A Direct Method for Power System Corrective Control to Relieve Current Violation in Transient with UPFCs by Barrier Functions

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Abstract: A direct method based new control scheme for corrective control with unified power flow controllers (UPFCs) is proposed to avoid mal-operation of transmission-line relay trips caused by current violation so as to prevent possible cascaded element failure by overloads of the parallel transmission-line. Undesirable relay trips are avoided by decreasing the dynamic currents near the thresholds thanks to the barrier function penalty, because the value of the barrier function approaches infinity as current of the transmission line approaches pre-set threshold. A control Lyapunov function (CLF), which consists of an energy function and barrier functions, is introduced to derive the control law that exploits the non-positive time derivative of the energy function and the large value of the barrier function as trajectories approach the boundary of constraints. In

particular, the feedback control exercised by the UPFC results in the negative time derivative of the Control Lyapunov Function in the reshaped attraction region, with the consequence that the state trajectory converges to the desired stable equilibrium point (SEP). Ultimately, the resulting reshaped attraction region for the desired operating point is analyzed, and illustrative numerical simulations are carried out using a 3-bus and the IEEE 162 Bus test system, demonstrating the effectiveness of the proposed method.

Key words: corrective control, transient analysis, unified power flow controller, direct method, current overload.

I Introduction

For one century, the electrical power plays an ever significant role in modern society and the power grid has been growing increasingly complex. The highly complex power network may be vulnerable to non-stationary disturbances, which may result from extreme weather conditions, inappropriate control actions and sudden loss of load, etc. [1]. Such disturbances may interrupt the energy balance of the power grid and may lead to serious transient events, which can disrupt normal operation and may even result in relay tripping of the transmission line due to the line overloads. Even though power systems are designed N-1 security, failure of a small number of transmission lines in a heavily loaded electric grid can create overloads in parallel lines, with the possibility for global, system-wide failures. Therefore, Investigating corrective control methods for ensuring reliability and security of a large-scale power system are of great importance.

As a means to enhance the security of the electrical power grid, flexible AC

transmission (FACT) systems have many attractive characteristics for corrective/preventive control. Among numerous FACT devices, the unified power flow controller (UPFC) is one of the most promising devices proposed in the FACT concept, with the potential to control power flow, improve the flexibility and stability of the transmission system[2]. A UPFC can rapidly control real power, reactive power, and bus voltage on a transmission line, thereby regulating multiple key quantities for the power system. However, given the UPFC's ability to rapidly vary such key quantities, its control must be designed with ample consideration of the impact on the protective relaying of traditional transmission lines — for example, one may wish to avoid the possibility that such control action might trigger undesired relay trips elsewhere in the system. This work provides a first step towards such corrective control design methodologies by direct approach and imposing a penalty function to force the control action of the UPFC to avoid regions that would trigger relay trips.

In the research on corrective/preventive control for power system with UPFC, a methodology to assess preventive control actions through adjustments of UPFC reference signals is proposed in [3], in which control actions are evaluated through a nonlinear optimization process. In [4], corrective actions are evaluated by a nonlinear programming methodology to improve the dynamic security of power systems when transient instability is detected. And due to the requirements of both speed and accuracy for online corrective control, a new linear programming (LP)—based on an optimal power flow (OPF) algorithm for corrective FACT control—was proposed in [5]. However, even with significant efforts to improve computational speed, the high on-line

computational cost of these optimal-control methods restricts their applications in large scale systems.

Among many studies on corrective control in transient, so far as we know, most of their control methods are based on optimization method with simulation/prediction (linear programming, nonlinear programming, etc.) [3, 6], artificial intelligence (neural network, pattern recognition, etc.) [7, 8] or sensitivity analysis [9, 10]. However the work presented here has a different flavor. In this paper we derive a novel corrective control scheme based on the direct method of Lyapunov with barrier function, which has low on-line computational cost in stability estimation [11] and more theoretical guarantee. Our goal is to develop a control scheme that can leverage UPFCs to mitigate branch transient current overflow and to facilitate the power system converging to a specified stable equilibrium point (SEP). In particular, we envision a power system where UPFC devices are installed on power transmission lines for corrective control in transient. In addition, a control algorithm is designed to avoid undesired current overloads that may trigger a chain reaction of circuit breaks leading to system-wide cascaded failure.

We develop a novel Control Lyapunov Function (CLF) based control scheme for UPFCs for removing mal-operation of over-current protection. Rather than avoiding the function of over-current protection, our proposed control scheme decrease the dynamic currents near the thresholds, so that the branches will not trip by mal-operation of over-current protection caused by current violation in dynamic process. Via a favorable trajectory without undesired transmission line trips, the control law derived by the CLF

will guide the system state into a prescribed SEP where the impacts of the power system transient may be mitigated. The CLF consists of an energy function [11] and one or more barrier functions. By employing the barrier function, a high penalty is imposed as the system state approaches line current constraints so that the current overload should be avoided. In addition, the energy function has a simple closed-form expression that ensures a negative time derivative property, which brings many benefits to the controller design. Moreover, the attraction region is analyzed.

The following material is organized in five sections. In Section II we discuss the transient model of electric power grids, including the physical model of UPFC devices and preliminaries. Then the energy function of the power system and the barrier function for avoiding current overload are given in Section III. The design of controllers, based on the proposed CLF, is described in Section IV, and a brief analysis of the attraction region is presented in V. The results of the simulation are provided in Section VI, and Section VII draws a conclusion.

II Electric power grid transient model and preliminaries

The power system used here is modeled as follows: the buses are separated into three groups—slack bus (infinite bus), generator buses, and load buses. The active power and the reactive power that flow among those buses are controlled by unified power flow controllers (UPFC). This section shows how we integrate the UPFC model into the dynamic model of power grid and present the preliminaries used in the following sections.

A. Power grid transient model

For a generator/swing bus i , we assume ω_i is the generator angular velocity and δ_i is the corresponding generator bus terminal voltage angle. We use M_i as generator inertia and D_{gi} as generator damping. At the generator bus, the mechanical power input of the generator is represented as P_{Mi} , while the network injection at the bus is P_i^E . The internal voltage of generator is held to be constant, and its corresponding angle equals to frequency deviation when differentiated by time. The sign convention here treats power absorbed by the transmission system (i.e., "injection" into to electrical bus) as positive. The equations for generator/swing at bus i are

$$\begin{aligned}\dot{\omega}_i &= -M_i^{-1}(D_{gi}\omega_i - P_{Mi} + P_i^E) \\ \dot{\delta}_i &= \omega_i\end{aligned}\tag{1}$$

The first equation in (1) is typically known as the generator "swing equation". It describes how the imbalance between mechanical power input and grid electric power absorption affects the time rate of change for generator angular frequency deviation. Apart from the generator swing equation, the model also includes the relationship between the generator internal voltage angle and the angular frequency deviation, as shown in the second equation in (1).

For a load bus i , we assume V_i and δ_i represent load voltage magnitude and angle respectively, D_{Li} is a coefficient describing frequency dependence of active power demand at a load bus, and ε_i is a coefficient describing reactive power dependence on the time rate of change of voltage magnitude. P_i^0 and Q_i^0 are load active power and reactive power absorption, P_i^E and Q_i^E are load active power and reactive power injection. The dynamic equations of the load bus i are

$$\begin{aligned}\dot{\delta}_i &= -D_{Li}^{-1}(P_i^0 + P_i^E) \\ \dot{V}_i &= -\varepsilon_i^{-1}V_i^{-1}(Q_i^0 + Q_i^E)\end{aligned}\tag{2}$$

Here, the two equations in (2) both have the singular perturbation form that are suggested in [12]. Viewed from the singular perturbation perspective, ε_i is a small positive parameter that controls the speed with which trajectories of voltage magnitude move towards values satisfying reactive power balance, and likewise D_i is a small parameter that controls the how fast the angle will move towards values satisfying active power balance.

On the electric power grid branches, we assume the power branch connected bus j and bus i denoted as (i,j) or k , it belongs to the set of power transmission line N_{br} . The set of branches connected to bus i is C_i , so we have $C_i \subseteq N_{br}$. The branch is assumed lossless, the admittance of branch k is jY_k . V_j and V_i are the voltage magnitudes of bus i and j , δ_i and δ_j are the voltage angle of bus i and j . The injected power from bus j to bus i are [12]

$$\begin{aligned}P_k^i + jQ_k^i &= V_i e^{j\delta_i} \left[Y_k e^{j\pi/2} (V_j e^{j\delta_j} - V_i e^{j\delta_i}) \right]^* = V_i Y_k V_j e^{j(\delta_i - \delta_j - \pi/2)} - V_i^2 Y_k e^{j(-\pi/2)} \\ P_k^i &= Y_k V_i V_j \sin(\delta_i - \delta_j), \\ Q_k^i &= -Y_k V_i V_j \cos(\delta_i - \delta_j) + V_i^2 Y_k.\end{aligned}\tag{3}$$

The symbol P_k^i and Q_k^i represents the active and reactive power flow from bus j to bus i . If P_i^E and Q_i^E are active and reactive power injected to bus i from other buses, they can be written as

$$P_i^E = \sum_{k \in C_i} P_k^i \text{ and } Q_i^E = \sum_{k \in C_i} Q_k^i.$$

B. UPFC Model

Unified Power Flow Controller (UPFC) is an electrical device for providing

fast-acting active power flow control and reactive power compensation on high-voltage electricity transmission networks. In the proposed scheme, UPFCs are installed on branches to control electric grid for avoiding a branch trip caused by the violation of branch current constraint. An equivalent circuit model of UPFC installed in an electric power branch is shown in Fig. 1. It consists by a series connected voltage source and a shunt connected current source^[13].

A phasor diagram is shown in Fig. 2. In the equivalent circuit, $\tilde{V}_i' = \tilde{V}_{sek} + \tilde{V}_i$. The equivalent series connected voltage source $\tilde{V}_{sek}$ of UPFC is controllable both in magnitude and phase angle, based on

$$\tilde{V}_{sek} = r_k \tilde{V}_i e^{j\alpha_k}, \quad (4)$$

where $0 \leq r_k \leq r_k^{\max}$ and $0 \leq \alpha_k \leq 2\pi$. The additional current on branch k induced by UPFC is

$$\begin{aligned} \tilde{I}_{sek} &= Y_k e^{j\pi/2} \left(V_i e^{j\delta_i} + r_k V_i e^{j\gamma_k} - V_j e^{j\delta_j} \right) - Y_{ij} e^{j\theta_{ij}} \left(V_i e^{j\delta_i} - V_j e^{j\delta_j} \right), \\ &= r_k Y_k e^{j(\pi/2 + \gamma_k)} V_i \end{aligned} \quad (5)$$

where $\gamma_k = \delta_i + \alpha_k$. The shunt connected current source is

$$\tilde{I}_{shk} = (I_{Tk} + jI_{qk}) e^{j\delta_i} \quad (6)$$

I_{Tk} and I_{qk} are shunt current associated with the active power and reactive power provided by UPFC shunt connected current source on branch k, where $|I_{qk}| \leq I_{qk}^{\max}$. The electric active power and reactive power of UPFC injected to bus i and j are

$$\begin{aligned} P_{upfc_k}^i &= \text{Re} \left[\tilde{V}_i \left(-\tilde{I}_{sek}^* \right) \right] + I_{Tk} V_i = -r_k V_i^2 Y_k \sin(\delta_i - \gamma_k) + I_{Tk} V_i \\ Q_{upfc_k}^i &= \text{Im} \left[\tilde{V}_i \left(-\tilde{I}_{sek}^* \right) \right] + I_{qk} V_i = r_k V_i^2 Y_k \cos(\delta_i - \gamma_k) + I_{qk} V_i, \\ P_{upfc_k}^j &= \text{Re} \left(\tilde{V}_j \tilde{I}_{sek}^* \right) = r_k V_i V_j Y_k \sin(\delta_j - \gamma_k) \\ Q_{upfc_k}^j &= \text{Re} \left(\tilde{V}_j \tilde{I}_{sek}^* \right) = -r_k V_i V_j Y_k \cos(\delta_j - \gamma_k) \end{aligned} \quad (7)$$

where $P_{upfc_k}^i$ and $P_{upfc_k}^j$ are the UPFC active power injected to bus i and j. A constraint

$$P_{upfc_k}^i + P_{upfc_k}^j = 0 \quad (8)$$

is imposed on UPFC to ensure its power balance (i.e., passive device). $Q_{upfc_k}^i$ and $Q_{upfc_k}^j$ are the UPFC reactive power injected to bus i and j. The adjustable power $P_{upfc_k}^i$, $P_{upfc_k}^j$, $Q_{upfc_k}^i$ and $Q_{upfc_k}^j$ injected by UPFC provide the possibility to control the power flow and regulate current in the power system.

C. The power system with UPFC

If one swing bus, m generator buses, and n load buses are in the power system, the dynamic system described above can be formulated as the following standard form

$$\dot{x} = f(x, u) \quad (9)$$

Where the state

$$x = (\omega_1, \delta_1, \dots, \omega_{m+1}, \delta_{m+1}, \delta_{m+2}, V_{m+2}, \dots, \delta_{m+n+1}, V_{m+n+1})^T,$$

As I_{Tk} is determined by γ_k and r_k according to the UPFC power balance principle described in (8), it is omitted in u . the control variable u is

$$u = \left[\left(\gamma_k, r_k, I_{qk} \right)^T \right]_{|N_{br}| \times 1}, k \in N_{br}$$

the vector $(\gamma_k, r_k, I_{qk})^T$ in u means we can control the active power and reactive power of

UPFC installed on branch k. For swing bus or generator buses, $1 \leq i \leq m+1$, $i \in N^+$, the

$2i-1^{th}$ and $2i^{th}$ rows of function f are

$$\begin{cases} -M_i^{-1} \left(D_{gi} \omega_i - P_{Mi} + P_i^E + \sum_{k \in C_i} P_{upfc_k}^i \right) \\ \omega_i \end{cases}$$

For load buses, $m+2 \leq i \leq m+n+1$, $i \in N^+$ the corresponding $2i-1^{th}$ and $2i^{th}$ rows of f are

$$\begin{cases} -D_{Li}^{-1} \left(P_i^0 + P_i^E + \sum_{k=(i,j) \in N_{br}, j \in D_i} P_{upfc_k}^i \right) \\ -\varepsilon_i^{-1} V_i^{-1} \left(Q_i^0 + Q_i^E + \sum_{k \in C_i} Q_{upfc_k}^i \right) \end{cases}$$

The objective of this research is to derive a control law for u and to shape attraction region under the constraints of UPFC, so that the branch trip caused by current overload cannot happen in the transient process if the state x_0 at fault clearing time (the initial state in our discussion) is in the attraction region of the controlled system. In mathematical form, if the desired stable equilibrium point is x_{ref} , then the problem can be state as: finding a control law for control variable $u(t)$, while

$$u = (u_k)_{|N_{br}| \times 1} = \begin{bmatrix} \gamma_k(x) \\ r_k(x) \end{bmatrix}_{|N_{br}| \times 1}, k \in N_{br}. \quad (10)$$

and

$$u_k = \begin{pmatrix} \gamma_k(x) \\ r_k(x) \end{pmatrix}, k \in N_{br} \quad (11)$$

for each UPFC, so that the system state $x \rightarrow x_{ref}$ when $t \rightarrow +\infty$, and for any $t \in [0, +\infty]$, $k \in N_{br}$, $I_k(t) \leq I_k^{\max}$, where I_k is the current magnitude in branch k and $I_k^{\max}$ is the allowed maximum magnitude in branch k . Consequently, the branch trip is avoided and the risk of cascade failure is eliminated.

The detail of control scheme will be shown in the next two sections.

D. Region of Attraction and control process

A region of attraction here is the region that the system can converge to a SEP start at any state in this region and will not violate the constraint of power system in infinite time. In mathematical form, we have the following definition under the consideration of

current constraint.

Definition 1. Let $\Gamma(t, x_0)$ be the solution of (9) that starts at state x_0 at fault clearing time, the region of attraction is defined as the set of all points x_0 such that $\lim_{t \rightarrow \infty} \Gamma(t, x_0) = x_{ref}$ and $|I_k(t)| \leq I_k^{max}, k \in N_{br}$ where x_{ref} is desired SEP which we expect the state converge to.

A centralized UPFCs control scheme is proposed to implement corrective control for avoiding current overload on the transient time scale. The proposed scheme assumes that an analysis is performed to detect particular disturbance threatening the transient security of the system. An analysis based on the classical “N-1” security paradigm is performed in advance. Contingencies, such as line tripping or generator outages, are considered as the causes of disturbances. Thus the state x_0 at the fault clearing time may be an unstable point lead to transient events and the model described in subsection A may cause line trip or may not converge from x_0 to the desired SEP x_{ref} without control effects, which mean the state x_0 may not be in the attraction region of power system. It may lead to more branch trips caused by line overloads in transients.

Therefore, our control objective is to enlarge or reshape the region of attraction defined in definition 1 by UPFC, so that the modified attraction region can cover the states x_0 . However, because the enlarging of the attraction region is restricted by the constraints of UPFC, the attraction region cannot be enlarged to incorporate any state point, but it can be manually changed by adjusting the parameters of our controller with purposes of including the specified state x_0 within the enlarged attraction region.

In our control scheme, the controller will be turned on when the system has

detected that the state x_0 at fault clearing time is not in the attraction region of the uncontrolled power system. With the known state x_0 and the system model, a pre-designed control strategy is selected to incorporate the state x_0 in the attraction region of controlled system. The control process is illustrated in Fig. 3.

III Energy function and barrier function for power system control design

In this section, the energy function and barrier function of the power system is introduced. The energy function is employed to construct a CLF and the barrier function is proposed for avoiding current constraint violation. They are employed to derive a control law which will be depicted in the next section.

A. Energy function of power system

Energy storage plays an intimate role in determining the structure of the state equations. The review in [14] depicted the importance of energy storage and energy-shaping techniques in the analysis and design of a control system. By energy function, the energy value can be evaluated for a given state at any point in time to obtain a direct measure of the relative stability of the system. Meanwhile, the attraction region of the power system can be figured out by find the unstable equilibrium points (UEP) in the energy function surface. If we assume the set of generator buses is N_g , the set of load buses is N_L , then a energy function for the electric power grid model described in the above section is [15]

$$\begin{aligned} \Phi = & \sum_{i \in N_g} \frac{1}{2} M_i \omega_i^2 + \sum_{i \in N_L} Q_i^0 \ln V_i + \sum_{i \in N_g \cup N_L} \left[\frac{1}{2} V_i^2 \sum_{k \in C_i} Y_k + P_i \delta_i \right] \\ & - \sum_{k=(i,j) \in N_{br}} Y_k V_i V_j \cos(\delta_i - \delta_j) \end{aligned} \quad (12)$$

Where Q_i^0 is reactive power demand for load bus. P_i equals to $-P_{Mi}$ for the generator

bus, and equals to P_i^0 for the load bus. It can be known that the time derivative of energy function (12) is semi-negative definite, and the energy function can be written in the form of $d\Phi(x)/dt = \nabla_x \Phi^T A \nabla_x \Phi$, where $A \leq 0$. For a generator bus i , the derivatives of energy function (12) respect to its voltage angular velocity and voltage angle are

$$\begin{pmatrix} \frac{\partial \Phi}{\partial \omega_i} \\ \frac{\partial \Phi}{\partial \delta_i} \end{pmatrix} = \begin{pmatrix} M_i \omega_i \\ P_i + \sum_{k=(i,j) \in C_i} Y_k V_i V_j \sin(\delta_i - \delta_j) \end{pmatrix}$$

For a load bus i , the derivatives of energy function (12) with respect to voltage angle and voltage magnitude are

$$\begin{pmatrix} \frac{\partial \Phi}{\partial \delta_i} \\ \frac{\partial \Phi}{\partial V_i} \end{pmatrix} = \begin{pmatrix} P_i + \sum_{k=(i,j) \in C_i} Y_k V_i V_j \sin(\delta_i - \delta_j) \\ Q_i^0 / V_i + \sum_{k \in C_i} Y_k V_i + \sum_{k=(i,j) \in C_i} Y_k V_j \cos(\delta_i - \delta_j) \end{pmatrix}. \quad (13)$$

Therefore, the transfer function of the power system can be expressed by the gradient of the energy function as

$$\dot{x} = A \nabla_x \Phi(x) \quad (14)$$

A is $\text{diag}[A_1, \dots, A_{m+1}, B_1, \dots, B_n]$, where A_i is

$$\begin{pmatrix} -M_i^{-1} D_{gi} M_i^{-1} & -M_i^{-1} \\ M_i^{-1} & 0 \end{pmatrix}$$

for swing bus and generator bus, B_i is

$$\begin{pmatrix} -D_{Li}^{-1} & 0 \\ 0 & -\epsilon_i^{-1} \end{pmatrix}$$

for load bus. As A is semi-negative definite, the time derivative of this energy function

$$d\Phi(x)/dt = \nabla_x \Phi^T A \nabla_x \Phi \leq 0.$$

B. Barrier function for avoiding current overload

For a stable power system, the system state will eventually converge to a local minimal point of energy function(12) due to the property of semi-negative definite time derivative of energy function. However, the most basic of relay functions—over-current protection—imposes a current magnitude constraint on the branches. In particular, if the branch current magnitude exceeds threshold, the relay acts and the branch is disconnected immediately. In order to avoid the current exceeding the user-defined threshold, barrier functions are applied [17]. Barrier function is a continuous function whose value on a point increases to infinity as the point approaches the boundary of the feasible region[16]. It is used as a penalizing term for violations of constraints. Though the barrier function is usually used in constrained optimization, it is also employed in constraint control [25].

In order to construct a barrier function whose value on a point increases to infinity as the point approaches the boundary of the branch current constraint, the boundary of the current constraint is analyzed. By employing the AC circuit principle, If the UPFC is installed on the side of bus i as shown in Fig. 1, the current flowed on branch $k=(i, j)$ ($k \in N_{br}$) can be represented as

$$\begin{aligned}
 \tilde{I}_k &= jY_k(\tilde{V}_i + \tilde{V}_{sek} - \tilde{V}_j) \\
 |\tilde{I}_k|^2 &= \tilde{I}_k \tilde{I}_k^* = \chi_{k1}(\tilde{V}_i, \tilde{V}_j) + \chi_{k2}(\tilde{V}_i, \tilde{V}_j, \tilde{V}_{sek}) \\
 \chi_{k1} &= Y_k^2 \left[|\tilde{V}_i|^2 + |\tilde{V}_j|^2 - 2|\tilde{V}_i||\tilde{V}_j|\cos(\delta_i - \delta_j) \right] \\
 \chi_{k2} &= Y_k^2 \left[|\tilde{V}_{sek}|^2 - 2|\tilde{V}_{sek}||\tilde{V}_j|\cos(\delta_j - \gamma_k) + 2|\tilde{V}_{sek}||\tilde{V}_i|\cos(\delta_i - \gamma_k) \right]
 \end{aligned} \tag{15}$$

From (15) it is known that the formula of current magnitude in branch k consists of the un-adjustable term χ_{k1} and the adjustable term χ_{k2} . The inequality of branch current constraint is

$$\chi_{k1} + \chi_{k2} \leq \left(I_k^{\max}\right)^2, \quad (16)$$

and $\chi_{k2}=0$ if $r_k=0$. so the above inequality can always be satisfied when

$$\chi_{k1} \leq \left(I_k^{\max}\right)^2. \quad (17)$$

Thus, a barrier function for constraint (17) is established on the boundary of inequality (17), as

$$w_k(\tilde{V}_i, \tilde{V}_j) = \frac{1}{2} d_k \ln \frac{a_k^2}{a_k^2 - (\chi_{k1} - \chi_{k1}^e)^2}, \quad (18)$$

where

$$\chi_{k1}^e = \chi_{k1}(\tilde{V}_i^e, \tilde{V}_j^e),$$

$$a_k = \left(I_k^{\max}\right)^2 - \chi_{k1}^e.$$

$\tilde{V}_i^e$ and $\tilde{V}_j^e$ are the voltage on the desired SEP x_{ref} . d_k is the non-negative real coefficient of the barrier function, while one group of d_k corresponds to one control strategy. This coefficient is important because it can directly determine the shape of the attraction region. Barrier function w_k is positive definite and continuous under (16), thus it is a valid Lyapunov function candidate. When χ_{k1} approaches the boundary value $\left(I_k^{\max}\right)^2$, the barrier function w_k will go to positive infinity.

Under the constraint of inequality (17), it should be noted that the parameter r_k and γ_k in the UPFC should satisfy

$$Y_k^2 \left[r_k^2 |\tilde{V}_i|^2 - 2r_k |\tilde{V}_i| |\tilde{V}_j| \cos(\delta_j - \gamma_k) + 2r_k |\tilde{V}_i|^2 \cos(\delta_i - \gamma_k) \right] + \chi_{k1} < \left(I_k^{\max}\right)^2, \quad (19)$$

to hold inequality (16). In order to obtain the feasible region of parameters r_k and γ_k under the inequality(19), $(r_k \cos \gamma_k, r_k \sin \gamma_k)$ is substituted by (x, y) , transforming the inequality (19) into an eclipse form:

$$\begin{aligned}
& \left(2|\tilde{V}_i|\sin\delta_i - 2|\tilde{V}_j|\sin\delta_j\right)y + \left(2|\tilde{V}_i|\cos\delta_i - 2|\tilde{V}_j|\cos\delta_j\right)x \\
& + |\tilde{V}_i|x^2 + |\tilde{V}_j|y^2 < \frac{\left(I_k^{\max}\right)^2 - \chi_{k1}}{Y_k^2|\tilde{V}_i|}.
\end{aligned} \tag{20}$$

On the branch k , the feasible region of parameter r_k and γ_k can be easily obtained from solving inequality(20).

IV. Controller design

In order to deal with the problem introduced in part D of section II, this design considers the scenario in which the given state x_0 at fault clearing time may evolve in a direction that leads to current overload in the vicinity of the desired SEP x_{ref} . The design objective is to reshape the attraction region with the proposed barrier function so that the system state will converge to the desired SEP x_{ref} without reaching state values that would trigger branch current overload under the control of UPFCs. The detail of the attraction region and its analysis will be discussed in the next section. In this section a Control Lyapunov Function(CLF) is introduced, and a control law is derived from the CLF.

A. Proposed Control Lyapunov Function and System Stability

In control theory, a control-Lyapunov function $V(x,u)$ is a generalization of the notion of Lyapunov function $V(x)$. It is used to check whether a control system is feedback asymptotically stable. That is, for any state x , whether there exists a control input $u(x,t)$, so that the system starting in a state $x \neq 0$ in some domain D will remain in D , or it will eventually return to $x = 0$.

Lemma 1 [18]. The control system is locally asymptotic stable, if there exists a CLF $V(x,u)$, for any x belongs to the attraction region D , that satisfy,

- 1) The CLF $V(x,u)$ is continuous, positive definite, and $V(x,u) \rightarrow \infty$ as $x \rightarrow \infty$;
- 2) For any $x \neq 0, x \in D$, there exist u , so that $dV(x,u)/dt \leq 0$.

Lemma 1 depicts the sufficient condition for locally feedback asymptotic stable; within region D the control system satisfies $dV(x,u)/dt \leq 0$. Then the Control Lyapunov Function decreases monotonically along the time. With the concept of CLF and locally feedback asymptotic stability, the control problem of this paper can be formulated as a constraint control. Therefore, a CLF $\Omega(x,u)$ is proposed to be consisted by an energy function(12) Φ , a barrier function(18) W and an offset K_{ref} , it is

$$\Omega = \Phi + W + K_{ref} \quad (21)$$

where

$$W = \sum_{i \in N_{br}} d_i w_i, d_i \geq 0, K_{ref} = -\Phi_{ref}.$$

Φ_{ref} is the energy value on the desired SEP x_{ref} . When the system state converges to x_{ref} , the value of Ω will converge to 0. The x_{ref} is also a SEP of the CLF(21) which is equivalent to the original $x=0$ in the Lemma 1, because x_{ref} is a local minimal point both in the energy function Φ and barrier function W . As we know from the previous section, if the current magnitude approaches the user-defined threshold, the barrier function goes to positive infinity. With this proposed CLF, we have the following Lemma.

Lemma 2 For a given state x_0 at fault clearing time, if a system with CLF (21) satisfies $d\Omega/dt \leq 0$, then the constraint (17) will not be violated.

Proof: Assuming $\Omega(x_0) = a < +\infty$ at $t=0$, then for any $t > 0$, $\Omega \leq a < +\infty$ as $d\Omega/dt \leq 0$. From barrier function (18), we know if the system state approaches the boundary of set $\{x | \chi_{kl} \leq I_k^{max2}, k \in N_{br}\}$, the CLF goes to infinity or $\Omega \rightarrow +\infty$ when $x \rightarrow \cup_{k \in N_{br}} \{x | \chi_{kl} = I_k^{max2}\}$.

That is inconsistent with $\Omega \leq a < +\infty$ for any $t > 0$, thus the constraint (17) will not be violated.

The Lemma 2 indicates that the barrier function in (21) makes the system state trajectory stay in the constraint if the time derivative of CLF is negative or semi-negative definite. However, resulting from the addition of function W in CLF, the time derivative of Ω may be positive in some regions. Thus, a control scheme is needed to make the CLF satisfy the conditions in Lemma 1 and Lemma 2, i.e. $d\Omega/dt \leq 0$.

B. Controller design

In order to obtain the control input $u(x)$ in the power system satisfying conditions in Lemma 1-3, the time derivative of CLF with the effect of UPFC is derived. The state transfer function with UPFC is written in the form of $dx/dt = A \nabla_x \Phi + U$, where U consists of $(-D_i^{-1} P_{upfc}^i \quad -\varepsilon^{-1} V_i^{-1} Q_{upfc}^i)^T$ for load buses and $(-M_i^{-1} P_{upfc}^i \quad 0)^T$ for generator buses. The P_{upfc}^i and Q_{upfc}^i are the active power and reactive power injected into bus i , provided by UPFC, which depend on the control u , written as

$$P_{upfc}^i = \sum_{k \in C_i} P_{upfc_k}^i \quad \text{and} \quad Q_{upfc}^i = \sum_{k \in C_i} Q_{upfc_k}^i ,$$

The derivative of Ω respect to time can be divided into an un-adjustable part and adjustable part, which is

$$\dot{\Omega} = \nabla_x \Phi^T A \nabla_x \Phi + \nabla_x W^T A \nabla_x \Phi + (\nabla_x W^T + \nabla_x \Phi^T) U . \quad (22)$$

In equation (22), the adjustable part is $(\nabla_x W^T + \nabla_x \Phi^T) U$ and $\nabla_x \Phi^T A \nabla_x \Phi + \nabla_x W^T A \nabla_x \Phi$ is the un-adjustable terms. To guarantee the system stability, the time derivative of CLF should be negative under the control $u(x)$ which is introduced in section II. Then, according to the system model equations(1)(2)(7), barrier function (18) and energy

function (12), the adjustable part in (22) can be rearranged in the form of the expression with control variables γ_k , r_k and I_{qk} , $k \in N_{br}$.

$$(\nabla_x W^T + \nabla_x \Phi^T)U = - \sum_{k \in N_{br}} r_k \Delta_{br}^k \begin{pmatrix} \cos \gamma_k \\ \sin \gamma_k \end{pmatrix} + \sum_{k \in N_{br}} o_{br}^k I_{qk}, \quad (23)$$

Where Δ_{br}^k is a 1×2 row vector and o_{br}^k is a scalar, they are functions of bus voltages and frequency deviations. To hold the stability condition $d\Omega/dt \leq 0$ in Lemma 1, the calculated control u should minimize the adjustable part $(\nabla_x W^T + \nabla_x \Phi^T)U$ under constraints (20), $0 \leq r_k \leq r_k^{\max}$, $0 \leq \gamma_k \leq 2\pi$ and $|I_{qk}| \leq I_{qk}^{\max}$, $k \in N_{br}$. By applying equation(23), it can be obtained by solving the problem,

$$\begin{pmatrix} r_k \\ \gamma_k \\ I_{qk} \end{pmatrix} = \arg \max_{(r_k, \gamma_k, I_{qk})^T} \left[\Delta_{br}^k \begin{pmatrix} r_k \cos \gamma_k \\ r_k \sin \gamma_k \end{pmatrix} - o_{br}^k I_{qk} \right], \quad (24)$$

If r_k^c is the maximal value of r_k on the boundary of constraint (20) for avoiding current overload caused by UPFC on branch k . The result of problem (24) is

$$u_k = \begin{pmatrix} \gamma_k \\ r_k \\ I_{qk} \end{pmatrix} = \begin{pmatrix} \text{angle}(\Delta_{brk}) \\ \min \{ r_k^{\max}, r_k^c \} \\ -\text{sign}(o_{br}^k) I_{qk}^{\max} \end{pmatrix} \quad (25)$$

With the control $u_k = (\gamma_k, r_k, I_{qk})^T$ in (25) under the constraints of UPFC and branch current, the minimal value of $(\nabla_x W^T + \nabla_x \Phi^T)U$ is obtained.

The controlled system is not globally asymptotic stable, the situation $d\Omega/dt > 0$ may exist under control law (25), which may lead to instability. However, in an attraction region with the constraints, the system can be locally asymptotically stable. The attraction region will be discussed in the next section.

V The attraction region analysis

From the previous section, it is known that the CLF is not globally positive-definite. Furthermore, the nature of the barrier function in CLF is such that the time derivative of CLF may become positive if r_k or I_{qk} are not large enough. Thus, it is necessary to establish a region about the stable equilibrium for which the CLF is locally positive definite, and its time derivative along trajectories is non-positive (mostly negative, without of generality); this region will then be a subset of the attraction region. Such analysis is performed in this section.

For the built barrier on the border of maximal current constraint, the state of the system in transient must be in the region $O_s = \{x \mid |I_k(x)| \leq I_k^{\max}, k \in N_{br}\}$, which is surrounded by the barrier if all r_k or I_{qk} can be infinitely large at any state point. However, under the constraint of r_k , I_{qk} and I_k , the attraction region C_s must be the subset of O_s , and it is a compact set in which the given state point x_0 at fault clearing time and the desired SEP are included. According to the design objective, for a given state x_0 , the system state should converge to the desired SEP under the control of UPFC, and Ω becomes 0 when converged. If we denote the connected component of a set H that includes a point x as H^x , then under the constraint of UPFC, the set C_s can be expressed as

$$C_s = \left\{ x \mid \Omega(x) < l, \dot{\Omega}(x) \leq 0 \right\}^{x_{ref}}, \quad (26)$$

where l is a user-defined maximal value, which will be derived in the following discussion. It is possible that multiple SEPs may exist in O_s since Ω is nonlinear. With the Claim A1 and Lemma A1 stated in [24], Theorem 1 is introduced below so that one may determine whether the system state will converge to a specific SEP x_{ref} .

Theorem 1. Let us define a control Lyapunov function Ω and a set $D_\delta = \{x | \Omega(x) < \delta\}$ for the power system, D_δ^{ref} is the component of D_δ containing the desired SEP x_{ref} , $\delta \in R$. select a scalar $\xi \in R$ small enough, and let δ increased from ξ . Then for any δ , $\xi \leq \delta \leq \delta^{\text{ue}}$ and $\xi \leq \delta \leq \delta^{\text{ua}}$, where δ^{ue} and δ^{ua} are the smallest real value such that D_δ^{ref} first contain a point $x^{\text{ue}, \min} \neq x_{\text{ref}}$ at which $\nabla \Omega(x^{\text{ue}, \min}) = 0$, and $x^{\text{ua}, \min} \neq x_{\text{ref}}$ at which $d\Omega(x^{\text{ua}, \min})/dt > 0$ respectively. Then the system start at x_0 would converge to x_{ref} if the following two conditions are satisfied:

- 1) Check that $\Omega(x_0) < \Omega(x^{\text{ue}, \min})$ and $\Omega(x_0) < \Omega(x^{\text{ua}, \min})$;*
- 2) Ensure that there exist a one dimensional path connecting x_0 to x_{ref} such that Ω is non-increasing along the path.*

Proof: As $\nabla \Omega(x_{\text{ref}}) = 0$ and $d\Omega(x_{\text{ref}})/dt = \nabla_x WU = 0$. For a sufficiently small $\xi > 0$, the fact that $\{x | d\Omega(x)/dt \leq 0, \Omega(x) \leq \xi\}^{\text{ref}}$ must be a bounded set, contained in the region of attraction of x_{ref} . When $\Omega(x) < \Omega(x^{\text{ue}, \min})$ and $\Omega(x) < \Omega(x^{\text{ua}, \min})$, it is obvious that $d\Omega(x)/dt \leq 0$. Then we have to check whether the post-fault state lies in the attraction region of the aimed SEP, and that is why we need condition 2). We may prove it by contradiction. Suppose post-fault system state x_0 satisfies 1) and 2), yet x_0 does not lie in the region of the attraction of x_{ref} . Then if we calculate the energy trajectory of x_0 connects to x_{ref} , since $\Omega(x_0) < \Omega(x^{\text{ue}, \min})$ and $\Omega(x_0) < \Omega(x^{\text{ua}, \min})$, then it is impossible to have Ω nonincreasing along the path. From definition of the region of the attraction, all paths connecting to x_{ref} have to go through the energy value of $\Omega(x^{\text{ue}, \min})$ or $\Omega(x^{\text{ua}, \min})$, however $\Omega(x_0) < \Omega(x^{\text{ue}, \min})$ or $\Omega(x^{\text{ua}, \min})$, which means all paths connecting x_0 to x_{ref} do not have

non-increasing Ω along the path. Then we have a contradiction for 2) and the theorem is proved. Readers may refer to Claim A.1 and Lemma A.1 [19] for more detail of this theorem.

From *Theorem 1* it is known that one may have to identify $x^{\text{ue},\min}$ and $x^{\text{ua},\min}$ to judge whether the system state will converge to the desired SEP or whether the state x_0 is in the region of attraction C_s . For the calculation of $x^{\text{ue},\min}$, some methods are proposed in the previous research, such as the dynamic gradient approach[20], singular fixed-point homotopy method[21], and minimal energy path(MEP) method[22]. In this paper, a MEP method—Nudged Elastic Band algorithm[23]—is applied. It is expected that the NEB algorithm may locate several type-1 UEPs along the MEP between two SEPs, in which $x^{\text{ue},\min}$ may possibly be included. In addition, if we find a non-increasing MEP from the point x_0 to the desired SEP x_{ref} , the second condition of theorem 1 is satisfied. On the other hand, $x^{\text{ua},\min}$ can be solved by the following constrained optimization problem

$$\min \Omega(x) \quad (27)$$

where

$$\dot{\Omega}(x) = 0, x \neq x_{\text{ref}}.$$

Assuming x^{ua} are the points which satisfies the above constraint, $x^{\text{ua},\min}$ is the minimal one. Above all, for a given state x_0 at fault clearing time, the design problem can be solved by finding a column vector d consisting of element $d_i > 0, i \in N_{br}$, to satisfy the constraint

$$\Omega(x_0) - \Omega(x^{\text{u},\min}) < 0, x_0 \in C_s \quad (28)$$

Where $x^{u,\min} = \text{argmin}[\Omega(x^{ua,\min}), \Omega(x^{ue,\min})]$ and there exists a one dimensional path connecting x_0 to x_{ref} such that Ω is non-increasing along the path. If a vector d satisfies the above conditions, then this design is feasible, and the l for the C_s in (26) is $\Omega(x^{u,\min})$. For different states x_0 at fault clearing time, which is out of the attraction region of uncontrolled system, the controller may select control strategies with different vector $d=(d_1, \dots, d_k)$. The vectors can be obtained by solving the optimization problem described above. In the runtime, the control system will select a control strategy with a vector $(d_1, \dots, d_k)$, of which the corresponding attraction region covers the given x_0 to guarantee that the system state will converge to x_{ref} without current overload on the branches.

VI Simulation results

This section includes two examples: the first 3 bus system case includes 1 generator bus, 1 slack bus, and 1 load bus. The following IEEE 162 bus power case includes 1 slack bus, 11 generator buses and 150 load buses. Since both systems are in heavy loaded condition, the risk of the transmission line tripping in which is high if a large disturbance is applied. The effectiveness of the proposed control schemes are demonstrated in both cases. The simulations are carried out in MATLAB 2012b.

A. 3 Bus case with branch trip

This simple example aims to prove the effectiveness of avoiding the branch trip with the proposed controller. In this case, readers may refer to Table I for detailed information of the test 3 bus system.

Bus 1 is the generator bus where the generator inertia is $M_g=0.0531$. The admittance of all branches are assumed to be 10 p.u. . The current thresholds of all

branches are set to 5 p.u.. The structure of the 3 bus power system is shown in Fig. 4.

We assume the state x_0 at fault clearing time is an unstable point, the generator frequency deviation $\omega_g=0.1\text{rad/s}$, the phase of the generator is $\delta_g=0.584\text{rad}$, the load phase is $\delta_l=-1.126\text{rad}$, and the load voltage is $V_l=1.0194\text{p.u.}$

Without control in this case, the branch 1 trips at $t=0.1\text{s}$ (Fig. 5), followed by the successive tripping of branch 2 and branch 3, i.e. the system fails in cascade. These trips are due to over-current protection, because the trip of branch 1 lead to the power flow shifting from branch 1 to branch 2 and 3, then the current on these branch are increased due to the power flow shifting along the path on branch 2 and 3, so as to carry the power flow which should have on branch 1. It demonstrates that the loss of a single branch may lead to cascade failure, and it will result in instability of the system. Thus a controller is required.

In the design of the proposed controller, assuming the largest r_k^{max} , $k \in N_{br}$ are 0.1 in all UPFCs in (4), we select the SEP $x_{ref}=(0, 0, 0, 0.034, 0.30, 1.01)^T$ as the desired SEP. According to the principle depicted in section V, we select the barrier functions coefficients $d_k=10$, $k=1, 2, 3$. the CLF value on the x_0 is $\Omega(x_0) = 2.2861$, the energy value on x_{ref} is -0.8290 , and no other stable equilibrium point (SEP) is in the region O_s constrained by the maximal branch current threshold. Three x^{ua} s are founded on the surface of CLF, their CLF values are 2.4338, 2.5218, 2.4340 and 2.4536. The CLF value of all these points are higher than that of x_0 . None of unstable equilibrium point (UEP) is found in the region O_s . Therefore, the point x_0 is in the attraction region of the designed control system, and this controller design problem is feasible.

With the proposed controller, the simulation result is shown in Fig. 6. The controller drives the system back to stable without tripping of any transmission line. The largest current magnitude is in branch 1, in the dynamic process. The current magnitude of branch 1 is very close to the threshold 5 p.u., and then the increasing current magnitude is prevented by the proposed controller, finally converged to the current on desired SEP x_{ref} . The contour maps of the CLF surface are illustrated in Fig. 7 and Fig. 8. They are contour maps are respected to load bus voltage magnitude - generator bus voltage phase and generator bus voltage phase - load bus voltage phase, with other dimensions set to the value of x_{ref} . As can be seen in the plot, barrier functions build a high wall on the current constraint boundaries, and reshape the surface of the energy function, when the state approaches the constraints, the CLF value increases dramatically.

B.162 Bus case simulation with branch trip

The IEEE 162 Bus System is used here to demonstrate the effectiveness of the proposed control scheme in a relatively large scale power system. This case was provided by courtesy of Professor Vijay Vittal and Roger Treinen of Iowa State University. It was previously distributed by Iowa State on diskette, and it has been the subject of an IEEE Committee report. This case has 11 generators on bus 1-11, 150 loads bus 12-161 and 1 swing bus 0. There are 280 branches in the power system. We have a relatively weak connection for branch connected bus 59 and bus 7, bus 159 and bus 57, bus 59 and 57. The fault (the branch connected load bus 57 and generator bus 5 is tripped) happens at 0.02s and then is cleared at 0.03s. The simulation results without

control are shown in Fig. 9 and Fig. 10. The fault leads to cascaded failure of two branches (159-57, 59-57), as shown in Fig.9. However, from Fig. 10 it is known that the bus frequency deviation increases when the fault happens and decreases after the fault is cleared.

For the state x_0 at fault clearing time 0.03s, the MEP from the state x_0 to the desired SEP x_{ref} is shown in Fig. 11. The curve is monotonously decreasing, which means the point x_0 is in the component of D_δ^{ref} where $\delta=\Omega(x_0)$. the values of barrier function coefficients $d_k, k \in N_{br}$ are all selected to be 1, the $r_k^{\text{max}}, k \in N_{br}$ of UPFCs are assumed to be 0.01, and the parameter set satisfies the attraction region analysis in section VI, i.e. our control design is theoretically feasible. The dynamic process simulation results with control are illustrated in Fig. 12.

From Fig. 12 it can be seen that the proposed controller successfully prevents the branch trip. The current magnitude respect to the branch between load bus 59 and load bus 7 is dramatically increased during time 0.04s and 0.07s from 0.13p.u. to 0.23p.u., and then damped during the time 0.07s and 0.3s. Afterward, the system state converged to the x_{ref} . Regard to current of branches between load bus 59 and load bus 57, load bus 159 and load bus 57 the current magnitude increased during time 0.03 and 0.04, then damped. In addition, from Fig. 12 it is apprent that the curves peaks are shaved during 0.03s and 0.07s, which are results from the control effect imposed on the system as the state point approaching the current constraint on the branch between swing bus 0 and load bus 61. Above all, the proposed control scheme is effective in dealing the problem of avoiding a branch trip caused by the violation of the current magnitude constraint

during the transient process.

VII Conclusion

A new corrective control scheme for power systems equipped with unified power flow controller (UPFC) devices is proposed. The control objective is to avoid cascaded transmission over-current tripping. The control schemes are directly derived by a CLF which consists of energy function and one or more barrier functions. The current overloads are avoided by employing barrier functions. With the control effect of UPFC, the attraction region of the system is reshaped according to the required operating points. Finally, we demonstrate the effectiveness of the proposed method with a 3-bus system case and an IEEE 162 bus test system case. However, because equipping each branch with a UPFC is a costly proposition at present, we are currently investigating optimal UPFC deployment strategies that seek balance between a system operating performance and the practical deployment costs and considering apply event-triggered control strategy to reduce the cyber cost[26].

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Appendix

Claim A.1[19]. Let us define $D_m = \{x/\Phi(x) - \Phi(x^{ref}) < m\}$ for the power system and D_m^{ref} is the component of D_m containing the point x_{ref} , $m \in \mathbb{R}$, Φ is the energy function. select a scalar $\zeta \in \mathbb{R}$ small enough, and let m is increased from ζ . Then for any m , $\zeta \leq m \leq \hat{m}$, where $\hat{m}$ = smallest real value such that D_m^{ref} first contain a point $x^{u,min} \neq x_{ref}$ at which $\nabla\Phi(x^{u,min})=0$. Then the point $x^{u,min}$ is the closest UEP and $\Omega(x^{u,min})=\hat{m}$.

Proof: Consider behavior of sets of form Dm , as m is increased from zero.

For a sufficiently small $\zeta > 0$, the fact that $\Phi(x_{ref})=0$ and $\nabla^2\Phi(x_{ref}) > 0$ result in $D_{\zeta,ref}$ must be a bounded set, contained in region of attraction of x_{ref} .

Moreover, for models constructed in (14), the constant matrix A is negative semi-definite. Therefore, for any $x_{ref} \in$ boundary of set $D_{e,xref}$, we may have

$$\left\langle \frac{dx}{dt} \Big|_{t_0} = A\nabla\Phi(x_0), \nabla\Phi(x_0) \right\rangle = \frac{1}{2} \nabla\Phi(x_0)^T (A + A^T) \nabla\Phi(x_0) \leq 0$$

Which indicates: if $\left\langle \frac{dx}{dt} \Big|_{t_0} = A\nabla\Phi(x_0), \nabla\Phi(x_0) \right\rangle \leq 0$, then all trajectories of the system

cross the boundary of $D_{e,x0}$ transversally, from “outside” of $D_{e,x0}$ towards its interior.

Hence the first point contained $x^{u,min} \neq x_{ref}$ at which $\nabla\Phi(x^{u,min})=0$ is the closest UEP.

Lemma A.1[19, 24]. For energy function Φ , A practical test to guarantee that the system start on the state x_0 at fault clearing time would converge to the desired SEP x_{ref} might

be constructed as:

- 1) Check that $\Phi(x_0) < \Phi(x^{u,min})$;
- 2) Ensure that there exist a one dimensional path connecting x_0 to x_{ref} such that Φ is non-increasing along the path.

Proof: The proof for the proposition follows the definition of the closest UEP method [24]. For the system state x_0 , if we could find the UEP $x^{u,min}$ such that the above two conditions are both satisfied, then we may claim that the path we found belongs to a connected component of the set $\{x: \Phi(x) < \Phi(x^{u,min})\}$ containing the stable equilibrium point x_{ref} , by the definition of the closest UEP. Then the convergence is guaranteed.

Reference

- [1] P. Kundur, Power System Stability and Control: McGraw-Hill Education, 1994.
- [2] M. A. Sayed and T. Takeshita, "All Nodes Voltage Regulation and Line Loss Minimization in Loop Distribution Systems Using UPFC", IEEE Transactions on Power Electronics, vol. 26, pp. 1694-1703, 2011.
- [3] S. Bruno and M. La Scala, "Unified power flow controllers for security-constrained transmission management", IEEE Transactions on Power Systems, vol. 19, pp. 418-426, 2004.
- [4] S. Bruno, E. De Tuglie, M. La Scala, and P. Scarpellini, "Dynamic security corrective control by UPFCs", IEEE Transactions on Power Systems, vol. 16, pp. 490-497, 2001.
- [5] S. Wei and V. Vittal, "LP-Based OPF for Corrective FACTS Control to Relieve Overloads and Voltage Violations", IEEE Transactions on Power Systems, vol. 21, pp. 1832-1839, 2006.
- [6] I. Genc, R. Diao, V. Vittal, S. Kolluri, and S. Mandal, "Decision Tree-Based Preventive and Corrective Control Applications for Dynamic Security Enhancement in Power Systems", Power

- Systems, IEEE Transactions on, vol. 25, pp. 1611-1619, 2010.
- [7] X. Yan, D. Zhao Yang, G. Lin, Z. Rui, W. Kit Po, and L. Fengji, "Preventive Dynamic Security Control of Power Systems Based on Pattern Discovery Technique", Power Systems, IEEE Transactions on, vol. 27, pp. 1236-1244, 2012.
- [8] A. A. Abou El Ela and S. R. Spea, "Optimal corrective actions for power systems using multi-objective genetic algorithms", Electric Power Systems Research, vol. 79, pp. 722-733, 2009.
- [9] Z. Feng, V. Ajjarapu, and D. J. Maratukulam, "A comprehensive approach for preventive and corrective control to mitigate voltage collapse", IEEE Transactions on Power Systems, vol. 15, pp. 791-797, 2000.
- [10] T. B. Nguyen and M. A. Pai, "Dynamic security-constrained rescheduling of power systems using trajectory sensitivities", Power Systems, IEEE Transactions on, vol. 18, pp. 848-854, 2003.
- [11] C. L. DeMarco, "A phase transition model for cascading network failure", Control Systems, IEEE, vol. 21, pp. 40-51, 2001.
- [12] C. L. DeMarco and T. J. Overbye, "An energy based security measure for assessing vulnerability to voltage collapse", IEEE Transactions on Power Systems, vol. 5, pp. 419-427, 1990.
- [13] K. M. Son and R. H. Lasseter, "A Newton-type current injection model of UPFC for studying low-frequency oscillations", Ieee Transactions on Power Delivery, vol. 19, pp. 694-701, Apr 2004.
- [14] R. Ortega, A. J. Van Der Schaft, I. Mareels, and B. Maschke, "Putting energy back in control", Control Systems, IEEE, vol. 21, pp. 18-33, 2001.
- [15] Z. Honghao and C. L. DeMarco, "A bi-stable branch model for energy-based cascading failure analysis in power systems", in North American Power Symposium (NAPS), 2010, pp. 1-7, 2010.
- [16] S. Wright and J. Nocedal, Numerical optimization vol. 2: Springer New York, 1999.

- [17] K. P. Tee, S. S. Ge, and E. H. Tay, "Barrier Lyapunov Functions for the control of output-constrained nonlinear systems", *Automatica*, vol. 45, pp. 918-927, 2009.
- [18] S. Sastry, *Nonlinear systems: analysis, stability, and control* vol. 10: Springer New York, 1999.
- [19] H. Zheng, "Lyapunov Approaches to Power System Cascading Failure Analysis", Ph.D. Dissertation, Electrical and Computer Engineering, University of Wisconsin-Madison, in preparation.
- [20] L. Jaewook, "Dynamic gradient approaches to compute the closest unstable equilibrium point for stability region estimate and their computational limitations", *IEEE Transactions on Automatic Control*, vol. 48, pp. 321-324, 2003.
- [21] L. Jaewook and C. Hsiao-Dong, "A singular fixed-point homotopy method to locate the closest unstable equilibrium point for transient stability region estimate", *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 51, pp. 185-189, 2004.
- [22] W. E. W. Ren, and E. Vanden-Eijnden, "Simplified and improved string method for computing the minimum energy paths in barrier-crossing events", *The Journal of Chemical Physics*, vol. 126, pp. 164103-1-164103-8, 2007.
- [23] G. Henkelman, B. P. Uberuaga, and H. Jónsson, "A climbing image nudged elastic band method for finding saddle points and minimum energy paths", *The Journal of Chemical Physics*, vol. 113, pp. 9901-9904, 2000.
- [24] C. Hsiao-Dong, F. F. Wu, and P. P. Varaiya, "Foundations of direct methods for power system transient stability analysis", *IEEE Transactions on Circuits and Systems*, vol. 34, pp. 160-173, 1987.
- [25] J. Sun, Y. Chai, Y. H. Hu, H. H. Zheng, R. Ling, K. Zhang, "UPFCs control design for avoiding generator trip of electric power grid with barrier function", *International Journal of Electrical Power & Energy Systems*, vol. 68, pp. 150-158, 2015.

- [26] J. Sun, Y. H. Hu, Y. Chai, R. Ling, H. H. Zheng, G. H. Wang, Z. Z. Zhu, "L-infinity event-triggered networked control under time-varying communication delay with communication cost reduction", *Journal of the Franklin Institute*, Available online 12 August 2015.

Fig. 1 Equivalent circuit of UPFC

Fig. 2 Phasor diagram

Fig. 3 The control process

Fig. 4 3 bus system structure

Fig. 5 Current magnitude curve without control in 3 bus test system

Fig. 6 Current magnitude curve with control in 3 bus test system

Fig. 7 Contour map of control Lyapunov function respect to load bus voltage magnitude
and angular of generator bus voltage

Fig. 8 Contour map of control Lyapunov function respect to angular of generator bus voltage and angular of load bus voltage

Fig. 9 Branch current magnitude curves without control

Fig. 10 Frequency deviation curves without control

Fig. 11 Minimal energy path from state x_0 to desired SEP x_{ref} on the CLF surface

Fig. 12 Branch current magnitude curves with control

Table I System Info of 3 Bus Test System (Initial Condition)

Bus (Type)	P	Q	V	Damp
1 (gen)	6	N/A	1.06	0.05
2 (ref)	-3.5	N/A	1.005	N/A
3 (load)	-2.5	-0.04	1.019	0.05

Note: P/Q means injected active/reactive power, voltage on bus 1, 2 is held constant. All units are in per unit.