A Novel Reliability Technique for Implementation of Performance-Based Seismic Design of Structures

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ABSTRACT

A unified Performance-Based Seismic Design (PBSD) procedure is proposed and successfully implemented. It provides an alternative to the currently used life safety design requirement. To successfully develop the concept, structures are represented by finite elements and excited by the seismic loading in time domain. A novel reliability evaluation **procedure** is proposed for such representation. An improved response surface based procedure is proposed by combining it with the First-Order Reliability Method (FORM) and the appropriate response surfaces are constructed by **combining** the saturated design and the central composite design sampling schemes. Performances are defined in terms of Collapse Prevention (CP), Life Safety (LS), and Immediate Occupancy (IO), as commonly used in the profession. The corresponding risks are evaluated by exciting a 9-story steel frame designed by experts satisfying all post-Northridge seismic design requirements. It was excited by 20 earthquake time histories for each

performance level and the corresponding probabilities of failure were estimated. It took around 300 deterministic evaluations. The **accuracy** of the method was established using 600,000 cycles of Monte Carlo simulations. The probabilities of failure estimated using the proposed algorithm are very similar to that of simulations indicating that it is accurate. The probability of failure for two serviceability limit states (overall and inter-story drifts) are very similar, indicating that the procedure will satisfy the basic intent of PBSO. They are also similar to the values used in developing the Load and Resistance Factor Design (LRFD) guidelines used in many current design codes. Designing a structure using multiple time histories, as suggested in recent design guidelines, is a step in the right direction. From the results and the observations made in this study, the authors believe that they proposed a robust, efficient, and accurate unified PBSO procedure and documented its implementation potential.

Keywords: Performance-based seismic design; structural reliability; seismic risk; finite element method; response surface method; serviceability performance functions; first-order reliability method; Monte Carlo Simulation.

1. Introduction

Enormous amount of property damages caused by the Northridge Earthquake of 1994 prompted the profession to find an alternative to the currently used design criteria of life safety. Although the life safety was not severely compromised during the earthquake, the structural damage was enormous. Several beam-to-column connections in steel frames fractured in a brittle manner making the buildings unusable following the earthquake. Similar damages caused by previous earthquakes indicate rooms for improvements in the current prescriptive design guidelines based on the safety factor concept as used in the Allowable Stress Design (ASD) or in the Load and Resistance Factor Design (LRFD) provisions to protect life. To address the

situation, under the sponsorship of the Federal Emergency Management Agency (FEMA), a major study by SAC [a joint venture of the Structural Engineers Association of California (SEAOC), Applied Technology Council (ATC), and California Universities for Research in Earthquake Engineering (CUREE)] was initiated in the late nineties. One of the main objectives of this study was to develop recommendations for more robust design and construction of steel structures and to propose alternative design criteria to avoid adverse economic consequences. The major findings were published in a series of reports (FEMA-350, 351, 352, 353, 355C, and 355F) [1-6].

One of the major outcomes of the SAC study was the introduction of a new design concept commonly known as the Performance-Based Seismic Design (PBSD), an alternative to the life safety design concept currently practiced. PBSD is essentially a more advanced risk-based design procedure currently used in Load and Resistance Factor Design (LRFD) [7]. The dynamic responses of a structure containing many uncertainty-filled load and resistance-related design parameters are also extremely difficult to predict. At present, the required time history of the seismic loading at a given site cannot be predicted with any confidence. The authors believe that numerous sources of uncertainty cannot be completely eliminated in any realistic seismic design and thus the absolute safety of a structure cannot be assured. The SAC study made similar conclusions and advocated that the associated risk needs to be managed appropriately. To achieve this intent, the risk of real structures to satisfy specific performance requirements need to be estimated by modeling them as appropriately as possible, applying loads the best way possible, and exciting them by seismic loading in time domain. As expected, the PBSD concept is certainly a little advanced.

To achieve the basic objectives, SAC proposed to correlate different performance levels with the corresponding risks and let the designers and/or owners decide the level of underlying risk they are willing to accept. FEMA 350 [1] defined a performance level as: “the intended post-earthquake condition of a building; a well-defined point on a scale measuring how much loss is caused by earthquake damage”. To appropriately implement the PBSD concept, it is essential that the underlying risk or probability of failure of not satisfying a prescribed performance must be explicitly quantified. Obviously, just before failure, a structure is expected to develop several sources of nonlinearities. Thus, for the successful implementation of PBSD, it will be necessary to estimate the corresponding risk by applying the seismic loading in time domain in the presence of multiple sources of nonlinearity and uncertainty. Unfortunately, SAC did not recommend any specific procedure for the risk estimation and the authors believe that no such procedure is currently available.

PBSD involves a set of procedures by which a structural system is designed in a controlled manner. The concept can be implemented by following five sequential steps: (1) select performance objectives, (2) develop preliminary design, (3) assess performance capability, (4) check performance capability with allowable values, in terms of the associated risks, and (5) if risks are not acceptable, revise the initial design. The available literature on PBSD is very limited, FEMA 355F [6] suggested that PBSD guidelines can be developed by incorporating the following six items: (1) to account for uncertainty in performance associated with unanticipated events, (2) to set realistic expectations for performance, (3) to assess performance variables in similar buildings located nearby, (4) to develop a reliability framework, (5) to set representative performance levels for various seismic hazards, and (6) to quantify local and global structural behaviors leading to collapse. The authors believe that a new reliability analysis technique is

required to incorporate the above items. And only then the appropriate PBSB guidelines can be developed and implemented.

Limited number of studies addressing different aspects of PBSB are reported in the literature [8-11]. However, the authors are not aware of any comprehensive study addressing all the above-mentioned 6 items. This study is expected to fill such knowledge gap. Even though the reliability approach presented in this paper is little advanced, if implemented properly, it will produce more economical and seismic-damage tolerant structures by identifying the most damage-prone structural elements and by satisfying all major failure modes or performance requirements satisfying similar pre-assigned risk levels. The authors believe that other similar studies will help to develop the appropriate design guidelines to be used by engineers in the near future.

2. Challenges in PBSB

Some of the important challenges in implementing the PBSB concept discussed above need further elaboration. The selection of performance levels and the appropriate mathematical models to represent such behavior could be very challenging. The structure is expected to develop various sources of nonlinearities following the load path to failure and the mathematical model to capture such behavior can be very demanding. To study such nonlinear behavior realistically, the structure is generally represented by Finite Elements (FEs). Considering accuracy and efficiency, representing real large complicated structural systems by FEs could be very complicated and challenging. To study the dynamic response behavior caused by the seismic loading, several methods with various degrees of sophistication are suggested in the current design guidelines [12, 13]. They include pseudo-static, modal, and time domain

application of the excitation. The most sophisticated analysis requires that a structure to be represented by nonlinear FEs and the dynamic seismic loading must be applied in time domain.

Since SAC did not recommend any specific risk evaluation procedure, the authors believe that the unavailability of such procedure is a major knowledge gap in realistically implementing the PBSO concept explicitly considering all major sources of nonlinearity and uncertainty. Risk is always estimated with respect to a limit state or performance function. When the Limit Performance Function (LPF) is explicit in nature, i.e., it can be expressed in terms of all Random Variables (RVs) present in the formulation and the performance requirements, the First-Order and/or Second-Order Reliability Methods (FORM/SORM) [14] can be used to extract the corresponding risk, since the derivatives of the LPF with respect to the RVs will be readily available. However, for the implementation of the PBSO concept in the presence of nonlinearity and the excitation in time domain, LPFs are expected to be implicit in nature. A considerable amount of expertise is required to extract reliability information using FORM/SORM for implicit LPFs, as reported by Haldar and Mahadevan [14]. As an alternative, the basic Monte Carlo Simulation (MCS) can also be used to estimate the reliability [15]. Unfortunately, the use of basic MCS for the reliability analysis of a real structure excited by the seismic loading applied in time domain can be impractical since one deterministic nonlinear FE-based analysis will require several hours of computing time.

Based on the above discussions, two major objectives of this paper are: (1) to propose a novel reliability analysis concept to estimate risk of a nonlinear structure represented by FEs and excited by the seismic loading applied in time domain, and (2) to showcase the implementation of PBSO to satisfy the intents of SAC.

3. A Unified PBSO Procedure

Some of the most basic requirements of PBSO are the deterministic evaluation of required performances as accurately as possible, the incorporation of major sources of nonlinearities in the deterministic formulation, and then estimate the risk satisfying the appropriate performance requirements. These requirements are discussed in the following sections.

3.1. Finite Element Representation of Structures

In the most sophisticated deterministic response evaluations, structures are generally represented by FEs. In this representation, the major sources of nonlinearities can be incorporated as the structure passes through several phases before reaching the failure state. As mentioned earlier, the time domain nonlinear response analysis required to implement PBSO can be very challenging. To overcome this issue, the authors decided to use the assumed stress-based FE method in developing the deterministic FE code using concentrated plasticity model to capture material nonlinearity, specifically applicable for steel frame structures [16, 17]. In this representation, fewer elements are required to represent the large deformed configuration and the tangent stiffness matrix can be expressed in an explicit form without integration at the every time step of the time domain analysis. The authors and their team members helped to develop and document the accuracy, efficiency, and capabilities of the software [18-22]. They developed a sophisticated computer program to implement the concept. For the ease of discussion, the essential features of the approach are briefly discussed below.

Without losing any generality, the nonlinear dynamic equilibrium equation of motion for large structural systems can be expressed in the matrix form at time $t+\Delta t$ as:

$$\mathbf{M}^{t+\Delta t} \ddot{\mathbf{D}}^{(n)} + \mathbf{C}^{t+\Delta t} \dot{\mathbf{D}}^{(n)} + \mathbf{K}^{(n)} \Delta \mathbf{D}^{(n)} = {}^{t+\Delta t} \mathbf{F}^{(n)} - {}^{t+\Delta t} \mathbf{R}^{(n-1)} - \mathbf{M}^{t+\Delta t} \ddot{\mathbf{D}}_g^{(n)} \quad (1)$$

where $\mathbf{M}$ is the mass matrix, ${}^t\mathbf{C}$ is the viscous damping coefficient matrix at time t , ${}^t\mathbf{K}^{(n)}$ is the global stiffness matrix of the n^{th} iteration at time t , ${}^{t+\Delta t}\Delta\mathbf{D}^{(n)}$ is the incremental displacement vector of the n^{th} iteration at time $t+\Delta t$, ${}^{t+\Delta t}\mathbf{F}^{(n)}$ is the external load vector of the n^{th} iteration at time $t+\Delta t$, ${}^{t+\Delta t}\mathbf{R}^{(n-1)}$ is the internal force vector of the $(n-1)^{\text{th}}$ iteration at time $t+\Delta t$, and ${}^{t+\Delta t}\ddot{\mathbf{D}}_g^{(n)}$ is the seismic ground acceleration vector of the n^{th} iteration at time $t+\Delta t$.

The amount of damping energy to be generated will depend mainly on two conditions: (1) the hysteretic behavior if the material yields, and (2) the non-yielding and yielding states of the structural elements. In structural engineering, the effect of non-yielding energy dissipation is commonly represented by viscous damping varying between 0.1 and 7 percent of the critical damping [23]. Since the mass and stiffness matrixes are available from the FE formulation, Rayleigh-type damping is utilized in this study to represent damping. It can be expressed in matrix form as:

$${}^t\mathbf{C} = \alpha\mathbf{M} + \gamma {}^t\mathbf{K} \quad (2)$$

where α and γ are proportional constants which can be evaluated from the natural frequencies of the structure [24], and ${}^t\mathbf{K}$ is the tangent stiffness matrix.

The step-by-step direct integration procedure using the Newmark method [25], with parameters $\eta = 1/2$ and $\beta_N = 1/4$, is utilized to solve Eq. (1). The iterative scheme can be represented as:

$${}^t\mathbf{K}_D {}^{t+\Delta t}\Delta\mathbf{D}^{(n)} = {}^{t+\Delta t}\mathbf{F}_D^{(n)} - {}^{t+\Delta t}\mathbf{R}^{(n-1)} \quad (3)$$

where

$${}^{t+\Delta t}\mathbf{F}_D^{(n)} = {}^{t+\Delta t}\mathbf{F}_D^{(n-1)} + {}^{t+\Delta t}\Delta\mathbf{F}_D^{(n)} \quad (4)$$

and ${}^t\mathbf{K}_D$ represents the dynamic tangent stiffness matrix which can be expressed as:

$${}^t\mathbf{K}_D = f_1\mathbf{M} + f_2 {}^t\mathbf{K} \quad (5)$$

In Eq. (4), ${}^{t+\Delta t}\mathbf{F}_D^{(n-1)}$ and ${}^{t+\Delta t}\Delta\mathbf{F}_D^{(n)}$ are the modified external force vector and its incremental vector, respectively. The modified external force vector can be represented as:

$${}^{t+\Delta t}\mathbf{F}_D^{(n-1)} = {}^{t+\Delta t}\mathbf{F}^{(n-1)} + {}^{t+\Delta t}\mathbf{P}^{(n-1)} - \mathbf{M} {}^{t+\Delta t}\ddot{\mathbf{D}}_g^{(n-1)} \quad (6)$$

${}^{t+\Delta t}\mathbf{R}^{(n-1)}$ in Eq. (3) is the internal force vector of the system. The term ${}^{t+\Delta t}\mathbf{P}^{(n-1)}$ in Eq. (6) is the modified force vector contributed by the displacement, velocity, and acceleration at time t ; and the displacement at time $t+\Delta t$. It can be expressed as:

$${}^{t+\Delta t}\mathbf{P}^{(n-1)} = \mathbf{M}[f_1 {}^t\mathbf{D} + f_3 {}^t\dot{\mathbf{D}} + f_4 {}^t\ddot{\mathbf{D}} - f_1 {}^{t+\Delta t}\mathbf{D}^{(n-1)}] + {}^t\mathbf{K}[f_5 {}^t\mathbf{D} + f_6 {}^t\dot{\mathbf{D}} + f_7 {}^t\ddot{\mathbf{D}} - f_5 {}^{t+\Delta t}\mathbf{D}^{(n-1)}] \quad (7)$$

The incremental external force term ${}^{t+\Delta t}\Delta\mathbf{F}_D^{(n)}$ in Eq. (4) is represented as:

$${}^{t+\Delta t}\Delta\mathbf{F}_D^{(n)} = {}^{t+\Delta t}\Delta\mathbf{F}^{(n)} - \mathbf{M} {}^{t+\Delta t}\Delta\ddot{\mathbf{D}}_g^{(n)} \quad (8)$$

The coefficients f_i are constants which can be evaluated in terms of α , γ , η , β_N , and Δt as [26]:

$$\begin{aligned} f_1 &= \frac{1}{\beta_N\Delta t^2} + \frac{\eta\alpha}{\beta_N\Delta t}, \quad f_2 = \frac{\eta\gamma}{\beta_N\Delta t} + 1, \quad f_3 = \frac{1}{\beta_N\Delta t} + \frac{\eta\alpha}{\beta_N} - \alpha, \\ f_4 &= \left(\frac{1}{2\beta_N} - 1\right) + \eta\alpha\left(\frac{1}{2\beta_N} - \frac{1}{\eta}\right)\Delta t, \quad f_5 = \frac{\eta\alpha}{\beta_N\Delta t}, \quad f_6 = \frac{\eta\gamma}{\beta_N} - \gamma, \\ f_7 &= \left(\frac{\eta\gamma}{2\beta_N} - \gamma\right)\Delta t \end{aligned} \quad (9)$$

Equation (3) is solved using the modified Newton-Raphson method with arc-length control [27]. Following the above procedure, the deterministic responses in terms of displacements and forces in the structural members are estimated to implement PBSO.

3.2. A Novel Reliability Analysis Procedure

With the availability of the deterministic time domain nonlinear responses of structures available using the FE representation, it is now necessary to incorporate the information on uncertainties in the formulation. As mentioned earlier, for PBSD with implicit LPFs, the use of FORM/SORM can be very tedious. The authors also eliminated the basic MCS as an option because of its inefficiencies. The other alternative will be to approximately describe an implicit LPF using the Response Surface Method (RSM) [28, 29]. And then FORM can be used to extract the reliability information. To generate a Response Surface (RS), few deterministic nonlinear FE-based time domain response analyses need to be carried out around a center point following a specified sampling scheme, and a curve needs to be fitted to the generated response data. The authors believe that the risk can be estimated using few hundreds instead of several thousands or millions of deterministic evaluations using this approach filling a fundamental knowledge gap for the implementation of PBSD. However, the basic RSM concept currently utilized in the profession cannot be used for the structural reliability estimation. It has three major deficiencies: (1) it cannot incorporate the distributional information of RVs even when it is widely available, (2) if the responses are not evaluated in the failure region, which is expected to be unknown for large structural systems, the estimated risk may not be acceptable, and (3) the number of experimental sampling points necessary to generate a RS is an open question; the optimum number must be decided in an objective way. Since the distributional information on RVs and the coordinates of the Most Probable Failure Point (MPFP) or the failure region are iteratively estimated using FORM, the authors decided to integrate RSM and FORM. With this integration, the first two deficiencies of basic RSM can be removed very effectively. The authors decided to address the third deficiency by introducing an improved scheme for the selection of sampling points, as will be discussed in more detail later. To differentiate between a RS generated without

or with improvements, a RS generated with the above integrated approach will be denoted hereafter as the improved or integrated RS or simply IRS.

3.2.1. Generation of the IRS for Structural Reliability Evaluation

The basic concept discussed in the previous section can be implemented by addressing the following four major issues: (1) the integration of RSM with FORM, (2) the selection of the mathematical form of the IRS, (3) the selection of center points and experimental sampling schemes, and (4) an improved sampling scheme to generate the appropriate response information to fit an IRS. All four issues are discussed next.

3.2.1.1. Integration of RSM with FORM

To generate a RS following the basic RSM concept, the center point, the sampling points, and the sampling region are selected as:

$$X_i = X_i^C + hx_i\sigma_{X_i} \quad \text{where } i = 1, 2, \dots, k \quad (10)$$

where X_i is the region or bound of the i^{th} RV, X_i^C and σ_{X_i} are the coordinate of the center point and the standard deviation of the i^{th} RV, respectively, h is an arbitrary factor controlling the experimental sampling region, x_i is the coded variable which has values of 0, ± 1 or $\pm \sqrt[4]{2^k}$, depending on the coordinate of the experimental sampling points with respect to the center points, and k is the number RVs. Clearly, the formulation represented by Eq. (10) fails to incorporate the distribution information of RVs even when it is available and the selection of the center point is arbitrary in nature.

In order to include distribution information of RVs and to generate a RS in the failure region, the authors propose to integrate RSM and FORM in the unified PBSO procedure. The iterative process of FORM will capture the underlying distribution information of RVs and will locate the coordinates of the failure region iteratively. In the first iteration, the coordinates of the

center point will be assumed to be the mean values of all RVs, as is generally assumed to implement FORM. In the context of FORM, the reliability is estimated in the normal variable space. For the successful integration, all non-normal RVs need to be transformed to equivalent normal variables during the first iteration of FORM. The equivalent normal mean ($\mu_{X_i}^N$) and standard deviation ($\sigma_{X_i}^N$) can be calculated by equating the Probability Density Functions (PDFs) and Cumulative Distribution Functions (CDFs) of the original non-normal variables to those of the equivalent normal variables [14] as:

$$\mu_{X_i}^N = x_i^* - \Phi^{-1}[F_{X_i}(x_i^*)]\sigma_{X_i}^N \quad (11)$$

and

$$\sigma_{X_i}^N = \frac{\phi\{\Phi^{-1}[F_{X_i}(x_i^*)]\}}{f_{X_i}(x_i^*)} \quad (12)$$

where x_i^* is the checking point, $\Phi(\cdot)$ and $\phi(\cdot)$ are the CDF and PDF of the standard normal variable, respectively, and $F_{X_i}(x_i^*)$ and $f_{X_i}(x_i^*)$ represent the CDF and PDF of the original non-normal variables at the checking point x_i^* , respectively. Thus, the iteration process of FORM will be initiated by replacing X_i and σ_{X_i} in Eq. (10) by $\mu_{X_i}^N$ and $\sigma_{X_i}^N$, respectively. The next step will be the selection of the mathematical form of an IRS, as discussed next.

3.2.1.2 Mathematical form of an IRS

The selection of mathematical form of an IRS is the next challenge. Obviously, in the context of PBSO, it is not expected to be linear. It is reported in the literature that the selection of more than second-order polynomial for an IRS can be very problematic [28]. The authors decided to represent an IRS using a second-order polynomial without or with cross terms. The polynomials can be expressed as:

$$\hat{g}(\mathbf{X}) = b_0 + \sum_{i=1}^k b_i X_i + \sum_{i=1}^k b_{ii} X_i^2 \quad (13)$$

$$\hat{g}(\mathbf{X}) = b_0 + \sum_{i=1}^k b_i X_i + \sum_{i=1}^k b_{ii} X_i^2 + \sum_{i=1}^{k-1} \sum_{j>1}^k b_{ij} X_i X_j \quad (14)$$

where X_i ($i=1,2,\dots,k$) is the i^{th} RV, k is the total number of RVs involved in the problem, b_0 , b_i , b_{ii} and b_{ij} are the unknown coefficients to be determined, and $\hat{g}(\mathbf{X})$ is the approximate representation of the original unknown LPF $g(\mathbf{X})$. The number of coefficients required to define Eqs. (13) and (14) are $2k + 1$, and $(k + 1)(k + 2)/2$, respectively. When $k = 6$, the number of unknown coefficients required to define Eqs. (13) and (14) will be 13 and 28, respectively. However, when k is large, say 60, as expected for a realistic structure, the corresponding numbers will be 121 and 1891, respectively, indicating a significant increase in the computational time if Eq. (14) is used. Additionally, the selection of experimental sampling points to generate response information plays an important role in the efficiency of the procedure.

3.2.1.3 Selection of Center Points and Experimental Sampling Schemes

In the context of IRS, the coordinates of the initial center point will be mean values of RVs, as in FORM, and the location of the center point will be updated as the iteration continues. To generate response information in developing a required IRS, several deterministic nonlinear time domain FE analyses need to be conducted around a center point at specific points commonly known as “experimental sampling points” following a sampling scheme. The sampling scheme is also expected to influence the accuracy and efficiency of the algorithm. Two commonly used sampling schemes reported in the literature are Saturated Design (SD) and

Central Composite Design (CCD) [28]. SD is less accurate, but more efficient since it requires only as many deterministic FE analyses as the total number of unknown coefficients necessary to define the polynomial by solving a set of linear equations. Also, a second-order polynomial without and with cross terms can be used to generate an IRS using the SD scheme, requiring $2k + 1$ and $(k + 1)(k + 2)/2$ number of deterministic FE analyses, respectively. On the other hand, CCD is more accurate but less efficient since it can generate only a second-order polynomial with cross terms and a regression analysis is needed to obtain the unknown coefficients. CCD consists of one center point, two axial points located on the axis of each RV at a distance equal to $\sqrt[4]{2^k}$ measured from the center point, and 2^k factorial points. Hence, the total number of deterministic FE analyses in CCD is equal to $2^k + 2k + 1$. More comprehensive discussions on SD and CCD can be found in the literature [28, 30, 31].

In general, to implement PBSO for large realistic structural systems, the total number of RVs is expected to be large. For the sake of discussion, suppose $k = 40$. The Total Number of Sampling Points (TNSPs) or deterministic nonlinear time domain FE analyses required to define Eq. (13) using the SD scheme will be $2 * 40 + 1 = 81$ and to define Eq. (14) will be $(40 + 1)(40 + 2)/2 = 861$. If CCD is used, it will require $2^{40} + 2 * 40 + 1 = 1,099,511,627,857$ TNSPs. Obviously, CCD cannot be used for large structural systems. Considering both the efficiency and accuracy, the authors propose a two-stage approach. In the first stage, they plan to reduce the total number of RVs in the formulation. In the second stage, they plan to combine the desirable features of SD and CCD, as discussed next.

3.2.1.4 Improved Sampling Scheme

It is obvious that when k is large, the CCD scheme cannot be implemented. It is known to the profession that the propagation of uncertainties from the parameter to system level is very

non-uniform. A measure called the sensitivity index can be used to quantify the influence of each RV [14]. The sensitivity of a variable X is the directional cosines $\alpha(X)$ of the unit normal variable at the checking or design point. The gradient vector of the LPF in the standard normal variable space can be used for this purpose and will be available from the FORM analysis. RVs with low sensitivity indexes at the end of the first iteration can be treated as deterministic at their mean values in the subsequent iterations. The authors observed that the reduction of the total number of RVs significantly improves efficiency without compromising accuracy. The reduced number of RVs will be denoted as k_r in this paper. Suppose, that out of a total of 40 RVs, only 7 are sensitive, the required TNSPs will be $2^7 + 2 * 7 + 1 = 143$, a significant reduction. This will be demonstrated with the help of examples later.

In proposing an Improved Sampling Scheme (ISS), in the second stage, the authors propose to use SD without cross terms (Eq. 13) in the intermediate iterations to increase efficiency and CCD with cross terms (Eq. 14) in the final iteration to increase accuracy, as shown in Fig. 1. For example, if $k = 40$ and $k_r = 7$, the TNSPs required to implement the proposed ISS considering three iterations will be $(2 * 40 + 1) + (2 * 7 + 1) + (2^7 + 2 * 7 + 1) = 239$. Hence, the ISS captures the efficiency of SD and the accuracy of CCD in one single scheme in the proposed reliability evaluation procedure.

3.2.2. Performance Levels

The selection of performance levels and the corresponding risks according to specific LPFs are the essence of PBSO. In constructing a specific LPF, the information on generated IRS as discussed in Section 3.2.1 needs to be integrated with the performance level. FEMA 350 [1] defined three performance levels; they range from immediate occupancy to collapse prevention. The Immediate Occupancy (IO) represents the performance of a building that has sustained

minimal or no damage to its structural elements and only minor damage to its nonstructural components. The Life Safety (LS) level represents a structure with extensive damage; the risk to life is low, but repairs may be required before re-occupancy. The Collapse Prevention (CP) level is represented by a building that has suffered extensive structural damage; total collapse is avoided, but structural repairs are uneconomical. PBSD criteria are usually established on the basis of multiple target performance levels. FEMA 350 [1] proposed the performance levels for CP, LS, and IO in terms of earthquake return period, probability of exceedence, and allowable drifts. The information is summarized in Table 1. The allowable drift is a function of H ; it represents the total height of the building for the overall allowable drift at the top of the roof. For the allowable inter-story drift, H represents the story height. The information provided in Table 1 is essentially for serviceability LPFs.

At present, a structure is analyzed and designed for strength first. It is then checked for serviceability-related issues like overall lateral and inter-story stiffness to assure that it does not develop excessive lateral and inter-story deformations. From the past studies [32, 33], the research team members of the authors observed that the structural members are generally designed conservatively but the structure may not be able to satisfy the serviceability limit states, particularly when lateral load caused by the seismic excitation controls the design. Most of these structures fail by developing excessive lateral or inter-story deformation. The authors believe that the SAC project also made similar observations and suggested PBSD design criteria for serviceability-related performance functions only. This prompted the authors to study only the serviceability-related issues in this paper.

3.2.2.1. Serviceability Performance Evaluation

The most commonly used serviceability LPFs related to the overall top roof displacement and inter-story drift are generally expressed as:

$$g(\mathbf{X}) = \delta_{allow} - y_{max}(\mathbf{X}) = \delta_{allow} - \hat{g}(\mathbf{X}) \quad (15)$$

where δ_{allow} can be selected based on the performance requirements, as summarized in Table 1 and $y_{max}(\mathbf{X})$ is the appropriate IRS which can be represented as $\hat{g}(\mathbf{X})$, where $\hat{g}(\mathbf{X})$ is the explicit polynomial function constructed using IRS, as discussed in Section 3.2.1.

3.2.3. Structural Reliability Estimation

At this stage, explicit expressions for appropriate LPFs are available and FORM will be used to extract the reliability information. A required LPF will be generated using the performance requirement and the corresponding IRS. To construct an IRS, the necessary response information will be generated at the sampling points, by calculating the maximum responses caused by an earthquake time history applied in time domain using a nonlinear FE algorithm. As in FORM, the first iteration will be initiated at the mean values of all RVs in the normal variable space. The LPF of interest will be generated using a second-order polynomial without cross terms (Eq. 13) and SD sampling schemes. In the process of implementing FORM, the direction cosines (α_i) or the sensitivity index of all the RVs will be generated. As discussed earlier, RVs with relatively smaller sensitivity indexes will be considered deterministic at their mean values and a new LPF will be generated. With the reduced number of RVs and modified IRS, the LPF will be reconstructed. Following the FORM strategy, the iteration will continue until the direction cosines of all remaining RVs converged to a pre-determine tolerance level. For this study, a tolerance level of 0.005 is used. It may take few iterations [14, 15]. Then, the first estimate of the reliability index β will be made using the standard FORM procedure. The coordinates of the new checking point (x_i^*) or center point will be recalculated as:

$$x_i^* = \mu_{X_i}^N - \alpha_i \beta \sigma_{X_i}^N \quad (16)$$

The overall updating will continue until β converges satisfying a pre-determine tolerance level. In this study, the tolerance level for β is considered to be 0.001. In the context of ISS, in the last iteration, the CCD sampling scheme and a second-order polynomial with cross terms (Eq. 14) will be used to generate an IRS and the corresponding LPF. In most cases, it may take 3 to 5 iterations to complete the iterative process in determining the final β value. Once β converges, the coordinates of the last checking point $\mathbf{x}^*$ representing the MPFP can be calculated as:

$$\beta = \sqrt{(\mathbf{x}^*)^t (\mathbf{x}^*)} \quad (17)$$

The corresponding probability of failure (p_f) can be estimated as:

$$p_f = \Phi(-\beta) = 1.0 - \Phi(\beta) \quad (18)$$

A flowchart of the unified PBSO procedure is shown in Fig. 2.

4. Uncertainty in Load and Resistance-Related RVs

The success of the novel reliability technique for the implementation of PBSO will depend on how the uncertainty associated with the load and resistance-related RVs is incorporated in the formulation. In implementing PBSO, the following three issues need to be specifically addressed: (1) how to quantify the uncertainty in the seismic acceleration time histories during the life time of the structure, (2) how to quantify uncertainty in the gravity loads, and (3) how to quantify uncertainty in the resistance related parameters.

4.1. Uncertainty in the seismic acceleration time histories

The incorporation of uncertainties in the time domain seismic analysis is still evolving. Conceptually, structures need to be excited by the two horizontal and one vertical acceleration time histories to produce the most critical performance of interest. The uncertainty associated

with the intensity level and frequency contents also need to be addressed. To incorporate uncertainty in the intensity, a factor (g_e), is considered in this study. It is considered to be a RV and represented by a Type I distribution with a coefficient of variation (COV) of 0.2. It will be discussed further in the example section.

Consideration of uncertainty in the frequency content of an earthquake time history is still under development. In recent design guidelines at least seven probable time histories expected for the location under consideration are suggested [12, 13]. In the absence of other alternatives, the authors believe that this is a step in the right direction. For PBSD with multiple performance requirements, the information on the Probability of Exceedence (PE) can be related to the performance level. During the second phase of the SAC project, Somerville et al. [34] suggested three sets of ground motion time histories corresponding to 2%, 10%, and 50% PE in 50 years for the Los Angeles (LA) area and correlated them with the performance levels of CP, LS, and IO, respectively, as summarized in Tables 1 and 2. For each performance level, 10 ground motion events with two horizontal orthogonal components providing 20 time histories were suggested. They used scale factors (SFs) to match target spectral values, on an average, for the periods of 0.3, 1.0, 2.0, and 4.0 seconds for site category of S_D (firm soil), suggested by the United States Geological Survey (USGS). The information is summarized in Table 2. Thus, for each performance level of CP, LS, and IO, 20 time histories are available, as summarized in Tables 3, 4, and 5, respectively. The authors used these time histories to verify the PBSD guidelines incorporating uncertainties in the intensity and the frequency contents of earthquake time histories, satisfying the intent of FEMA 350 [1].

4.2. Uncertainty in the Gravity Loads

For the seismic analysis, the gravity loads are considered to be Dead Load (DL) and Live Load (LL), in most design guidelines [12, 13]. The uncertainties associated with them are widely reported in the literature [7, 14] and the authors used the same information in this study also. DL and LL are represented by normal and Type I distributions with COV of 0.10 and 0.25, respectively, as shown in Table 6.

4.3. Uncertainty in the resistance related parameters

The information on uncertainty associated with the resistance related parameters widely used in the profession is also used in this study [7, 14]. Young's modulus (E), yield stress of columns (F_{yc}) and girders (F_{yg}), the cross sectional area (A), and moment of inertia (I) of W-sections used for structural elements are considered to be RVs with a lognormal distribution with COV of 0.06, 0.10, 0.10, 0.05, and 0.05, respectively, as shown in Table 6.

5. Implementation and Demonstration of the Proposed Unified PBSD Procedure

At this stage, since all the elements of the proposed unified PBSD procedure are available, it is essential to demonstrate the implementation potential of the approach with the help of an illustrative example. As part of the SAC project, several steel buildings were designed by experts for further investigations [1-6]. Among many structural systems reported in FEMA-355C [5], a 9-story building designed for LA area is studied to demonstrate the proposed procedure. **The structure was designed by expert practicing engineers by satisfying all post-Northridge design criteria.** The current code requirements are expected to follow the similar guidelines. The building consists of two-dimensional steel frames made with W-sections. A typical frame with size of columns and girders and the distributed loads acting on it is shown in Fig. 3. W_{D1} and W_{D2} represent DL at the roof and floor levels, respectively. W_{L1} and W_{L2} are the LL for roof and floor levels, respectively. **The main intent of this example is to document that**

structures designed using PBSD will satisfy similar reliability requirements for different performance levels.

The statistical information on all RVs is summarized in Table 6. One can observe that the total number of RVs for both overall top roof and inter-story drift LPFs is 68. To reduce the total number of RVs, each LPF is considered separately and the corresponding LPF is constructed using IRS with the SD scheme and without the cross terms (Eq. 13). Then, using FORM, the direction cosines or the sensitivity indexes of RVs are estimated. For both LPFs, 7 RVs are found to be the most sensitive and are considered to be RVs in the subsequent analyses. All others are considered to be deterministic at their mean values.

The probability of failure, p_f , for serviceability conditions is evaluated using the proposed reliability analysis procedure using ISS. The serviceability LPFs are considered to be the overall drift at the top of the frame (LPF1) and the inter-story drift at the 4th floor level (LPF2). The allowable drifts (δ_{allow}) for LPF1 are considered to be 188.1, 94.05, and 26.33 cm for CP, LS, and IO performance levels, respectively. The corresponding allowable drifts for LPF2 are considered to be 19.8, 9.9, and 2.7 cm, respectively. The frame is then excited by 20 time histories for each of the CP, LS, and IO performance level. The reliability indexes β for two LPFs are then calculated and reported in Tables 3-5. The mean reliability indexes ($\bar{\beta}$) and the corresponding probabilities of failure, p_f , are summarized in Table 7.

To establish the accuracy of the proposed algorithm, the information on the probability of failure is extracted by using the classical MCS. Obviously, it will take an enormous amount of time to estimate p_f considering all time histories, performance levels, and LPFs. For this reason, the probabilities of failure for the two LPFs when excited by only the earthquake time history no.

36 (Table 4) are given in this paper. The TNSPs or total number of deterministic evaluations required to generate them using the ISS and using 600,000 MCSs are summarized in Table 8.

Several important observations can be made from the results obtained from the study. The results given in Table 8 clearly indicate the probabilities of failure estimated using the proposed algorithm are very similar to that of MCS indicating that it is accurate. However, the information on p_f is extracted using only 310 TNSPs; not 600,000 simulations of MCS. Obviously, the proposed method not only is accurate but also efficient. The p_f for the two serviceability LPFs are very similar and of the order of 10^{-6} . The results are very exciting for several reasons. Since the results for two LPFs are very similar, they satisfy the intent of PBSD, i.e., p_f should be similar for different LPFs. The authors also observed that the p_f values are similar to the values used in developing the LRFD requirements used in many currently used design guidelines. The reliability indexes versus the earthquake time histories are plotted in Figs. 4 and 5. The β values obtained for the two components of the same earthquake are different. It clearly indicates that designing a structure for one design earthquake time history is inadequate. Designing a structure using multiple time histories, as suggested in recent design guidelines [12, 13], is a step in the right direction. The mean values of β for CP, LS, and IO performance levels for LPF1 are 7.06, 6.96, and 6.57, respectively and the corresponding values for LPF2 are 6.22, 6.32, and 6.05. From the above results, the authors believe that the allowable values suggested in the design guidelines [1] are reasonable. The β values fall within a range, satisfying the intent of the code. It is also interesting to note the distinct separations of performance levels of CP, LS, and IO in Figs. 4 and 5. It indicates that the SFs to match target spectral values at certain frequencies by Somerville et al. [34] are reasonable. The same procedure can be used for the

PBSD for the time being. The authors are in the process of developing another alternative. It will be reported in the near future.

From the results and the observations made in this study, the authors believe that they proposed a robust, efficient, and accurate reliability analysis procedure for nonlinear structures represented by finite elements and excited by the seismic loading in time domain. Using the procedure, they developed a unified PBSd procedure, successfully implemented it, and demonstrated how to use it with the help of an illustrative example.

6. Conclusions

Issues related to developing and implementing PBSd are presented in this paper. Since a required performance level cannot be assured for a specific design, it must be assessed in terms of the associated risk or reliability. In the implementation of PBSd, the required performances need to be assessed as accurately as possible similar to the procedures followed by the deterministic community. Some of the major building blocks of the proposed procedure are: (1) structures are represented by finite elements and excited by earthquake time histories applied in time domain to produce the critical performance level of interest, (2) since the required limit or performance functions are implicit for the class of problems under consideration, they are formulated using the response surface concept but removing its three major deficiencies, (3) the required performance information needed to explicitly generate a response surface, saturated design and central composite design sampling schemes are used and their various alternatives are proposed, (4) FORM is used to extract reliability information, and (5) several efficiency improvements schemes are proposed. It is capable of generating a response surface with hundreds of deterministic nonlinear finite element analyses. The proposed method is verified by estimating risk of a 9-story steel building designed by expert professional engineers satisfying

the post-Northridge design requirements. It was excited by 20 earthquake time histories for each performance level of CP, LS, and IO, and the corresponding reliability indexes were estimated using the proposed method. It took around 300 deterministic evaluations using the improved sampling scheme proposed in the paper. The accuracy of the method was established using 600,000 cycles of simulations. Several important observations can be made from this study including: (1) the probabilities of failure estimated using the proposed algorithm are very similar to that of MCS indicating that it is accurate, (2) the p_f for the two serviceability LPFs are very similar; of the order of 10^{-6} , (3) since the results for two LPFs are very similar, they satisfy the intent of PBSD, i.e., they should be similar for different LPFs, and (4) results are also similar to the values used in developing the LRFD guidelines used in many current design codes. The study confirms that designing structures using multiple time histories, as suggested in recent design guidelines, is a step in the right direction. The authors believe that the study documented how to develop and implement the PBSD guidelines with the help of an informative example.

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Table 1. Performance level, earthquake return period, probability of exceedence, and allowable drift relations.

Performance Level	Earthquake Return Period	Probability of Exceedence	Allowable Drift (δ_{allow})
CP	2475-year	2% in 50 years	$0.050 * H$
LS	475-year	10% in 50 years	$0.025 * H$
IO	72-year	50% in 50 years	$0.007 * H$

Table 2. Target response spectra values in terms of gravitational acceleration (g).

Probability of exceedence	Period (sec.)			
	0.3	1.0	2.0	4.0
2% in 50 years (CP)	1.610	1.190	0.540	0.190
10% in 50 years (LS)	1.070	0.680	0.330	0.123
50% in 50 years (IO)	0.514	0.288	0.149	0.069

Table 3. Set 1: 2475-year return period time histories for CP performance level.

EQ	Record Name	SF	PGA (g)	Excitation Time (sec)	LPF1		LPF2	
					β	TNSP	β	TNSP
1	1995 Kobe	1.15	1.282	25.0	5.24	295	4.55	295
2	1995 Kobe	1.15	0.920	25.0	6.39	295	5.74	295
3	1989 Loma Prieta	0.82	0.418	20.0	9.58	295	8.92	280
4	1989 Loma Prieta	0.82	0.473	20.0	6.88	295	6.58	295
5	1994 Northridge	1.29	0.868	14.0	6.79	310	6.44	280
6	1994 Northridge	1.29	0.943	14.0	5.44	295	4.59	295
7	1994 Northridge	1.61	0.926	15.0	7.20	310	8.60	310
8	1994 Northridge	1.61	1.329	15.0	5.76	295	5.06	295
9	1974 Tabas	1.08	0.808	25.0	10.95	295	9.91	295
10	1974 Tabas	1.08	0.991	25.0	8.35	310	7.62	295
11	Elysian Park (simulated)	1.43	1.295	18.0	6.28	280	7.22	280
12	Elysian Park (simulated)	1.43	1.186	18.0	6.13	295	4.33	295
13	Elysian Park (simulated)	0.97	0.782	18.0	7.78	295	7.16	280
14	Elysian Park (simulated)	0.97	0.680	18.0	6.53	310	5.99	310
15	Elysian Park (simulated)	1.1	0.991	18.0	8.97	310	4.30	310
16	Elysian Park (simulated)	1.1	1.100	18.0	5.02	310	4.09	310
17	Palos Verdes (simulated)	0.9	0.711	25.0	6.89	310	6.41	310
18	Palos Verdes (simulated)	0.9	0.776	25.0	7.34	295	4.29	295
19	Palos Verdes (simulated)	0.88	0.500	25.0	7.59	295	6.96	310
20	Palos Verdes (simulated)	0.88	0.625	25.0	6.16	310	5.66	310

Table 4. Set 2: 475-year return period time histories for LS performance level.

EQ	Record Name	SF	PGA (g)	Excitation Time (sec)	LPF1		LPF2	
					β	TNSP	β	TNSP
21	Imperial Valley, 1940	2.01	0.461	25.0	7.11	295	6.42	295
22	Imperial Valley, 1940	2.01	0.675	25.0	7.67	310	7.06	295
23	Imperial Valley, 1979	1.01	0.393	15.0	6.98	295	6.38	280
24	Imperial Valley, 1979	1.01	0.488	15.0	10.51	310	9.84	280
25	Imperial Valley, 1979	0.84	0.301	15.0	7.94	295	7.36	310
26	Imperial Valley, 1979	0.84	0.234	15.0	10.94	310	10.64	310
27	Landers, 1992	3.2	0.421	30.0	7.67	310	7.03	295
28	Landers, 1992	3.2	0.425	30.0	7.46	295	6.98	295
29	Landers, 1992	2.17	0.519	30.0	4.44	310	4.00	295
30	Landers, 1992	2.17	0.360	30.0	6.14	310	5.53	310
31	Loma Prieta, 1989	1.79	0.665	16.0	6.97	310	6.23	295
32	Loma Prieta, 1989	1.79	0.969	16.0	9.55	295	8.87	310
33	Northridge, 1994, Newhall	1.03	0.678	15.0	5.90	295	5.17	295
34	Northridge, 1994, Newhall	1.03	0.657	15.0	5.03	280	4.13	280
35	Northridge, 1994, Rinaldi	0.79	0.533	14.0	5.61	295	4.45	295
36	Northridge, 1994, Rinaldi	0.79	0.579	14.0	4.34	310	3.80	310
37	Northridge, 1994, Sylmar	0.99	0.569	15.0	7.84	295	7.39	310
38	Northridge, 1994, Sylmar	0.99	0.817	15.0	4.97	295	3.96	280
39	North Palm Springs, 1986	2.97	1.018	16.0	7.57	310	7.12	310
40	North Palm Springs, 1986	2.97	0.986	16.0	4.66	295	3.99	280

Table 5. Set 3: 72-year return period time histories for IO performance level.

EQ	Record Name	SF	PGA (g)	Excitation Time (sec)	LPF1		LPF2	
					β	TNSP	β	TNSP
41	Coyote Lake, 1979	2.28	0.589	12.0	6.68	310	5.96	280
42	Coyote Lake, 1979	2.28	0.333	12.0	5.88	295	5.43	280
43	Imperial Valley, 1979	0.4	0.143	15.0	6.56	295	6.08	295
44	Imperial Valley, 1979	0.4	0.112	15.0	9.03	310	8.60	295
45	Kern, 1952	2.92	0.144	30.0	6.99	295	6.38	295
46	Kern, 1952	2.92	0.159	30.0	6.95	295	6.34	295
47	Landers, 1992	2.63	0.337	25.0	6.28	295	5.92	295
48	Landers, 1992	2.63	0.307	25.0	9.77	310	8.57	310
49	Morgan Hill, 1984	2.35	0.318	20.0	5.90	295	5.45	310
50	Morgan Hill, 1984	2.35	0.546	20.0	6.25	295	5.75	280
51	Parkfield, 1966, Cholame	1.81	0.780	15.0	5.52	295	5.09	280
52	Parkfield, 1966, Cholame	1.81	0.631	15.0	7.63	310	7.11	310
53	Parkfield, 1966, Cholame	2.92	0.693	15.0	5.79	280	5.16	295
54	Parkfield, 1966, Cholame	2.92	0.790	15.0	6.14	295	5.95	295
55	North Palm Springs, 1986	2.75	0.517	20.0	4.57	295	3.96	280
56	North Palm Springs, 1986	2.75	0.379	20.0	4.49	295	4.00	295
57	San Fernando, 1971	1.3	0.253	20.0	7.32	295	7.04	310
58	San Fernando, 1971	1.3	0.231	20.0	6.07	295	5.65	310
59	Whittier, 1987	1.27	0.269	15.0	6.69	310	6.16	295
60	Whittier, 1987	1.27	0.167	15.0	6.90	310	6.34	295

Table 6. Random variables utilized for the serviceability risk evaluation.

Random Variable (RV)	Number of RVs	Distribution	Mean Value	COV
E (kN/m ²)	1	Lognormal	1.9994X10 ⁸	0.06
F _{yc} * (kN/m ²)	1	Lognormal	3.4474X10 ⁵	0.10
F _{yg} ** (kN/m ²)	1	Lognormal	2.4822X10 ⁵	0.10
A (m ²)	30	Lognormal	***	0.05
I _x (m ⁴)	30	Lognormal	***	0.05
W _{D1} (kN/m)	1	Normal	32.9457	0.10
W _{D2} (kN/m)	1	Normal	32.9457	0.10
W _{L1} (kN/m)	1	Type I	2.9188	0.25
W _{L2} (kN/m)	1	Type I	2.9188	0.25
g _e	1	Type I	1.00	0.20

*Yield stress of column sections (SAC/FEMA value).

**Yield stress of girder sections (SAC/FEMA value).

***Since A and I_x are taken as random for each section, the mean values are not reported. However, they can be completely found in steel construction manuals. The w-sections utilized for the analysis are illustrated in Figure 3.

Table 7. Mean values of reliability index $\bar{\beta}$ for each set of ground motions.

ISS	Set 1 (CP): 2475-Year Return Period		Set 2 (LS): 475-Year Return Period		Set 3 (IO): 72-Year Return Period	
	LPF1	LPF2	LPF1	LPF2	LPF1	LPF2
$\bar{\beta}$	7.06	6.22	6.96	6.32	6.57	6.05
p_f	8.326×10^{-13}	2.486×10^{-10}	1.701×10^{-12}	1.308×10^{-10}	2.516×10^{-11}	7.242×10^{-10}

Table 8. Accuracy verifications: serviceability evaluations using MCS and ISS for EQ 36.

Scheme	MCS		ISS	
	LPF1	LPF2	LPF1	LPF2
β	4.42	3.83	4.34	3.80
p_f	4.935×10^{-06}	6.407×10^{-05}	7.124×10^{-06}	7.235×10^{-05}
TNSP	600,000	600,000	310	310

● A center point ⊙ $2k$ axial points ○ 2^k factorial points

Figure 1. Improved sampling scheme for three random variables ($k=3$): X_1 , X_2 , and X_3

Figure 2. Flowchart of the unified PBSO procedure.

Figure 3. 9-story building (E.C. =Exterior Columns; I.C. =Interior Columns).

Figure 4. Overall roof drift reliability evaluation (LPF1).

Figure 5. Inter-story (4th floor) drift reliability evaluation (LPF2).