High-Fidelity CFD Simulation of Wave Run-Up for Single/Multiple Surface-Piercing Cylinders in Regular Head Waves

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Abstract

The capabilities of CFDShip-Iowa V4.5 and V6.2 simulations for wave run-up for single/multiple surface-piercing cylinders in regular waves are assessed in conjunction with the ITTC OEC Workshop on VIV and Wave Run-up held in Nantes, France October 17-18, 2013. Statistical convergence and grid/domain/turbulence model sensitivity studies show uncertainty U_I values are less than 1% for both the experiment/simulation data and grid/domain/turbulence model sensitivities are less than the facility bias U_{FB} . Simulations are qualitatively validated using U_{FB} and error E between experiment and simulation. Errors for single/four cylinder(s) are comparable with U_{FB} and average error $\bar{E}$ for the other CFD data of the workshop participants. Results from both V4.5 and V6.2 agree well with experiment and also show capabilities to apply for complex/large-scale problem and to describe detailed wave breaking/spraying for V4.5 and V6.2, respectively. Study on run-up profile shows mean wave elevations are similar to free surface elevations for a cylinder in a steady current. For wave steepness/wave length effect, 2nd to 1st harmonic ratio on the cylinder weather face decreases with steepness for short wave, but increases for long wave. Wave diffraction analysis shows that 2nd / 3rd to 1st harmonic ratio in diffracted wave contour reach up to 18~41% / 9~16% respectively. Due to interaction between cylinders, crest/trough heights

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increase 5-17% / 4-10% for 4C, respectively. Study on vortex shedding shows short wave has insufficient time to develop periodic vortex shedding compared to long wave.

Key words: CFD, run-up, wave diffraction, vortex shedding

1. Introduction

Accurate estimation of run-up height and forces induced by ocean waves is essential to the design of offshore platforms. Attention has focused on interactions between steep waves and floating platform systems, for which a typical geometry consists of single/multiple surface-piercing cylinders.

Potential flow methods are the common numerical approach to study the run-up height and forces. Since McCamy and Fuchs [1] presented an analytic solution for a bottom-mounted circular cylinder, further advances based on linear potential theory have been made [2]. For steep waves, the linear assumption significantly under-predicts crest heights around columns [3]. Second-order and higher-order effects have been studied [4], but non-linear tools based on potential theory have limitations due to inability to model wave breaking and viscous effects in simulating strongly non-linear wave-body interaction flows [5].

CFD simulations which include viscous effects have recently been used to study the non-linear mechanism of wave run-up [6] [7], which showed that fully nonlinear CFD simulation can capture highly nonlinear wave-structure interaction effects, such as jet-like run-up that may occur on the weather side of platform columns, which can not be predicted by potential. In order to assess the capabilities of state-of-the-art CFD codes for wave run-up simulation around cylinders, the ITTC Ocean Engineering Committee (OEC) held Workshop on VIV and Wave Run-up in Nantes, France October 17-18, 2013 for blind submissions for benchmark tests [8]. Numerical results were compared with experimental data measured from MOERI [9] and MARINTEK [10] for single (1C) and four circular cylinders (4C), respectively. For 1C the CFD results at the weather side of cylinder agreed well with the experimental results for the 1st harmonics of wave elevation, but not for mean and the 2nd harmonics. Overall trends of experimental

results by MOERI and MARINTEK [4] for 1C were also similar for the 1st harmonic of wave elevation, but mean and the 2nd harmonics were different in values and in trend.

Herein, the capabilities of CFDShip-Iowa V4.5 [11] and V6.2 [12] simulations for wave run-up for single/multiple surface-piercing cylinders in regular waves are assessed. Pre (blind) and post workshop results were submitted to the OEC. The simulations are for full-scale conditions, as specified by the OEC. Quantitative verification and validation [13] studies are beyond the scope since the grid sizes are large and experimental uncertainties are not available. However, statistical convergence and grid, domain and turbulence model sensitivity are evaluated and the simulations are qualitatively validated using comparison error $E=(D-S)\%D$ where D and S are the experimental and simulation values, respectively. Wave elevations and horizontal/vertical forces experimental data was provided post workshop by the OEC. The facility biases U_{FB} for wave elevations based on the differences between the MOERI [9] and MARINTEK [4] data for 1C short and long waves for 0th, 1st and 2nd harmonics are compared with the sensitivity studies and validation results. Variables are grouped into first (1st harmonics) and second (0th and 2nd harmonics) order terms for separate assessment capabilities both for the experiments and CFD. The effects of wave steepness and length, wave diffraction, interaction between cylinders, and vortex shedding are studied.

2 Experimental Validation Data

The ITTC OEC Workshop used experimental data from MOERI [9] wave basin (56×30×4.5m) for six wavelengths ($\lambda/d=4.7, 6.2, 7.9, 9.7, 14.0, 21.9$) and four wave steepness ($H/\lambda=1/50, 1/30, 1/16, 1/10$) conditions where λ and H are the wave length and height, respectively, and d is the cylinder diameter. Table 1 provides the 1C model and full-scale geometries. The model scale (1/50) and aspect ratio (Draft/Diameter=1.5) are small. The OEC provided time histories of wave elevations $\eta(t)$ and horizontal $X(t)$ and vertical $Z(t)$ forces and their 0th, 1st and 2nd harmonics for selected test cases. Figure 1(a) illustrates the locations of the wave gages near the cylinder. Ten gages are located at $r=0.513d$ and $r=1.0d$

at angles $\theta=0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ$. Regular/deep-water (water depth/wavelength $> 1/2$) waves with zero heading $\beta=0$ are generated as the incident waves for all tests. Table 1 includes the $U_{FB}=(D_1-D_2)\%D_1$ where D_1 and D_2 are the MOERI [9] and MARINTEK [4] wave elevation data, which is 4.7% and 118% for the 1st and 2nd order variables, respectively.

The ITTC OEC Workshop used experimental data from MARINTEK [10] Towing tank III (80×10.5×10m) for four wavelengths ($\lambda/d=4.7, 7.9, 14.0, 21.9$) and three wave steepness ($H/\lambda=1/30, 1/16, 1/10$) conditions for the 4C validation. Table 1 provides the 4C model and full-scale geometries. The model scale (1/49) and aspect ratio (1.5) are small. The OEC provided time histories of wave elevations $\eta(t)$ and their 0th, 1st and 2nd harmonics for selected test cases. The arrangement of the four cylinders and the locations of wave probes are shown in Figure 1(b) and 1(c). Wave elevations around down wave cylinder (denoted as A) were measured using twelve gages. Four gages are located along each radial line at angles $180^\circ, 225^\circ$, and 270° relative to the incoming waves. The incoming waves are regular waves with $\beta=0^\circ$ and $\beta=45^\circ$ heading and with deep-water condition.

3 Computational Methods, Conditions and Grids

The CFDShip-Iowa V4.5 is an overset, block structured CFD solver using either absolute or relative inertial non-orthogonal curvilinear coordinate system for arbitrary moving but non-deforming control volumes. Turbulence models include blended $k-\varepsilon/k-\omega$ based isotropic and anisotropic RANS, and DES approaches with near-wall or wall functions. A single-phase level-set method is used for free-surface capturing with air/water interface modeled using a semi-coupled method. Captive, semi-captive, and full 6DOF capabilities for multi-objects with parent/child hierarchy are available.

Table 2 summarizes the V4.5 simulations. For 1C, the shortest $\lambda/d=4.7$ and the longest $\lambda/d=21.9$ waves with $H/\lambda=1/30, 1/16$ and $1/10$ were used. For 4C, the shortest $\lambda/d=4.7$ wave with $H/\lambda=1/30$ and $\beta=0^\circ$ was used. Table 2 includes the full-scale Reynolds $Re = \bar{u}d/\nu$ and Froude $Fr = \bar{u}/\sqrt{gd}$ numbers based on the wave-induced mean velocity (defined later) and cylinder diameter. In context of ship

hydrodynamics: the Re is model scale; the Fr is very small; and the wavelength and steepness are large. In context of single-phase cylinder flow all Re are in the post-critical regime.

The results submitted to the ITTC OEC Workshop used grids G1, G2 and G8 for the 1C short and long λ/d and 4C short λ/d test cases, respectively. G3 (fine) and G4 (medium) with same domain size (medium) but different grid resolution were used to study grid sensitivity: $\Delta G\%G3$. G4 has similar grid resolution as G1 and G2. Domain size study used G1 (D1), G4 (D2) and G5 (D3), which have large, medium and small upstream lengths, respectively, but the same domain sizes in the other directions: $\Delta D\%D1$. $\lambda/\Delta x$ and $H/\Delta z$ are more than 75 and 20 respectively for all cases and the same resolutions are maintained for domain size study to eliminate grid resolution effects. G6 used for comparison of URANS and DES turbulence models: $\Delta URANS\%DES$. The total number of grid points of G6 is almost the same with G1 and G2, but it has much smaller domain size and fine enough grid resolution for the DES model.

Figure 2 shows overset grid system and boundary condition for V4.5 simulation. The cylinder surface is a no-slip wall. Zero gradient boundary conditions are applied on the top, bottom and both sides of the domain. Overset grids are used for grid clustering. The grid system consists of a ‘boundary layer grid’, ‘1st refinement grid’, ‘2nd refinement grid’ and a ‘background grid’. From the ‘boundary layer grid’ to the ‘background grid’, grid sizes increase gradually, ensuring tolerable numerical errors in overset interpolations. The 1C and 4C grid sizes are large ranging from 13-26M and 65M, respectively.

At the inlet and outlet, 2nd order Stoke’s waves are imposed for non-dimensional free surface elevation, velocities and pressure. For the deep-water condition, the equations are:

$$\begin{aligned}
 \eta_1(x, t) &= A_1 \cos \theta + \frac{A_1^2 k}{2} \cos 2\theta \\
 u_1(x, z, t) &= \frac{A_1 k}{Fr \sqrt{k}} e^{kz} \cos \theta + \frac{6A_1^2 k^2}{Fr \sqrt{k}} e^{2k(z-d_w)} \cos 2\theta \\
 w_1(x, z, t) &= \frac{A_1 k}{Fr \sqrt{k}} e^{kz} \sin \theta + \frac{3A_1^2 k^2}{4 Fr \sqrt{k}} e^{2k(z-d_w)} \sin 2\theta \\
 p_1(x, z, t) &= \frac{A_1}{Fr^2} e^{kz} \cos \theta + \frac{6A_1^2 k}{Fr^2} e^{2k(z-d_w)} \cos 2\theta - \frac{1}{2} \frac{A_1^2 k}{Fr^2} e^{2kz}
 \end{aligned} \tag{1}$$

where $\theta = kx - \omega t$. A_1 , k , ω and d are the linear part of incident wave amplitude, wave number, wave angular frequency and water depth, respectively.

CFDSHIP-IOWA V6.2 utilizes the filtered incompressible Navier-Stokes equations fully transformed from Cartesian coordinates to orthogonal curvilinear coordinates. Extended to multi-block topology, V6.2 has been improved for its capability to handle realistic geometries remarkably. V6.2 is equipped with a dynamic SGS turbulence model for LES computation. In present study, however, no SGS model was used for comparison with URANS/DES model in V4.5. The air/water interface is calculated by VOF method with a level-set based sharp interface treatment.

As shown in Table 2, V6.2 simulations were conducted using G7 for the short wave $\lambda/d=4.7$ with $H/\lambda=1/16$ and $1/10$ conditions. Since V6.2 is based on an orthogonal curvilinear grid solver, cylindrical grid system is required as shown in Fig. 3. G7 has a cylindrical topology with radius of 2λ . The far boundary in the radial direction (i_{max} plane) is split into two parts, inlet and outlet, and then the incoming wave and convective boundary condition are applied for inlet and outlet, respectively. Bottom/top sides are slip walls. Periodic boundary condition is imposed at a plane shared by j_{min} and j_{max} planes in azimuthal direction. The 1C grid size is 11M, which is 54% smaller than G1. Due to different solver characteristics for V4.5 and V6.2, computational domain and boundary condition are not same with those of V4.5. However, preliminary tests without cylinder have shown the validity of present domain size/grid resolution/boundary condition for each V4.5 and V6.2 solver in steep wave conditions, although not presented herein.

In order to describe highly steep incident wave more accurately, numerical tests for the incident wave boundary condition in V6.2 were performed. Without cylinder in a domain ($L \times H \times W = 3\lambda \times 2\lambda \times \lambda$), 2nd and 5th order Stoke's wave were imposed as incident wave boundary condition and the CFD wave elevation at the center of the domain were compared with the elevations from the corresponding incident wave formula. For $H/\lambda=1/16$ and $1/10$ cases, 2nd and 5th order Stoke's wave formula showed differences in crest height and phase. In addition, for $H/\lambda=1/10$ case, the differences in crest height / phase between the CFD

data and the corresponding incident wave formula were 8% / 18%360° and 3.8% / 1.9%360° for 2nd and 5th order Stoke's wave, respectively. Thus, in order to minimize difference between incident wave and actual CFD computation, 5th order Stoke's wave is used in V6.2 for the inlet elevation and velocities. Whereas V4.5 requires pressure boundary condition for incident wave, V6.2 utilizes homogeneous Neumann boundary condition without explicit description of pressure. For deep-water condition, free surface elevation and velocities of incident wave are given by:

$$\begin{aligned}\eta_1(x, t) &= \frac{1}{k} \sum_{n=1}^5 \epsilon^n b_n \cos(n\theta) \\ u_1(x, z, t) &= C_0 \sqrt{\frac{g}{k}} \sum_{n=1}^5 \epsilon^n n a_n \cosh(nkz) \cos(n\theta) \\ w_1(x, z, t) &= C_0 \sqrt{\frac{g}{k}} \sum_{n=1}^5 \epsilon^n n a_n \sinh(nkz) \sin(n\theta)\end{aligned}\tag{2}$$

where $\epsilon = kA_1$ and $\theta = kx - \omega t$. A_1 , k and ω are the linear part of incident wave amplitude, wave number and wave angular frequency, respectively. Coefficients a_n and b_n are given in [14].

4 Statistical Convergence, Harmonic Analysis and Grid, Domain and Turbulence Model Sensitivity

Simulations were conducted for 10 wave periods, as specified by the OEC. Figure 4 and 5 compare experiments and simulation time histories for wave elevation and horizontal/vertical forces for 1C and $\lambda/d=4.7$, $H/\lambda=1/30$ and $\lambda/d=21.9$, $H/\lambda=1/10$, respectively, including running mean and running RMS, which are typical for all the results. Statistical convergence uncertainty is based on the magnitude of the running mean and running RMS oscillations $U_I = \left| \frac{1}{2} (S_U - S_L) \right|$, where S_U and S_L are the maximum and minimum value, respectively [13]. The running RMS U_I for the experiment / CFD are 0.48% A_1 , 0.29% X_1 , 0.54% Z_1 / 0.54% A_1 , 0.28% X_1 0.66% Z_1 for $\lambda/d=4.7$ and 0.58% A_1 , 0.75% X_1 , 0.74% Z_1 / 0.24% A_1 , 0.23% X_1 , 0.17% Z_1 for $\lambda/d=21.9$. The U_I values are less than 1% for both the experiments and simulations.

Fast Fourier transform (FFT) was conducted to have harmonics of wave elevation (η_0 , η_1 , η_2) and horizontal (X_0 , X_1 , X_2) / vertical forces (Z_0 , Z_1 , Z_2). For harmonic analysis, in order to exclude transition

period, the last 7 and 5 wave periods out of the 10 cycles were chosen for short and long wave cases, respectively.

As shown in Table 3, the grid sensitivity $\Delta G\%G3$ for the 1st and 2nd order variables is 2% and 45%, respectively. The domain size sensitivity $\Delta D\%D1$ for 1st and 2nd order variables for D2 is 1% and 4%, respectively, and for D3 is 7% and 22%, respectively. The turbulence model sensitivity $\Delta URANS\%DES$ for 1st and 2nd order variables is 1% and 8%, respectively. The grid, domain and turbulence model sensitivities are less than the respective U_{FB} for first and second order variables, as shown in Table 1. Nonetheless, the 2nd order variable sensitivity is large indicting the need for finer grids to resolve 2nd order terms. The results indicate that both the large and medium domains should be sufficient for the validation studies, as used by G1, G2, G7 and G8. The turbulence model sensitivities are small such that the URANS turbulence model is sufficient for the validation studies.

5 Validation

For 1C, the OEC provided validation data for the wave elevation and horizontal/vertical force time histories for short wave with $H/\lambda=1/30$ and long wave with $H/\lambda=1/10$, respectively, and their harmonics for short and long waves with all three H/λ . Short wave elevation time histories for $H/\lambda=1/16$ were extracted from [15]. For 4C, the OEC provided validation data for the wave elevation time histories and harmonics for the short wave with $H/\lambda=1/30$. In order to compare total elevations of the experiment and CFD, wave elevations at the crest $\bar{\eta}_c$ and trough $\bar{\eta}_t$ are calculated by averaging the maximum and minimum run-up heights for the last 10 cycles, respectively.

5.1 Single circular cylinder (1C)

Figure 6 shows that the 1st harmonics of horizontal force decreases for increasing steepness, while the 2nd harmonics increases slightly for the short wave. Therefore, the 2nd to 1st harmonics ratio increases by increasing wave steepness. Long wave has lower 1st harmonics, but higher 2nd harmonics compared to

short waves. Thus, nonlinearity in horizontal force is dominant for long wave cases. For vertical force, the 1st and 2nd harmonics are fairly independent of steepness. Table 4 shows that the E%D values are 72 and 23/2%D for 2nd order and 1st order amp/phase variables, respectively. The errors are mostly larger for the vertical than the horizontal force. The X/Z phases lead the wave phase by small and large amounts for the short and long wave conditions. For the long wave condition, the X/Z phases are nearly 180° out of phase with the wave phase. Unfortunately, U_{FB} for forces is not available for comparison.

Figure 7(a) shows the mean wave profile along the center plane from upstream, around cylinder, to downstream for $\lambda/d=4.7$ and $H/\lambda=1/30$, $1/16$ and $1/10$ conditions. The mean wave profiles of both experiment and CFD have higher run-up height on the front surface of cylinder than depression depth on the rear surface. Both results from V4.5 and V6.2 are reasonably accurate, even though discrepancies are observed on the weather and leeward faces. Along the cylinder shoulder, V6.2 predicts mean wave elevation for $H/\lambda=1/10$ case more accurately than V4.5. For V4.5, the elevations are below the calm water condition at upstream, but slightly higher than the calm water level at downstream locations. On the weather face of the cylinder, run-up heights for $H/\lambda=1/30$ and $1/16$ are similar, but increases for $H/\lambda=1/10$. Along the cylinder shoulder, depression pattern of V4.5 is similar for all three steepness. On the leeward face of the cylinder, the mean profiles of V4.5 are similar, but the elevation of V6.2 for $H/\lambda=1/10$ increases and oscillates at downstream likely due to wave breaking and no turbulence model.

Figure 7(b) shows the mean wave elevations along the center plane from upstream, around cylinder, to downstream for $\lambda/d=21.9$ and $H/\lambda=1/30$, $1/16$ and $1/10$ conditions. Compared to the experimental data, results from V4.5 are reasonably accurate except on the weather face for $H/\lambda=1/16$ and $1/10$ cases. For V4.5, the elevations are slightly higher than the calm water level at upstream and downstream locations. On the weather face of the cylinder, the run-up heights increase with increasing steepness and the height for $H/\lambda=1/10$ case is much higher than that for short wave. Along the shoulder and on the leeward face of the cylinder, the mean profiles are also similar for all three steepness.

The mean wave profiles for both small and large wavelengths and all wave steepness resemble wave profiles for surface piecing cylinders for steady Fr in calm water, as previously investigated using CFDShip-Iowa V6.2 [16] albeit for much larger $0.2 \leq Fr \leq 1.24$. Ref. [16] uses experimental [17] and empirical [18] data to validate run-up height and depression depth. Bernoulli equation along the free surface from upstream to the cylinder stagnation point estimates the run-up height as

$$\eta_{RU}/d = Fr_B^2/2, \quad (3)$$

$$Fr_B = \sqrt{\overline{V^2}}/\sqrt{gd}. \quad (4)$$

Fr_B is based on the square root of the time mean of the total velocity vector $V(t) = (\bar{u} + u', \bar{w} + w')$ squared,

$$\overline{V^2} = \overline{u^2} + \overline{u'^2} + \overline{w'^2}. \quad (5)$$

$\bar{u}$ is the wave-induced mean velocity, as used in defining Fr and Re shown in Table 2. u' and w' are the wave induced velocity oscillations. Figure 7 includes η_{RU} estimated using the simulation velocities on the free surface at 2λ upstream of the cylinder. The estimated η_{RU} values using the simulation velocities and u_I / w_I in Eq. (1) and (2) were compared with the experimental η_0 . The error is 24% for the estimation from CFD simulation velocities and 62% for u_I / w_I , respectively. Figure 8 shows $\bar{u}$ along vertical locations in water at 2λ upstream of the cylinder. While steady current is constant at any depth, $\bar{u}$ is a function of vertical location z . The decaying rate is approximately proportional to e^{15kz} and e^{18kz} for short and long wave respectively, which is much faster than the decaying rates e^{kz} for 1st order velocity component.

Figure 9(a) shows the comparisons for +/- 1st harmonics of wave elevation and wave elevations at the crest $\bar{\eta}_c$ / trough $\bar{\eta}_t$ for $\lambda/d=4.7$ and $H/\lambda=1/30$, $1/16$ and $1/10$ conditions. Compared to the experimental results, V4.5 and V6.2 shows good accuracy in value and trend. In the results of V4.5, 1st harmonics values at upstream are already more than 1.5 and keep increasing up to 2.0 on the weather face ($\theta=180^\circ$)

for the steepest case. Along the shoulder and leeward side, 1st harmonics decreases. After a depression point at $\theta=45^\circ$, the values increase at $\theta=0^\circ$ and remain almost same at downstream. The elevations at crest/trough are not symmetric with respect to x -axis for all cases and the asymmetry increases for steeper wave, which can be seen by comparing crest/trough to $\pm$ 1st harmonic values, respectively. Crest height is similar at upstream and downstream, but trough height is lower at upstream than at downstream. On the weather face $\theta=180^\circ$, crest height is the highest $\bar{\eta}_c/A_I=1.7\sim 2.1$ and trough height is the lowest $\bar{\eta}_t/A_I\sim -1.5$. While trough height increases along the shoulder and leeward face, crest height shows a depression on the leeward side $\theta=45^\circ$.

Figure 9(b) shows comparisons of $\pm$ 1st harmonics of wave elevation and wave elevations at the crest $\bar{\eta}_c$ / trough $\bar{\eta}_t$ for $\lambda/d=21.9$ and $H/\lambda=1/30$, $1/16$ and $1/10$ conditions. V4.5 shows good agreement with the experiment data, even though the crest height on the weather face is over-estimated. Compared to short wave cases, 1st harmonic values are nearly level with the incident wave amplitude, indicating small diffraction by the cylinder. The elevations at crest/trough are also asymmetric with respect to x -axis for all cases. The profiles of crest and trough heights are level, compared to short wave cases. At upstream and downstream, crest/trough heights are almost same. On weather face, crest height is the highest $\bar{\eta}_c/A_I=1.2\sim 1.7$ at $\theta=180^\circ$ and trough height is the lowest $\bar{\eta}_t/A_I\sim -1.1$ at $\theta=135^\circ$. On leeward side, crest/trough heights are similar for all cases.

The 1st harmonic wave profiles are similar in appearance to those for the mean wave profiles. The 1st harmonic wave phases are shown in Table 5. The average errors for phases are 4.2% and 2.3% for short and long wave, respectively. For short waves, the 1st harmonic wave phase shows small lead ($\approx 10^\circ$) on the cylinder face and large lag ($\approx 120^\circ$) on the cylinder back, whereas for long waves the 1st harmonic wave phase shows fairly large nearly constant lead ($\approx 40\sim 70^\circ$).

Figure 10(a) shows comparisons of the 2nd harmonics of wave elevation for $\lambda/d=4.7$ and $H/\lambda=1/30$, $1/16$ and $1/10$ conditions. V4.5 and V6.2 show reasonably accurate agreement with the experiment except for the case of $H/\lambda=1/10$. In the results of V4.5, the 2nd harmonic values at downstream are higher than

those at upstream, which indicates stronger non-linearity behind the cylinder. On the cylinder surface at $\theta=45^\circ$ and 135° , two depressions are shown. In contrast with mean and 1st harmonics, 2nd harmonic value is the highest on the shoulder of the cylinder ($\theta=90^\circ$) and the highest value increases with increasing wave steepness ($\eta_2/A_1=0.2\sim0.4$).

Figure 10(b) shows comparisons of the 2nd harmonics of wave elevation for $\lambda/d=21.9$ and $H/\lambda=1/30$, 1/16 and 1/10 conditions. Compared to the experiment, results of V4.5 are reasonably accurate, but show some discrepancies for $H/\lambda=1/10$ case. In contrast to short wave case, the 2nd harmonic values of V4.5 is higher at upstream than at downstream, indicating stronger non-linearity in front of the cylinder. On the weather side ($\theta=180^\circ$), 2nd harmonic value is the highest and increases for steeper wave condition. Along the shoulder, 2nd harmonic value decreases and shows a depression point.

Validation study shows harmonics of wave elevations η_0, η_1, η_2 are generally in the range of $-0.1 \leq \eta_0/A_1 \leq 0.3$, $0.5 \leq \eta_1/A_1 \leq 2.0$ and $0.05 \leq \eta_2/A_1 \leq 0.4$, respectively. 1st harmonics shows dominant values which are almost one order higher than mean and 2nd harmonics, but maximum values of mean and 2nd harmonics reach up to 15% and 20% of 1st harmonics respectively, indicating severe non-linearity effect for steep wave.

Table 5 summarizes the errors of wave elevation harmonics, 1st harmonic wave phase and crest/trough heights for $\lambda/d=4.7$, $H/\lambda=1/30$ and $\lambda/d=21.9$, $H/\lambda=1/10$. For short wave case, the average E for the 1st order amp/phase and the 2nd order variables are 5.5 / 4.2 and 92%D, respectively. The errors increase from weather to shoulder to leeward side and are larger for off-body locations compared to on-body locations. For long wave case, the average E for the 1st order amp/phase and the 2nd order variables are 5.0 / 2.3 and 84%D, respectively. The errors at weather side are large compared to short wave case. As shown in Fig. 11, energy dissipation caused by wave breaking was not described by the single-phase LS method of V4.5, which in turn could be the reason for large errors at the weather side for long wave.

The errors and differences for V4.5 and V6.2 for $\lambda/d=4.7$, $H/\lambda=1/16$ are also compared in Table 5. The errors for most of 1st and 2nd order variables are smaller for V6.2 than for V4.5, but they are

comparable and agree well with experiment. The differences between V4.5 and V6.2 are larger for the 2nd order than the 1st order variables and also show larger differences for on-body locations than for off-body locations. Even though V6.2 used coarse spatial/temporal resolution, it predicted wave run-up accurately due to high order numerical method and sharp interface treatment. V6.2, however, needs more expensive CPU time per iteration for the high order method and VOF function evaluation. As a result, overall computing costs for V4.5 and V6.2 are comparable in present study.

Table 5 also includes the averaged errors $\bar{E}=(D-S_{OEC})\%D$ where D and S_{OEC} are the MOERI [9] wave elevation data and the other CFD data of ITTC workshop participants provided by OEC, which is 5.6% and 141% for the 1st and 2nd order variables, respectively. Averaged E for all 1C cases are 5.8%D and 67%D for 1st and 2nd order variables, respectively, whereas U_{FB} is 4.7% and 118%. Thus, error for the 1st order variables is slightly higher than U_{FB} and $\bar{E}$ but still comparable. The error for the 2nd order variables is less than half of $\bar{E}$.

Figure 11 shows photograph of experiment and instantaneous V4.5 wave pattern for $\lambda/d=21.9$, $H/\lambda=1/10$ case. The experiment exhibits wave breaking and spray on the weather and shoulder sides of the cylinder and a broken unsteady wake with air entrainment on the leeward side of the cylinder and wake. The V4.5 wave pattern is qualitatively similar but does not capture wave breaking/bubbly wake flow. Figure 12 shows comparison of wave pattern of V4.5 and V6.2 for $\lambda/d=4.7$, $H/\lambda=1/10$ case. Even though experimental photograph is not available, the comparison confirms that V6.2 has capability to describe wave breaking/spraying even with coarse grid resolution compared to V4.5 simulation.

5.2 Four circular cylinders (4C)

The errors of wave elevation harmonics, 1st harmonic wave phase and crest/trough heights for 4C, $\lambda/d=4.7$, $H/\lambda=1/30$ case are summarized in Table 5. The 1st harmonic wave phases show lead/lag with respect to the wave phase for short/long waves. The errors do not show clear trend regarding weather/shoulder faces of the cylinder, but shows that errors for crest/trough heights at the weather side

are relatively high even under short wave condition. Since wave gages are located around the downwave cylinder, the interaction between cylinders could be related with the deterioration of accuracy at the weather side. Average E for η_1 is relatively low at the weather side, which suggests that majority of the errors for crest/trough heights at the weather side are not caused by the 1st harmonics, but by the 2nd or higher harmonics. In most cases, the errors for on-body locations are higher than for off-body locations. For 4C case, the average E for the 1st and 2nd order terms are 7.4% and 118%, which seems reasonably accurate compared to 1C case, even if U_{FB} and $\bar{E}$ for 4C are not available.

6 Discussion

Stansberg and Kristiansen [5] categorized different contributions of non-linear effect to total wave amplification as follows

- (i) 2nd-and higher-order components in the incident wave
- (ii) 2nd- and higher-order components due to interaction with the column
- (iii) viscous effects ('blocking' due to flow separation)
- (iv) local dissipation and breaking due to interaction with the columns
- (v) thin 'jets' along columns

The non-linear contribution from factor (i) can be approximately up to 20% and is established well for regular undisturbed wave [5] for which the present results are consistent, as discussed in Section 6.1. The contribution from factor (ii) is discussed in terms of wave diffraction analysis in Section 6.2 and then interaction between cylinders is studied in Section 6.3, comparing the results of 1C and 4C. In Section 6.4, factor (iii) is also studied in terms of vortex structure. CFDSHIP-IOWA V6.2 is capable of studying factor (v), but grids with billions of points are required.

6.1 Effect of wave steepness and wave length

According to linear potential theory, the incident wave is not distorted for long wave ($\lambda/d > 15.7 \sim 20.9$), while for short wave (approximately $\lambda/d < 3.9 \sim 5.2$), the amplification factor is higher than 1.6. When a linear wave reflects perfectly from a vertical wall, it approaches to asymptotic value of 2.0. For short $\lambda/d = 4.7$ waves, normalized wave crests are higher than 1.6 at weather side $\theta = 180^\circ$. The steepest wave $H/\lambda = 1/10$ produces even higher crest than 2.0 at $r = 0.513d$, $\theta = 180^\circ$. The amplifications in long wave cases $\lambda/d = 21.9$ are much less than those in short wave cases, but still exceeds 1.2 even for $H/\lambda = 1/30$. In Figure 4 and 5, horizontal force is dominant in short wave case, while both of horizontal and vertical forces are important in long wave case, which confirms that wave length is major parameter for wave induced loads compared to wave steepness.

The harmonic analysis of wave run-up for present results confirms about 20-30% of non-linearity for the steepest waves. Figure 13 shows mean, 1st and 2nd harmonics of wave elevations for short and long wave. The harmonic values were normalized by 1st order wave amplitude A_1 in order to investigate non-linear effect according to wave steepness. In terms of the normalized harmonic values, variation amount according to wave steepness indicates the effect of wave steepness. For linear wave with small wave steepness, mean and 2nd harmonics are almost zero and 1st harmonics are almost constant.

Observations from Fig. 13 can be organized into four categories: (1) upstream vs. downstream, (2) weather side, (3) leeward side, (4) shoulder.

- (1) at upstream, harmonic values are almost same for short wave case, but difference between the harmonic values increases for long wave. At downstream, on the other hand, 1st and 2nd harmonics shows similar values for both short and long wave case.
- (2) at weather side ($\theta = 180^\circ$ and 135°), 1st and 2nd harmonics for short wave increase with steepness, which is especially noticeable for the steepest wave. For long wave case, 1st harmonics shows similar values, but mean and 2nd harmonic values substantially increase with increasing steepness suggesting non-linear effect.

- (3) at leeward side ($\theta=45^\circ$ and 0°), 1st harmonics for short wave decrease with increasing steepness, but 2nd harmonic values increase with steepness. Long wave case also shows similar trend.
- (4) at shoulder ($\theta=90^\circ$), mean and 1st harmonic values for short wave decrease slightly, but 2nd harmonics increase with increasing steepness. For long wave case, on the other hand, 1st and 2nd harmonics shows almost same for different wave steepness, but mean values increase with increasing steepness.

Based on the above, the 2nd to 1st harmonic ratio on the weather side decreases with steepness for short waves, but increases for long waves. For both of wave lengths, the 2nd to 1st harmonic ratio at leeward side increases by increasing steepness. At shoulder, the 2nd to 1st harmonic ratio increases with steepness for short wave, but remains almost constant for long wave.

6.2 Wave diffraction analysis

In order to study the sources of the non-linearity in total wave elevation, wave diffraction analysis was performed. As already mentioned in Section 3.2, the incident wave profile η_I given in Eq. (1) or Eq. (2) can be different with those inside the domain of actual CFD simulation due to strong non-linear and viscous effect. Thus, we performed additional CFD simulations with no cylinder(s) to obtain CFD wave elevations without cylinder diffraction, which is denoted by η_{NC} . With η_{NC} , diffracted wave elevation (η_D) is defined as:

$$\eta_D = \eta_T - \eta_{NC} \quad (6)$$

where η_T is total wave elevation. The harmonics of the diffracted wave elevation $\eta_{D0}, \eta_{D1}, \eta_{D2}$ was obtained using FFT.

Harmonic contours of diffracted wave for 1C short wave ($\lambda/d=4.7$) are presented in Fig. 14. For $H/\lambda=1/30$ case, mean values are higher than calm water level right in front of the cylinder, indicating run-up in mean wave profile. At upstream, mean values are partially higher and lower than calm water level

but they converge to calm water level asymptotically. The 1st harmonic component spreads toward upstream widely, but propagates along relatively narrow path behind the cylinder. The 2nd harmonics shows spreading pattern like bird's footprint behind the cylinder. Meanwhile, the 3rd harmonics is mostly observed in the vicinity of the cylinder.

For steeper wave condition $H/\lambda=1/10$, mean and 1st harmonic values have similar pattern to the case of $H/\lambda=1/30$, but stronger diffractions are observed in wide region around the cylinder. The pattern of 2nd harmonics is also similar to $H/\lambda=1/30$ case, but it spreads further with strong magnitude behind the cylinder. For the 3rd harmonics, diffraction waves with stronger pattern are observed in much wider region behind the cylinder and the pattern is similar to that of 2nd harmonics. The spreading pattern of diffraction wave confirms the observation previously in Section 6.1 that non-linearity for short wave decreases at weather side, but increases at leeward side by increasing steepness.

Harmonic contours for 1C long wave ($\lambda/d=21.9$) were obtained, but the cylinder does not diffract the wave field compared to the short wave case. Especially for $H/\lambda=1/30$ case, 1st harmonics doesn't show clear pattern for diffraction wave. The 2nd and 3rd harmonics spread toward upstream, but the diffraction strength is weak. For $H/\lambda=1/10$ case, diffraction strength becomes stronger, but strong diffraction is mainly staying near the cylinder, even though 2nd and 3rd harmonics show mild diffraction toward upstream.

Mean wave elevation contours in Fig. 14 are similar to those from experiment and CFD simulation for a vertical cylinder in steady current [19]. For present case, however, Kelvin wave pattern behind the cylinder is less clear and diffraction from the cylinder spreads further towards upstream, which is attributed to high Re and low Fr compared to $Re=2.7 \times 10^4$ and $Fr=0.8$ in [19]. Compared to diffraction pattern for ship in head waves [20] [21], 1st harmonics of diffracted wave in Fig. 14 shows significantly wider spreading pattern toward upstream. In addition, 2nd harmonics behind the cylinder is also much clearer than that behind the ship in head waves. The most important factor causing the differences is steady horizontal velocity generated by towing the ship. In [20] and [21], wave induced current is less than 1% of the steady horizontal velocity and thus the diffraction pattern is affected mainly by the steady

horizontal velocity. In addition, for present study, wave steepness is much higher and cylinder has blunt geometry rather than sharp shape for ship, which leads quite different diffraction pattern from that for ship in head waves.

The highest values in Fig. 14 and the ratios of them to 1st harmonics were investigated, which shows that the normalized 1st harmonics of diffracted wave reaches up to 1.01. The 2nd and 3rd harmonics have much smaller magnitude than that of 1st harmonics, but still exert their influences on the diffraction. Especially, 2nd harmonics for short wave is 18% of 1st harmonics for $H/\lambda=1/30$ and increases up to 41% by steepness increase. The 3rd to 1st harmonic ratio also increases from 8.8% to 16% by steepness increase, suggesting strong non-linearity. For long wave case, the 2nd to 1st harmonic ratio is 14% for $H/\lambda=1/30$ and increases up to 34% for $H/\lambda=1/10$. The 3rd to 1st harmonic ratio also increases from 9.1% to 20% by steepness increase. Nevertheless, diffraction pattern for long wave was observed only near the cylinder.

Figure 15 shows harmonic contours of diffracted wave for 4C. The incident wave condition is $\lambda/d=4.7$ and $H/\lambda=1/30$, which is same with the case of Fig. 14(a). The pattern of mean and the 3rd harmonics seems like superposition of diffractions from each cylinder, which is especially true around two upwave cylinders. The 2nd harmonics shows quite complicated pattern with increased magnitude, which is considered due to strong interaction between diffracted waves. Moreover, the 1st harmonic diffraction is distributed at the center of the four cylinders as well as in-between upwave and downwave cylinders, which can be also considered as interaction effect. Even if the highest 1st harmonic values are almost same for 1C and 4C, the 2nd to 1st harmonic ratio is increased and reaches up to 25%. The 3rd to 1st harmonic ratio is also increased to 12%.

6.3 Interaction between cylinders

As observed in previous subsection, the magnitude and spreading pattern of diffracted waves are affected by interaction between cylinders. Wave elevations at crest/trough are compared in Table 6 for 1C and 4C, which also includes the experimental results $\Delta EFD_{4C}\%EFD_{1C}$ (or $\Delta CFD_{4C}\%CFD_{1C}$). The difference is caused only by interaction between cylinders, since the other conditions including incident

wave, grid resolution etc. are controlled. For crest height, the averaged difference between experiment data for 1C and 4C ($\Delta EFD_{4C}\%EFD_{1C}$) increases with increasing θ (8.82% $\rightarrow$ 13.39% $\rightarrow$ 25.26%) and CFD result also shows the same tendency (5.25% $\rightarrow$ 12.69% $\rightarrow$ 17.34%). For trough height, the average of $\Delta EFD_{4C}\%EFD_{1C}$ decreases with increasing θ (16.93% $\rightarrow$ 7.21% $\rightarrow$ 5.81%), but CFD does not capture the tendency (4.32% $\rightarrow$ 13.34% $\rightarrow$ 10.46%). The interaction effect on the crest height is stronger at shoulder ($\theta=270^\circ$) than at weather side ($\theta=180^\circ$). On the other hand, the trough height is more affected by the interaction at weather side ($\theta=180^\circ$) than at shoulder ($\theta=270^\circ$).

A distinct feature of wave run-up for 4C is near-trapping phenomenon [22]. When four cylinders are located at the corners of a square, near-trapping can be one of the main contributing factors which give rise to high localized wave elevations inside the square region. Although near-trapping seems to make a partial effect on the crest height at shoulder, intensive near-trapping has not been observed in present test case. It is thought that the less sheltered geometry ($\beta=0^\circ$) and much lower d/l parameter prevent near-trapping for present test [22]. On the other hand, in the case with $\beta=45^\circ$, even with same d/l parameter and incident wave, near-trapping condition ($\lambda \approx \sqrt{2}l - d$) [23] is almost satisfied, which motivates future works to investigate near-trapping phenomenon in detail.

6.4 Vortex shedding

The relevant variables for analysis of vortex shedding are the number of shedding cycles T_k per wave half period $(T/2)/T_k$, where $T_k = 1/f_k$ and f_k is the shedding frequency, and Keulegan-Carpenter number $K_c = u_\zeta \cdot T/d$, where u_ζ is the maximum velocity ($u_\zeta = \bar{u} + |u'|_{max}$) evaluated on free surface $z = \zeta$ at $x = 2\lambda$ upstream of the cylinder. f_k can be estimated using single phase cylinder Strouhal number $St = \frac{f_k d}{u_\zeta} = S(Re) \sim 0.25$ for post critical Re. Table 7 shows $(T/2)/T_k$ and K_c values. It also shows A_l/T_d , λ/T_d , u_{Td} and u_{Td}/u_ζ values where u_{Td} is the maximum velocity at $z = T_d$ at $x = 2\lambda$ upstream of the cylinder. A_l/T_d values are large and increase with wave steepness, reaching up to 0.7 for steepest long wave case. λ/T_d also shows large values, especially for long wave cases. u_ζ and u_{Td} increase

with increasing wave steepness/wave length and u_{Td}/u_z shows that velocity decays 4.6 times faster over depth for short waves than for long waves. $(T/2)/T_k$ values are less than 0.18 for short waves, but reaches up to 0.83 for long wave, which indicates insufficient time to develop vortex shedding for short wave cases. K_c values are less than 1.5 for short wave, but the highest K_c reaches up to 6.9 for long wave. Sarpkaya [24] used various ranges of K_c for harmonically oscillating single phase flows to describe the evolution of vortices: (a) $K_c < 4$; no vortex, (b) $4 < K_c < 8$; symmetric vortex shedding, (c) $K_c > 8$; asymmetric vortex shedding. In case of incoming wave for surface-piercing cylinder, $K_c \approx 6$ was considered as a criterion for onset of separated vortices by Mei [25], but it has been argued that vortex shedding has also been observed at lower K_c numbers [26].

Figure 16 and 17 show instantaneous air/water interface and Q-criterion contour for short and long wave with $H/\lambda=1/10$, respectively. Positive Q-criterion value indicates vortex. Small and weak vortices are shown even for $K_c=1.5$ (Fig. 16) and vortices grow in size and strength significantly for $K_c=6.9$ (Fig. 17). The structure of vortices doesn't show regular pattern such as periodic shedding. Figure 18 presents mean air/water interface and Q-criterion contour for short and long wave with $H/\lambda=1/10$. In Fig. 18(a), small vortices are mainly observed near air/water interface. Due to positive/negative horizontal velocity field of incident wave at crest/trough, vortices are generated at upstream as well as at downstream. In Fig. 18(b), relatively strong vortices are observed in wide region, but the structure of vortices still shows irregular pattern.

In Table 7, C_d and C_m values are also presented. By using the time histories of horizontal force, horizontal velocity and its time derivative extracted from CFD simulation, C_d and C_m can be calculated through Morison equation and have been compared with those in the study of Sarpkaya [24]. C_d / C_m for CFD results are 0.33 / 1.3 and 0.42 / 1.3 for short and long wave respectively. The C_d / C_m values for short and long wave are 49% / 26% and 35% / 26% lower than Sarpkaya's data for the corresponding Reynolds number, respectively, even if K_c for present study is in the much lower range. It should be noted that Sarpkaya's data is for two-dimensional harmonic single phase flow generated by harmonically

oscillating cylinder in deep water, whereas present CFD handles surface-piercing truncated cylinder due to incoming wave. As mentioned by Sarpkaya [24], the differences seem to be caused by (a) vertical velocities, (b) vertical gradient of horizontal velocity amplitude, (c) non-linear harmonic component of the horizontal velocity and (d) free surface effect.

Based on parameters in Table 7 and Q-criterion contours, vortex shedding can be discussed in two perspectives: (1) size/strength of the vortices, (2) irregularity,

(1) Size/strength of the vortices are functions of $(T/2)/T_k$ or K_c , where $(T/2)/T_k = St \cdot K_c/2$ and

St is almost constant for post critical Re . Short wave period reduces the size/strength of vortices by breaking them up into small and weak pieces.

(2) The vortices do not show regular structure such as periodic shedding. Incident wave keeps changing velocity direction for crest/ trough, which disturbs a development of regular pattern.

Therefore, for cases with same wave period, higher wave steepness produces higher K_c values and in turn induces strong vortices.

7 Conclusions

The capabilities of CFDShip-Iowa V4.5 and V6.2 simulations for wave run-up for single/multiple surface-piercing cylinders in regular waves were assessed. The simulations were conducted for various wave conditions including $\lambda/d=4.7$ and $\lambda/d=21.9$ with $H/\lambda=1/30$, $1/16$ and $1/10$. Statistical convergence and grid, domain and turbulence model sensitivity were evaluated. The simulations were qualitatively validated using facility bias U_{FB} and error E between the experiment and simulation. Variables were grouped into first (1st harmonics) and second (mean and 2nd harmonics) order terms for separate assessment capabilities of each both for the experiments and CFD. Relation between wave induced current and run-up height in mean wave profile was analyzed and the effect of wave steepness/wave length, wave diffraction, interaction among cylinders, and vortex shedding structure were discussed.

Statistical convergence uncertainty U_I values were less than 1% for both the experiment and simulation data. The grid, domain and turbulence model sensitivities were less than the respective U_{FB} for 1st and 2nd order variables. The results indicated that both the large and medium domains should be sufficient for the validation studies. The turbulence model sensitivities were also small such that the URANS turbulence model was sufficient for the validation studies.

For horizontal/vertical forces, the validation studies showed that the errors E were 72 and 23/2%D for 2nd order and 1st order amp/phase variables, respectively. For wave elevations, the detailed study of the wave run-up profile showed that the mean wave field elevations were similar to the free surface elevations for a cylinder in a steady current due to the large wave induced current (up to 18% of the horizontal velocity for the steepest wave). Investigation on wave induced current and run-up height in mean wave profile showed that the maximum run-up (η_{RU}) in mean wave elevation can be estimated accurately using Bernoulli equation. The maximum values of mean and 2nd harmonics reached up to 15% and 20% of 1st harmonics respectively, indicating severe non-linearity effect for steep wave. The error E for all 1C cases was 5.8%D / 67%D for 1st / 2nd order variables, respectively, whereas U_{FB} was 4.7% / 118% and averaged errors $\bar{E}$ for the other CFD data provided by OEC was 5.6% / 141%. Thus, error for the 1st order variables is slightly higher than U_{FB} and $\bar{E}$ but still comparable. The error for the 2nd order variables is less than half of $\bar{E}$. The average E of 4C for the 1st and 2nd order terms are 7.4% and 118%, which seemed reasonably accurate compared to 1C case, even if U_{FB} and $\bar{E}$ for 4C were not available.

Comparison between V4.5 and V6.2 showed that results from both agreed well with experiment, even if V6.2 had slightly smaller error than V4.5 for most of 1st and 2nd order terms. The comparison also showed the merits of each version. V4.5 is a generalized/robust code to guarantee relatively easy application for complex/large-scale problem, whereas V6.2 has capability to describe more detailed wave breaking/spraying utilizing high order method and VOF sharp interface treatment.

For wave steepness/wave length effect, wave length was major factor for wave induced loads compared to wave steepness. The 2nd to 1st harmonic ratio on the weather side decreased with steepness for short waves, but increased for long waves. For both of wave lengths, the 2nd to 1st harmonic ratio at

leeward side increased by increasing steepness. The wave diffraction analyses showed that 2nd / 3rd to 1st harmonic ratio in diffracted wave contour were 18~41% / 9~16% for short wave, suggesting stronger non-linearity for steeper wave. Kevin wave in mean wave elevation contours was similar but less clear due to high Re and low Fr, compared to that for a vertical cylinder in steady current. Compared to diffraction pattern for ship in head waves, 1st harmonics of diffracted wave showed significantly wider spreading pattern toward upstream due to different dominance of wave induced current, wave steepness and body geometry. The highest 1st harmonic values were almost same for 1C and 4C, but the 2nd / 3rd to 1st harmonic ratio increased up to 25% / 12% for 4C case.

Comparison of wave elevations at crest/trough for 1C and 4C showed that the interaction of cylinders increased trough height for 4-10% whilst crest height increased about 5-17%. The strongest interaction effect on the crest height was found at the shoulder side of the cylinders. Intensive near-trapping was not observed in present test case. Studies on vortex shedding showed that size/strength of the vortices were functions of $(T/2)/T_k$ or K_c . Short wave case had insufficient time to develop vortex shedding, compared to long wave case. Regular structure such as periodic shedding was not observed since velocity direction change for wave crest/trough disturbed regular pattern developing, especially for low K_c value.

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Figure 1 Locations of wave probes around circular cylinders: green circles (●) indicate wave probes, (a) single circular cylinder, (b) arrangement of four circular cylinders, (c) wave probes for four circular cylinders

Figure 2 Overset grid system and boundary condition for V4.5 simulation:
(a) computational domain and boundary conditions, (b) side view for 1C, (c) top view for 4C

Figure 3 Cylindrical grid system and boundary condition for V6.2 simulation:
(a) computational domain and boundary conditions, (b) surface mesh

Figure 4 Time history results for $\lambda/d=4.7$, $H/\lambda=1/30$ (black symbol: EFD, red line: CFD V4.5):
(a) wave elevation at $r/d=0.513$, $\theta=180^\circ$ and forces (b) running mean, (c) running root mean square

Figure 5 Time history results for $\lambda/d=21.9$, $H/\lambda=1/10$ (black symbol: EFD, red line: CFD V4.5):
(a) wave elevation at $r/d=0.513$, $\theta=180^\circ$ and forces (b) running mean, (c) running root mean square

Figure 6 Mean, 1st and 2nd harmonics of horizontal and vertical forces (1C)

Figure 7 Mean wave profile for 1C: (a) short wave case, (b) long wave case; η_0 calculated along centerline ($y=0$) of computational domain for off-body

Figure 8 Mean horizontal velocity distribution according to vertical depth

Figure 9 1st harmonics of wave elevation for 1C: (a) short wave case, (b) long wave case

Figure 10 2nd harmonics of wave elevation for 1C: (a) short wave case, (b) long wave case

Figure 11 Comparison of instantaneous wave field between EFD and $\text{CFD}_{V4.5}$ ($1C$, $\lambda/d=21.9$, $H/\lambda=1/10$): photo from experiment [5]; (upper) side view, (lower) upwave view

Figure 12 Comparison of instantaneous air/water interface profiles of $\text{CFD}_{V4.5}$ and $\text{CFD}_{V6.2}$ ($1C$, $\lambda/d=4.7$, $H/\lambda=1/10$): (a) $V4.5$, (b) $V6.2$; $t=T$ and $t=T+T/2$ when wave crest and trough meets cylinder, respectively

Figure 13 Mean, 1st and 2nd harmonics of wave elevation for different wave steepness (1C):
(a) short wave cases, $\lambda/d=4.7$, (b) long wave cases, $\lambda/d=21.9$

Figure 14 Contours of harmonics for diffracted wave, $\eta_D/A_1 = (\eta_T - \eta_{NC})/A_1$ (1C, $\lambda/d=4.7$):
(a) $H/\lambda=1/30$, (b) $H/\lambda=1/10$; twenty contour levels determined to be suitable for the min. and max. values

Figure 15 Contours of harmonics for diffracted wave, $\eta_D/A_1 = (\eta_T - \eta_{NC})/A_1$ (4C, $\lambda/d=4.7$, $H/\lambda=1/30$) ; twenty contour levels determined to be suitable for the min. and max. values

Figure 16 Instantaneous air/water interface and Q-criterion contour near the cylinder (1C, $\lambda/d=4.7$, $H/\lambda=1/10$)

Figure 17 Instantaneous air/water interface and Q-criterion near the cylinder (1C, $\lambda/d=21.9$, $H/\lambda=1/10$)

Figure 18 Mean air/water interface and Q-criterion near the cylinder: (a) $\lambda/d=4.7$, $H/\lambda=1/10$, (b) $\lambda/d=21.9$, $H/\lambda=1/10$

Table 1 Main particulars of single/four cylinders and facility biases uncertainty (U_{FB})

Description	Symbol [unit]	Prototype	Single Cylinder Model (MOERI)	Single/Four Cylinder Model (MARINTEK)
Diameter	d [m]	16	0.318	0.327
Draft	T_d [m]	24	0.477	0.490
Draft/Diameter	T_d/d	1.5	1.5	1.5
Scale	Γ	-	1/50.31	1/48.93
Facility biases uncertainty U_{FB} for single cylinder cases				
		$H/\lambda=1/30$	$H/\lambda=1/16$	$H/\lambda=1/10$
η_0	$\lambda/d=4.7$	104.38	70.89	62.81
	$\lambda/d=21.9$	666.08	43.90	62.41
	Avg.	168.41		
η_1	$\lambda/d=4.7$	7.79	3.58	4.27
	$\lambda/d=21.9$	4.03	4.45	4.20
	Avg.	4.72		
η_2	$\lambda/d=4.7$	93.63	73.09	64.81
	$\lambda/d=21.9$	70.78	31.87	68.13
	Avg.	67.05		
1 st order variables: η_1		4.72		
2 nd order variables: $(\eta_0+\eta_2)/2$		117.73		

Table 2 Test matrix for CFD simulations

Body/Code/ Turbulence model	Grid			λ/d	T [s]	f [Hz]	H/λ	Re	Fr	Wave resolution				
		Domain size	Total grid points							Grid type	$\lambda/\Delta x$	$H/\Delta z$	$T/\Delta t$	
1C / V4.5 / RANS	G1	Large $-3.0\lambda < x < 3.0\lambda$ $-1.5\lambda < y < 1.5\lambda$ $-2.0\lambda < z < 2.5d$	24M Medium resolution	4.7	7	0.143	1/30	7.34E+05	0.0048	Background	76	30	548	
							1/16	2.58E+06	0.0168	Background	76	56	548	
							1/10	6.60E+06	0.0431	Background	76	90	548	
	G2	Large $-3.0\lambda < x < 3.0\lambda$ $-1.5\lambda < y < 1.5\lambda$ $-2.0\lambda < z < 6.0d$	23M Medium resolution	21.9	15	0.067	1/30	1.57E+06	0.0103	Background	76	20	1174	
										Refinement 2	154	20		
										Refinement 1	336	22		
							1/16	5.53E+06	0.0361	Background	76	58	1174	
										Refinement 2	154	58		
										Refinement 1	336	64		
							1/10	1.42E+07	0.0924	Background	76	92	1174	
										Refinement 2	154	94		
										Refinement 1	336	104		
	G3	Medium $-2.0\lambda < x < 3.0\lambda$ $-1.5\lambda < y < 1.5\lambda$ $-2.0\lambda < z < 2.5d$	25M Fine resolution	4.7	7	0.143	1/30	7.34E+05	0.0048	Background	82	58	548	
										Refinement 1	119	132		
	G4	Medium $-2.0\lambda < x < 3.0\lambda$ $-1.5\lambda < y < 1.5\lambda$ $-2.0\lambda < z < 2.5d$	21M Medium resolution	4.7	7	0.143	1/30	7.34E+05	0.0048	Background	76	30	548	
	G5	Small $-\lambda < x < 3.0\lambda$ $-1.5\lambda < y < 1.5\lambda$ $-2.0\lambda < z < 2.5d$	13M Medium resolution							Background	76	30	548	
	1C / V4.5 / RANS	G6	Very small $-0.6\lambda < x < 1.2\lambda$ $-0.7\lambda < y < 0.7\lambda$ $-1.4\lambda < z < 0.4d$	26M Very fine resolution	4.7	7	0.143	1/16	2.58E+06	0.0168	Background	159	60	548
	1C / V4.5 / DES							1/16			Background	159	60	2192
1C / V6.2 / No Turbulence model	G7	Medium $-2.0\lambda < x < 2.0\lambda$ $-2.0\lambda < y < 2.0\lambda$ $-3.0\lambda < z < 3.0d$	11M Coarse resolution	4.7	7	0.143	1/16	2.58E+06	0.0168	Background	128	22.8	210	
							1/10	6.60E+06	0.0431	Background	128	36.4	520	
4C / V4.5 / RANS	G8	Medium $-2.0\lambda < x < 3.0\lambda$ $-1.5\lambda < y < 1.5\lambda$ $-2.0\lambda < z < 2.5d$	65M Medium resolution	4.7	7	0.143	1/30	7.34E+05	0.0048	Background	82	58	548	
										Refinement 1	119	132		

Table 3 Effect of computational parameters on wave elevation and force

		Grid sensitivity ΔG ($\lambda/d=4.7$, $H/\lambda=1/30$)	Domain size ΔD ($\lambda/d=4.7$, $H/\lambda=1/30$)		Turbulence model ΔTM ($\lambda/d=4.7$, $H/\lambda=1/16$)
		$\Delta G4\%G3$	$\Delta D2\%D1$	$\Delta D3\%D1$	$\Delta URANS\%DES$
Averaged wave height (Avg. among 10 locations)	$\bar{\eta}_c$	2.19	0.84	6.26	3.00
	$\bar{\eta}_t$	4.13	1.66	5.15	1.35
Harmonics of wave elevation (Avg. among 10 locations)	η_0	118.26	3.62	19.73	7.98
	η_1	3.20	0.92	8.76	0.57
	η_2	37.22	1.05	14.94	5.12
Harmonics of horizontal force	X_0	3.80	4.65	29.07	6.38
	X_1	0.18	0.97	7.39	0.33
	X_2	20.00	6.12	24.49	12.00
wave height: $(\bar{\eta}_c + \bar{\eta}_t)/2$		3.16	1.25	5.71	2.18
0 th harmonics: $(\eta_0 + X_0)/2$		61.03	4.14	24.4	7.18
1 st harmonics: $(\eta_1 + X_1)/2$		1.69	0.95	8.08	0.45
2 nd harmonics: $(\eta_2 + X_2)/2$		28.61	3.59	19.72	8.56
1 st order variables:	$(\bar{\eta}_c + \bar{\eta}_t + \eta_1)/3$	3.17	1.14	6.72	1.64
	$(\bar{\eta}_c + \bar{\eta}_t + \eta_1 + X_1)/4$	2.43	1.10	6.90	1.32
2 nd order variables:	$(\eta_0 + \eta_2)/2$	77.74	2.34	17.34	6.55
	$(\eta_0 + \eta_2 + X_0 + X_2)/4$	44.82	3.87	22.06	7.87

Table 4 Harmonics of horizontal and vertical forces for 1C

Short wave case ($\lambda/d=4.7$)										
		$H/\lambda=1/30$			$H/\lambda=1/16$			$H/\lambda=1/10$		
		EFD	CFD	E%D	EFD	CFD	E%D	EFD	CFD	E%D
Mean	$X_0/(pgd^2 A_1/4)$	0.639	0.486	23.940	0.603	0.590	2.224	0.730	0.480	34.146
	$Z_0/(pgd^2 A_1/4)$	-0.162	0.031	119.421	-0.034	-0.027	19.875	-0.099	-0.034	65.540
	average			71.681			11.049			49.843
1 st	$X_1/(pgd^2 A_1/4)$	5.583	4.881	12.586	5.069	3.727	26.489	5.146	4.773	7.241
	$Z_1/(pgd^2 A_1/4)$	0.401	0.256	36.105	0.384	0.304	20.626	0.407	0.303	25.513
	average			24.345			23.558			16.377
	X_{ei} [deg.] [‡]	-29.37	-42.50	3.65	-	-	-	-	-	-
	Z_{ei} [deg.]	-6.08	-7.90	0.51	-	-	-	-	-	-
	average			2.08			-			-
2 nd	$X_2/(pgd^2 A_1/4)$	0.461	0.374	18.720	0.380	0.414	-9.061	0.329	0.485	-47.428
	$Z_2/(pgd^2 A_1/4)$	0.259	0.446	-72.413	0.230	1.029	-348.143	0.262	0.183	30.009
	average			45.567			178.602			38.719
Long wave case ($\lambda/d=21.9$)										
		$H/\lambda=1/30$			$H/\lambda=1/16$			$H/\lambda=1/10$		
		EFD	CFD	E%D	EFD	CFD	E%D	EFD	CFD	E%D
Mean	$X_0/(pgd^2 A_1/4)$	0.078	0.105	-35.206	0.132	0.109	17.042	0.149	0.085	42.771
	$Z_0/(pgd^2 A_1/4)$	-0.080	-0.322	-301.408	-0.124	-0.111	9.806	-0.144	-0.124	14.378
	average			168.307			13.424			28.575
1 st	$X_1/(pgd^2 A_1/4)$	2.276	3.288	-44.482	2.229	2.079	6.732	2.198	1.828	16.848
	$Z_1/(pgd^2 A_1/4)$	1.965	2.827	-43.912	2.044	1.801	11.898	2.188	1.701	22.263
	average			44.197			9.315			19.555
	X_{ei} [deg.]	-	-	-	-	-	-	-134.15	-137.58	0.95
	Z_{ei} [deg.]	-	-	-	-	-	-	-127.66	-137.45	2.72
	average			-			-			1.84
2 nd	$X_2/(pgd^2 A_1/4)$	0.737	1.326	-80.000	0.702	0.410	41.504	0.598	0.291	51.295
	$Z_2/(pgd^2 A_1/4)$	0.027	0.105	-284.259	0.032	0.046	-46.137	0.038	0.031	19.306
	average			182.130			43.821			35.300

[‡] Error percentage for phase is defined as $E\%360^\circ = (D-S)\%360^\circ$

Table 5 Harmonics of wave elevations for 1C and 4C

				η_0/A_1	η_1/A_1	η_{el} [deg.] [§]	η_2/A_1	$\bar{\eta}_c/A_1$	$\bar{\eta}_t/A_1$
1C, $\lambda/d=4.7$, $H/\lambda=1/30$	E%D _{V4.5}	$r/d=0.513$ (on-body)	Avg _{weather}	33.84	1.97	3.79	51.19	3.96	0.22
			Avg _{leeward}	232.49	5.08	5.17	24.95	15.22	3.59
			Avg _{on-body}	122.77	3.66	4.57	34.34	9.02	1.98
		$r/d=1.0$ (off-body)	Avg _{weather}	49.95	6.57	5.44	41.56	4.75	8.45
			Avg _{leeward}	227.66	8.18	2.18	31.49	22.16	16.93
			Avg _{off-body}	173.71	7.39	3.77	36.34	14.51	10.86
		Total	Avg _{weather}	41.90	4.27	4.61	46.38	4.35	4.33
			Avg _{shoulder}	197.25	5.83	4.27	27.48	12.74	2.89
			Avg _{leeward}	230.07	6.63	3.67	28.22	18.69	10.26
			Avg _{total}	148.24	5.53	4.17	35.34	11.77	6.42
1C, $\lambda/d=21.9$, $H/\lambda=1/10$	E%D _{V4.5}	$r/d=0.513$ (on-body)	Avg _{weather}	10.75	4.06	0.91	142.50	24.31	12.33
			Avg _{leeward}	31.90	4.30	2.61	105.58	4.68	2.28
			Avg _{on-body}	21.06	3.83	1.69	135.48	14.46	7.03
		$r/d=1.0$ (off-body)	Avg _{weather}	68.18	10.74	2.91	60.12	4.13	6.65
			Avg _{leeward}	131.44	3.47	3.07	119.64	10.97	5.96
			Avg _{off-body}	87.85	6.24	2.86	91.40	10.38	6.06
		Total	Avg _{weather}	39.47	7.40	1.91	101.31	14.22	9.49
			Avg _{shoulder}	30.00	2.60	1.87	139.38	18.01	5.49
			Avg _{leeward}	81.67	3.89	2.84	112.61	7.83	4.12
			Avg _{total}	54.46	5.03	2.28	113.44	12.42	6.54
1C, $\lambda/d=4.7$, $H/\lambda=1/16$	E%D _{V4.5}	$r/d=0.513$ (on-body)	Avg _{on-body}	32.94	6.68	-	15.21	21.91	7.96
	E%D _{V6.2}	$r/d=0.513$ (on-body)	Avg _{on-body}	16.27	9.53	-	22.15	14.37	3.75
	ΔV6.2%V4.5	$r/d=0.513$ (on-body)	Avg _{weather}	11.91	10.44	-	37.72	9.14	9.72
			Avg _{leeward}	24.23	15.35	-	8.19	17.02	10.04
			Avg _{on-body}	22.16	15.51	-	23.44	16.06	11.20
		$r/d=1.0$ (off-body)	Avg _{weather}	18.79	6.33	-	14.51	4.55	6.51
			Avg _{leeward}	75.86	8.77	-	9.91	14.80	5.86
			Avg _{off-body}	54.66	9.35	-	11.51	12.03	7.91
		Total	Avg _{weather}	15.35	8.38	-	26.11	6.85	8.11
			Avg _{shoulder}	61.28	21.27	-	17.07	24.72	15.65
			Avg _{leeward}	50.05	12.06	-	9.05	15.91	7.95
			Avg _{total}	38.41	12.43	-	17.48	14.05	9.56
	E%D _{V4.5}	$\theta=180^\circ$	Avg $\theta=180^\circ$	78.66	4.87	5.72	37.91	10.66	14.31
		$\theta=225^\circ$	Avg $\theta=225^\circ$	500.59	8.05	1.18	24.01	7.26	8.54
		$\theta=270^\circ$	Avg $\theta=270^\circ$	54.58	9.35	3.37	14.67	3.22	13.14
		Total	Avg $r/d=0.503$	76.80	12.14	1.02	31.59	9.79	18.73
			Avg $r/d=0.592$	91.49	6.43	5.73	31.76	8.19	11.60
			Avg $r/d=0.797$	131.23	5.62	2.10	19.01	5.15	6.74
			Avg $r/d=1.0$	545.57	5.50	4.85	19.76	5.05	10.92
			Avg _{total}	211.27	7.42	3.43	25.53	7.05	12.00
				1 st order variables					2 nd order variables
				$(\bar{\eta}_c+\bar{\eta}_t)/2$	η_l	η_{el} [deg.]	$(\bar{\eta}_c+\bar{\eta}_t+\eta_l)/3$	$(\bar{\eta}_c+\bar{\eta}_t+\eta_l+\eta_{el})/4$	$(\eta_\theta+\eta_2)/2$
1C	E%D _{V4.5}	Total	Avg _{1C}	11.17	5.75	3.23	9.36	6.78	66.61
1C	E%D _{V6.2}	Total	Avg _{1C}	9.06	9.53	-	9.22	-	19.21
4C	E%D _{V4.5}	Total	Avg _{4C}	9.53	7.42	3.43	8.82	7.47	118.40
1C	OECD E%D E**	Total	Avg _{1C}	-	5.63	-	-	-	141.03

[§] Error percentage for phase is defined as $E\%360^\circ=(D-S)\%360^\circ$

** $\bar{E} = \Delta CFD_{average} \% EFD_{MOERI}$

Table 6 Comparison of wave elevations for 1C and 4C (Cyl.A, downwave; $\lambda/d=4.7$, $H/\lambda=1/30$)

	$\bar{\eta}_c/A_I$		$\bar{\eta}_t/A_I$	
	$\Delta EFD_{4C}\%EFD_{1C}$	$\Delta CFD_{4C}\%CFD_{1C}$	$\Delta EFD_{4C}\%EFD_{1C}$	$\Delta CFD_{4C}\%CFD_{1C}$
Avg $\theta=180^\circ$	8.82	5.25	16.93	4.32
Avg $\theta=225^\circ$	13.39	12.69	7.21	13.34
Avg $\theta=270^\circ$	25.26	17.34	5.81	10.46
Avg $r=0.503d$	15.60	14.45	11.70	7.68
Avg $r=1.0d$	16.05	9.07	8.27	11.06
Avg _{total}	15.82	11.76	9.98	9.37

Table 7 Summary of parameters for separated vortices

T [s]	λ/d	H/λ	A_I/T_d	λ/T_d	u_ζ [m/s]	u_{T_d} [m/s]	u_{T_d}/u_ζ [%]	T_k	$(T/2)/T_k$	K_c	C_d	C_m
7	4.7	1/30	0.055	3.19	1.14	0.16	13.9	58.5	0.06	0.50	0.33	1.33
		1/16	0.100		2.15	0.30		31.0	0.11	0.94	-	-
		1/10	0.160		3.43	0.48		19.4	0.18	1.50	-	-
15	21.9	1/30	0.245	14.65	2.45	1.60	65.1	27.2	0.28	2.30	-	-
		1/16	0.460		4.60	3.00		14.5	0.52	4.31	-	-
		1/10	0.730		7.36	4.79		9.1	0.83	6.90	0.42	1.33