Active Assist Device for Simultaneous Reduction of Heat and Vibration in Precision Scanning Stages

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Abstract

Scanning stages are used for precise positioning in a variety of advanced manufacturing processes, and must deliver high accelerations/decelerations at motion reversals to achieve high throughput. The resulting inertial forces cause excessive motor heating and residual vibration of the stage, both of which must be mitigated to preserve position accuracy. A novel scanning stage, having a permanent magnet based device that actively provides assist forces to the motor to simultaneously reduce motor heating and residual vibration, is presented. The active assist device is optimally designed to be versatile, allowing it to provide adequate assist for scan trajectories involving the widest range of scan strokes/positions. A basis-spline-based method is proposed for optimal control of the device, to maximize the assist it provides for scan trajectories with varying scan strokes/positions. Experiments conducted with a prototype stage involving scanning trajectories commonly employed in silicon wafer processing are used to demonstrate over 60% and 32% reductions in heat and vibration, respectively, when compared to cases without the proposed active assist device.

Keywords: Scanning stage, motor heating, residual vibration, trajectory optimization, assist device.

Nomenclatures

List of symbols

α	maximum repulsive force between two permanent magnets
β	distance decay rate of force between two permanent magnets
Δ	length difference between successive scan strokes
δ_c	critical distance between two permanent magnets ensuring no collision
δ_{CV}	critical distance between two permanent magnets during constant velocity scanning
ζ	damping ratio of isolation system
η	efficiency of ball screw system
θ	angular motion direction
τ	torque of rotary motor
Ω	rotation speed of rotary motor
ω_n	natural frequency of isolation system
ω_{res}	resonance frequency of isolation system
a_{PM}	acceleration of moveable permanent magnet assembly
a_B	acceleration of isolated machine base
a_T	acceleration of scan table
b_B	damping coefficient of isolation system for machine base

b_{PM}	equivalent viscous damping coefficient of moveable permanent magnet assembly
b_T	viscous damping coefficient of scan table assembly
d	distance between two permanent magnets
F_{LM}	linear motor force
F_{PM}	repulsive force between two permanent magnets
F_{RM}	equivalent force of rotary motor
g	knot of basis-spline curve
I_m	mass moment inertia of rotor (rotary motor)
I_s	mass moment inertia of ball screw shaft
J_{contr}	control objective function of magnet assisted scanning stage
J_{vers}	versatility objective function for ball screw system
k_B	spring stiffness of isolation system for machine base
k_{PM}	equivalent spring stiffness between two repulsive permanent magnets
l_s	lead of ball screw
l_T	length of scan table
m_n	mass of ball screw nut
m_{PM}	equivalent mass of moveable permanent magnet assembly
m_T	mass of scan table assembly
N	basis function of basis-spline curve
n	length of vector
p	control point of basis-spline curve
q	number of motion reversal regions included in each optimizing window

R	percentage increase in resonance frequency
r	degree of basis-spline curve
t	time
T_s	sampling interval of discrete-time domain
u	number of motion reversal regions
v_{PM}	speed of moveable permanent magnet assembly
v_T	speed of scan table
w	weight for bi-objective optimization
x	horizontal motion direction
x_B	horizontal displacement of isolated machine base
x_g	horizontal displacement of machine ground
x_{PM}	horizontal displacement of moveable permanent magnet assembly
x_T	horizontal displacement of scan table

List of acronyms and abbreviations

AAD	active assist device
acc	acceleration
AD	assist device
B-spline	Basis-spline
CAD	computer-aided design
CV	constant velocity
dec	deceleration

FRF	frequency response function
LM	linear motor
MR	motion reversal
PAD	passive assist device
PM	permanent magnet
PWM	pulse-width modulation
RM	rotary motor
RMS	root mean square
SF	safety factor

1. Introduction

Precision scanning stages are used to provide repetitive back and forth motions required in a variety of advanced manufacturing processes, such as laser patterning, 3D printing, silicon wafer patterning and inspection [1] – [4]. A schematic illustrating a typical scan profile, which can be divided into repeating step and scan motions, is shown in Figure 1. Each scan stroke is composed of one constant velocity (CV) and two motion reversal (MR) regions, located at either end of the CV region. The CV regions are where the actual manufacturing process (e.g., printing, patterning or inspection) takes place; so positioning within the CV regions must be extremely precise. On the other hand, the MR regions do not directly contribute to the actual manufacturing process; therefore, they need to be executed as fast as possible, i.e., with very high accelerations/decelerations (acc/dec) to increase throughput. The large inertial forces resulting from high acc/dec cause Joule heating of the motor, which is proportional to the square of the motor current (or motor force),

contributing increased thermal errors of the stage [5], [6]. Moreover, precision scanning stages are typically mounted on soft springs, to isolate the stage from the adverse effects of ground vibration [7]. However, when the isolated stage is subjected to large inertial forces, residual vibration is generated, which degrades the precision and/or achievable speed of the stage [8].

Various methods such as forced cooling [9], thermal compensation [10], [11], light weighting of moving parts [12], passive assist devices [13] and optimal control of motors [14] have been employed to mitigate thermal problems from motion stages. Similarly, the use of balance masses [15], counter motion devices [16], active control [17] – [19], tuned mass dampers [19], optimal placement of isolators [8], [20], and input-shaping [21] have been demonstrated for tackling residual vibration. While these methods, to greater or lesser extents, separately address the heat or vibration problems of precision scanning stages, they are not typically designed to address both problems simultaneously. Several benefits, e.g., cost, space and energy savings, can be obtained by tackling both problems simultaneously.

Prior work done by the present authors [22] demonstrated, for the first time, the simultaneous reduction of motor heating and residual vibration in precision scanning stages, using a permanent magnet (PM) based passive assist device (PAD). However, their PAD was not versatile; it was shown to be very effective for motion profiles with fixed scan strokes/positions, but was not suitable for scan profiles with varying strokes/positions (which are very common in practice). This paper (and its preliminary version [23]), addresses the shortcomings of the authors' prior work by proposing: (1) a PM based active assist device (AAD) for simultaneously reducing motor heating and residual vibration for scan trajectories with varying scan strokes/positions, and (2) a design and control

methodology for the AAD to maximize its versatility and effectiveness in reducing heat and vibration. The concept and optimal design of the proposed PM based AAD are presented in Section 2, followed by the optimal control method for the AAD in Section 3. The versatility and effectiveness of the proposed AAD in reducing heat and vibration are experimentally validated on a prototype stage in Section 4, using two wafer scanning case studies.

Figure 1. Typical scan profile comprising repeated step and scan motions. Each scan stroke can be divided into constant velocity (CV) and motion reversal (MR) regions.

2. Concept and Design Optimization of PM based Active Assist Device

2.1. Concept of PM based Assist Device for Precision Scanning Stages

Consider the schematic of the x -axis (i.e., the scanning axis) of a precision scanning stage shown in Figure 2 (a). The scanning table, actuated by linear motor force F_{LM} , is mounted on a rigid base. The base is isolated from ground vibration using very soft springs

in order to achieve the desired precision in the CV regions [22]. However, the presence of the soft springs causes unwanted horizontal (x) and rocking (θ) vibration of the base due to the large inertial forces present during acc/dec in the MR regions [22]. Upon arriving at the next CV region, the stage must wait for the residual vibration to settle before the manufacturing process resumes, thus slowing down the manufacturing process. Moreover, large inertial forces draw high electric currents from the motors, causing unwanted heat that compromises accuracy in the CV regions. Assist devices (ADs), shown as dotted springs in Figure 2 (a), can help reduce the magnitude of F_{LM} needed for acc/dec by storing and releasing some of the kinetic energy of the table when it is in the MR regions, thus reducing motor heating [13], [22]. Residual vibration can be reduced simultaneously by transmitting the reaction forces from the ADs directly to the ground so that they do not disturb the vibration-sensitive base of the machine [4], [22]. An ideal AD would store and release all of the kinetic energy of the scanning table. Additionally, it would disengage completely from the table upon entering the CV region to stop the transmission of ground vibration to the table, and to prevent the actuators from doing unnecessary work against the AD to maintain the stage at constant velocity.

As shown in Figure 2 (b), two pairs of repelling PMs, i.e., PM_i ($i = \{1, 2\}$), can be used to construct a set of ADs for the scanning axis of a precision scanning stage. For each pair, one of the PMs is mounted to the table of length l_T and position x_T , and the other PM is positioned in the MR region, via an actuator mounted to the ground. Accordingly, the PM based ADs are effectively disengaged from the table when the table is in the CV region (because, due to the large distance d_i between the PM pairs, the repulsion forces F_{PMi} between them are virtually zero [22]). On the other hand, much needed assist is provided as

the table enters either MR region, due to the decreased d_i between the corresponding PM_i pair which increases F_{PM_i} .

In the prior work by the authors [22], the positions x_{PM_i} of the MR-side PMs were controlled to be fixed at optimal locations, determined based on a given scan table trajectory $x_T(t)$ with a fixed scan stroke (note, t denotes time). In other words, the ADs were configured as passive assist devices (PADs). However, many applications of scanning stages involve x_T trajectories with varying scan strokes/positions, which render the assist provided by PADs suboptimal or ineffective. This paper proposes replacing PADs with active assist devices (AADs) for which x_{PM_i} are not fixed, but are instead actively controlled to vary in real time in order to provide versatility and optimal assist for x_T trajectories with varying scan strokes/positions.

with 600 N peak and 150 N continuous force (combined) is selected to drive the table; each motor is powered by a PWM servo drive (Kollmorgen, AKD-P00306) in current mode. The table position is measured using linear encoders (Renishaw, RGSZ20 scale and T1000 readhead) with 4.88 nm resolution post-interpolation. The scanning table sits on a 900 mm $\times$ 600 mm $\times$ 100 mm granite base suspended by four pneumatic isolators with automatic leveling (Bilz, BiAir[®] 0.5-ED). Each PM is constructed using two adjacent 40 mm $\times$ 40 mm 2-D Halbach arrays (see Figure 3 (b)), capable of providing the 600 N force needed to fully assist the LMs. The design and sizing of the Halbach arrays are detailed in [22]. The PMs are configured as AADs by actuating each MR-side PM using a ball screw-driven servo system, which is connected directly to the ground through a bridge. The design of the servo system, and particularly optimal sizing of the transmission ratio of the ball screw, is the focus of the rest of the section.

(a)

(b)

Figure 3. CAD schematic of (a) prototype PM assisted scanning stage and (b) MR-side PM mounted on bridge (connected to ground).

Table 1. Specifications of prototype scanning stage.

Max. travel of table	0.3 m
Max. speed of table	1 m/s
Max. acceleration of table	35 m/s ²

The precision requirements of the servo system of the AADs are not tight, since they are not directly involved in manufacturing; micron-level positioning resolution is sufficient for them. Therefore, as shown in Figure 3 (b), an inexpensive servo system, comprising a single linear guide (Misumi, SSEBZ13) and a 10 mm-diameter rolled ball screw (Misumi, BSSR1002) driven by a rotary stepper motor (Oriental Motor, NX620AC-3) is employed for each AAD. The AAD position is measured by a motor-built-in encoder providing 1 μm resolution. The selection of the motor is primarily based on size and torque constraints; it must have enough torque to support F_{PMi} while keeping a small enough profile to fit into the stage. The diameter of the screw is selected to adequately satisfy loading conditions.

Once the motor is selected, the power available to the servo system is fixed. However, the transmission ratio (i.e., lead) of the ball screw can be optimized to best allocate the power of the motor for the purpose of the AAD. A lot of work has been done on optimizing transmission ratios for load bearing, speed, acceleration, and etc. (e.g., [24] – [26]). However, in this paper, the optimization of ball screw lead for versatility (i.e., the ability of an AAD to accommodate the widest range of scan strokes/positions using the given motor) is sought. To further explain, let x_{Tj} and x_{Tj+1} ($j = \{1, 2, 3, \dots\}$) in Figure 4 (a)

represent any two successive scan positions of the table where assist is to be provided by a given PMi pair (e.g., PM1). To successfully provide the needed assist, the MR-side PM must be repositioned over distance $\Delta = |x_{T,j+1} - x_{T,j}|$ during the travel time of the table from $x_{T,j}$ to $x_{T,j+1}$. Assuming trapezoidal velocity profiles for the table and PM, as shown in Figure 4 (b), the following condition must be satisfied:

$$t_a + t_b + t_c + t_d + t_e + t_f = t_1 + t_2 + t_3 \quad (1)$$

The time segments for the trajectory of table are given by

$$\begin{aligned} t_a = t_c &= \min \left(\frac{v_{T,\max}}{a_{T,\max}}, \sqrt{\frac{x_{T,j}}{a_{T,\max}}} \right) \\ t_b &= \max \left(0, \frac{x_{T,j} - a_{T,\max} t_a^2}{v_{T,\max}} \right) \\ t_d = t_f &= \min \left(\frac{v_{T,\max}}{a_{T,\max}}, \sqrt{\frac{|x_{T,j} \pm \Delta|}{a_{T,\max}}} \right) \\ t_e &= \max \left(0, \frac{|x_{T,j} \pm \Delta| - a_{T,\max} t_d^2}{v_{T,\max}} \right) \end{aligned} \quad (2)$$

where $v_{T,\max}$ and $a_{T,\max}$ represent the maximum speed and acceleration of the table, which have been chosen because they represent the worst-case assist requirements of the table.

Similarly, the time segments for the PM are given by

$$\begin{aligned} t_1 = t_3 &= \min \left(\frac{v_{PM,\max}}{a_{PM,\max}}, \sqrt{\frac{\Delta}{a_{PM,\max}}} \right) \\ t_2 &= \max \left(0, \frac{\Delta - a_{PM,\max} t_1^2}{v_{PM,\max}} \right) \end{aligned} \quad (3)$$

with $v_{PM,max}$ and $a_{PM,max}$ representing the maximum speed and acceleration of the PMs.

They can be calculated based on the maximum rotational speed Ω_{max} and continuous torque τ_{max} of the rotary motor (RM), using the equations

$$v_{PM,max} = \frac{\Omega_{max} [\text{rpm}]}{60} l_s \quad (4)$$

$$a_{PM,max} = \left(\frac{\tau_{max} \eta}{SF} - \frac{F_{PM,max} l_s}{2\pi} \right) \frac{l_s}{2\pi} \left(m_n \frac{l_s^2}{4\pi^2} + I_s + I_m \right)^{-1} \quad (5)$$

where $F_{PM,max}$ represents the maximum assist force provided by the PM; l_s , SF , and η are respectively the lead, safety factor, and mechanical efficiency of the ball screw system; I_s and I_m represent the mass moments of inertia of the screw shaft and rotor (motor), respectively, and m_n is the mass of the nut (including the PM attached to it). One observes by substituting Eqs. (2) – (5) into Eq. (1) that the repositioning distance Δ is a function of l_s and $x_{T,j}$, which could assume any value between $x_{T,min}$ and $x_{T,max}$, representing the minimum and maximum strokes of the scanning table. Arising from the $\pm$ condition attached to Δ in Eq. (2), two solutions (i.e., Δ_1 and Δ_2) are available for a given l_s and $x_{T,j}$ set, because $|x_{T,j+1}|$ can either be larger or smaller than $|x_{T,j}|$. Accordingly, the optimal l_s is selected to maximize a versatility objective function, J_{vers} , defined as

$$\underset{l_s}{\text{maximize}} J_{vers} = \frac{1}{2(x_{T,max} - x_{T,min})} \int_{x_{T,min}}^{x_{T,max}} (\Delta_1(l_s, x_{T,j}) + \Delta_2(l_s, x_{T,j})) dx_{T,j} \quad (6)$$

representing the average value of Δ for all values of $x_{T,j} \in [x_{T,min}, x_{T,max}]$. Figure 5 shows a plot of J_{vers} vs. l_s , computed using the parameters of the prototype stage summarized in Table 2. Notice that the optimal lead is $l_s = 2.87$ mm, which corresponds to $J_{vers} = 63.0$ mm. However, the AADs shown in Figure 3 had to be constructed with $l_s = 2$ mm, the closest

lead to 2.87 mm that was available for the selected make and diameter of ball screws, resulting in $J_{vers} = 53.4$ mm (i.e., a 15% sacrifice of versatility compared to $l_s = 2.87$ mm).

Figure 4. (a) Scan trajectory with varying scan strokes/positions, (b) Velocity profiles of scan table and PM during two successive scan strokes.

Figure 5. Versatility objective function (J_{vers}) vs. ball screw lead (l_s).

Table 2. Parameters used for optimal design of PM based AAD.

$\omega_{\max} = 5500 \text{ rpm}$	$SF = 1.5$
$F_{PM,\max} = 600 \text{ N}$	$\eta = 0.9$
$m_n = 1.5 \text{ kg}$	$\tau_{\max} = 0.64 \text{ Nm}$
$a_{T,\max} = 35 \text{ m/s}^2$	$I_s = 3.01 \times 10^{-6} \text{ kg} \cdot \text{m}^2$
$v_{T,\max} = 1 \text{ m/s}$	$I_m = 1.63 \times 10^{-5} \text{ kg} \cdot \text{m}^2$
$x_{T,\min} = 0.05 \text{ m}$	$x_{T,\max} = 0.3 \text{ m}$

3. Optimal Control of Active Assist Device

3.1. System Modeling

The equations of motion of the table and PMs shown in Figure 2 (b) can be written as

$$m_T \ddot{x}_T(t) + b_T \dot{x}_T(t) = F_{LM}(t) - F_{PM1}(t) + F_{PM2}(t) \quad (7)$$

$$m_{PM1} \ddot{x}_{PM1}(t) + b_{PM1} \dot{x}_{PM1}(t) = F_{RM1}(t) + F_{PM1}(t) \quad (8)$$

$$m_{PM2} \ddot{x}_{PM2}(t) + b_{PM2} \dot{x}_{PM2}(t) = F_{RM2}(t) - F_{PM2}(t) \quad (9)$$

where m and b respectively denote equivalent masses and viscous damping coefficients, while the subscripts T and PM refer to the table and PMs, respectively; F_{RMi} represent the equivalent RM forces used to drive the respective PMi . Eqs. (7) – (9) are derived based on the assumption that the table and PMs (together with their ball screw actuators) are rigid, and that the motions of the base are negligible compared to those of the table and PMs. It is however of interest to consider the scanning direction (x -axis) motion of the base x_B caused

by horizontal motions of the ground x_g , particularly within the CV region where high precision is desired (see Figure 6). Even though the horizontal vibration isolators of combined stiffness k_B and damping coefficient b_B are designed to reduce the transmission of x_g to x_B , the PMs create another channel for x_g to reach x_B through their equivalent stiffness k_{PM} . It is important to ensure that the presence of k_{PM} does not subvert the isolation of x_B from x_g .

For the purpose of analyzing the transmissibility of x_g to x_B , without loss of generality, assume that the table is at rest relative to the base, and that the connection between the table and base is rigid, due to the high bandwidth controllers regulating the position of the table relative to the base. Accordingly, the equation of motion of the base is given by

$$\ddot{x}_B(t) + 2\zeta\omega_n\dot{x}_B(t) + \omega_n^2 x_B(t) = 2\zeta\omega_n\dot{x}_g(t) + \omega_n^2 x_g(t) \quad (10)$$

where ω_n and ζ are respectively the natural frequency and damping ratio of the isolation system (including the effect of the PMs), given by

$$\omega_n = \sqrt{\frac{k_B + k_{PM}}{m_B + m_T}} \quad (11)$$

$$\zeta = \frac{b_B}{2(m_B + m_T)\omega_n} \quad (12)$$

where m_B represents the mass of the base. Note that the equivalent stiffness provided by the two PM pairs is nonlinearly related to the distance d_i between each PM pair; it is given by

$$k_{PM} = -\frac{d}{d(x_B - x_g)}(-F_{PM1} + F_{PM2}) = \alpha\beta e^{-\beta \cdot d_1} + \alpha\beta e^{-\beta \cdot d_2} \quad (13)$$

based on the understanding that $d_1 = d_{1,0} - (x_B - x_g)$, $d_2 = d_{2,0} + (x_B - x_g)$, and the assumption that F_{PMi} can be expressed as [22]

$$F_{PMi}(d_i) = \alpha e^{-\beta d_i} \quad (14)$$

where α and β are positive constants and $d_{i,0}$ represent the initial values of d_i (i.e., when $x_B = x_g = 0$) at the position of the table where k_{PM} is evaluated. The worst-case degradation of transmissibility in the CV regions due to the AADs occurs when the table is sandwiched between the two PM pairs such that $d_1 = d_2 = \delta_{CV}$. In this case, the table experiences the maximum stiffness given by

$$k_{PM, \max} = 2\alpha\beta e^{-\beta \cdot \delta_{CV}} \quad (15)$$

where δ_{CV} represents the smallest allowable distance between any two PM pairs when the table is in the CV region. The resonance frequency of an isolation system, given by

$$\omega_{res} = \omega_n \sqrt{1 - 2\zeta^2} \quad (16)$$

is commonly considered as a figure of merit for its vibration isolation quality [7]. Therefore, δ_{CV} can be selected to ensure that the worst-case percentage increase in resonance frequency, R , given by

$$R = \left(\frac{\omega_{res}|_{k_{PM}=k_{PM, \max}} - \omega_{res}|_{k_{PM}=0}}{\omega_{res}|_{k_{PM}=0}} \right) \times 100\% \quad (17)$$

is acceptably small, meaning that transmissibility is not unduly sacrificed by the introduction of the AADs.

Figure 6. Representation of isolation system using a mass-spring-damper model.

3.2. Formulation of Optimal Control Problem

Optimal control of the AADs involves the determination of $x_{PMi}(t)$ for any given $x_T(t)$ such that F_{LM} is minimized, as a means to reduce heat and vibration. Note, however, that there are costs associated with providing assist using the AADs. The rapid motion of the PMs together with the larger F_{PMi} associated with providing assist for scan trajectories with variable strokes/positions increase F_{RMi} , which in turn generates heat in the RMs. Furthermore, the reaction forces conducted to the ground via F_{RMi} (which include F_{PMi}) may increase the amplitude of ground vibration.

Representing the respective motion trajectories of the table and PMs as vectors $\mathbf{x}_T = [x_T(1) \dots x_T(n)]^T$ and $\mathbf{x}_{PMi} = [x_{PMi}(1) \dots x_{PMi}(n)]^T$ in discrete-time domain, with sampling interval T_s [s], the equivalent motor force signals necessary to track the trajectories can be derived from Eqs. (7) – (9) as

$$\mathbf{F}_{LM} = m_T \ddot{\mathbf{x}}_T + b_T \dot{\mathbf{x}}_T + \mathbf{F}_{PM1} - \mathbf{F}_{PM2} \quad (18)$$

$$\mathbf{F}_{RM1} = m_{PM1} \ddot{\mathbf{x}}_{PM1} + b_{PM1} \dot{\mathbf{x}}_{PM1} - \mathbf{F}_{PM1} \quad (19)$$

$$\mathbf{F}_{RM2} = m_{PM2} \ddot{\mathbf{x}}_{PM2} + b_{PM2} \dot{\mathbf{x}}_{PM2} + \mathbf{F}_{PM2} \quad (20)$$

where the dot accents on the vectors represent the discretized derivatives of respective motion trajectories. $\mathbf{F}_{PM1}$ and $\mathbf{F}_{PM2}$ can be evaluated using Eq. (14) with the respective distance vectors, $\mathbf{d}_1$ and $\mathbf{d}_2$, given as

$$\mathbf{d}_1 = \mathbf{x}_{PM1} - \mathbf{x}_T - \frac{1}{2} l_T \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}_n \quad (21)$$

$$\mathbf{d}_2 = \mathbf{x}_T - \mathbf{x}_{PM2} - \frac{1}{2} l_T \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}_n \quad (22)$$

Note that each vector in Eqs. (18) – (22) is $n \times 1$ in size, where n is the number of data points in the discretized trajectories. Moreover, the equations assume that the servo dynamics of the LM and RMs can be ignored such that $\mathbf{x}_T$ is the same as the desired trajectory of the table, which is known, and $\mathbf{x}_{PMi}$ represent the desired trajectories of the PMs, which are to be determined. Hence the optimal control of the AADs boils down to optimal feed forward control (or optimal trajectory generation) for the PMs.

Considering the costs and benefits of the AADs, the optimal control objective for the AADs is formulated as

$$J_{contr} = (1 - w) \mathbf{F}_{LM}^T \mathbf{F}_{LM} + w (\mathbf{F}_{RM1}^T \mathbf{F}_{RM1} + \mathbf{F}_{RM2}^T \mathbf{F}_{RM2}) \quad (23)$$

where $w \in [0, 1]$ is a user-defined weight. It is reasonable to select $w \ll 0.5$ because the heat and vibration costs associated with the RMs are typically much smaller than those associated with the LM. This is because RMs are, in general, much more efficient than LMs [6], [27], hence they generate much less heat for the same magnitude of force. Moreover,

the heat generated by the RMs is distant from the manufacturing operation, compared to the heat generated by the LM which is directly conducted to the table. Similarly, vibration of the machine ground caused by the reaction forces borne by the AADs is attenuated by the vibration isolators, whereas the reaction forces caused by F_{LM} directly excite the machine base.

Based on the defined objective, $\mathbf{x}_{PM1}$ and $\mathbf{x}_{PM2}$ can be obtained by solving the constrained optimization problem formally given by

$$\begin{aligned}
& \text{minimize } J_{contr}(\mathbf{x}_{PM1}, \mathbf{x}_{PM2}) \\
& \text{s.t.} \\
& \delta_c - \min(\mathbf{d}_i) \leq 0 \\
& \delta_{CV} - \min(\mathbf{d}_{i,CV}) \leq 0 \\
& \max(|\dot{\mathbf{x}}_{PMi}|) - v_{PM, \max} \leq 0 \\
& \max(|\ddot{\mathbf{x}}_{PMi}|) - a_{PM, \max} \leq 0
\end{aligned} \tag{24}$$

where δ_c represents a safety gap (i.e., the minimum allowable distance between either PM pair to prevent collisions). The vectors $\mathbf{d}_{i,CV}$ are subsets of the vectors $\mathbf{d}_i$; they correspond to the distances between the associated PM pairs when the table is in the CV regions.

Accordingly $\mathbf{d}_{i,CV}$ must be constrained by δ_{CV} to ensure that transmissibility is not unduly sacrificed by the AADs. Using the operator $|\cdot|$ to denote the absolute value of each vector element, the last two constraints ensure that the kinematic limits of the AAD (found using Eqs. (4) and (5)) are not violated.

3.3. Solution of the Optimal Control Problem

3.3.1. Direct Solution

A direct approach can be taken to solve the optimization problem defined in Eq. (24) by treating every element of $\mathbf{x}_{PMi}$ as an optimization variable. In this case, $2n$ variables must be optimized simultaneously using numerical optimization algorithms. The number of optimization variables can be halved by realizing that $\mathbf{x}_{PM1}$ and $\mathbf{x}_{PM2}$ are practically decoupled. This is based on the premise that, by design, as shown in Figure 2, the MR regions are separated by intervening CV regions for which no assist is needed. Nonetheless, n could still be very large in practice, thus making the numerical solution of the optimization challenging. More computationally efficient solution approaches, which employ basis-splines to represent the trajectories, are proposed and evaluated in the rest of this paper.

3.3.2. Basis-spline-based Solution

Basis-splines (or B-splines), widely used for representing free-form curves, can be adopted to reduce the number of variables in solving for the optimal trajectories [28] – [31]. A B-spline curve is characterized by its degree r , knot vector $\mathbf{g}$, and control point vector $\mathbf{p}$ (see Appendix A for more details). Using B-splines, x_{PMi} and their time derivatives can be represented as, in discrete-time domain, [31]

$$\mathbf{x}_{PMi} = \mathbf{N}_{pi} \mathbf{p}_{PMi} \quad (25)$$

$$\dot{\mathbf{x}}_{PMi} = \mathbf{N}_{vi} \mathbf{p}_{PMi} \quad (26)$$

$$\ddot{\mathbf{x}}_{PMi} = \mathbf{N}_{ai} \mathbf{p}_{PMi} \quad (27)$$

where $\mathbf{p}_{PMi}$ are control point vectors (of size $n_{PMi} \times 1$) while $\mathbf{N}_{pi}$, $\mathbf{N}_{vi}$ and $\mathbf{N}_{ai}$ are basis function matrices for displacement, velocity and acceleration, respectively [30], [31] (see Appendix A for more details). The elements of $\mathbf{p}_{PMi}$ can be used as optimization variables in place of the elements of $\mathbf{x}_{PMi}$, thus drastically reducing the size of the optimization problem, based on the fact that $n_{PMi} \ll n$. Note that, when represented as B-splines, the time derivatives of x_{PMi} are guaranteed to be smooth up to the $(r - 1)$ th derivative. Consequently, x_{PMi} and their derivatives can be down-sampled to smaller vectors based on $\mathbf{N}_{pi}$, $\mathbf{N}_{vi}$ and $\mathbf{N}_{ai}$ matrices calculated using a coarser (i.e., larger) value of T_s without sacrificing smoothness, as long as $r \geq 3$. Once the optimal $\mathbf{p}_{PMi}$ vectors calculated using the coarser sampling are obtained, the original T_s can be employed to generate new $\mathbf{N}_{pi}$, $\mathbf{N}_{vi}$ and $\mathbf{N}_{ai}$ matrices, thereby up-sampling x_{PMi} and their derivatives back to the original resolution before implementation on the actual stage. The last two constraints of Eq. (24) are enforced using vectors of length n , which is computationally burdensome. Accordingly, the length of last two constraint vectors can be reduced by replacing $\dot{\mathbf{x}}_{PMi}$ and $\ddot{\mathbf{x}}_{PMi}$ by control point velocity and acceleration vectors, $\mathbf{p}_{vi}$ and $\mathbf{p}_{ai}$, of respective lengths $n_{PMi} - 1$ and $n_{PMi} - 2$, given by

$$\mathbf{p}_{vi} = \mathbf{D}_v \mathbf{p}_{PMi} \quad (28)$$

$$\mathbf{p}_{ai} = \mathbf{D}_a \mathbf{D}_v \mathbf{p}_{PMi} \quad (29)$$

where $\mathbf{D}_v$ and $\mathbf{D}_a$ are time derivative operators expressed as

$$\mathbf{D}_v = \begin{bmatrix} h_p(1) & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & h_p(n_{PMi}-1) \end{bmatrix}^{-1} \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix}_{(n_{PMi}-1) \times n_{PMi}} \quad (30)$$

$$\mathbf{D}_a = \begin{bmatrix} h_v(1) & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & h_v(n_{PMi}-2) \end{bmatrix}^{-1} \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix}_{(n_{PMi}-2) \times (n_{PMi}-1)} \quad (31)$$

with $h_p(\cdot)$ and $h_v(\cdot)$ being the temporal spacing between vector elements of $\mathbf{p}_{PMi}$ and $\mathbf{p}_{vi}$, respectively (see Appendix A for more details). Thus, with the adoption of B-splines, the constrained optimization problem of Eq. (24) simplifies to

$$\begin{aligned} & \text{minimize } J_{\text{contr}}(\mathbf{p}_{PM1}, \mathbf{p}_{PM2}) \\ & \text{s.t.} \\ & \delta_c - \min(\mathbf{d}_i) \leq 0 \\ & \delta_{CV} - \min(\mathbf{d}_{i,CV}) \leq 0 \\ & \max(|\mathbf{p}_{vi}|) - v_{PM,\max} \leq 0 \\ & \max(|\mathbf{p}_{ai}|) - a_{PM,\max} \leq 0 \end{aligned} \quad (32)$$

reducing the number of optimization variables from n to n_{PMi} .

3.3.3. Windowed B-spline-based Solution

Even with the reduced number of optimization variables facilitated by down-sampling and the adoption of B-splines, the size of the $\mathbf{p}_{PMi}$ vectors can still be problematic for numerical solvers. Fortunately, because of the local property of B-splines [30], a windowing technique can be applied to solve the optimization problem sequentially as smaller sub-problems. Let MR_i represent the MR regions in a scan trajectory for which the

acceleration direction of the table is the same (i.e., the MR regions associated with a given PM*i* pair), and let u_i represent the total number of MR*i* regions in a scan trajectory. Then, for $j = \{1, 2, \dots, u_i - q + 1\}$, a subset of the $\mathbf{p}_{PMi}$ vectors, covering the j th to $(j + q - 1)$ th MR*i* regions can be optimized at a time (or window), where q is the number of MR*i* regions being optimized in each window.

4. Numerical and Experimental Validation

4.1. System Identification of Prototype Stage

Figure 7 shows an in-house built prototype of the scanning stage with the proposed AADs whose CAD model is presented in Figure 3. The stage is controlled using dSPACE DS1007 real-time control and DS3002 and DS5203 data acquisition boards, at 10 kHz sampling frequency. The system parameters of the stage are identified using frequency response functions (FRFs), whose magnitude plots are shown in Figure 8. Each FRF is obtained by applying constant-amplitude-sinusoidal force input signals with varying frequencies to the stage and recording the corresponding displacement output magnitudes. System parameters m_T , m_{PM1} , m_{PM2} , b_T , b_{PM1} and b_{PM2} have been obtained via curve fitting Figures 8 (a), (b) and (c) using Eqs. (7) – (9), respectively, with emphasis on lower frequencies (i.e., < 200 Hz). Parameters m_B , b_B and k_B have been obtained by curve fitting Figure 8 (d) using the dynamics from F_{LM} to x_B , given by

$$m_B \ddot{x}_B(t) + b_B \dot{x}_B(t) + k_B x_B(t) = F_{LM}(t) \quad (33)$$

Note that, in generating each FRF, the distances d_i of the PMs have been kept sufficiently large such that $F_{PMi} \approx 0$. The identified parameters are summarized in Table 3 and the FRFs

resulting from the curve fit are overlaid on the corresponding measured FRFs in Figure 8. The discrepancies at low frequencies in the FRF from F_{LM} to x_B are due to the un-modeled nonlinear dynamics of the rubber diaphragms of the vibration isolators, whose stiffness decreases with larger displacement amplitudes occurring at low frequencies [32]. The purpose of the FRF from F_{LM} to x_B is to identify m_B , c_B and k_B , which are used to analyze the transmissibility of x_g to x_B . As discussed in Section 3.1, transmissibility is typically evaluated based on FRF magnitudes beyond the resonance frequency [7]. Notice that the modeled FRF in Figure 8 (d) provides an excellent fit to the measured FRF in the region beyond the resonance frequency; hence it is accurate enough for the purposes of this paper.

Figure 7. In-house built prototype of scanning stage with proposed AADs used for experimental validation. CAD schematic of stage is presented in Figure 3.

Figure 8. Measured and curve fitted FRFs from (a) F_{LM} to x_T , (b) F_{RM1} to x_{PM1} , (c) F_{RM2} to x_{PM2} , and (d) F_{LM} to x_B

Table 3. Identified system parameters of stage. (The subscripts associated with each parameter listed in the first column is provided in parentheses in the first row).

	Table (T)	PM1 ($PM1$)	PM2 ($PM2$)	Base (B)
m [kg]	13.8	265	271	287
b [Ns/m]	123	562	567	4.68×10^3
k [N/m]	-	-	-	4.26×10^5

The F_{PM} to d relationship for both PM pairs is identified by measuring the linear motor force (i.e., F_{LM}) – via motor current – needed to vary d ; the table is controlled to move at a constant speed while the MR-side PMs and granite base are locked into fixed positions. The measured relationship, which is almost identical for both PM pairs, is shown

in Figure 9. The identified curve is fitted with parameters $\alpha = 1.11 \times 10^3$ N and $\beta = 245$ m⁻¹, following the expression given in Eq. (14).

Figure 9. Measured PM repulsion force vs. distance between PMs relationship. The relationship is almost identical for both PM pairs.

4.2. Numerical Evaluation of Solution Approaches

The performance and computational costs associated with the three solution approaches presented in Section 3.3 are comparatively evaluated in simulations using a simple scan trajectory, x_T , with the number of MR regions related to PM1 and PM2 given by $u_1 = 5$ and $u_2 = 6$, respectively, as shown in Figure 10. Its scan velocity and maximum acceleration are respectively 0.8 m/s and 20 m/s². For the solution process, $l_T = 0$ is assumed, for simplicity, because it is a constant offset value that does not make a difference in obtaining the optimal trajectories. Based on the original sampling interval ($T_s = 0.1$ ms) of x_T , $n = 30,001$, which is prohibitively large for the direct solution. Therefore, x_T is down-sampled by a factor of 20 (i.e., $T_s = 2$ ms).

For the direct approach, the down-sampled x_T is used to solve for $\mathbf{x}_{PM1}$ and $\mathbf{x}_{PM2}$ vectors separately, after which the optimal solutions are up-sampled back to the original $T_s = 0.1$ ms resolution using cubic spline interpolation. For the B-spline-based approach, with and without the windowing, the same down-sampled x_T is used to solve for the optimal $\mathbf{p}_{PM1}$ and $\mathbf{p}_{PM2}$ vectors separately. For both cases, B-spline parameters $n_{PM1} = n_{PM2} = 301$ and $r = 4$, are employed; additionally, $q = 2$ is employed for the windowed B-spline technique. The optimal solutions found with the down-sampled x_T are up-sampled back to the original resolution with an $\mathbf{N}_p$ matrix generated using $T_s = 0.1$ ms. The numerical solutions for all three techniques are performed using the *fmincon* function of MATLAB®, running on a computer having a 2.3 GHz quad-core processor and 8 GB memory. The system parameters for the prototype stage, summarized in Table 3, are used for the simulations. Additionally, $w = 10^{-4}$, $\delta_c = 1$ mm and $\delta_{CV} = 14.3$ mm are employed in Eqs. (24) and (32). Note that $\delta_{CV} = 14.3$ mm is calculated using Eq. (17) based on a change of resonance frequency from 5.7 Hz to 5.8 Hz (i.e., $R = 2\%$), indicating negligible loss of transmissibility due to the AADs.

Figure 10 shows the optimal (up-sampled) x_{PMi} and their derivatives obtained by the three solution approaches, while the resulting computational times and optimal J_{contr} (based on the up-sampled vectors) are summarized in Table 4. Notice from Figure 10 that all x_{PMi} trajectories are very similar, and that the B-splines-based solutions are almost undistinguishable. However, the time derivative profiles of x_{PM1} show that while the kinematic constraints, $v_{PM,\max} = 175$ mm/s and $a_{PM,\max} = 3$ m/s², enforced during the optimization are satisfied by both B-spline-based solutions, they are not satisfied by the direct approach after up-sampling; similar behaviors are observed for x_{PM2} , but the plots are

omitted for the sake of brevity. Notice from Table 4 that, even though the direct solution approach yields an optimal J_{contr} value that is only 0.4% lower than the J_{contr} values obtained using B-splines, it comes at the expense of significantly longer computation time (and violation of constraints). Observe also that the B-spline technique with windowing achieves the same optimal J_{contr} values as the one without windowing, but its computational time is more than two times shorter. Consequently, the windowed B-spline solution is adopted for the experimental validation of the AAD presented in following section.

Figure 10. Optimal x_{PMi} and time derivative profiles of x_{PM1} solved by three optimization approaches (direct, B-splines without windowing, and B-splines with windowing) in the original sampling resolution with $T_s = 0.1$ ms.

Table 4. Comparison of computation times and objective function values (based on $T_s = 0.1$ ms) for the three optimization approaches (direct, B-splines without windowing, and B-splines with windowing).

	Direct	B-splines without windowing	B-splines with windowing
Computation time [s]	5,660	57	25
J_{contr} [N^2]	3.380×10^8	3.392×10^8	3.392×10^8

4.3. *Experimental Evaluation of AAD using Case Studies from Silicon Wafer Processing*

The proposed AAD with optimal control achieved using the proposed windowed B-splines is evaluated in comparison with the prior work done by the authors (i.e., PAD [22]) and with the No AD condition, using two case studies from silicon wafer processing that reflect typical usage of precision scanning stages. For both cases, $T_s = 0.1$ ms is used for the actual stage implementation of the motion trajectories, but the optimization is carried out after 10 times down-sampling (i.e., $T_s = 1$ ms). The values of $w = 10^{-5}$, $\delta_c = 1$ mm, $\delta_{CV} = 14.3$ mm (corresponding to $R = 2\%$) are employed for both case studies.

4.3.1. *Case Study I: Scanning for Photolithography*

The first case study aims to replicate the scanning trajectories used in photolithographic patterning processes of silicon wafers. A 200 mm diameter wafer with 25 mm $\times$ 25 mm dies, shown in Figure 11 (a), is assumed to be scanned using the step and

scan profile shown in the figure. The scan portion (x -axis) of the trajectory, x_T , shown in Figure 11 (b), has 0.5 m/s scan speed and 25 m/s² peak acceleration, with 25 ms settling time after each MR region; its step (y -axis) motions are not considered because they are outside the scope of this paper. The dotted lines inside the solid circle line of Figure 11 (a), representing the boundary of wafer, are used to mark the edges of the dies. The scan tool (e.g., the optical field) starts its scanning from the upper left quadrant and ends the process at the lower right quadrant of the wafer. The beginning and ending portions of the trajectory are highlighted with blue dashed lines, revealing varying scan strokes/positions, while the arrows are used to indicate the direction of scanning. The green colored squares indicate the dies being scanned during the trajectory portions highlighted in the figure.

The parameters for the windowed B-spline used to generate the optimal trajectories shown in Figure 11 (b) are $n_{PM1} = n_{PM2} = 901$ $r = 5$ and $q = 2$, to ensure continuity up to the fourth derivative of x_{PMi} (i.e., snap); the computation time for generating the optimal trajectories is 283 s. The optimal (constant) x_{PM1} and x_{PM2} for the PAD case, using the same optimization formulation as the AAD configuration (i.e., Eq. (32)) but with the PMs fixed, are 131.5 and -131.5 mm, respectively, as shown in Figure 11 (b).

(a)

(b)

Figure 11. (a) Step and scan profile and (b) Scan trajectory of table, as well as optimal PAD and AAD trajectories for photolithographic patterning of 200 mm silicon wafer.

4.3.2. Case Study II: Scanning for Inspection

The second case study aims at replicating optical inspection processes of silicon wafers. Again, a 200 mm wafer diameter is used, but this time, its full length is scanned in 5 mm steps, as shown in Figure 12 (a). Its scan trajectory, x_T , having 0.25 m/s scan speed, 10 m/s² peak acceleration with 50 ms settling time after each MR region is highlighted in Figure 12 (b). The inspection tool starts its process from the bottom of wafer and ends at the top of the wafer.

B-spline parameters, $n_{PM1} = n_{PM2} = 991$, $r = 5$ (for snap continuity) and $q = 2$, are used to generate the optimal trajectories (x_{PMi}) shown in Figure 12 (b), with total computation time of 692 s. The optimal (constant) x_{PM1} and x_{PM2} for the PAD case are 126.7 and -126.7 mm, respectively.

(a)

(b)

Figure 12. (a) Step and scan profile and (b) Scan trajectory of table, as well as optimal PAD and AAD trajectories for inspection of 200 mm silicon wafer.

4.3.3. Experimental Results

The effectiveness of the optimally controlled AAD is experimentally validated on the prototype stage of Figure 7 in comparison with the No AD and PAD scenarios based on

the two case studies discussed above. Note that the No AD scenario is achieved by positioning the PMs with d_i large enough that $F_{PMi} \approx 0$ for the entire duration of scan trajectories. Three performance metrics are used to evaluate the comparative effectiveness of the AADs; they are: (1) the RMS values of $\mathbf{F}_{LM}$ and $\mathbf{F}_{RMi}$ respectively measured via motor currents and force commands; (2) the RMS values of the horizontal acceleration, $\mathbf{a}_{B,x}$, and the rotational acceleration, $\mathbf{a}_{B,\theta}$, of the isolated machine base, measured using accelerometers (PCB Piezotronics, 393B05); and (3) the RMS value of the position error within the CV regions, $\mathbf{e}_{CV}$, measured using the linear encoders of the stage. A moving average filter with 10 ms width is utilized to filter the position error signal of table, as is common practice in wafer scanning applications [4]. Table 5 summarizes the experimental results for both case studies. For most metrics, the PAD does not show much improvement over the No AD scenario, because it cannot accommodate the varying strokes of x_T , thus rendering it ineffective. However, the AAD beats both the No AD and the PAD cases in all regards, except (obviously) in terms of $\mathbf{F}_{RMi}$.

The heat energy $E_{heat,LM}$ generated by the LM (note that two LM actuators in parallel are used to drive the scan table) per scan cycle is given by

$$E_{heat,LM} = \frac{\mathbf{F}_{LM}^T \mathbf{F}_{LM}}{n \cdot K_{m,LM}^2} T_{tot} \quad (34)$$

where $K_{m,LM}$ is the equivalent motor constant of the LM pair in $[\text{N}/(\text{W})^{0.5}]$, n is the total number of sampled data for each scan cycle, which is 90,001 and 330,001 for Case Studies I and II, and $T_{tot} = (n - 1)T_s$ is the total length of the trajectory, which is 9 s and 33 s for Case Studies I and II, respectively. The equivalent motor constant for the LM pair, based on the motor specification provided by its manufacturer, is $K_{m,LM} = 21.40 \text{ N}/(\text{W})^{0.5}$.

Accordingly, the LM heat generated per cycle for the No AD, PAD and AAD scenarios are 533 J, 510 J and 215 J, for Case Study I and 167 J, 150 J, and 90 J for Case Study II, respectively. In other words, the AAD shows a 60 % and 58 % reduction in LM heat (in Case Study I) and a 46% and 40% reduction in LM heat (in Case Study II) over the No AD and PAD scenarios, respectively. No information for calculating the heat of the RMs (stepper motors) is provided by their manufacturer. However, the relative impacts of the LM and RM heat can be visualized from the temperature distribution of the scan table and stage (shown in Figure 13), captured using an infrared thermometer (FLIR, E4 with 0.95 emissivity coefficient) after 40 consecutive scan cycles from thermal equilibrium at room temperature. It can be seen that, even though RMs heat up with repetitive assist motions of the AAD, their heat is locally confined and distant from the scan table (though no external cooling devices are employed). However, as seen, the relatively large LM heat of the No AD and the PAD scenarios (particularly for Case I), has a significant impact on the temperature distribution of the scan table, and hence could cause large thermal errors (and other thermal problems such as hysteresis, drift and creep) compared to the AAD scenario [5], [33].

With regard to vibration, the lower F_{LM} levels of the AAD scenario lead to smaller RMS values of $\mathbf{a}_{B,x}$ and $\mathbf{a}_{B,\theta}$, compared to the No AD and PAD scenarios, as shown in Figure 14 and Table 5. Consequently, with the AAD, the tracking performance of the table, measured by its RMS tracking error in the CV regions (i.e., $\mathbf{e}_{CV}$), is improved by 21% and 20% for Case I, compared to the No AD and PAD scenarios, respectively; for Case II, the same $\mathbf{e}_{CV}$ improvements are 34% and 29%. Note that slightly larger $\mathbf{a}_{B,\theta}$ are recorded for the PAD scenario than for the No AD scenario, because small misalignments of PMs occur

with the rocking of the base, in which case additional rocking moments are created by the repelling PMs.

Table 5. Experimental scan axis results of three stage configuration scenarios (No AD, PAD, and AAD).

		No AD	PAD	AAD
Case Study I: Scanning for Photolithography	J_{contr} [N ²]	-	2.331×10 ⁹	9.843×10 ⁸
	RMS $\mathbf{F}_{LM}$ [N]	164.7	161.0	104.6
	RMS $\mathbf{F}_{RM1}$ [N]	-	34.8	553.6
	RMS $\mathbf{F}_{RM2}$ [N]	-	25.1	526.5
	RMS $\mathbf{a}_{B,x}$ [m/s ²]	0.570	0.539	0.385
	RMS $\mathbf{a}_{B,\theta}$ [rad/s ²]	0.851	0.859	0.643
	RMS $\mathbf{e}_{CV}$ [nm]	57.6	56.4	45.4
Case Study II: Scanning for Inspection	J_{contr} [N ²]	-	6.863×10 ⁸	4.111×10 ⁸
	RMS $\mathbf{F}_{LM}$ [N]	48.1	45.6	35.3
	RMS $\mathbf{F}_{RM1}$ [N]	-	46.7	286.0
	RMS $\mathbf{F}_{RM2}$ [N]	-	22.4	290.0
	RMS $\mathbf{a}_{B,x}$ [m/s ²]	0.201	0.191	0.164
	RMS $\mathbf{a}_{B,\theta}$ [rad/s ²]	0.261	0.267	0.210
	RMS $\mathbf{e}_{CV}$ [nm]	20.7	19.2	13.6

Figure 13. Surface temperature distribution of scan table and stage after 40 cycles of processing (a) photolithographic patterning scan trajectory and (b) inspection scan trajectory.

Figure 14. Measured signals for three configurations (No AD, PAD and AAD) processing
(a) photolithographic patterning scan trajectory and (b) inspection scan trajectory.

5. Conclusions and Future Work

An active assist device (AAD) set which uses actuated permanent magnets (PMs) to simultaneously reduce heat and vibration in precision scanning stages with variable scan strokes/positions is proposed. It is important for the proposed AAD to be versatile – i.e., to accommodate a wide range of scan strokes and positions. Therefore, a method for sizing the transmission ratio (i.e., lead) of the ball screw actuator for the AAD to maximize its versatility is proposed. The actuated PMs of the AAD must also be controlled to provide maximum assist for a given scan trajectory with variable scan strokes/positions. This goal

boils down to a constrained optimal trajectory generation problem for the PMs, which is solved in a computationally efficient manner in this paper using a basis-spline approach with windowing. The superior performance of the proposed AAD in simultaneously reducing motor heat and residual vibration of a scanning stage is experimentally demonstrated using two practical case studies involving silicon wafer scanning with variable scan strokes/positions. Future work will investigate methods for simultaneously reducing heat and vibration for the stepping axes of precision scanning stages, which have not been considered in the present work.

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Appendix A: Parameterization of a Curve Using Basis-Splines (B-Splines)

A B-spline curve $c(\tau)$ of degree r , can be expressed as a linear combination of basis functions $N_{i,r}(\tau)$ and control points $p(i)$ ($i = \{1, 2, \dots, k\}$), as [30]

$$c(\tau) = \sum_{i=1}^k N_{i,r}(\tau) p(i) \quad (\text{A.1})$$

with $\tau \in [0,1]$ being the spline parameter [30]. For a uniformly spaced knot vector $\mathbf{g} = [g(1) \dots g(r+k+1)]$ given by

$$g(\gamma) = \begin{cases} 0 & 1 \leq \gamma \leq r+1 \\ \frac{\gamma-r-1}{k-r+1} & r+2 \leq \gamma \leq k+1 \\ 1 & k+2 \leq \gamma \leq r+k+1 \end{cases} \quad (\text{A.2})$$

$N_{i,r}(\tau)$ can be determined recursively from the expressions

$$N_{i,0}(\tau) = \begin{cases} 1 & g(i) \leq \tau < g(i+1) \\ 0 & \text{otherwise} \end{cases} \quad (\text{A.3})$$

$$N_{i,j}(\tau) = \frac{\tau - g(i)}{g(i+j) - g(i)} N_{i,j-1}(\tau) + \frac{g(i+j+1) - \tau}{g(i+j+1) - g(i+1)} N_{i+1,j-1}(\tau), \quad j = \{1, 2, \dots, r\}$$

If the spline parameter $\tau \in [0,1]$ is discretized into n equally spaced points, resulting in a vector $\boldsymbol{\tau} = [\tau(1) \dots \tau(n)]^T$, a basis function matrix, $\mathbf{N}_p$, can be defined as

$$\mathbf{N}_p = \begin{bmatrix} N_{1,r}(\tau(1)) & \dots & N_{k,r}(\tau(1)) \\ \vdots & \ddots & \vdots \\ N_{1,r}(\tau(n)) & \dots & N_{k,r}(\tau(n)) \end{bmatrix} \quad (\text{A.4})$$

Accordingly, Eq. (A.1) can be re-written in a matrix form as [31]

$$\mathbf{c} = \mathbf{N}_p \mathbf{p} \quad (\text{A.5})$$

where $\mathbf{c}$ is the $n \times 1$ discretized curve vector and $\mathbf{p} = [p(1) \dots p(k)]^T$ is the control point vector.

The derivative of $c(\tau)$ w.r.t. τ , denoted as $c'(\tau)$, is also a B-spline curve, but it has $k - 1$ control points, degree $r - 1$ and knot vector $\mathbf{g}_d = [g(2) \dots g(r+k)]$, obtained by dropping the first and last knots from the knot vector $\mathbf{g} = [g(1) \dots g(r+k+1)]$ of $c(\tau)$. The control point vector $\mathbf{p}_d = [p_d(1) \dots p_d(k-1)]^T$ of $c'(\tau)$ is related to $\mathbf{p}$ by the derivative expression using their vector elements [30], [31]

$$p_d(i) = r \frac{p(i+1) - p(i)}{g(i+r+1) - g(i+1)}, \quad i = \{1, 2, \dots, k-1\} \quad (\text{A.6})$$

Let us define $N_{i,r-1}(\tau)$ as the basis functions defined over the reduced knot vector $\mathbf{g}_d$, and assume that the spline parameter $\tau \in [0,1]$ represents time $t \in [0, T_{tot}]$ normalized by T_{tot} .

Accordingly, using the chain rule, $\dot{c}(\tau)$, the time derivative of $c(\tau)$, can be written with respect to the control point velocity $p_v = p_d/T_{tot}$ as

$$\dot{c}(\tau) = \frac{1}{T_{tot}} c'(\tau) = \frac{1}{T_{tot}} \sum_{i=1}^{k-1} N_{i,r-1}(\tau) p_d(i) = \sum_{i=1}^{k-1} N_{i,r-1}(\tau) p_v(i) \quad (\text{A.7})$$

whose discretized form is given by

$$\dot{\mathbf{c}} = \mathbf{N}_d \mathbf{p}_v = \mathbf{N}_d \mathbf{D}_v \mathbf{p} = \mathbf{N}_v \mathbf{p} \quad (\text{A.8})$$

where $\mathbf{N}_d$ and $\mathbf{D}_v$ are defined as

$$\mathbf{N}_d = \begin{bmatrix} N_{1,r-1}(\tau(1)) & \dots & N_{k-1,r-1}(\tau(1)) \\ \vdots & \ddots & \vdots \\ N_{1,r-1}(\tau(n)) & \dots & N_{k-1,r-1}(\tau(n)) \end{bmatrix} \quad (\text{A.9})$$

$$\mathbf{D}_v = \frac{r}{T_{tot}} \begin{bmatrix} g(r+2) - g(2) & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & g(r+k) - g(k) \end{bmatrix}^{-1} \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix}_{(k-1) \times k} \quad (\text{A.10})$$

Following the same approach, higher order time derivatives in discretized form (i.e., $\ddot{\mathbf{c}}$, $\dddot{\mathbf{c}}$ and so forth) can be generated. In [31], an additional time derivative case, i.e., generation of acceleration vector, is presented.

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Table 1. Specifications of prototype scanning stage.	13
Table 2. Parameters used for optimal design of PM based AAD.	17
Table 3. Identified system parameters of stage. (The subscripts associated with each parameter listed in the first column is provided in parentheses in the first row). .	28
Table 4. Comparison of computation times and objective function values (based on $T_s =$ 0.1 ms) for the three optimization approaches (direct, B-splines without windowing, and B-splines with windowing).	33
Table 5. Experimental scan axis results of three stage configuration scenarios (No AD, PAD, and AAD).	40

Figure 1. Typical scan profile comprising repeated step and scan motions. Each scan stroke can be divided into constant velocity (CV) and motion reversal (MR) regions.	7
Figure 2. Schematic of (a) Scanning stage with assist device, (b) Proposed PM based AAD.....	10
Figure 3. CAD schematic of (a) prototype PM assisted scanning stage and (b) MR-side PM mounted on bridge (connected to ground).	12
Figure 4. (a) Scan trajectory with varying scan strokes/positions, (b) Velocity profiles of scan table and PM during two successive scan strokes.	16
Figure 5. Versatility objective function (J_{vers}) vs. ball screw lead (l_s).	16
Figure 6. Representation of isolation system using a mass-spring-damper model.	20
Figure 7. In-house built prototype of scanning stage with proposed AADs used for experimental validation. CAD schematic of stage is presented in Figure 3.	27
Figure 8. Measured and curve fitted FRFs from (a) F_{LM} to x_T , (b) F_{RM1} to x_{PM1} , (c) F_{RM2} to x_{PM2} , and (d) F_{LM} to x_B	28
Figure 9. Measured PM repulsion force vs. distance between PMs relationship. The relationship is almost identical for both PM pairs.	29
Figure 10. Optimal x_{PMi} and time derivative profiles of x_{PM1} solved by three optimization approaches (direct, B-splines without windowing, and B-splines with windowing) in the original sampling resolution with $T_s = 0.1$ ms.	32
Figure 11. (a) Step and scan profile and (b) Scan trajectory of table, as well as optimal PAD and AAD trajectories for photolithographic patterning of 200 mm silicon wafer.	35

Figure 12. (a) Step and scan profile and (b) Scan trajectory of table, as well as optimal PAD and AAD trajectories for inspection of 200 mm silicon wafer.....	37
Figure 13. Surface temperature distribution of scan table and stage after 40 cycles of processing (a) photolithographic patterning scan trajectory and (b) inspection scan trajectory.....	41
Figure 14. Measured signals for three configurations (No AD, PAD and AAD) processing (a) photolithographic patterning scan trajectory and (b) inspection scan trajectory.....	42