GenCos are motivated to rethink and adjust their bidding strategies to gain the market competitiveness. In this section, we study how a GenCo turns itself into a strategic player to maximize its profit by conceiving a responsive bidding policy. To make the problem tractable, the following assumptions are made:

1). Since a bid above the marginal cost may reap extra profits, each GenCo has two bidding options: a high quotation (or bidding price) in the form of $\rho_k = \varphi_k(\alpha_k + \beta_k P_k)$ with $1 < \varphi_k \leq 1.5$, and a normal quotation in the form of $\rho_k = \alpha_k + \beta_k P_k$.

2). Besides the supply to demand ratio, another factor that influences the GenCos' decision is the ratio of the adjacent load to the system load. It is very likely that a GenCo may opt to place a high quotation when it is located near the load center owing to a lower transmission loss.

3). The probability for GenCo k to choose a bid b_k is denoted as $q_k^{(b_k)}$. Two bidding options are considered: $b_k=1$ represents a high quotation, and $b_k=2$ is for a normal quotation. A strategically behaving GenCo can deduce the bidding probabilities of its rivals based on market information and experiences[26].

4.1 Estimating posterior bidding probabilities

Given the historical bidding data, the immediate problem encountered by a strategically behaving GenCo is how to estimate the bidding probabilities of high and normal quotations of its rivals. We adopt the probabilistic support vector machine (SVM) technique to infer the posterior bidding probability. Following the method in [27], [28], the posterior probability can be obtained by using the following approach,

$$\Pr\{y=1 | f\} = [1 + \exp(Af + B)]^{-1}, \quad (37)$$

where f is an output score of the SVM decision function and y is a binary class label. If $y=1$, a high quotation bid is placed; and $y=2$, a normal quotation is placed by the GenCo. Note that A and B are model parameters which can be obtained by maximizing the following log-likelihood function,

$$g(A, B) = \sum_{i=1}^N \left[\frac{y_i + 1}{2} \log([1 + \exp(Af_i + B)]^{-1}) \right] + \sum_{i=1}^N \left[\frac{y_i - 1}{2} \log(\exp(Af_i + B) \times [1 + \exp(Af_i + B)]^{-1}) \right]. \quad (38)$$

Here $\{y_i, f_i\}$ are pairs of training data for $i=1, 2, \dots, N$ with N being the number of pairs. Using the SVM approach by Lin et al. [27] and Cheng et al. [28], the posterior probability of Gencos' bidding strategy can be estimated.

4.2 Modeling the strategically behaving GenCo

Each GenCo in the MAS framework can place a high bid (i.e. $b_k=1$) or a normal bid (i.e. $b_k=2$). Let $\mathbf{b}=[b_1, b_2, \dots, b_m]$ be the vector denoting the bidding outcome for m GenCos. For the strategically behaving GenCo k who strive to maximize its profit subject to its rivals' uncertain decisions, the optimal bidding strategy can be determined as follows based on Bayesian game theory:

$$\arg \max_{b_k} \{E[\pi_k^{(b_k)}]\}, \quad (39)$$

with

$$E[\pi_k^{(b_k)}] = \sum_{\substack{b_1=1 \\ \vdots \\ b_{m-1}=1}}^2 \cdots \sum_{b_m=1}^2 \left[\prod_{h=1, h \neq k}^m q_h^{(b_h)} (\rho_k(P_k(\mathbf{b}) + L_k(\mathbf{b})) - C_k) \right], \quad (40)$$

$$C_k = c_0(P_k(\mathbf{b}) + L_k(\mathbf{b})) + c_1(P_k(\mathbf{b}) + L_k(\mathbf{b}))^2. \quad (41)$$

Where, $q_k^{(b_k)}$ is the posterior probability for GenCo k to place a high ($b_k=1$) or a normal ($b_k=2$) bid, and $E[\pi_k^{(b_k)}]$ is the expected profit when GenCo k chooses b_k , and its rivals choose $b_1, \dots, b_{k-1}, b_{k+1}, \dots, b_m$. Note that $P_k(\mathbf{b})$ and $L_k(\mathbf{b})$ are, respectively, the power and the loss allocated to GenCo k when it chooses b_k , and its rivals choose $b_1, \dots, b_{k-1}, b_{k+1}, \dots, b_m$, respectively. Eq. (41) is the quadratic cost model where c_0 and c_1 are model parameters. In this paper, we leverage Eqs. (37) and (38) to compute the posterior probabilities, and the optimal b_k is derived by solving Eqs. (39)-(41).

5. NUMERICAL EXPERIMENTS

5.1 System description

The IEEE 14-bus system is chosen as the test bed to demonstrate the performance of the proposed bidding mechanism. Two load profiles are considered and the electric demand of individual buses is listed in Table 1. The total system demands in both profiles are the same with $D=186$ MW. One significant distinction between Profiles 1 and 2 is that the load on bus 3 in the former is 94.2 MW which is 128.64% higher than the latter. The purpose is to compare the bidding strategies when a GenCo is located in a close proximity to the load center.

Table 1: Load Demand for Different Buses (unit: MW)

Bus	Notation	Profile 1	Profile 2
2	D_2	21.7	21.7
3	D_3	94.2	41.2
4	D_4	17.8	17.8
5	D_5	4.6	7.6
6	D_6	1.2	11.2
9	D_9	19.5	39.5
10	D_{10}	9.0	9.0
11	D_{11}	3.5	3.5
12	D_{12}	6.1	6.1
13	D_{13}	3.5	13.5
14	D_{14}	4.9	14.9
Total	D	186	186

There are five GenCos scattered in the network, the active power capacity of GenCos 1, 2, 4, 5 are 46 MW, and that of GenCo 3 is 48.5 MW. The coefficients of their bidding curves are listed in Table 2. Eleven cases are investigated and the results are compared to show how the system performance is affected subject to different load profiles, bidding curves, and loss allotment coefficients. In all the cases, the unit of power is MW, and the units of other parameters are p.u.

Table 2: Coefficients of Bidding Curves in Different Cases

Bus	GenCo	Cases	1-3, 8	4	5	6	7
1	1	α_1	1.5	1.575	1.5	1.575	1.575
		$\beta_1 (\times 10^{-3})$	9	9.454	9	9.454	9.45
2	2	α_2	1.5	1.575	1.5	1.575	1.5

		$\beta_2 (\times 10^{-3})$	9	9.45	9	9.45	9
3	3	α_3	1.5	1.5	1.575	1.575	1.5
		$\beta_3 (\times 10^{-3})$	9	9	9.45	9.45	9
6	4	α_1	1.5	1.575	1.5	1.575	1.5
		$\beta_4 (\times 10^{-3})$	9	9.45	9	9.45	9
8	5	α_5	1.5	1.575	1.5	1.575	1.5
		$\beta_5 (\times 10^{-3})$	9	9.45	9	9.45	9

5.2 Impact of load profile on power allocation

In Case 1, we want to demonstrate that the proposed bidding mechanism is capable of lowering the overall system losses. The load Profile 1 is taken as the inputs and the results are listed in Table 3. When GenCos' losses are ignored (i.e. $l=1$), the virtual market clearing price is $\omega^{(1)}=1.835$ under the normal bidding curves. The allocated power of five GenCos is the same, namely 37.2 MW. The amount of the system losses is 1.983 MW. After taking into account GenCos' losses effects (i.e. $l=2$), $\omega^{(2)}$ becomes 1.854, and the allocated power for GenCos 1, 2 and 4 is reduced to 35.25, 36.81 and 36.54 MW while the allocated power for GenCos 3 and 5 increases, becoming 39.33 MW and 38.07 MW, respectively. As the direct benefit, the overall system losses decrease from 1.983 to 1.884 MW, resulting in a cost saving of $345.55-345.39=0.16$ pu. In this case, $l=2$ is also the last iteration.

Table 3: Results of Power Reallocation for Case 1 (n/a=not applicable)

GenCo k		1	2	3	4	5	Total
P_k	$l=1$	37.2	37.2	37.2	37.2	37.2	186
	$l=2$	35.25	36.81	39.33	36.54	38.07	186
L_k	$l=1$	0.768	0.466	0	0.519	0.23	1.983
	$l=2$	0.684	0.451	0	0.51	0.239	1.884
ρ_k	$l=1$	1.841	1.839	1.835	1.839	1.837	n/a
	$l=2$	1.823	1.835	1.854	1.833	1.845	n/a
Electricity Payment		Before power reallocation					345.55
		After power reallocation					345.39

Table 4: Results of Power Reallocation for Case 2

GenCo k		1	2	3	4	5	Total
P_k	$l=1$	37.2	37.2	37.2	37.2	37.2	186
	$l=2$	35.78	37.11	37.51	36.58	39.01	186
L_k	$l=1$	0.662	0.408	0.332	0.508	0.054	1.964
	$l=2$	0.608	0.406	0.342	0.5	0.059	1.915
ρ_k	$l=1$	1.841	1.838	1.838	1.839	1.835	n/a

	$l=2$	1.827	1.838	1.841	1.834	1.852	n/a
Electricity Payment	Before power reallocation						345.54
	After power reallocation						345.47

In Case 2, we use the load Profile 2 to test the loss reduction effect when the bus loads are more evenly distributed across the network compared to Profile 1, and the results are listed in Table 4. With no surprise, the total losses in Case 2 are also reduced under the new bidding mechanism. Case 2 differs from Case 1 in that a GenCo near the load center is more likely to be allocated with a larger quota of power. In Case 1, the load of bus 3 where GenCo 3 is sited is $D_3=94.2$ MW, and the allocated power of GenCo 3 increases from 37.2 to 39.33 MW under the loss consideration, or increased by 5.73%. While in Case 2, since D_3 is only 41.2 MW compared to 94.2 MW in Case 1, the power increment of GenCo 3 is only 0.83%, growing from 37.2 to 37.51 MW.

5.3 Impact of loss allotment coefficient on power allocation

In Case 3, we explore how the loss allotment coefficient λ influences the reduction of overall system losses. Without loss of generality, we use the load Profile 1 to analyze the relation between λ and the amount of losses reduced. Fig. 3 shows that the percentage of the loss reduction increases with λ . Up to 5.02% losses can be saved in the extreme situation with $\lambda=1$. In general, the larger the λ is, the higher the electricity purchase cost would be. For a high λ , the ISO has to implement a more aggressive loss reduction policy in order to lower the total electricity purchase cost.

Fig. 3 Percentage of system loss reduction versus λ

5.4 Impact of bidding curve on power reallocation

Cases 4-7 are designed to evaluate the impacts of the bidding curves on power reallocation, and the results of power and loss allocations are summarized in Table 5. Without loss of generality, we set $\lambda=1$ and use load Profile 1 for the comparative analysis. All the results are compared to Case 1 which serves as the baseline allocation where all GenCos choose the normal quotation. In Case 4, only GenCo 3 selects the normal quotation and its rivals choose the high quotation. It is found that the power allocated to GenCo 3 is 47.33 MW, i.e. 20.3% higher than the baseline case. In Case 5, we assume GenCo 3 selects the high quotation while all its rivals choose the normal quotation. It is found that the power allocated to GenCo 3 is only 31.59 MW, almost 20% less than the baseline case. In Case 6, we let all the GenCos place high quotation, and the resulting power allocation is the same as Case 1 where all of them place a normal quotation. In Case 7, if GenCo 1 is the only generator placing a high quotation, its allocated power drops significantly compared to all other cases. These different bidding outcomes show that the amount of power allocated to a specific GenCo is jointly determined by its own bid, its rivals' bid, and the relative distance to the load center. In all these cases, there are no losses incurred by GenCo 3 since it is located at Node 3 which shares a major portion of the system load.

Table 5: Comparison of Power Reallocation for Cases 1,4-7 (unit: MW)

Case No		1	4	5	6	7
GenCo 1	P_1	35.25	33.31	37.10	35.25	27.55
	L_1	0.684	0.614	0.764	0.684	0.518
GenCo 2	P_2	36.81	34.83	38.74	36.81	38.84
	L_2	0.451	0.395	0.515	0.451	0.452
GenCo 3	P_3	39.33	47.33	31.59	39.33	41.26
	L_3	0	0	0	0	0
GenCo 4	P_4	36.54	34.45	38.57	36.54	38.44
	L_4	0.510	0.468	0.559	0.510	0.537
GenCo 5	P_5	38.07	36.09	40	38.07	39.90
	L_5	0.239	0.192	0.295	0.239	0.267

5.5 Performance under multi-period bidding

In Case 8, a 48-hour simulation experiment is carried out to analyze the performance of the bidding mechanism subject to a multi-period bidding process. To mimic the system demand variability, D_3 varies as a uniform random variable between [94.2, 124.2] while the demand of other nodes remains constant. As a

result, the hourly demand D varies between [186, 216]. We compute the system losses for each hour using the proposed bidding mechanism. As shown in Fig. 4, the total system losses could be reduced upon the power adjustment.

Fig. 4 Variation of losses in 48 hours

Fig. 5 shows the hourly power allocation to each GenCo over a 48-hour period after incorporating the transmission losses. If we compare Figs. 5 to 4, it is interesting to see that the power output of individual GenCo resembles the same pattern of the system load profile.

Fig. 5 Variation of power output for GenCos with losses included

Fig. 6 shows the hourly data for GenCo's net power (excluding losses) over 48 hours. As can be seen, when the load on bus 3 becomes larger, more power is allocated to GenCo 3 due to its proximity to bus 3 even though all the GenCos have the same bidding curves.

Fig. 6 Variation of each GenCo's net power output

We further extend the experiment to a 30-day period, and the daily system losses are computed and aggregated based on the cumulative losses of every 24 hours. Fig. 7 depicts the daily system losses over 30 days, and shows that the proposed loss-based bidding mechanism is able to alleviate the transmission losses consistently by 4.8% over the entire horizon.

Fig. 7 Variation of losses in 30 days

5.6 Analyzing GenCos' responsive bidding strategy

For simplicity, it is assumed that GenCos 1 and 3 are the only two competitive players with identical bidding curves: the bidding curve under the normal quotation is $1.5+0.009P_k$ and under high quotation is $1.575+0.00945P_k$ for $k=1$ and 3. GenCo 1 behaves strategically and a set of 30-day market data are used to analyze its responsive bidding strategy. The cost functions of GenCos 1 and 3 are $C_1=1.38(P_1+L_1)+0.002(P_1+L_1)^2$ and $C_3=1.35(P_3+L_3)+0.002(P_3+L_3)^2$, respectively. To keep the system stability, GenCos 2, 4 and 5 are assigned 35 MW power output, respectively. Except for bus 3, the loads of

other buses remain the same as those in Profile 1. For bus 3, the hourly demand D_3 randomly changes between $[74.2, 124.2]$. Consequently, the hourly system load D randomly changes between $[166, 216]$. The total active power capacity limit is 232.5 MW.

GenCo 3 is located in bus 3 which shares the largest portion of the system load. Hence, the forecasted load of bus 3 would substantially influence GenCo 3's bidding strategy. Fig. 8 depicts the profit difference between $E[\pi_3^{(1)}]$ and $E[\pi_3^{(2)}]$ (i.e. $E[\pi_3^{(1)}] - E[\pi_3^{(2)}]$) over the 30-day period, and shows that the profit difference slightly decreases after power reallocation. This implies that GenCo 3 gains a little bit of motivations to adopt more aggressive bidding strategy under the proposed bidding mechanism

Fig. 8 Profit difference of GenCo 3 over a 30-day period

It is assumed that GenCo 3 chooses the bidding curve based on the following rules: if the load share ratio of bus 3, which is between $[34\%, 58\%]$, is above 50% and the ratio of the supply to the demand, which is between $[1.08, 1.4]$, is below 1.3, it opts to place a high quotation with probability $q_3^{(1)}=0.9$; otherwise it places the normal quotation with $q_3^{(2)}=0.9$.

Leveraging the historical bidding data and the algorithm mentioned in [27, 28], GenCo 1 can derive corresponding posterior probability that GenCo 3 chooses $b_3^{(1)}$ or $b_3^{(2)}$. After synthesizing the posterior information, GenCo 1 can place the bid to maximize its profit. Due to the space limitation, Table 6 only shows the results of the first five days pertaining to the strategic behavior of GenCo 1.

Table 6: Bidding Decisions of GenCo 1

day	1	2	3	4	5
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$\pi_1^{(1,1)}$	302.25	496.8	510.59	297.81	522.59
$\pi_1^{(1,2)}$	376.45	598.75	614.12	373.7	627.75
$\pi_1^{(2,1)}$	294.8	496.6	510.62	292.3	471.64
$\pi_1^{(2,2)}$	375.2	603.54	619.29	372.38	575.47
$q_3^{(1)}$	0.896	0.104	0.104	0.896	0.105
$E[\pi_1^{(1)}]$	308.17	588.14	603.45	575.17	616.71
$E[\pi_1^{(2)}]$	303.16	592.42	607.99	578.15	621.7
b_k	1	2	2	1	2

To further illustrate the efficiency of the Bayesian bidding strategy, Cases 9-11 are designed to compare the long-term profitability of GenCo 1 under three different bidding strategies described as following. In Case 9, GenCo 1 is a strategically behaving player who places bids based on Bayesian game theory. In Case 10, it is a conservative player who always chooses the normal quotation, while in Case 11 it is an aggressive player and always chooses the high quotation. We compute the profit difference over a 30 day period, and the results are depicted in Figs. 9 and 10, respectively. It can be seen that the bidding decision based on Bayesian game theory and SVM enables GenCo 1 to reap more profit in a long run.

Fig. 9 Profit difference over 30 days

5.7 Performance discussion

The most extensive computation of the proposed bidding mechanism is the allocation and re-allocation of power and losses in each bilateral transaction. Since the RS-based power flow tracing method does not require buses re-grouping, matrix inversions, and other complex operations, it offers more flexibility and scalability in handling large networks compared to other methods. For instance, cases 9-11 are carried out on the 14-bus system that involves the bidding analysis of 30 days (i.e. $30 \times 24 = 720$ hours).

We implemented the single bidding mechanism procedure coded by Matlab, and the elapsed computation time is only 46.41 seconds. Then, the single bidding mechanism procedure is packaged and implanted into the multi-agent Java platform. It is found that the total solution time became longer, yet we still able to obtain the solution in about 71 seconds.

We conclude this section by summarizing three major findings from the numerical study of eleven cases:

- 1) The proposed bidding mechanism is able to reduce the overall transmission losses while motivating the GenCos to adjust their bidding strategies to maximize the profit.
- 2) When a GenCo is close to the load center, more power will be assigned due to the lower transmission losses.
- 3) As the loss allotment coefficient λ increases, the ISO becomes more eager to reduce the transmission losses via power reallocation.

6. CONCLUSION

This paper proposes a multi-agent bidding mechanism to reduce the system transmission losses based on generators' power utilization ratios. A simple, yet effective, reachable set power flow tracing method is developed for allocating the losses to individual GenCos. The optimal power allocation scheme is determined through an iterative adjustment procedure. The actual transaction consists of the net power and the line losses, yet the bid decisions made by the GenCos are solely based on the net power. Support vector machine is used to infer the posterior probability of the GenCos' bidding strategy, and the optimum bidding policy is further obtained based on the Bayesian game theory. Eleven cases are tested on the IEEE 14-bus system to examine the performance of the new bidding mechanism as well as the responses of the market participants. The results show that the optimal power allocation is strongly correlated with the loss allotment coefficient, the bidding curve, and the proximity to the load center. For future research, we will extend the method to transmission networks where variable power sources such as wind and solar

generation are involved. Another important aspect is to perform an in-depth comparison between the proposed method and those existing methods in terms of computation time and robustness.

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Appendix

Suppose the value of L_k/P_k for GenCo k (for $k=1, 2, \dots, m$) is fixed, then the competitive effects of the bidding curves in the following two cases are equivalent from the perception of the ISO. In Case 1, if GenCo k submits a bid $\rho_k = \alpha_k + \beta_k P_k$, the ISO must pay an amount of $(\alpha_k + \beta_k P_k)P_k$ plus an additional fee of $\lambda(\alpha_k + \beta_k P_k)L_k$ in order to purchase P_k from GenCo k . In Case 2, if GenCo k submits a bid $\rho_k = \gamma_k(\alpha_k + \beta_k P_k)$ which has taken into account the compensation for transmission losses, the ISO makes a lump sum payment of $\gamma_k(\alpha_k + \beta_k P_k)P_k$ to GenCo k for purchasing power P_k . Obviously it is more convenient for the ISO to deal with the bidding processes in the second case, because of the cost consolidation. Therefore, if the ratio of L_k/P_k for GenCo k is fixed, it is not difficult to obtain the loss adjustment factor which is defined as follows.

$$\gamma_k = \frac{(\alpha_k + \beta_k P_k)(P_k + \lambda L_k)}{(\alpha_k + \beta_k P_k)P_k} = 1 + \lambda \frac{L_k}{P_k}, \text{ for } k=1, 2, \dots, m.$$

Where λ is loss allotment coefficient. Technically, γ_k reflects the level of the GenCo's contribution to the system losses.

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