ANALYTICAL STUDY OF IN-PLANE BUCKLING OF LONGITUDINAL BARS IN REINFORCED CONCRETE COLUMNS UNDER EXTREME EARTHQUAKE LOADING

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1. ABSTRACT

Seven full-scale reinforced concrete (RC) columns were tested at the Multi-Axial Subassemblage Testing (MAST) Laboratory of the University of Minnesota to investigate their performance under extreme seismic events. The specimens were designed according to seismic provisions of *ACI 318-11* [1] and incorporated closely-spaced transverse hoops at their base. However, at large drift ratios during these tests, longitudinal bars were observed to buckle parallel to the face of the columns with transverse ties having little effect. This previously unobserved bar buckling phenomenon is investigated numerically to gain a better understanding of the column characteristics that affect it. First, a bar-spring mechanical model is used to understand the conditions needed to prevent buckling of longitudinal bars by means of restraints with finite stiffness. Second, a three-dimensional (3D) nonlinear finite element analysis of the lower portion of the specimen subjected to monotonic loading was formulated in ABAQUS and validated with test data. The analysis reveals that columns with larger cross-sectional dimensions that incorporate larger longitudinal bar sizes (No. 8 and above) and lower strength concrete are more prone to in-plane bar buckling.

Keywords: reinforced concrete; column, bar buckling, in-plane buckling, finite element modeling

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2. INTRODUCTION

Reinforcing bar buckling is a complex phenomenon that can reduce ductility of RC columns and lead to significant stiffness reduction and strength loss. The complexity of this phenomenon is attributed to the fact that the reinforcing bar buckling in RC columns involves both material and geometric nonlinearities and depends on several parameters including tie spacing and effectiveness, spalling of cover concrete, expansion of concrete core, and loading history. Due to the complexity of the problem, many prior experimental and analytical studies were carried out on isolated bars under compression to investigate bar buckling behavior. Experimental investigations showed that the behavior of bars in compression is generally different from that in tension [2], [3] and hence analytical formulations were proposed to capture the constitutive relations of compression bars including buckling [4]–[6]. Additionally, the investigations revealed that the post-yield behavior of reinforcing bars under compression is generally controlled by the ratio of their unsupported length to bar diameter (s/d_b) as well as their eccentricity to bar diameter ratio (e/d_b) [3], [7]. Berry and Eberhard [8] utilized experimental results of a dataset of RC columns to develop an equation for estimation of the drift ratio at the onset of bar buckling. However, their proposed equation seems to overestimate the drift ratio at the onset of bar buckling as they calibrated their equation with reported drift ratios corresponding to initiation of bar buckling that were mostly based on visual inspections at or near the end of the tests [9], [10]. Considering the major role of transverse ties in controlling the bar buckling, some research studies idealized the bar buckling problem as a system of bar and springs with the springs representing the axial stiffness introduced by transverse ties [2], [11]. Kashani et al. [12] used this concept to calculate the buckling length of longitudinal bars and employed a nonlinear fiber element modeling technique to investigate bar buckling phenomenon in RC bridge piers. Using fiber-based model of circular RC columns, Feng et al. [13] investigated the strain limits at to the onset of bar buckling.

In all the former studies of bar buckling in RC columns, it is assumed that the bars will buckle outward, that is perpendicular to the nearest column face, and the transverse ties are essential in restraining them from buckling. However, recent tests carried out at the MAST Lab in the Department of Civil, Environmental and Geo- Engineering of the University of Minnesota on full scale RC columns subjected to extreme loading conditions similar to what the columns experience during extreme seismic events revealed another buckling mode in which the bars buckled parallel to the compression face of the columns (in-plane bar buckling). In several instances, the ties were observed to still be effective in restraining the bars from outward buckling. However, after extensive damage to the concrete surrounding the bars, the reinforcing bars buckled over a height equal to twice the tie spacing and exhibited large lateral deformations parallel to compression face of the columns.

The main restraint against in-plane buckling is the concrete surrounding the bars, especially the concrete residing between adjacent bars (i.e. the concrete “teeth”). This concrete, while being within the column core, tends to be the portion of the core that gets damaged earlier and to a greater extent during a load test. Extensive damage to the concrete surrounding the bars would typically happen under large seismic loading conditions during which concrete exhibits severe inelastic behavior. The unique control and loading capabilities of the MAST Lab provided the opportunity to continue loading of the specimens far beyond the extent of loading in previous tests on RC columns. During these tests, the loading continued until the specimens lost more than 80% of their lateral load capacity whereas in almost all previous tests, loading was terminated after a 20% strength loss. In the authors’ opinion, such an extended loading regime provided the chance to observe a failure mechanism in RC columns that was not addressed in previous tests.

In this study, first the required lateral restraint to prevent bar buckling is investigated numerically using FEM modeling of isolated bars. In practice, the lateral restraint can be provided by transverse

ties in the case of outward deformation, or by concrete surrounding the bars (i.e. the “teeth”) in the case of in-plane deflection. Results from analysis of the isolated bars are then utilized in a nonlinear finite element model in ABAQUS/Explicit to investigate the observed failure mechanism in the longitudinal bars of tested column specimens. The model was validated with experimental data and employed to explore the influence of several parameters that can affect in-plane buckling of the bars.

3. EXPERIMENTAL PROGRAM

The columns tested at the MAST Lab represent the lower portion of typical columns on the ground floor of a moment resisting frame in a high-rise building. The column specimens were designed according to seismic provisions of *ACI 318-11* [1] and featured two distinct cross sectional dimensions: 36×28 in. and 28×28 in. (914×711 mm. and 711×711 mm.). The specimens were built using normal-strength, normal-weight concrete with 28-day nominal compressive capacity (f'_c) of 5000 psi (34.47 MPa), *ASTM A706* [14] Grade-60 No. 8 and No. 9 steel reinforcing bars confined by *ASTM A615* [15] Grade-60 No. 5 steel hoops. Transverse hoops were placed at close spacing in the base of the columns to prevent outward bar buckling and ensure ductile behavior (**Fig. 1**).

At the beginning of each test, a constant axial load was applied on top of the specimens (**Fig. 1(a)**) and kept constant during the tests. To simulate behavior at near-collapse conditions during extreme seismic excitation, the column specimens were then subjected to either a monotonic or one of several progressively increasing displacement reversals until they lost more than 80% of their lateral loading capacity. Further details regarding these tests can be found elsewhere [16].

4. TEST OBSERVATIONS

When the lateral loading was applied to the column specimens, flexural cracks formed on tensile face of the columns (**Fig. 1**). Flexural cracking initiated at approximately 0.2% drift level, which was equal to 6 mm of the cross-head displacement. Additionally, flexure-shear cracks formed in the lower

Fig. 1. Specimen geometry and detailing (unit: in., 1 in. =25.4 mm.)

portion of the other two faces of the columns and longitudinal cracks were observed along the corners on the compression sides. Following concrete cracking, the longitudinal bars started to yield at approximately 0.5% drift ratio (15 mm of cross-head displacement). However, the columns exhibited further lateral load capacity due to confining pressure from the transverse reinforcement and strain hardening of longitudinal reinforcement. An increase in lateral load was accompanied by more crack developments and propagations. Finally, the specimens started to lose flexural strength due to crushing of the cover concrete and buckling of the longitudinal bars. The columns exhibited severe strength loss by the end of the test (more than 80% of their peak lateral load capacity) following the fracture of the buckled bars and extensive damage to the concrete around the perimeter.

While it is commonly assumed that longitudinal bars in an RC column would buckle in the outward direction (i.e. perpendicular to the compression face of the column), investigation of the

buckled bars at the end of the tests conducted at the MAST lab indicated that this assumption is not necessarily true in all cases. Instead, it was observed in all tested specimens that some of the bars buckled parallel to the compression face of the columns (**Fig. 2**). More specifically, when only the two middle bars on compression faces of the columns in monotonic and cyclic uniaxial tests at the MAST Lab are considered, 10 out of 24 bars (i.e. 42% of the total observed buckling cases) revealed either a complete or a dominant in-plane buckling mode. A buckling mode is considered to be completely in-plane when no out of plane translation is observed in the buckled region whereas if a small outward translation is present in conjunction with large in-plane translation, it is considered a dominant in-plane buckling mode.

Another observation is related to the axis of bending in the buckled bars. Since reinforcing bars incorporate ribs along their length, they have strong and weak axes of bending. In practice, longitudinal bars are typically oriented in a way that their outward deformation (i.e. deflection perpendicular to the column face) mobilizes flexural rigidity corresponding to the weak axis while their in-plane deflection (i.e., deflection parallel to the column face) occurs about their strong axis of

Fig. 2. In-plane bar buckling during tests at the MAST lab for the column specimen under monotonic loading (left), and a column specimen subjected to cyclic loading (right).

bending. In many of the observed cases of buckled bars, in-plane buckling occurred about the strong axis of bending of the bars, which is contrary to the common assumption, that is supported by experimental results from isolated bars, that the bars would only buckle about their weak axis.

5. ANALYTICAL INVESTIGATION USING BAR-SPRING MODEL

A bar and spring model is used to investigate the demands on lateral restraints that have finite stiffness to prevent buckling of longitudinal reinforcing bars. For outward (i.e. out-of-plane) buckling, these restraints are the hoops or ties that are also used as shear reinforcement and reinforcement for confinement. For in-plane buckling, the restraints are provided by the concrete that bears against the longitudinal reinforcement.

5.1. General Description of the Bar-Spring System

The buckling behavior of bars can be simply modeled with a bar-spring system as illustrated in **Fig. 3** where springs represent the axial stiffness of the ties preventing the bars from buckling. Dhakal and Maekawa [2] employed such model to estimate the required tie stiffness for a stable buckling mode and to estimate the buckling length of the bars. Based on their investigation, Dhakal and Maekawa [2] proposed Eq. (1) to estimate the required stiffness (K_n) for the n^{th} stable buckling mode as:

$$K_n = \alpha_n \frac{\pi^4 EI}{s^3} \quad (1)$$

where according to their definition, n is the number of tie spacings within the buckled zone, s is the tie spacing, EI is the reduced flexural rigidity of the bars that depends on their yield strength (f_y), and α_n is a constant that depends on the stable buckling mode. For the case shown in **Fig. 3**, in which the buckled length equals two tie spacings, the required tie stiffness to prevent the bars from buckling is

calculated as 5.8, 29.1, and 71.5 kN/mm for No. 6, No. 9, and No. 11 bars, respectively, with α_n given as 0.1649 and assuming a yield strength (f_y) of 510 MPa for longitudinal bars of the test specimens.

5.2. Finite Element Model of the Bar-Spring System

To further investigate the behavior of the buckled bars in the bar-spring system, a 3D finite element model of the system represented in **Fig. 3** was built in ABAQUS using 2-node shear flexible beam-in-space (B31) elements representing the longitudinal bar and a linear spring at its mid-height whose stiffness corresponds to the axial stiffness of the tie leg. The length of the bar (l) was selected equal to two times the tie spacing that was used in tested RC columns (i.e. $s=127$ mm and $l=254$ mm.).

Behavior of the steel bar was modeled using a trilinear curve fitting the stress-strain curve of longitudinal bars in tested specimens with yield and ultimate strengths of 510.2, and 772.1 MPa, respectively (**Fig. 4**) while the elastic stiffness of spring was taken as constant during each analysis. A number of analyses with different bar sizes and values for spring stiffness were conducted. An initial imperfection was introduced to perturb the bar geometry and mobilize buckling. In practice, this imperfection can be caused by misalignment or initial crookedness of the bars. To minimize the effect

Fig. 3. Bar-Spring system to simulate bar buckling behavior

Fig. 4. Stress-strain behavior of longitudinal bars

of the initial imperfection on the buckling load, a separate series of analyses were performed and the imperfection magnitude was selected sufficiently small such that estimated critical loads had a maximum error of 1% compared to the exact analytical solutions in the case of elastic bar. To simulate fixed boundary conditions as assumed by Dhakal and Maekawa [2], all degrees of freedoms (DOFs) at the top and bottom of the bar (Points A and B in **Fig. 3**) were fixed except the vertical translation on the top (Point B). During each analysis, the vertical load on top of the bar increased gradually using Modified Riks (arc-length) method [17]–[19] until the bar buckled.

5.3. Static Analysis Using Modified Riks (Arc-Length) Method

The modified Riks method was used to obtain static equilibrium solutions for this geometrically and materially nonlinear problem with emphasis on the effect of buckling on the descending branch of the force-displacement. The loading on the system is governed by a single scalar parameter, namely the load proportionality factor (λ), an arc-length and an iterative procedure were used to determine a set of loads and displacements at each increment that satisfy static equilibrium. The method is neither displacement nor load control, and the arc length includes components of both. The magnitude of loading at a specified increment is defined as follows:

$$P_{total} = P_o + \lambda (P_{ref} - P_o) \quad (2)$$

where P_o is the dead load (i.e. any load applied to the structure prior to the Static Riks step), λ is the load proportionality factor that changes in each load increment, and P_{ref} is the reference load (i.e. the load that is applied during Static Riks step). A unit load was applied to the bar during the Static Riks step ($P_{ref}=1$). Since, there are no other loads prior to the applied unit load, P_o was equal to zero, hence the magnitude of the load proportionality factor, λ , represents the evolution of loading (P_{total}) during analysis.

5.4. Results from Bar-Spring Analysis

For each bar size in the study, a series of finite element analyses was carried out with spring stiffness as the variable parameter. The applied axial loads for such analyses on No. 9 bars and various spring stiffness coefficients is represented in **Fig. 5**, which indicates that the magnitude of critical load increases with a rise in spring stiffness. However, beyond a stiffness of 8.8 kN/mm, the rate of increase in critical load with addition of stiffness drops significantly such that a spring that is 10 times stiffer would only slightly increase the critical load (i.e. less than 3%). Adding more stiffness will eventually cause the bar to exhibit other buckling shapes that are inconsistent with the observed behavior of buckled bars during test of RC columns and the one shown in **Fig. 3**. A comparison of the estimated critical loads associated with different spring stiffness with those having stiffness as proposed by Dhakal and Maekawa [2] reveals that their proposed equation overestimates the required spring stiffness to prevent bar buckling (**Table 1**). Specifically, this analysis shows that a spring stiffness of 17.5 kN/mm for No. 11 and 8.8 kN/mm for No.9 bars would suffice to deliver the axial load capacity within 1% and 2.5% of those obtained from their proposed spring stiffness (i.e. 71.5 and 29.1 kN/mm, respectively). In other words, providing 25% and 30% stiffness as of those estimated by Eq. (1) for No. 11 and No. 9 bars, respectively, can prevent the first mode of bar buckling with only a small reduction

Fig. 5. FEM analysis results for a No. 9 bar

Table 1. Outward (out-of-plane) buckling loads from FEM analysis of bar-spring system

K_x (kN/mm)	<i>Bar Size</i>		
	No. 6	No. 9	No. 11
	$d_b = 19$ mm	$d_b = 28.6$ mm	$d_b = 35.8$ mm
0	141.7	349.8	616.3
0.9	143.0	361.4	626.8
4.4	201.9	415.3	662.7
5.8	212.0*	440.4	679.7
8.8	217.0	482.7	737.5
17.5	219.0	493.6	772.9
29.1		494.5*	776.5
52.5		494.8	777.4
71.5		494.9	777.5*
87.6		495.0	777.6
140.1			777.7

* Represents buckling load calculated based on Eq. (1)

in the magnitude of the critical load (i.e. less than 3%). The bar-spring system analogy can also be applied to the case of in-plane buckling of the bars. While longitudinal bars in RC columns are restrained by transverse hoops to prevent outward buckling, it is only the concrete surrounding the bars, as well as the friction between the bars and transverse hoops, that prevent them from buckling sideways (i.e. parallel to the compression face of the column). In this case, the spring stiffness would

represent an initial stiffness provided by the surrounding concrete to restrain the bars from buckling. Assuming that the preventive effect of concrete on the in-plane bar buckling can be idealized by a system of discrete springs, **Table 1** implies that when concrete with the same properties are used (i.e. similar spring stiffness) columns incorporating larger bar sizes are more vulnerable to in-plane buckling as their reinforcing bars require stiffer surrounding concrete (i.e. stiffer spring) to restrain the occurrence of in-plane buckling.

6. FINITE ELEMENT MODELING OF RC COLUMN SPECIMENS

A three-dimensional, nonlinear finite element model is described here and used to simulate the observed in-plane buckling of the longitudinal reinforcement in the RC column specimens tested in the MAST Lab. The model focuses on the portion of the column adjacent to the base of the member, and it offers a more realistic representation of this portion of the columns than does the bar-spring model in the preceding section. The goal of the FEM analysis is to simulate the phenomenon of in-plane buckling by means of a qualitative assessment, and to give an indication of the variables that may affect it.

6.1. General Description of the FEM Model

A 3D finite element model representing the test columns was built in ABAQUS to investigate qualitatively the in-plane buckling of reinforcing bars in the tested specimens. Since the in-plane bar buckling was observed during test of the first specimen (SP1) under monotonic loading, SP1 was considered for model calibration of in-plane buckling behavior. Also, to reduce analysis cost, only the lower portion of column specimen SP1 (635 mm. from the base) was modeled because in-plane buckling was observed in this region (**Fig. 1**).

The FEM model was 635 mm. tall and featured a cross-sectional dimension of 36×28 in. (914×711 mm.) (depth×width) (**Fig. 6**). Fixed boundary conditions were applied to the nodes at the column base

Fig. 6. General Description of FEM model: (a) Modeled section (b) mesh of concrete material (c) modeling of buckling bars within concrete elements

whereas all translational and rotational Degrees of Freedom (DOFs) of the nodes on top of the column were constrained to a reference point at the centroid of the cross section. Constraining the nodes to the reference point eliminates singularity issues in the problem, while the loads are applied to the reference point. Concrete was modeled using 8-node brick elements with reduced integration and hourglass control (C3D8R). The maximum and minimum mesh size for concrete elements were 2.64 and 2.125 in. (67 and 54 mm.). Longitudinal bars were modeled with 2-node shear flexible beam-in-space (B31)

elements. The bars, except those on the compression face, were defined within the concrete solid using embedment technique. In this approach, a perfect bond between embedded elements (reinforcing bars) and the host element (surrounding concrete) is assumed. The interaction between cracked concrete and longitudinal bars (i.e. bond-slip) is defined by adjusting the strain softening behavior of concrete in tension (i.e. tension stiffening). When embedded, all translational DOFs at the nodes of embedded (reinforcing bar) elements are eliminated and these nodes are constrained to the interpolated values of corresponding DOFs at adjacent nodes of the host (concrete) element. As a result, bar buckling cannot be captured when the embedment technique is used since buckling involves a sudden lateral deformation of the bars free from the surrounding concrete. Thus, on the compression face where bar buckling is likely to occur, the bars were modeled as independent B31 elements. These independent elements were connected to the reference point on top of the specimen to move in accordance with the translation and rotation of the specimen top. On the other hand, restraining effects of the ties and surrounding concrete on the buckling bars were modeled using linear spring elements.

6.2. Estimation of Spring Stiffness

Given the proximity of most longitudinal bars to 90° bends in the ties, the spring stiffness associated with ties were defined on the basis of the axial stiffness of tie legs as follows:

$$K_{tie} = \frac{E_t A_t}{L_t} \times \frac{n_l}{n_b} \quad (3)$$

where E_t , A_t , and L_t are elastic modulus (200000 MPa), cross sectional area and length of the tie leg, respectively. Also, n_l and n_b represent the number of tie legs in the direction of restraint and the number of bars that are restrained, respectively.

In the case of in-plane buckling, the restraining effect depends on the performance of the concrete surrounding the bars. An analogy of the preventive concrete pressure on the rebar to the force created

in a discrete system of springs can be made in order to relate spring stiffness to the characteristics of the physical problem. For the concrete to act as an axial element, it is assumed that the concrete between adjacent longitudinal bars has begun to crack off from the rest of the core (i.e., micro-crack generation reducing stress transfer with the rest of the core) (**Fig. 7**). In that case, the secant stiffness can be idealized as follows:

$$K_s = \frac{P_{eff}}{\Delta_{eff}} = \frac{\sigma_{eff} A_{eff}}{\Delta_{eff}} \quad (4)$$

where Δ_{eff} is the maximum lateral displacement of the bar when it buckles, σ_{eff} is the concrete strength when the buckling starts, and A_{eff} is the effective contact area between bars and surrounding concrete which prevents the bars from in-plane buckling.

The maximum lateral displacement at the onset of in-plane buckling (Δ_{eff}) depends on many parameters (e.g. characteristic properties of the concrete surrounding the bars and its performance as well as that of the longitudinal bars, loading history, core concrete expansion and friction between ties and longitudinal bars) and its accurate measurement requires a careful instrumentation and monitoring of the longitudinal bars during the test. The bars which buckled in the in-plane mode during the MAST tests were observed to have a maximum lateral displacement between d_b and $3d_b$ by the end of the tests. Therefore, for the purpose of this study and because the onset of buckling is considered here, Δ_{eff} was assumed to be equal to d_b .

It is further assumed that the effective strength of concrete at buckling (σ_{eff}) would be only a fraction of the compressive strength of core concrete since the concrete surrounding the bars will

Fig. 7. Preventive effect of surrounding concrete from in-plane buckling after cover spalling

soften as the specimen is loaded in compression and flexure. The state of strain at the onset of in-plane buckling is unknown, thus the stress level is indeterminate. A lower bound estimate for the stress level is approximated assuming that the concrete is deformed well into the post-peak range of compression behavior and its strength has degraded to the “residual strength” level that the Kent-Park concrete model [20] quantifies as $0.2\lambda f'_c$, where λ is the factor that represents the strength enhancement due to confining reinforcement. Thus, Eq. (4) can be rewritten as follows:

$$K_s \cong \frac{(0.2\lambda f'_c)[(\beta s)d_b]}{d_b} = 0.2\lambda f'_c \beta s \quad (5)$$

where βs is the dimension along the length of the bar in which the concrete is effective in restraining the buckling bar and λ is suggested by Kent and Park as follows:

$$\lambda = 1 + \frac{\rho_s f_{yh}}{f'_c} \quad (6)$$

where ρ_s is the volumetric ratio of the hoops and f_{yh} is the yield strength of the hoops. From experimental observations, the in-plane buckling length was typically equal to two times the tie spacing ($2s$). Therefore, assuming a half-sinusoidal shape over the observed buckling length, the deflected shape of the bar (before buckling) is given by

$$\frac{\Delta(x)}{\Delta_{max}} = \sin\left(\frac{\pi x}{2s}\right) \quad \text{for } 0 \leq x \leq 2s \text{ and } \Delta = \Delta_{max} \text{ at } x = s \quad (7)$$

The concrete is assumed to provide restraint over the portion of the buckled bar with a lateral deflection $\Delta \geq 0.5\Delta_{max}$, and at the location x_o , $\Delta = 0.5\Delta_{max}$ which can be determined as follows:

$$\frac{\Delta(x)}{\Delta_{max}} = 0.5 = \sin\left(\frac{\pi x_o}{2s}\right) \quad \text{or} \quad \frac{x_o}{s} = \frac{2}{\pi} \sin^{-1}(0.5) \approx 1/3 \quad (8)$$

The resulting effective length factor is

$$\beta = (2s - 2x_o)/(2s) = 2(1 - 1/3) = 4/3 \quad (9)$$

When substituted in Eq. (5), the effective (axial) stiffness of concrete springs is as follows:

$$K_s = \frac{4}{15} \lambda f'_c s \quad (10)$$

In the case of *SPI* with $f'_c = 4.86$ ksi (33.5 MPa), $f_{yh} = 60$ ksi (413.7 MPa), $s = 5$ in. (127 mm), and $\rho_s = 0.0148$, λ would be 1.18 from Eq. (6) and the stiffness K_s will be equal to 7.65 kip/in. (1.35 kN/mm).

The behavior of the springs representing the axial stiffness of ties and restraining effect of concrete was modeled differently in tension and compression. In the inward direction (i.e. -Y direction

in **Fig. 6(c)**), the concrete is assumed incompressible and hence the bars cannot bend inward whereas the tie stiffness will be activated when the bars deflect in the outward direction (i.e. +Y direction). In the case of lateral deformation (i.e. parallel to column face), the behavior would depend on the geometry of the bar and transverse hoops. For the bar shown in **Fig. 6(c)**, it is only the concrete stiffness that restrains the bar from deflection in the -X direction (K_{x1}) whereas the tie legs would limit its deformation in the +X direction (K_{x2}). Distinct spring stiffness corresponding to the deflection of this bar in the +X and -X directions is represented in **Fig. 8**. As represented in **Fig. 8** for the bar in **Fig. 6(c)**, the spring stiffness in the +X direction (K_{x2}) that corresponds to the effect of transverse ties and concrete is larger than that in the -X direction (K_{x1}) due to the effect of only concrete teeth. The four bars on the compression face (i.e. buckling bars) in **Fig. 6** were also connected to their adjacent concrete elements using stiff linear springs along Z direction to simulate strong bond conditions as was assumed for the other embedded bars.

Fig. 8. Spring stiffness parallel to the column face

6.3. Modeling of Concrete Behavior

Concrete behavior is modeled using Concrete Damage-Plasticity (CDP) implemented in ABAQUS/Explicit. The CDP model uses the yield function proposed by Lubliner et al. [21] with

modifications proposed by Lee and Fenves [22] to capture two main limit states for concrete as cracking in tension and crushing in compression. The potential plastic flow is based on the Drucker-Prager hyperbolic function and the flow rule is non-associated.

For the RC columns modeled herein, the elastic modulus of concrete was estimated as 3,974 ksi (27400 MPa) for a concrete with compressive strength of 4.86 ksi (33.5 MPa) according to Sec. 8.5.1 of *ACI 318-11* [1]. Except the dilation angle which was assumed as 30° , the default values were used for the other parameters in the CDP model. These default values are 1.16 for the ratio of the initial equibiaxial compressive yield stress to initial uniaxial compressive yield stress, $2/3$ for the ratio of the second stress invariant on the tensile meridian to that on the compressive meridian, 0.1 for eccentricity and 0 for viscosity.

The stress-strain behavior of concrete in compression was assumed linear up to $0.45f'_c$ and defined according to the model proposed by Kent and Park thereafter. A different stress-strain behavior was considered for core concrete as compared to the one for cover concrete as illustrated in **Fig. 9** to account for the confining effect of rectangular hoops. The uniaxial tensile behavior of concrete was assumed linear up to a tensile yield stress (f_{t0}) of 0.522 ksi (3.6 MPa) calculated according to Sec. 9.5.2.3 of *ACI 318-11*[1]. The post-peak behavior of concrete in tension (tension stiffening) was defined by specifying the fracture energy (G_f , the energy required to form a unit area of crack) as a material property. This model assumes a linear softening behavior after cracking in tension until a complete loss of tensile strength at a cracking displacement equal to $u_{t0} = 2G_f/f_{t0}$. Although suggested values for fracture energy of plain concrete are typically low, low values of this parameter often lead to numerical issues. Moreover, fracture energy is much larger in reinforced concrete due to the presence of distributed reinforcing bars [23]. In this study, a value for fracture energy equal to 0.001 kip/in.

(1.76×10^{-4} kN/mm) was used for the concrete. Also, since calibration was done using only the RC column specimen under monotonic loading, damage parameters were not needed.

Fig. 9. Stress- strain relationship of concrete

6.4. Explicit Dynamic Analysis of the Model

The FE model of the bottom of RC column specimen SP1 was analyzed using the Explicit Dynamic procedure in ABAQUS, which is an efficient way of solving dynamic problems. Unlike Static Riks (Arc-Length) method that is sensitive to parameters that define behavior of concrete and encounters convergence issues, Explicit Dynamic analysis is more robust, due to smoothing of the solutions by inertia effects, and can handle problems for which the former method fails to converge. However, the central difference operator used in Explicit Dynamic analysis is only conditionally stable. By default, estimation of a stable time limit and time incrementation scheme are automatically carried out in ABAQUS. The total number of increments can be estimated as $n=T/\Delta t$ where T represents total analysis time.

To minimize inertia effects during Explicit Dynamic analysis of the model, the loading was applied smoothly. To eliminate inertia effects, it is usually enough to set the loading time 10 to 50 times larger than the fundamental period of system (i.e. period associated with the lowest natural

frequency of the system) [23]. In this analysis, the lowest frequency was estimated as 19.41 Hz resulting in a natural period of 0.052 sec. Therefore, the total analysis time was set to 2 sec. and subdivided into two equally spaced windows. During the first window, the axial load was applied and increased as a ramp function to its final value of 756 kips (3362.9 kN) at the end of the first loading window (i.e. 1 sec. from start point). The magnitude of the axial load was kept constant during the rest of the analysis. Following the first loading window, a top rotation was applied to the column whose magnitude increased as a ramp function to a final rotation of 0.05 radian at the end of the step time (i.e. 2 sec). The loading time during each window was equal to 1 sec., which is approximately 20 times the fundamental period of the system. To estimate inertia effects and evaluate accuracy of the analysis, total strain energy (E_I) and total kinetic energy (E_K) of the system were calculated. Using the above loading rate, the ratio of the kinetic energy to total strain energy (E_K/E_I) remained small (less than 0.1%) indicating that inertia effects are negligible.

6.5. Model Validation

The 3D model described in the previous section was analyzed using Explicit Dynamic analysis and its results were compared against test observations. The FEM model could capture the incidence of in-plane buckling in the bars located on the compression face of the column (**Fig. 10**). The two middle bars represent in-plane buckling towards the centerline of the column that resembles what was observed during test of specimen SP1. These middle bars were located at the corners of the middle hoop that restrained them from in-plane buckling in the opposite direction as shown in **Fig. 2**.

Variation of the base moment with rotation at a section located 25 in. (635 mm.) above the column base during the test is represented in **Fig. 11** and compared to the top moment in the FEM model. To obtain the rotation at this section during the test, the curvature profile of the lower portion of the column was estimated based on displacement readings with the LVDTs located on the compression

Fig. 10. In-plane buckling of bars in the FEM model

and tension faces of the column (**Fig. 1**) and at a distance equal to 12, 24, and 36 in. (305, 610, and 914 mm.) above the column base. The estimated curvature profile was integrated to get rotations at 24 and 36 in. (610 and 914 mm.), which were then interpolated to obtain the rotation at a distance of 25 in. (635 mm.) from the column base. The test specimen exhibited larger rotations at the column base due to deflection at the column-footing interface generated by bar slip of the bars within the footing, and this behavior is not captured in the FEM model because it does not include representation of the footing. Additionally, the model underestimates the peak strength and strain-hardening behavior of the tested specimen due to the idealization regarding spring stiffness representing the restraining effect of the concrete. In other words, the spring stiffness due to the effect of concrete surrounding the bars is estimated based on the assumption that concrete has reached its residual capacity. This limit state has been chosen to ensure that conditions for bar buckling are present during the FEM analysis. The spring stiffness and thus, the amount of restraint on the bars located on the compression face of the column, is much larger at initial stages of the test and its magnitude deteriorates during loading of the specimen. An underestimation of the initial restraining effect of the concrete on reinforcing bars can lead to

differences in strength and post-yield stiffness as observed in **Fig. 11**. However, for the purpose of a “qualitative” study of buckling behavior of the bars located on the compression face, these differences are considered acceptable.

Fig. 11. Moment-rotation behavior of tested and modeled specimens

6.6. Parametric Study

The FEM model discussed in previous section (benchmark model) was employed in a parametric study to investigate the influence of several parameters on the in-plane buckling of middle and corner bars. The parameters include concrete compressive strength, longitudinal bar size and spacing, tie spacing, and cross sectional dimensions. A list of the models is given in **Table 2**. Maximum in-plane deformation of middle and corner bars of the benchmark model were compared to corresponding values of the other models to estimate the influence of the parameters under study.

7. RESULTS AND OBSERVATIONS

The FEM models listed in **Table 2** were subjected to an axial load and top rotation similar to those applied to the benchmark model. For each parameter, results from three models (including the benchmark model) were compared.

Table 2. Parametric study matrix

Model	Parameter	b×h (mm)	Bar Size			Tie Spacing (mm)			f_c (MPa)			Bar Spacing (mm)		
			6	9	11	102	127	152	24.1	34.5	55.2	150	200	300
1	Base Model	914×711												
2	Concrete Strength													
3														
4	Bar Size													
5														
6	Bar Spacing													
7														
8	Tie Spacing													
9														
10	Column Size	1067×813										235		
11	Column Size	508×406										100		

7.1. Effect of Concrete Compressive Strength (f'_c)

Models 2 and 3 in **Table 2** were defined with properties similar to the benchmark model except that they feature concrete compressive strengths (f'_c) of 3.5 and 8 ksi (24.1 and 55.2 MPa), respectively. The spring stiffness associated with concrete surrounding the bars (K_s) in Models 2 and 3 were calculated from Eq. (1) as 5.85 and 11.85 kip/in. (1.02 and 2.07 kN/mm), respectively. In-plane buckling was observed at the lower portions of middle and corner bars in both models. The maximum in-plane displacement for corner and middle bars of these two models are compared to those from the benchmark model in **Fig. 12(a) and 12(b)**. As the concrete compressive strength (f'_c) decreases, these figures reveal that initiation of in-plane buckling occurs earlier and the bars end up with larger lateral deformations. In other words, in-plane bar buckling is expected to start earlier in RC columns that are built with normal and low strength concrete due to the reduced effective stiffness associated with lower

Fig. 12. Effect of concrete compressive strength on buckling displacement in: (a) Corner bars, (b) Middle bars

concrete strength. Moreover, an RC column with a lower f'_c is likely to undergo earlier cracking and crushing under the effect of axial and flexural loads. Therefore, assuming that the outward buckling is completely prevented by transverse hoops, the support from concrete to prevent in-plane deformation of the bars would vanish faster in these columns, which reduces the resistance to in-plane buckling of the longitudinal bars. Thus, the sudden reduction of strength due to bar fracture (from kinking of the buckled region) can occur earlier in these columns. As shown in **Fig. 13**, the embedded bars in Model 3 that incorporates concrete with f'_c of 8 ksi (55.2 MPa), started to lose axial load capacity at a larger top rotation than did bars in models with f'_c of 3.5 and 5 ksi (24.1 and 34.5 MPa) (i.e. Model 2 and benchmark model, respectively).

7.2. Effect of Longitudinal Bar Size and Spacing

As suggested by FEM analysis of simple bar-spring models, larger bar sizes require more restraining force from the surrounding concrete to resist buckling. However, as the specimen is subjected to axial and flexural loads, longitudinal bars exhibit lateral deformations leading to in-plane buckling when the

Fig. 13. Variation of axial loads of the reinforcing bars during FEM analysis in: (a) Corner bars, (b) Middle bars

concrete softens. Results from FEM analysis of models with different bar sizes (Models 4 and 5) are compared with the benchmark model in **Figs. 14 and 15** for corner and middle bars in the compressive face. As noted in these figures, larger bar sizes (i.e. No. 11 and No. 9) experience larger final deformations in the buckling zone. Final lateral in-plane deformation for a No. 11 bar is approximately 50% larger than that for a No. 6. However, all bar sizes start to lose their axial strengths and buckle at approximately similar top rotations. Unlike bar size, the effect of longitudinal bar spacing seems to be insignificant on the in-plane buckling of the bars. A comparison of the results from Models 6, 1 and 7 with No. 9 longitudinal bars that were spaced at 150, 200 and 300 mm., respectively, shows that in-plane buckling initiates at similar loading level and leads to approximately the same lateral deformations in these models. It should be noted that longitudinal bar spacing may have a role in the extent of damage and stiffness deterioration in the concrete surrounding the bars during the test. However, at a specified concrete damage state implemented in these FEM models where concrete reaches its residual compressive strength, bar spacing is shown to be insignificant for in-plane buckling of the bars.

(a)

(b)

Fig. 14. Effect of bar size on buckling displacement in: (a) Corner bars, (b) Middle bars

(a)

(b)

Fig. 15. Effect of bar spacing on buckling displacement in: (a) Corner bars, (b) Middle Bars

7.3. Effect of Tie Spacing

While transverse ties are effective in preventing outward buckling of bars, they may also help the bars to resist in-plane buckling by enhancing concrete properties through confinement. Closely spaced transverse hoops will reduce softening rate of concrete and postpone its stiffness deterioration.

Therefore, longitudinal bars in columns with closely spaced ties can resist in-plane buckling longer as

the surrounding concrete that is effective in restraining them from in-plane buckling will degrade at a slower rate. However, the beneficial effect of lateral ties in limiting the in-plane buckling will disappear once the concrete crushes and loses significant amounts of strength. Three columns with transverse ties spaced at 4, 5 and 6 in. (102, 127, and 152 mm.) (Models 8, 1 and 9) which were designed in accordance with seismic provisions in *ACI 318-11* [1], are compared in **Fig. 16**. As the spacing of transverse ties increase, these columns lose their axial capacity faster and undergo larger lateral deformations under the applied loads (**Fig. 16**). However, these variations are minor since the concrete is considered to have degraded to its residual capacity in these FEM models

7.4. Effect of Column Cross Sectional Size

Models 10 and 11 in **Table 2**, respectively, represent typical perimeter moment frame columns that in multi-story buildings that satisfy *ACI 318-11* [1] seismic provisions. Model 10 is a column featuring a 42×32 in. (1067×813 mm) cross sectional dimension that is built with 4 No. 11 bars spaced at 9 ¼ in. (235 mm) on the compression face (i.e. 16 No. 11 bars within the whole cross section) whereas Model 11 represents a column, 20×16 in. (508×406 mm) in cross section that includes 4 No. 6 bars spaced at 3 15/16 in. (100 mm) on the compression side. Both columns incorporate No. 5 transverse hoops located at 5 in. (127 mm) spacing along the columns and concrete with f'_c of 5 ksi (34.5 MPa). The axial load on the models prior to the application of top rotation was 1008, and 240 kips (4483.8 and 1067.6 kN) for models 10 and 11, respectively, which is equivalent to an axial load ratio ($P/f'_c A_g$) of 15%. FEM analysis of these models reveals that in-plane buckling of longitudinal bars in Model 10 with larger bars embedded in a bigger cross section is more critical than in Models 1 and 11. In-plane buckling of bars in the column with a bigger cross section (i.e. Model 10) initiates earlier and these bars lose their axial load capacity faster (**Fig. 17**). Therefore, larger columns incorporating larger bar sizes appear to have higher propensity to in-plane buckling than smaller columns that are constructed with similar concrete properties but which incorporate smaller bar sizes.

(a)

(b)

Fig. 16. Effect of tie spacing on buckling displacement in: (a) Corner bars, (b) Middle bars

(a)

(b)

Fig. 17. Effect of cross sectional size on buckling displacement in: (a) Corner bars, (b) Middle bars

8. CONCLUSIONS

This paper presents results from a computational study of a failure mechanism observed during tests of seven full-scale RC columns at the MAST Lab subjected to a constant axial load and various lateral loading protocols. During failure of these columns, longitudinal bars were observed to buckle parallel to the face of the columns with their surrounding concrete completely crushed at the end of the

tests. Based on observations from these tests and a set of 3D FEM analysis of the lower portion of the column specimen the following results can be drawn:

1. In-plane bar buckling was observed at a stage of loading where the cover concrete as well as the concrete residing between adjacent longitudinal bars (i.e., concrete teeth) are completely crushed. This stage of loading is typically occur at large lateral drift ratios (at least 8% for monotonic and 2% for progressively increasing symmetric cyclic loading protocols) and well past the deformation capacity of RC columns under which they exhibit significant (i.e., more than 20%) strength loss.
2. The loading capabilities at the MAST Lab enabled testing of full scale column specimens at larger drift ratios compared to most of the previous tests on RC columns until they approximately lost 80% of their peak lateral loading capacities which can be regarded as one reason for not reporting the in-plane buckling in previous tests.
3. Concrete compressive strength (f'_c) is found to be an important factor in preventing the incidence of in-plane bar buckling. Concrete with a larger f'_c helps to postpone the in-plane mode of buckling and reduces the rate of axial strength loss in longitudinal bars.
4. Unlike longitudinal bar spacing, the effect of longitudinal bar size on the in-plane bar buckling is significant. Larger bar sizes (No.8 and above) are recognized to be more likely to experience in-plane buckling as their restraint demands are higher.
5. Transverse tie spacing (s) is also found to have minor beneficial effect on in-plane buckling of the bars.
6. Column cross sectional size can also be considered a controlling factor in the occurrence of in-plane bar buckling as larger columns typically incorporate larger longitudinal bars (No. 8 and above). The fact that the columns which were tested at the MAST Lab featured cross

sectional dimensions larger than those in nearly all previous tests can be regarded as another reason for not reporting the in-plane bar buckling mode in previous tests of RC columns.

7. Further study is required to investigate methods to prevent the occurrence of in-plane buckling in longitudinal bars. Crossties with 180 degree end hooks and grouted sleeve connections may have beneficial effects. However, their effects in preventing the in-plane bar buckling need to be studied.

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REFERENCES

- [1] ACI Committee 318, *Building Code Requirements for Structural Concrete (ACI 318-11) and Commentary*. Farmington Hills, MI: American Concrete Institute, 2011.
- [2] R. P. Dhakal and K. Maekawa, "Reinforcement Stability and Fracture of Cover Concrete in Reinforced Concrete Members," *J. Struct. Eng.*, vol. 128, no. 10, pp. 1253–1262, Sep. 2002.
- [3] S. Bae, A. M. Mises, and O. Bayrak, "Inelastic buckling of reinforcing bars," *J. Struct. Eng.*, vol. 131, no. 2, pp. 314–321, 2005.
- [4] G. Monti and C. Nuti, "Nonlinear cyclic behavior of reinforcing bars including buckling," *J. Struct. Eng.*, vol. 118, no. 12, pp. 3268–3284, 1992.
- [5] A. Gomes and J. Appleton, "Nonlinear cyclic stress-strain relationship of reinforcing bars including buckling," *Eng. Struct.*, vol. 19, no. 10, pp. 822–826, Oct. 1997.
- [6] M. E. Rodriguez, J. C. Botero, and J. Villa, "Cyclic stress-strain behavior of reinforcing steel including effect of buckling," *J. Struct. Eng.*, vol. 125, no. 6, pp. 605–612, 1999.
- [7] O. Bayrak and S. A. Sheikh, "Plastic hinge analysis," *J. Struct. Eng.*, vol. 127, no. 9, pp. 1092–1100, 2001.

- [8] M. P. Berry and M. O. Eberhard, "Practical performance model for bar buckling," *J. Struct. Eng.*, vol. 131, no. 7, pp. 1060–1070, 2005.
- [9] A. Nojavan, "Performance of Full-Scale Reinforced Concrete Columns Subjected to Extreme Earthquake Loading," UNIVERSITY OF MINNESOTA, 2015.
- [10] D. A. Bournas and T. C. Triantafillou, "Bar buckling in RC columns confined with composite materials," *J. Compos. Constr.*, vol. 15, no. 3, pp. 393–403, 2010.
- [11] M. Papia, G. Russo, and G. Zingone, "Instability of longitudinal bars in RC columns," *J. Struct. Eng.*, vol. 114, no. 2, pp. 445–461, 1988.
- [12] M. M. Kashani, L. N. Lowes, A. J. Crewe, and N. A. Alexander, "Nonlinear fibre element modelling of RC bridge piers considering inelastic buckling of reinforcement," *Eng. Struct.*, vol. 116, pp. 163–177, 2016.
- [13] Y. Feng, M. J. Kowalsky, and J. M. Nau, "Effect of Seismic Load History on Deformation Limit States for Longitudinal Bar Buckling in RC Circular Columns," *J. Struct. Eng.*, vol. 141, no. 8, p. 4014187, 2014.
- [14] ASTM A706/A706M-03a, "Standard Specification for Low-Alloy Steel Deformed and Plain Bars for Concrete Reinforcement," ASTM International, West Conshohocken, PA, 2003.
- [15] ASTM Standard A615/A615M-03a, "Specification for Deformed and Plain Carbon-Steel Bars for Concrete Reinforcement," ASTM International, West Conshohocken, PA, 2003.
- [16] A. Nojavan, A. E. Schultz, C. Haselton, S. Simathathien, X. Liu, and S.-H. Chao, "A New Data Set for Full-Scale Reinforced Concrete Columns under Collapse-Consistent Loading Protocols," *Earthq. Spectra*, vol. 31, no. 2, pp. 1211–1231, 2015.
- [17] M. A. Crisfield, "A fast incremental/iterative solution procedure that handles 'snap-through,'" *Comput. Struct.*, vol. 13, no. 1, pp. 55–62, 1981.
- [18] E. Ramm, *Strategies for tracing the nonlinear response near limit points*. Springer, 1981.

- [19] G. Powell and J. Simons, "Improved iteration strategy for nonlinear structures," *Int. J. Numer. Methods Eng.*, vol. 17, no. 10, pp. 1455–1467, 1981.
- [20] D. C. Kent and R. Park, "Flexural members with confined concrete," *J. Struct. Div.*, vol. 97, no. 7, pp. 1969–1990, 1971.
- [21] J. Lubliner, J. Oliver, S. Oller, and E. Onate, "A plastic-damage model for concrete," *Int. J. Solids Struct.*, vol. 25, no. 3, pp. 299–326, 1989.
- [22] J. Lee and G. L. Fenves, "Plastic-damage model for cyclic loading of concrete structures," *J. Eng. Mech.*, vol. 124, no. 8, pp. 892–900, 1998.
- [23] B. Mercan, A. E. Schultz, and H. K. Stolarski, "Finite element modeling of prestressed concrete spandrel beams," *Eng. Struct.*, vol. 32, no. 9, pp. 2804–2813, 2010.