Flutter characteristics of twin-box girder bridges with vertical central stabilizers

Yongxin Yang^a; Rui Zhou^a; Yaojun Ge^a; Xiaojie Zou^a; Lihai Zhang^b

^a State Key Lab for Disaster Reduction in Civil Engineering, Tongji University, Shanghai 200092, China

^b Department of Infrastructure Engineering, University of Melbourne, VIC 3010, Australia

Abstract:

It has been well known that Vertical Central Stabilizers (VCS) have the potential of improving flutter performance of long-span bridges. However, the fundamental flutter mechanisms of VCS are still not fully understood so far. In this study, a series of wind-tunnel tests involving the combination of six representative heights and four types of VCS were conducted to fundamentally investigate the influence of VCS on flutter performance of twin-box girders with various Slot Width Ratios (SWRs). Experimental results show that the flutter instability of 20% SWR is significantly sensitive to the height change of VCS, whereas the VCS have little effect on the flutter performance for 80% and 100% SWR. In addition, the results from Two-Dimensional Three Degree of freedom (2D-3DOF) flutter analysis demonstrates that aerodynamic damping Part A with reference of flutter derivative A_2^* makes the greatest contribution to the flutter instability for a 0.8 h/H VCS, while the role of Part D with reference of $A_1^* H_3^*$ becomes critical for a short VCS (i.e. the ratio of h/H is less than 0.2). Besides, the results of Computational Fluid Dynamics simulation indicate that the geometry of VCS influence the transforming vortices' structures and pressure distribution under the central slotting. Finally, the modified Selberg formula presented in this study has the capability of predicting the critical flutter speed of twin-box girders with various SWRs and VCS.

Keywords: Twin-box girder; vertical central stabilizer; slot width ratio; flutter instability; aerodynamic damping; flow structures; Selberg formula.

1. Introduction

Twin-box girders with a center gap between two girders has been proven to be one of the effective aerodynamic countermeasures for improving the aerodynamic performance of

long-span cable-supported bridges, and so its implementation becomes increasingly popular for long-span, or even super long-span bridges [1,2,3], *e.g.*, Xihoumen Bridge with the main span of 1650 m (China) and Gwangyang Bridge with the main span of 1545 m (Korea). Nevertheless, twin-box girders with various Slot Width Ratios (SWRs, for example, the 20% SWR refers to $D/B_s = 0.2$, where D is slot width and B_s is the width of two decks) may exhibit different flutter performance [4,5]. Although the maximum growth rate of Critical Flutter Wind Speed (U_{cr}) could reach more than 20% after slotting for twin-box girders, the aerodynamic stability of the bridges becomes uncertain when the length of the growing span of the bridge is over certain limit [4,6]. Therefore, the implementation of practicable aerodynamic countermeasures (*e.g.* Vertical Central Stabilizers (VCS)) becomes necessary for further enhancing the flutter performance of super long-span bridges.

VCS could play an important role in the airflow separation and have been applied in many long-span bridges with different geometrical shapes to improve flutter performance. For example, the Akashi Kaikyo Bridge (the length of the longest span is 1991 m) with a 2.15m height VCS installed in the centerline of the truss-type stiffening girder [7] and the Runyang Yangtze River Bridge (the length of the longest span is 1490 m, China) with a 0.65m height VCS on the top of the closed box girder [8]. Based on the numerical simulation, VCS have been verified to be an effective aerodynamic measure to improve the flutter performance of long-span bridges either with an open cross section or with a streamlined box cross section [9]. The results from wind tunnel tests also confirm that both the VCS on the top of two-isolated-girder section and box girders with cantilevered slabs could increase the U_{cr} by approximately 11% [10]. Moreover, the flutter performance of twin-box girders could be significantly improved when installing a reasonable scheme of VCS [11,12,13].

Over the last decades, a lot of research work has been carried out to investigate the control effectiveness of VCS on the flutter performance of long-span bridges with box girders. It has been found that the flutter control effect of VCS is closely related to the height and location of VCS. The flutter performance of box girders will decrease if the height of VCS exceeds a critical limit [10]. The installation of central stabilizer on the top

of a box girder (Type A) appears to be one of the best ways of stabilizing box girders aerodynamically, while the optimal height of VCS depends on the types of VCS (three types of stabilizers were studied in present study: Type A-only one VCS installed on the top of deck; Type B - only one VCS installed on the bottom of deck; Type AB – one VCS installed on the top of deck, while another one installed below the bottom of deck) [8]. Flutter performance of a multi-box girder section gradually improves with the increase of the height of VCS on the top of girder section, whereas the U_{cr} decreases when the height of VCS is greater than $0.5h/H$ [14]. As for the twin-box girders, Type C has the best flutter-controlling effect for a twin-box girder among three basic types of Type A, B and C (i.e. Type C – installing VCS under the central slotting), and the combination schemes of VCS (Type A+B and Type C+B) are more effective with the about 15% growth rate of U_{cr} [10]. However, the control effect of VCS with various positions and configurations flutter performance for twin-box girder bridges with various SWR is still not well understood. Therefore, further research work on identifying the optimal height and location of VCS for twin-box girder bridges with various SWR is required.

To further understand the role of VCS in the improvement of flutter performance, the present study mainly focuses on the flutter instability taking into consideration flutter derivatives and the coupling effects of heaving and torsional degree, as well as the effect of flow structures around girders. Matsumoto et al. [15] proposed a new approach in studying the flutter instability of flutter branches. Their results suggested that the controlling flutter derivatives is a way of controlling the flutter mechanism. Chen, et al. [16] found that the aerodynamic damping with the reference of flutter derivative A_2^* is the most important stabilizing term among five flutter derivatives, and the use of VCS could produce a higher level of heaving degree participation and greater critical wind speed for a truss- girder section. In addition, many studies have successfully employed the Computational Fluid Dynamics (CFD) technique for simulating the flow structures around oscillating bridge sections with the aim of exploring the flutter mechanism and predicting the critical flutter wind speed [17-23]. CFD simulation results based on Random Vortex Method also showed that the U_{cr} could be increased by using VCS since the strength of the large vortices' structure become weakened and its rhythmic motion is destroyed [9].CFD

simulation results showed that the VCS control the stream around the girder as well as the vortex generation [16]. However, the relevant studies on the flutter mechanism of twin-box girders with VCS are limited and need to be further investigated. In the present study, we aim to investigate the flutter characteristics of twin-box girders with various schemes of SWRs and VCS. Firstly, wind tunnel tests were firstly conducted to demonstrate the influence of bridge geometry in the flutter performance of twin-box girders with six representative height and four types of VCS. The measured U_{cr} corresponding to these schemes of aerodynamic countermeasures, such as different combination of five SWRs and VCS, were analyzed and compared, respectively. Secondly, a Two-Dimensional Three Degree of Freedom (2D-3DOF) flutter analysis method has been adopted to quantify the flutter mechanism with regard to aerodynamic damping and DOF participation level of three types of VCS. Accordingly, the velocity field and pressure distributions from CFD simulations were used to further understand the aerodynamic behavior of twin-box girder bridges with various VCS. In addition, both the Lorentz function and Sine function were employed to estimate the correction coefficient of the modified Selberg formula, which is of importance for estimating the U_{cr} of twin-box girders with various SWRs and VCS.

2. Flutter performance of twin-box girders with VCS

2.1 Experimental set-up

The measurement of the U_{cr} of twin-box girders with different combination schemes of SWR and VCS is the purpose of this experimental investigation. As shown in Fig. 1(a) and (b), a series of 1:80 simplified section models of twin-box girders without consideration of the deck secondary structures were performed in the TJ-1 Wind Tunnel of Tongji University to accommodate the size of sectional models and testing section. Since the SWR plays an important role in the aerodynamic performance of twin-box girder bridges [4,5], spring-supported rigid sectional models with five representative cases of SWR (i.e. $D/B_s = 0.2, 0.4, 0.6, 0.8$, and 1.0) were used in the testing to assess the control effectiveness of VCS on the flutter instability in twin-box girders bridges. In addition, to systematically investigate the influence of different heights of VCS on the flutter instability of the bridges, six representative relative height (h/H) of VCS (i.e. $h/H = 0, 0.2, 0.4, 0.6$,

0.8 and 1.0) were selected for Types A, B, AC and BC (Fig. 1(c) and Table 2). The values of structural parameters used in the testing, such as geometric dimensions, mass characteristics, fundamental frequencies and structural damping, are given in Table 1. It should be mentioned that, the adjustable additional mass on the section models were adopted to make the equivalent mass characteristics consistent in all testing cases.

Fig. 1. Flutter tests of twin-box girders with central stabilizers: (a) Sectional model diagram (unit: m); (b) Setup of springs-supported sectional models; and (c) four types of VCS

Table 1

Parameters of sectional model of twin-box girders

Properties		Slot ratio					
		0%	20%	40%	60%	80%	100%
Geometric scale	Length L/m	1.7	1.7	1.7	1.7	1.7	1.7
	Width B/m	0.338	0.406	0.473	0.541	0.608	0.676
	Height H/m	0.042	0.042	0.042	0.042	0.042	0.042
Equivalent mass	Mass/unit length m /kg/m	4.406	4.406	4.406	4.406	4.406	4.406
	Mass moment of inertia/unit length I_m / kg·m ² /m	0.244	0.244	0.244	0.244	0.244	0.244
Frequency	Vertical f_h /Hz	1.343	1.343	1.343	1.343	1.343	1.343
	Torsional f_t /Hz	2.660	2.660	2.660	2.660	2.660	2.660

Damping ratio	Vertical $\zeta_h/\%$	5	5	5	5	5	5
	Torsional $\zeta_t/\%$	5	5	5	5	5	5

Table 2 shows a total of 240 test cases under three wind attack angles (i.e. $+3^\circ$, 0° and -3°), in which Type A and Type B are two basic types which are the first focus of this study. The smaller SWR (i.e. 20% and 40%) are common ratios in the existing twin-box girders bridges and their flutter performance are better than those with larger SWR (i.e. 80% and 100%) [4]. Therefore, two combination types of Type AC (both on the top and under the central slotting) and Type BC (both below the bottom and under the central slotting) are mainly focus on two small SWR (i.e. 20% and 40%). In this study, a naming convention was adopted for all testing cases, for example S02_A02 denotes case Type A with 20% SRW and 0.2 h/H of VCS.

Table 2

Testing cases of flutter performance of twin-box girders with various SWR and VCS

Name	SRW	Location of VCS	Relative height of VCS (h/H)	Height of VCS h (m)	Slot width D (m)	Total width B (m)	Wind attack angle ($^\circ$)
S02_A00	20%	Type A, Type B, Type AC	0	0	0	0.338	$0, \pm 3$
S02_A02			0.2	0.0084	0.068	0.406	$0, \pm 3$
S02_A04			0.4	0.0168	0.135	0.473	$0, \pm 3$
S02_A06			0.6	0.0252	0.203	0.541	$0, \pm 3$
S02_A08			0.8	0.0336	0.270	0.608	$0, \pm 3$
S02_A10			1.0	0.0420	0.338	0.676	$0, \pm 3$
S04_A00	40%	Type A, Type B, Type AC	0	0	0	0.338	$0, \pm 3$
S04_A02			0.2	0.0084	0.068	0.406	$0, \pm 3$
S04_A04			0.4	0.0168	0.135	0.473	$0, \pm 3$
S04_A06			0.6	0.0252	0.203	0.541	$0, \pm 3$
S04_A08			0.8	0.0336	0.270	0.608	$0, \pm 3$
S04_A10			1.0	0.0420	0.338	0.676	$0, \pm 3$
S06_A00	60%	Type A, Type B	0	0	0	0.338	$0, \pm 3$
S06_A02			0.2	0.0084	0.068	0.406	$0, \pm 3$
S06_A04			0.4	0.0168	0.135	0.473	$0, \pm 3$
S06_A06			0.6	0.0252	0.203	0.541	$0, \pm 3$
S06_A08			0.8	0.0336	0.270	0.608	$0, \pm 3$
S06_A10			1.0	0.0420	0.338	0.676	$0, \pm 3$
S08_A00	80%	Type A, Type B	0	0	0	0.338	$0, \pm 3$
S08_A02			0.2	0.0084	0.068	0.406	$0, \pm 3$
S08_A04			0.4	0.0168	0.135	0.473	$0, \pm 3$
S08_A06			0.6	0.0252	0.203	0.541	$0, \pm 3$
S08_A08			0.8	0.0336	0.270	0.608	$0, \pm 3$
S08_A10			1.0	0.0420	0.338	0.676	$0, \pm 3$
S10_A00	100%	Type A,	0	0	0	0.338	$0, \pm 3$
S10_A02			0.2	0.0084	0.068	0.406	$0, \pm 3$
S10_A04			0.4	0.0168	0.135	0.473	$0, \pm 3$

S10_A06	Type B	0.6	0.0252	0.203	0.541	0,±3
S10_A08		0.8	0.0336	0.270	0.608	0,±3
S10_A10		1.0	0.0420	0.338	0.676	0,±3
S02_BC00	20% Type BC	0	0	0	0.338	0,±3
S02_BC02		0.2	0.0084	0.068	0.406	0,±3
S02_BC04		0.4	0.0168	0.135	0.473	0,±3
S02_BC06		0.6	0.0252	0.203	0.541	0,±3
S04_BC00	40% Type BC	0	0	0	0.338	0,±3
S04_BC02		0.2	0.0084	0.068	0.406	0,±3
S04_BC04		0.4	0.0168	0.135	0.473	0,±3
S04_BC06		0.6	0.0252	0.203	0.541	0,±3

2.2 Effects of VCS on critical flutter speed

The objective of this testing phase was to study the influence of different schemes of VCS on the minimum U_{cr} of twin-box girders with various SWR, since the dominant factor of flutter instability is the minimum value of three U_{cr} corresponding to the attack angles of $+3^\circ$, 0° and -3° , respectively. The lowest U_{cr} under three wind attack angles and their corresponding growth rates (β) for twin-box girders with six SWR (including the 0% slot ratio) and six h/H of VCS are illustrated in Fig. 2. It is noted that the growth rates β of U_{cr} are calculated by using the equation: $\beta = (U_{cr} - U_{cr0}) / U_{cr0} \times 100\%$, where U_{cr0} is the value of U_{cr} without consideration of VCS. It should be noted that the wind attack angle of $+3^\circ$ results in the lowest U_{cr} , and thus represents the most unfavorable condition under wind loading.

Fig. 2. U_{cr} and β of different SWR and VCS: (a) U_{cr} of Type A, (b) β of Type A, (c) U_{cr} of Type B, (d) β of Type B, (e) U_{cr} of Type AC and BC, (f) β of Type AC and BC

2.2.1 Height of central stabilizers

For Type A and Type B, U_{cr} and their β of twin-box girders with 0% SWR were obtained from the study of Ge et al. 2009, which are defined as two cases (i.e. s00_A and s00_B in Fig.2), respectively. It is interesting that the 0% SWR of Type B has a rapidly decline along with the increase of the h/H of VCS, and thus the Type B has disadvantage contribution to the flutter instability of a twin-box girder with 0% SWR.

It can be seen from Fig. 2 (a) to (d) that the stabilizing effectiveness of twin-box girders with a certain Type of VCS firstly increases with the increase of h/H, reaches to an optimal value and then gradually decreases. Noteworthy, the optimal value of h/H for different types of VCS is different, e.g. about 0.5 for Type A and 0.2 for Type B. In addition, the growth rate β of Type A are negative when the ratio of h/H is over 0.6, and the β for Type B are also negative when the ratio of h/H is greater than 0.4. As shown in the Fig. 2 (e) and (f), for Type AC, the U_{cr} and their β firstly increase with the increase of

the h/H of VCS. In addition, for Type AC with 20% SWR, the value of β could reach as high as 42% for both 0.4 and 0.6 h/H ratios. The relationship between β and h/H demonstrates that the optimal h/H for Type BC is around 0.4. As a result, the best flutter performance for four types VCS is the case of s02_A06, s04B_04, s02AC_04, s04BC_04, respectively. In general, the optimal height of VCS is around 0.4 h/H , which could effectively improve the flutter performance of a twin-box girder bridge.

2.2.2 Influence of slot width ratios

As for the influence of SWR on the flutter instability, both the U_{cr} and β resulted from 20% SWR are significantly sensitive to the heights change of VCS among six SWR. As shown in Fig. 2 (b), (d) and (f), the 20% SWR leads to the largest positive growth rate for relatively small h/H , such as the 25.8% of the 0.6 h/H Type A and 16% of the 0.2 h/H Type B. The largest growth rates of Type A and B with the 1.0 h/H are about -30% when the SWR is 20%. In addition, the U_{cr} of 60% slot ratio rapidly decreases after the ratio of h/H of Type A is over 0.6. Particularly, Types A and B have limited influence on the flutter performance when the SWR is 80% or 100%, regardless of the relative height of VCS.

For Type AC, the values of β for the 20% SWR are positive and larger than that of the 40% SWR. This indicates that Type AC is the best choice for the 20% SWR, especially when h/H ratio of VCS is 0.4 or 0.6. However, for Type BC, the values of U_{cr} of the 40% SWR are much larger than those with the 20% SWR. This indicates that Type BC is the best choice for the 40% SWR. In a word, the control effects of twin-box girders with VCS are closely related to the SWR, but the influence of SWR on flutter instability of twin-box girders gradually decreases with the increase of SWR.

2.2.3 Types of central stabilizers

The flutter control effectiveness of twin-box girders with four types of VCS are different from each other. Fig. 3 (a) and (b) show the growth rate β of the 20% and 40% SWR with four types of VCS, respectively.

Regarding to the 20% SWR, the control effects of Types AC and A are better than those of Types B and BC for higher ratios of h/H . Especially, Type AC can significantly enhance the U_{cr} when the h/H is larger than 0.2. However, the β of Types B and BC with the 0.2 h/H are larger than those of Types AC and A, respectively. Consequently, a smaller

h/H (i.e. 0.2) of VCS installed at the bottom of central slotting could improve the flutter instability, and a high h/H (i.e. 0.4 to 1.0) of VCS installed at the top of slotting has positive effects. On the other hand, the β values of Type AC are generally larger than that of Type A, while the β values of Type BC are larger than that of Type B. This indicates that the control effects of Types AC and BC are better than that of Types A and B. In summary, the Type AC produces the best flutter performance for a twin-box girder with 20% SWR, followed by Type A, while Type B results in the worst outcomes.

As for the 40% SWR, the β value of Type BC is the largest among the four types when the ratios of $h/H < 0.6$, while the β value of Type AC becomes the largest when the ratios of $h/H > 0.6$. Moreover, the control effect of Type BC is also better than that of Type B, while the effect of Type AC is better than that of Type A, except for the 0.2 h/H of Type AC. Thus, two combination types (i.e. Type AC and Type BC) produces better outcomes than two basic types (i.e. Type A and Type B). Therefore, Type AC with a higher ratio of h/H is a recommended flutter countermeasure for twin-box girders bridges.

Fig. 3. β of different types of VCS: (a) 20% slot ratio, and (b) 40% slot ratio

3. Theoretical analysis of stabilizing mechanism

A two-dimensional three-degree-of-freedom (2D-3DOF) flutter analysis method was developed to quantify the effects of flutter derivatives, aerodynamic damping, and flutter modality on flutter performance [4, 8]. In this study, 2D-3DOF was applied to theoretically analyze the stabilization mechanism of twin box girders under various VCS. Three typical types, including the 20% SWR for Types A and B and the 40% SWR for Type AC, were

given as the example, and the stabilization mechanism of the three types with different h/H ratios were mainly investigated in this study.

3.1 Total damping ratios and Flutter derivatives

Regarding to heaving-torsional coupled flutter, the most important aerodynamic damping component is torsional damping ratio. The total damping ratios and typical eight flutter derivatives of twin-box girders using different VCS were calculated under the unfavorable wind attack angle of $+3^\circ$ [24-27]. The total torsional damping ratios (ξ_t) and A_2^* are compared in Fig. 4.

It can be seen that all of total torsional damping ratios initially increase with the increase of wind speed, peak values, then rapidly decrease, and finally shift from the positive to negative territory on the flutter onset. Especially, the values of ξ_t of the 20% SWR with 0.4 and 0.6 h/H (Type A) are smaller than those without VCS when the wind speed (U) is less than 14 m/s. However, when U is greater than 14 m/s, the values of ξ_t of these two h/H ratios (i.e. 0.4 and 0.6) are larger than those without VCS. The ξ_t of the 20% SWR with 0.2 h/H (Type B) are larger than those without VCS, when $U > 12$ m/s. The variation curves of the ξ_t of the 40% SWR with 0.4 h/H (Type AC) constantly increase to a peak value at $U=17$ m/s which exceeds the value without VCS, and then becomes negative at a higher wind speed.

The value of A_2^* generally decreases with the increase of wind speed. After installing a relatively short VCS (i.e. 0.2 to 0.6 h/H), the absolute value of A_2^* becomes smaller than those without VCS. Different from the trend of A_2^* for small h/H ratios, all the absolute values of A_2^* for the Type A with 0.8 h/H are larger than those without VCS.

Fig. 4. Total torsional damping ratios and flutter derivatives of three types VCS: (a) ξ_t of Type A, (b) ξ_t of Type B, (c) ξ_t of Type AC, (d) A_2^* of Type A, (e) A_2^* of Type B, and (f) A_2^* of Type AC

3.2 Aerodynamic damping ratios

For two-degree-of-freedom (2-DOF) vibration, aerodynamic damping ratios in torsion can be represented by the summation of five parts in Eq. 1 [4, 8], where Ω_{ij} is the dimensionless equivalent frequency between two motions; θ_{ij} is phase lag between two motions; m_h and I are mass and mass moment of inertia in the corresponding DOF including heaving (h), swaying (p) and torsion (α). It should be noted that the swaying DOF are not considered in the 2-DOF vibration. Fig. 5 shows the change curves of these five parts (labeled A to E) for twin-box girders with different h/H ratios of VCS.

$$\begin{aligned}
 A &\Leftrightarrow -1/2 \cdot \rho B^4 / I \cdot A_2^* \\
 B &\Leftrightarrow -\rho^2 B^6 / 2m_h I \cdot \Omega_{ha} \cdot A_1^* H_2^* \cos(\theta_{ah} + \frac{3}{2}\pi) \\
 C &\Leftrightarrow \rho^2 B^6 / 2m_h I \cdot \Omega_{ha} \cdot A_4^* H_2^* \sin(\theta_{ah} + \frac{3}{2}\pi) \\
 D &\Leftrightarrow -\rho^2 B^6 / 2m_h I \cdot \Omega_{ha} \cdot A_1^* H_3^* \cos \theta_{ah} \\
 E &\Leftrightarrow \rho^2 B^6 / 2m_h I \cdot \Omega_{ha} \cdot A_4^* H_3^* \sin \theta_{ah}
 \end{aligned} \tag{1}$$

Fig. 5. Aerodynamic damping parts of VCS: (a) 0 h/H Type A, (b) 0.4h/H Type A, (c) 0.8h/H Type A, (d) 0 h/H Type B, (e) 0.2 h/H Type B, (f) 0.6 h/H Type B, (g) 0 h/H Type AC, (h) 0.2 h/H Type AC, and (i) 0.4 h/H Type AC

It can be seen from Fig. 5 that Part D (with a reference of coupling flutter derivatives $A_1^* H_3^*$) which has a negative value, monotonically decrease with the increase of U , and so has the greatest negative effect on aerodynamic stability among five parts, especially for the h/H ratios 0.2 and 0.4. Consequently, the negative aerodynamic damping resulting from Part D mainly drives the overall oscillating structure into flutter onset condition when installing smaller h/H of VCS. In contrast, Part B, Part C and Part E also have little influence on aerodynamic stability for smaller h/H of VCS. Part A (with a reference of flutter derivatives A_2^*) which is a positive component, generally increases with the increase of U , and therefore makes the greatest contribution to the aerodynamic stability for the 0.8 h/H of VCS. The absolute values of Part D are the larger than those of Part A for the smaller height VCS (i.e. 0 h/H and 0.2 h/H), while the absolute values of Part A are close to those of Part D for the larger height VCS (i.e. 0.8 h/H).

Fig.5 (c) and (f) show that Part B and Part C generally increase with the increase of h/H of VCS. Although Part B and C are more likely to have some positive effect on the aerodynamic stability for relatively high VCS due to the increase of flutter derivatives H_2^* , the energy of the positive aerodynamic damping produced by Part C could balance the energy of the negative aerodynamic damping produced by Part B. Therefore, both Part A and Part D in torsional motion also play an important role in maintaining the flutter stability of a girder with VCS, most importantly Part A and Part D become critical in the flutter instability for larger and smaller h/H of VCS, respectively.

3.3 DOF participation

The participation level of heaving and torsional DOF at the point of flutter onset, namely the flutter modality, has been used to investigate the role of each DOF plays in structural motion and the coupling effect of DOFs in the flutter oscillation. Based on the 2D-3DOF method, the flutter modality vectors of twin-box girders were calculated and shown in Fig. 6. The numbers from 1 to 3 in Fig. 6 refer to three different h/H ratios of VCS, respectively.

Fig. 6. DOF participation level of three types VCS: (a) Type A, (b) Type B, and (c) Type AC

The position of the flutter modality vector for twin-box girders with VCS indicates that the participation level of heaving motion is considerably high, resulting in strong coupling effects between torsional and heaving motions. With the increase in the h/H of Type A and Type B for the 20% SWR causes the flutter modality vector to move away from the axis of torsional DOF, and approaches to the mid-line between the two axes, indicating stronger coupling effect in flutter oscillation. However, when the h/H of Type

A and Type B for the 20% SWR increase to 0.8 and 0.6, respectively, the modality vector further approaches the axis of torsional DOF, indicating that the participation level of heaving motion decreases and the coupling effect becomes weaker.

In addition, for the 40% SWR with the 0.2 h/H (Type AC), the torsional motion becomes dominant in the coupling effect of DOF participation level. Nevertheless, with the h/H ratio become to 0.4, the modality vector further approaches the axis of heaving DOF, implying the heaving participation level and the DOF coupling effect become stronger and stronger. Therefore, the participation level of heaving DOF, which may not monotonic increase with the increase of the h/H ratio, has a significant influence in the flutter performance of twin-box girders with VCS. In addition, the increase of contribution from heaving motion associated with negative aerodynamic damping leads to higher critical flutter wind speed when an optimal h/H ratio is adopted.

4. Flow structures simulation

In this section, the aerodynamic instability mechanism of the VCS was further investigated through CFD simulation in term of instantaneous velocity field and vortex structures.

4.1 Numerical model

To determine the flow structures for twin-box girders with VCS, the Smagorinsky subgrid-scale models for fluid–structural coupling and the free oscillation method [5] were used in this study. Based on a finite volume method (FVM), the unstructured grid system incorporating large eddy simulations (LES) modeling was used to discretize the governing Navier-Stokes equations [28]. A stagger mesh discretization method was adopted in the CFD simulation using C# based on the unstructured triangular mesh generation technique. Based on the Quadratic Upwind Interpolation of Convective Kinematics of scheme and multidimensional interpolation algorithm, a multidimensional quadratic mixed interpolation scheme in space was adapted. A time marching procedure was performed by the fourth-order Runge-Kutta method.

The overall unstructured grid system with the computational region of $75B \times 50B$ rectangle is described in Fig.7. The wall function is adopted for the wall boundaries. The maximum Courant number C_{conv} is about 0.4 for the 20% SWR with $0.6h/H$ VCS. The minimum and maximum values of y^+ , representing the points close to the structure and the wall, are in the range of about 1.0 and 50, respectively. The time step was set at 0.001s. The Reynolds number was around 5000 at the flutter onset. A steady uniform flow velocity was applied at the inlet boundary, and an opening pressure condition was given at the outlet boundary. In addition, symmetry condition was used for top and bottom surfaces of the domain. The comparison results of U_{cr} between experimental observations and numerical predictions are summarized in Table 3, and the U_{cr} of experimental observations agree well with the results of CFD simulation. Moreover, the static coefficients comparison of twin-box girders with various VCS between the wind tunnel tests and CFD simulation were presented in Fig.8. For three representative cases of S02_A00, S02_A06 and S02_A08, the static coefficients of experimental results are close to those of CFD simulation, which further show that the CFD simulation are reasonable.

Fig. 7. Overview of computational region: (a) The whole computational region, (b) zoom around the girder section

Table 3

Comparison U_{cr} obtained from wind tunnel tests and CFD simulation

Case	Experimental (unit: m/s)	CFD (unit: m/s)	Case	Experimental (unit: m/s)	CFD (unit: m/s)
------	-----------------------------	--------------------	------	-----------------------------	--------------------

S02_A00	15.5	15.4	S04_B04	18.4	18.8
S02_A06	19.5	19.2	S02_AC04	22.3	22.0
S02_A08	14.8	15.0	S04_BC04	19.9	20.2
S08_A04	15.8	16.2			

342

343

344

Fig. 8. Comparison of static force coefficients: (a) S02_A00, (b) S02_A06, (c) S02_A08

4.2 Velocity fields comparison

Fig. 9 shows the velocity vector maps of some typical testing cases of twin-box girders with VCS under the $+3^\circ$ wind attack angle when the structure undergoes the flutter onset.

352

353

354

Fig.9 Instantaneous velocity maps: (a) S02_A00, (b) S02_A06, (c) S04_B04 , (d) S02_AC04 , (e) S04_BC04 , (f) S02_A08, and (g) S08_A04

The simulation results show that there is a significant change in the flow structures under the central slotting and round the leeward girder after installing the VCS. The comparison of the fluctuating velocity field of Type A and Type AC (or Type B and Type BC) shows that the VCS under the central slotting could potentially inhibit the formation of large vortices around the surface of the leeward girder. Meanwhile, only small vortices are generated in the rear of the VCS for Type A and Type B, while large vortices appear in most regions of the central slotting and completely isolated by the VCS for Type AC and Type BC.

Although the large vortices in central slotting could be found in the rear of the VCS with the 0.8 h/H (Type A), they only appear in relatively small areas, which are far away from the upper surface of the leeward girder (Fig. 9(f)). With the increase of SWR, large vortices gradually occupy most of the central slotting area of a twin box with 80% SWR, and the vortices in the rear of the Type A are alternated with other vortices under the central slotting. The comparison of Fig. 9(a), (b),(d) and (f) shows that, for the 20% SWR without VCS, large vortices could appear under the central slotting, around the surface and the wind fairing of the leeward girder. In summary, the VCS have significant impact on the flutter performance of twin-box girders through altering the vortices' structures under the central slotting and around the leeward girder.

4.3 Comparison of pressure distribution

Fig. 10 compared the corresponding pressure distributions around the girders of the above mentioned cases (Fig.9) at the flutter onset. The red and the blue vortices denote the largest positive and negative pressure amplitude, respectively.

382

383

384

385

386

387

Fig. 10. Surface pressure around girder section: (a) S02_A00, (b) S02_A06, (c) S04_B04, (d) S02_AC04, (e) S04_BC04, (f) S02_A08, and (g) S08_A04

Fig.10 demonstrates that the installation of VCS has great influence on the pressure distributions under the central slotting and round the leeward girder of a twin-box girder. Nevertheless, the change of instantaneous pressure distribution of the windward girder could be neglected and large positive pressure always happens at the bottom edge of the wind fairing of the windward girder. It can be seen that larger negative pressures appear in the rear of VCS. Both the Type A and Type AC mainly affect the pressure distribution of the upper surface of the leeward girder, while the pressure distribution of the lower surface of the leeward girder have change significantly after installing the Type B or Type BC. In addition, the region with negative pressure distribution under the central slotting becomes smaller for the twin-box girder, regardless of the Type A and Type B.

The large negative pressures rarely happen under the central slotting of the twin-box girder with the 0.8 h/H VCS (Type A). However, the large negative pressures appear at the upper surface and beneath the wind fairing of the windward girder. Fig. 9(g) and Fig. 10(g) show the velocity and pressure distribution of 80% SWR with the 0.4h/H VCS, respectively. The comparison results show that most of the regions in the front of the VCS under the central slotting of a twin box with 80% SWR are subjected to high positive pressures, while only high negative pressures are developed in front of the leeward girder. Therefore, the large vortex structures could potentially lead to the formation of negative

pressure zone under the central slotting and round the leeward girder. The transformation of pressure distribution owing to the installation of VCS determine the effect of the aerodynamic forces on the girder section, ultimately modify the flutter instability performance of the twin-box girder.

5. The development of modified Selberg formula

5.1 Empirical expression

Selberg formula is widely utilized to calculate the critical flutter wind speed of streamlined cross sections for design purpose [29]. A modified Selberg formula was further developed to calculate the U_{cr} of a closed box girders with Type A of VCS, wherein Lorentz peak-value function was utilized [8]. Subsequently, the Lorentz peak-value function was also used to estimate the U_{cr} of twin-box girders with various SWR [4]. In order to quantitatively evaluate the control effects of VCS on the flutter performance of twin-box girders with various SWR, both the Lorentz peak-value function and Sine function were used to estimate the U_{cr} in this study. The U_{cr} of twin-box girders with different h/H ratios and types of VCS could be estimated by using a correction coefficient of η_{CS} . Based on the correction coefficient η_{TB} of twin-box girders with various SWR (Eq.3), the revised critical flutter wind speed U_{cr}^{TBCS} of twin-box girders with various SWR and VCS (Eq.2) may be predicted from a modified Selberg formula. Both the Lorentz peak-value function and Sine function of η_{CS} were fitted to the measured U_{cr} in the case of the angle of attack of $+3^\circ$ using the nonlinear least squares method, as shown in Eq.4 and Eq.5, respectively.

$$U_{cr}^{TBCS} = \eta_{TB} \eta_{CS} K B f_{\alpha} \left\{ \frac{mr}{\rho B^3} \times \left[1 - \left(\frac{\omega_h}{\omega_{\alpha}} \right)^2 \right] \right\}^{1/2} \quad (2)$$

$$\eta_{TB} = 0.91 + \frac{0.12}{0.4 + 4 \times (0.48 - D/B_s)^2} \quad (3)$$

$$\eta_{CS}^L = a_1 + \frac{a_2}{a_3 + 4 \times \left(\frac{h}{H} - a_4 \right)^2} \quad (4)$$

$$\eta_{CS}^S = a_1 + a_2 \times \sin \left(a_3 \times \left(\frac{h}{H} \right) + a_4 \right) \quad (5)$$

where η_{TB} is correction coefficient of the U_{cr} of twin-box girders and basic box girders (0% SWR), as reported in the study of Yang et al [10]; η_{CS} is correction coefficient of the

U_{cr} of twin-box girders with VCS; the m is the effective mass per unit span; r is the mass radius of gyration; ω_h and ω_a are the circular frequencies of the vertical and torsional modes, respectively; ρ is air density; D/B_s is slot width ratio; and K is shape factor of sectional model with $K = 4 - 8.37 \times \left(\frac{H}{B}\right)^{0.766}$; η_{CS}^L and η_{CS}^S are using the Lorentz function and Sine function, respectively. As shown in Eq. (3) to (5), the η_{TB} is correction coefficient of the U_{cr} of twin-box girders which is related to the critical factor of slot width ratio D/B_s , while η_{CS} is correction coefficient of the U_{cr} of twin-box girders with VCS which is related to the critical factor of relative height (i.e. h/H) of VCS.

5.2 Comparison of correction coefficient

The correction coefficient (η_{CS}^L and η_{CS}^S) of twin-box girders with four types of VCS using Lorentz function and Sine function were calculated according to the testing cases of Table 2. The typical cases of four types of VCS by two functions were compared at the angle of attack of $+3^\circ$ in Fig. 10, and the specific parameters of η_{CS}^S for five SWR by Sine function are listed in the Table 4.

By comparing all the η_{CS} fitted by Lorentz function and Sine function, it can be found that the fitted η_{CS}^L has maximum value of 1.258 when the SWR is 20% and the h/H ratio of VCS is 0.6 for Type A, and a minimum value of 0.709 when the h/H ratio approaches 1.0. As shown in Fig. 11(a) and (b), for 60% SWR, there is a difference in the fitting curves by using the Lorentz function and Sine function, where the largest residual of the Lorentz function and Sine function are 1.2% and -0.8%, respectively. The fitting curves of η_{CS} for Type B using the Lorentz function are similar to those using the Sine function, expect for the 20% SWR. All the η_{CS} for Type B were fitted reasonably well by these two functions and the largest residual value of Type B is 1.76% at the 80% SWR. The largest residual value of the 40% SWR for Type AC reduces to -8.794% using Lorentz function and -4.22% using Sine function, respectively. Moreover, the residual value of the Type BC using the Lorentz function (i.e. 5.762%) is still larger than that using the Sine function (i.e. 3.286%).

All the largest residual values of η_{CS} fitted by the Sine function are smaller than 5%, as shown in the Table 4. In particular, the largest residual values of the 20% SWR with

464 Type B using the Lorentz function and Sine function are 3.0% and 6.2%, respectively. It is
 465 found that only the Lorentz function could perfectly describe the relationship of different
 466 heights of VCS and η_{CS} of the 20% SWR, since its distribution is more likely to be a singly
 467 peaked function. Therefore, the Lorentz peak-value function is not suitable for the case of
 468 twin-box girders with various SWR and VCS (except for the 20% SWR of Type B), and
 469 the fitted function of correction coefficient based on the Sine function agrees reasonably
 470 well with the change in characteristic of the U_{cr} under various schemes of VCS.

Fig. 11. Correction coefficient η_{CS} of four types: (a) Lorentz function of 60% slot ratio with Type A, (b) Sine function of 60% slot ratio with Type A, (c) Lorentz function of 20% slot ratio with Type B, (d) Sine function of 20% slot ratio with Type B, (e) Lorentz function of 40% slot ratio with Type AC, (f) Sine function of 40% slot ratio with Type AC, (g) Lorentz function of 40% slot ratio with Type BC, and (h) Sine function of 40% slot ratio with Type BC.

Table 4

Parameters of correction coefficient η_{CS} by Sine function

Type	D/B _s	η_{CS}	a_1	a_2	a_3	a_4	Largest residual
A	20%	$\eta_{CS}^{s02,A}$	1.123	0.159	7.814	0.282	-4.257%
	40%	$\eta_{CS}^{s04,A}$	0.281	0.802	1.524	-0.717	2.082%
	60%	$\eta_{CS}^{s06,A}$	1.021	0.031	8.513	0.256	-0.812%
	80%	$\eta_{CS}^{s08,A}$	0.900	0.112	2.041	-0.524	0.623%
	100%	$\eta_{CS}^{s10,A}$	0.988	0.033	3.775	-0.098	-1.065%
B	20%	$\eta_{CS}^{s02,B}$	0.897	0.201	4.282	-0.167	6.272%
	40%	$\eta_{CS}^{s04,B}$	1.03	0.0257	7.854	0	2.143%
	60%	$\eta_{CS}^{s06,B}$	1.007	0.055	4.254	0.049	0.716%
	80%	$\eta_{CS}^{s08,B}$	1.025	0.025	6.014	0.244	-1.245%
	100%	$\eta_{CS}^{s10,B}$	0.985	0.077	2.650	-0.077	0.454%

AC	20%	$\eta_{CS}^{s02_AC}$	1.251	0.225	6.980	0.290	3.316%
	40%	$\eta_{CS}^{s04_AC}$	1.037	0.114	6.446	0.416	4.422%
BC	20%	$\eta_{CS}^{s02_BC}$	1.055	0.158	7.854	0	4.194%
	40%	$\eta_{CS}^{s04_BC}$	1.081	0.069	7.854	0.2	3.286%

6. Conclusions

In this study, the flutter characteristics of twin-box girders bridges with the combination schemes of various slot width ratios and vertical central stabilizers were studied through experimental investigation in conjunction with theoretical analysis. The major conclusions are as follows:

- The height variation of VCS significantly affects the U_{cr} and β of a twin-box girder bridge with 20% SWR. However, under a relatively large SWR (e.g. >60%), the change of the height of VCS has little impact on the flutter stabilization. Besides, among four types, Type AC and Type BC is the best choice of the flutter performance for the 20% SWR and 40% SWR, respectively.
- Using 2D-3DOF analysis, we demonstrated that aerodynamic damping Part A with reference of flutter derivative A_2^* makes the greatest contribution to the flutter instability for a VCS with a h/H ratio of 0.8, while the role of Part D with reference of $A_1^* H_3^*$ becomes critical for a relatively short VCS (i.e. h/H is less than 0.2). In addition, the increase of heaving DOF participation level with the VCS with an optimal h/H ratio of 0.4 leads to a higher U_{cr} .
- The results from computational Fluid Dynamics (CFD) simulations indicate that the geometry and position of VCS could potentially change the evolution of vortices' structures and pressure distribution under the central slotting, and so affect the flutter performance.
- The proposed modified Selberg formula based on the Sine function has the capability of predicting the U_{cr} of twin-box girders with various SWRs and VCS.

It should be mentioned that further investigation of stabilization mechanism of VCS are required in order to fundamentally understand the critical factors (e.g. the flow structures, aerodynamic forces and structural responses). In addition, future research work should focus on the overall aerodynamic performance of twin-box girders bridges with vertical central stabilizers (e.g. stationary aerodynamic performance, vortex-induced vibration and buffet performance).

Acknowledgments

The authors gratefully acknowledge the support for the research work jointly provided by the National Key Basic Research Program of China (973 Program) (No. 2013CB036300), the National Science Foundations of China (No.91215302, 51078276, and 51678436).

References

- [1] Sato H, Hirahara N, Fumoto K, Hirano S. and Kusuvara S. Full aeroelastic model test of a super long-span bridge with slotted box girder. *J Wind Eng Ind Aerodyn* 2002;90:2023-2032.
- [2] Yang YX, Zhou R, Ge YJ, Zhang LH. Experimental studies on VIV performance and countermeasures for twin-box girder bridges with various slot width ratios. *J Fluid Struct*, 2016;66:476-489.
- [3] Trein CA, Shirato HM, Matsumoto M. On the effects of the gap on the unsteady pressure characteristics of two-box bridge girders. *Eng Struct* 2015;82(1):121-133.
- [4] Yang YX, Zhou R, Ge YJ, Mohotti D, Mendis P. Aerodynamic instability performance of twin-box girder for long-span bridges. *J Wind Eng Ind Aerodyn* 2015;145:196-208.
- [5] Miranda SD, Parruno L, Ricci M, Ubertini F. Numerical study of a twin box bridge deck with increasing gap ratio by using RANS and LES approaches. *Eng. Struct*, 2015;99(15):546-558.
- [6] Andersen MS, Johansson J, Brandt A, Hansen SO. Aerodynamic stability of long span suspension bridges with low torsional natural frequencies. *Eng Struct* 2016; 120(1):82-91.
- [7] Ueda T, Yasuda M. Nakagaki, R. Mechanism of aerodynamic stabilization for long-span suspension bridge with stiffening truss-girder. *J Wind Eng Ind Aerodyn* 2000;33(1-2):333-340.

- [8] Ge YJ, Zou XJ, Yang YX. Aerodynamic stabilization of central stabilizers for box girder suspension bridges. *Wind Struct* 2009;12(4):285-298.
- [9] Chen AR, Zhou ZY, Xiang HF. On the mechanism of vertical stabilizer plates for improving aerodynamic stability of bridges. *Wind Struct* 2006; 9(1):59-74.
- [10] Yang YX, Ge YJ, Xiang HF. Aerodynamic flutter control for typical girder sections of long-span cable-supported bridges. *Wind Struct* 2009;12, 205–217.
- [11] Sato H, Kusahara S, et al. Aerodynamic characteristics of super long-span bridges with slotted box girder. *J Wind Eng Ind Aerodyn* 2000;88(2): 297-306
- [12] Miyata T. Significance of aero-elastic relationship in wind-resistant design of long-span bridges. *J Wind Eng Ind Aerodyn* 2002;90:1479-1492.
- [13] Fumoto K, Hata K, Matsuda K, Murakami T, Saito Y. and Shirai S. Aerodynamic Improvement of Slotted One-Box Girder Section for Super Long Suspension Bridge. In: *Proceedings of the 6th Asia-Pacific Conf. on Wind Engineering*, Seoul, Korea; 2005. pp. 1222-1236.
- [14] Li YL, An WS, Li CJ, Liao HL, Aerodynamic optimization of three-box-girder deck by CFD method. *China Civil Eng J* 2013;46(1):.61-68. (In Chinese)
- [15] Matsumoto M, Mizunoc K. et al. Flutter instability and recent development in stabilization of structures. *J Wind Eng Ind Aerodyn* 2007;95(9-11):888-907.
- [16] Chen ZQ, Ouyang KJ, Niu HW, Hua XG. Aerodynamic Mechanism of Improvement of Flutter Stability of Truss-girder Suspension Bridge Using Central Stabilizer. *China J Highway and Trans* 2009;22(6):53-59. (In Chinese)
- [17] Shirai S, Ueda T. Aerodynamic simulation by CFD on flat box girder of super-long-span suspension bridge. *J Wind Eng Ind Aerodyn* 2003;91:279-290.
- [18] Bruno L, Khris S. The validity of 2D numerical simulations of vortical structures around a bridge deck. *Math Comput Model* 2003;37(7): 795-828.
- [19] Bruno, Luca, et al. 3D flow around a rectangular cylinder: a computational study. *J Wind Eng Ind Aerodyn* 2010;98 (6): 263-276.
- [20] Bruno L, Salvetti MV, and Ricciardelli F. Benchmark on the aerodynamics of a rectangular 5: 1 cylinder: an overview after the first four years of activity. *J Wind Eng Ind Aerodyn* 2014;126: 87-106.
- [21] Starossek U, Aslan H, and Thiesemann L. Experimental and numerical identification of flutter derivatives for nine bridge deck sections. *Wind Struct* 2009; 12 (6): 519.
- [22] Miranda SD, Patruno L, Ubertini F, Vairo G. On the identification of flutter derivatives of bridge decks via RANS turbulence models: Benchmarking on rectangular prisms. *Eng Struct* 2014; 76: 359-370.

- [23] Patruno, L. Accuracy of numerically evaluated flutter derivatives of bridge deck sections using RANS: effects on the flutter onset velocity. *Eng Struct* 2015; 89: 49-65.
- [24] Scanlan RH, Tomko JJ. Airfoil and bridge deck flutter dervatives. *J Eng Mech* 1971;97(6):1717-1737.
- [25] Agar, T.J.A. Aerodynamic flutter analysis of suspension bridges by a modal technique. *Eng. Struct* 1989;11(2): 75-82.
- [26] Gu M, Qin XR. Direct identification of flutter derivatives and aerodynamic admittances of bridge decks. *Eng Struct* 2004; 26 (14) : 2161–2172.
- [27] Seo D W, Caracoglia L. Estimation of torsional-flutter probability in flexible bridges considering randomness in flutter derivatives. *Eng Struct* 2011;33 (8) : 2284–2296.
- [28] Yoshikawa M, Tamura T. LES for wind load estimation by unstructured grid system. In: BBAA VII international colloquium on bluff bodies aerodynamics & applications. Shanghai, China; 2012.
- [29] Selberg A. Aerodynamic Effect on Suspension Bridges. *Proc. of Int. Symposium on Wind Effects on Buildings and Structures*, Teddington, England, 1963;2: 462-486.