Probabilistic Damage Identification of a Designed 9-Story Building using Modal Data in the Presence of Modeling Errors

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Abstract

Validity and accuracy of model based identification techniques such as linear finite element (FE) model updating are sensitive to modeling errors. Models used for the design and performance assessment of civil structures often contain large modeling errors for certain frequency ranges of response. In other words, modeling errors have unequal effects on different vibration modes of structures. Therefore, the performance of FE model updating for damage identification is sensitive to the type and the subset of data used and to the residual weight factors. This study proposes a process to mitigate the effects of modeling errors by selecting the optimal subset of modes and the optimal modal residual weights. Multiple model updating classes are defined based on different subsets of modes and different weight factors. Structural damage is then identified using Bayesian model class selection and model averaging techniques over the results of all the considered model updating classes. In addition, a new likelihood function is defined to allow damage identification without the need for calibrating a reference FE model. Performance of the proposed damage identification process and the new likelihood function is evaluated numerically at multiple levels of modeling errors and structural damage on

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the SAC 9-story steel moment frame. It is shown that the structural damages can be identified with negligible bias when the proposed likelihood and updating process is implemented.

Keywords: Probabilistic damage identification; modeling error; Bayesian FE model updating; Bayesian model class selection.

1. Introduction

In the structural health monitoring research community, damage identification is defined as the process of determining: (1) existence of damage; (2) location of damage; (3) severity of damage; and (4) remaining useful life of structures [1]. Among many methods that have been proposed in the past two decades [2-4], finite element (FE) model updating methods are popular for damage identification [5-10] because they provide information about the existence, location, and extent of the damage, and because in some cases the updated FE models can be used for response prediction and damage prognosis. These methods have been applied for damage identification of several civil structures in recent years [11-14]. In the FE model updating methods, a set of structural model parameters, usually stiffness of substructures (groups of finite elements), are adjusted so that the model predicted quantities of interest best match those obtained from the test data. Note that the identifiability of structural damage depends on the sensitivity of measured data (or data features) to the damage [15]. Despite the fact that the natural frequencies of civil structures often have small sensitivity to local damage, they are commonly used for assessment of structural health and performance. This is mainly due to the facts that (1) they provide a global measure of structural dynamic properties and (2) they can be easily extracted from ambient or operational vibration measurements. Damage identification through FE model updating is usually performed in two steps: a baseline/reference model is

calibrated from the initial FE model in the first step to match the data at the undamaged state of the structure, and in the second step, a second model is calibrated to represent the data of the structure at its current (potentially damaged) state. The difference between the two models indicates the location and extent of damage [12, 13, 16-18]. This damage identification process is summarized in Fig. 1.

The identification results obtained from the process shown in Fig. 1 depend on (1) the accuracy and completeness of the identified modal parameters used in the identification process, and (2) the accuracy of the initial FE model. The accuracy of the identified modal parameters are mainly affected by measurement noise, estimation uncertainties, and changing environmental conditions [19, 20]. The effects of these sources of uncertainty on model updating and damage identification results are well-documented in the literature. The effects of noisy measurements can be alleviated by using multiple sets of data and averaging as shown in [16, 17]. Damage identification under environmental variability is discussed in [21-23]. Among the aforementioned sources of uncertainty, modeling errors can be regarded as the most influential factor and its effects on model updating results has been the subject of a few studies in the past [24-26]. Goller and Schueller [24] showed that FE modeling errors will increase the variance of prediction error parameters and will result in bias for model-predicted modal parameters even after updating. They suggested that the bias should be included in predictions from the updated FE model. Goulet and Smith [25] proposed model falsification instead of identification to overcome the difficulties of addressing modeling error uncertainties. They also discussed the biased-predictions of Bayesian model updating methods. Ching and Beck stated that the poor damage identification results of their experimental test are mostly due to modeling error effects. Haukass and Gardoni [27] discuss different sources of modeling errors such as discretization of

finite elements, linear assumptions for material properties, geometrical uncertainties, or simplifications in boundary condition modeling.

This paper first investigates the effects of modeling errors on the damage identification results, and then proposes a two-step damage identification process to mitigate the effects of modeling errors. In the first step, the optimal subset of modes to be included in the FE model updating are selected using Bayesian model class selection technique [28, 29] by defining multiple model classes each with different subsets of modes. In the second step, different weight factors are defined between the eigenvalue and mode shape errors. A set of damage identification results is estimated for each weight factor, and the final estimation is obtained by averaging all the results using Bayesian model averaging technique. The effect of weight factors between the eigenvalue errors and the mode shape errors were previously studied in [30, 31].

The proposed first step is based on the hypothesis that using an optimal subset of modes for model updating will provide more accurate damage identification results compared to when using all contributing modes [32, 33]. This can be justified by the fact that modeling errors (e.g., due to discretization) have different effects on different modes, i.e., FE models are non-uniformly valid within the frequency domain. Therefore, including certain modes in the updating process will negatively affect the updating and damage identification results. This hypothesis has often (almost always) been used in an ad-hoc manner in real-world applications of model updating for complex structures. In these applications, the higher vibration modes – where generally the modeling errors are larger – or certain identified modes that cannot be paired with the initial model are not included in the updating process. The proposed first step will exclude the modes with large estimation errors. It is worth noting that the influence of modes with large

errors can be alternatively mitigated by considering relatively large standard deviations for their corresponding error functions in the likelihood.

Another challenge in formulating the damage identification process shown in Fig. 1 is the fact that modeling errors cause biased error functions, i.e., the updated model will not be an unbiased representation of the true system. To address this challenge, a new likelihood function is proposed that uses both data sets in the damaged and the undamaged conditions. By using this likelihood function, the structural damage can be identified using the initial FE model directly and there is no need to create the baseline/reference FE model. In practice, the initial models are created based on the best level of engineer's knowledge and material test data and therefore, significant modifications of the initial models to match the measured data at the undamaged state of the structure are unrealistic. These modifications can cause certain physical parameters of the model (e.g., stiffness, mass) fall outside acceptable ranges. This shortcoming is also addressed by using the proposed likelihood function.

The proposed model updating process and likelihood function are used for damage identification of a 9-story building based on its numerically simulated dynamic response. The considered test bed structure is the 9-story steel moment frame SAC building, designed for Los Angeles, California [34]. The original model is modified to reflect different levels of realistic modeling errors. Performance of the proposed identification process is evaluated considering three levels of structural damages and three levels of modeling errors.

2. Proposed likelihood function

This section presents a summary of the Bayesian FE model updating formulations used in this study. More detailed formulations of the Bayesian model updating process can be found in

seminal publications on this topic [18, 26, 32, 35-39]. According to the Bayes theorem, conditional posterior probability distributions of updating parameters $\boldsymbol{\theta}$ (vector of structural model parameters) and $\boldsymbol{\sigma}$ (vector of standard deviations of the error functions) given the measured data $\mathbf{D}$ (identified modal parameters in this study) and the model class M can be obtained by:

$$p(\boldsymbol{\theta}, \boldsymbol{\sigma} | \mathbf{D}, M) = cp(\mathbf{D} | \boldsymbol{\theta}, \boldsymbol{\sigma}, M)p(\boldsymbol{\theta}, \boldsymbol{\sigma} | M) \quad (1)$$

where $p(\mathbf{D} | \boldsymbol{\theta}, \boldsymbol{\sigma}, M)$ is the likelihood function and $p(\boldsymbol{\theta}, \boldsymbol{\sigma} | M)$ is the prior probability distribution of the updating parameters. The likelihood function is the probability of observing the measured data given the updating model parameters. The normalization constant c ensures that the posterior probability density function (PDF) integrates to one, and its value is important for model selection.

In this study, the error functions are defined based on the differences between changes from the undamaged to the damaged modal parameters computed from the model and those obtained from the measured data as shown in Equations (2) and (3).

$$e_{\lambda_m} = \frac{\lambda_m^d(\boldsymbol{\theta})}{\tilde{\lambda}_m^d} - \frac{\lambda_m^{initial}}{\tilde{\lambda}_m^h} \sim N(\mu_{\lambda_m}, \sigma_{\lambda_m}^2) \quad (2)$$

$$\mathbf{e}_{\Phi_m} = \left(\frac{\tilde{\boldsymbol{\Phi}}_m^d}{\|\tilde{\boldsymbol{\Phi}}_m^d\|} - \frac{\tilde{\boldsymbol{\Phi}}_m^h}{\|\tilde{\boldsymbol{\Phi}}_m^h\|} \right) - \left(a_m^d \frac{\boldsymbol{\Gamma}\boldsymbol{\Phi}_m^d(\boldsymbol{\theta})}{\|\boldsymbol{\Gamma}\boldsymbol{\Phi}_m^d(\boldsymbol{\theta})\|} - a_m^h \frac{\boldsymbol{\Gamma}\boldsymbol{\Phi}_m^{initial}}{\|\boldsymbol{\Gamma}\boldsymbol{\Phi}_m^{initial}\|} \right) \sim N(\boldsymbol{\mu}_{\Phi_m}, \boldsymbol{\Sigma}_{\Phi_m}) \quad (3)$$

where $N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ denotes a Gaussian distribution with mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Sigma}$. The superscript d denotes the damaged state and the superscript h refers to the undamaged state of structure. In Equations (2) and (3), $\lambda_m^d(\boldsymbol{\theta})$ and $\boldsymbol{\Phi}_m^d(\boldsymbol{\theta})$ are the model-calculated eigenvalue and mode shapes

of mode m in the damaged state. Similarly, $\lambda_m^{initial}$ and $\Phi_m^{initial}$ are the model-calculated eigenvalue and mode shape of mode m from the initial FE model. The identified eigenvalue and mode shape of mode m are shown by $\tilde{\lambda}_m^d$ and $\tilde{\Phi}_m^d$ at the damaged condition and $\tilde{\lambda}_m^h$ and $\tilde{\Phi}_m^h$ at the undamaged condition. The matrix Γ picks the observed DOFs from the model-calculated mode shapes. The scaling factors a_m^h and a_m^d are defined in Equations (4) and (5):

$$a_m^h = \frac{(\tilde{\Phi}_m^h)^T \cdot \Gamma \Phi_m^{initial}}{\|\Gamma \Phi_m^{initial}\|^2} \quad (4)$$

$$a_m^d = \frac{(\tilde{\Phi}_m^d)^T \cdot \Gamma \Phi_m^d(\theta)}{\|\Gamma \Phi_m^d(\theta)\|^2} \quad (5)$$

Further details about the derivation of the proposed error functions in Equations (2) and (3) can be found in the Appendix.

If the error functions are assumed to have zero means, the likelihood function can be written as:

$$p(\mathbf{D} | \theta, \sigma_{\lambda_{lm}}, \sigma_{\Phi_{lm}}) = \frac{1}{(\sqrt{2\pi})^{N_m(N_s+1)}} \prod_{m=1}^{N_m} \sigma_{\lambda_m} \prod_{m=1}^{N_m} \sigma_{\Phi_m} \exp \left(-\frac{1}{2} \sum_{m=1}^{N_m} \frac{(e_{\lambda_m})^2}{\sigma_{\lambda_m}^2} - \frac{1}{2} \sum_{m=1}^{N_m} \frac{e_{\Phi_m}^T e_{\Phi_m}}{\sigma_{\Phi_m}^2} \right) \quad (6)$$

where N_m and N_s are the number of identified modes and the number of identified mode shape components (number of sensors), respectively. Considering the following simplifying assumptions on the standard deviation terms of the error functions:

$$\sigma_{\lambda_1} = \sigma_{\lambda_2} = \dots = \sigma_{\lambda_m} = \sigma_{\lambda} \quad (7)$$

$$\sigma_{\Phi}^2 = w N_s \sigma_{\lambda}^2 \quad (8)$$

Equation (6) can be written as

$$p(\mathbf{D}|\boldsymbol{\theta}, \sigma_\lambda) = \frac{1}{\left(\sqrt{2\pi}\right)^{N_m(N_s+1)} \sigma_\lambda^{N_m(N_s+1)} \sqrt{wN_s}^{N_mN_s}} \exp\left(-\frac{1}{2\sigma_\lambda^2} \sum_{m=1}^{N_m} (e_{\lambda_m})^2 + \frac{1}{wN_s} e_{\Phi_m}^T e_{\Phi_m}\right) \quad (9)$$

where w is the weigh factor. The proposed likelihood function of Equation 9 can be evaluated using measurements at the two states of the structure and the modal parameters predicted by the initial model and the updated model. This will eliminate the need for calibration of a reference model as illustrated in Fig. 1.

For small to moderate levels of structural damage, the effects of modeling errors are expected to be similar on both healthy and damaged models. Therefore, even in the presence of modeling errors, the error functions of Equations (2) and (3) are expected to be almost unbiased, justifying the zero-mean assumption. The formulation assumes that the model prediction errors for the mode shape components within a given mode are uncorrelated in space. Also, based on the Equations (7) and (8), the variances for all model prediction errors for either the natural frequencies or the mode shapes are assumed to be the same. These assumptions have been widely used in the literature [16-18, 24, 26, 29, 30, 38]. In particular, the effect of spatial correlation on the model prediction errors of the mode shape components have been considered in [40] and shown to affect inference for relatively dense sensor arrays. In relation to the wave length of the mode, our proposed approach can readily be extended to include such spatial correlations. In a recent study [41], the authors compared the performance of the considered likelihood function for damage identification with two other likelihood functions that are commonly used in the literature. From that comparison, similar accuracy was observed for damage identification results from the considered likelihood functions. However, the likelihood function of Eq. (2-3) simplifies the identification process and reduces the computational cost

since no calibration is required at the reference/undamaged state. Moreover, the estimation of unrealistic structural parameters at the undamaged state is avoided.

3. Proposed damage identification process

A two-step damage identification process is proposed to mitigate the effects of modeling errors on different error functions (residuals). In the first step, the optimal subset of modes will be selected to minimize the effects of modeling errors on model updating residuals. Multiple model classes ($M_j \in \mathbf{M}$) are defined based on different subsets of the contributing modes, and the probability of each model class will be calculated by Equation (10).

$$p(M_j | \mathbf{D}, \mathbf{M}) = \frac{p(\mathbf{D} | M_j) p(M_j | \mathbf{M})}{p(\mathbf{D} | \mathbf{M})} = \frac{p(\mathbf{D} | M_j) p(M_j | \mathbf{M})}{\sum_{j=1}^{N_M} p(\mathbf{D} | M_j) p(M_j | \mathbf{M})} \quad (10)$$

The model class with the largest posterior probability will be selected. It should be noted that all model classes include exactly the same parameters in the parameter set $\boldsymbol{\theta}$.

In the second step, the effects of residual weights on the identification results will be addressed. Another set of model classes are defined with different weight factors w (see Equation 8). The optimal subset of modes from the first step will be used in the second step. It should be noted that in this step, more than one model class may have high posterior probability. Therefore, Bayesian Model Averaging [42, 43] is used to estimate the probability of a quantity of interest $\boldsymbol{\theta}$ (stiffness reduction factor):

$$p(\boldsymbol{\theta} | \mathbf{D}, \mathbf{M}) = \sum_{j=1}^{N_M} p(\boldsymbol{\theta} | \mathbf{D}, M_j) p(M_j | \mathbf{D}) \quad (11)$$

In particular, the expected value of $\boldsymbol{\theta}$ is

$$E[\boldsymbol{\theta} | \mathbf{D}, \mathbf{M}] = \sum_{j=1}^{N_M} \hat{\boldsymbol{\theta}}_j p(M_j | \mathbf{D}) \quad (12)$$

where $\hat{\boldsymbol{\theta}}_j = E[\boldsymbol{\theta} | \mathbf{D}, M_j]$. A measure of the uncertainty in $\boldsymbol{\theta}$ can also be obtained by extending Equation (12) to estimate the second moment and then the standard deviation from the first and second moments [42]. The posterior probability of the model classes and the maximum a-posteriori (MAP) can be estimated through numerical sampling techniques or analytical approximation, as discussed in the next two subsections.

3.1. Posterior probability estimation through sampling

This section reviews the process of estimating the posterior probability of each model class using the Markov Chain Monte Carlo (MCMC) sampling technique. As the first step of this process, N_{MC} samples are generated from the posterior probability distribution function of Equation (1) for each of the model classes. The posterior probability of each model class is then calculated using Equation (10). In this equation, the prior probability of different model classes $p(M_j | \mathbf{M})$ are assumed to be equal and thus are taken as $1/N_M$, where N_M is the number of considered model classes. The evidence can be estimated using the N_{MC} samples of the posterior probability distribution as follows [44-46]:

$$p(\mathbf{D} | M_j) = N_{MC} \left(\sum_{k=1}^{N_{MC}} \frac{g(\boldsymbol{\theta}^k)}{p(\mathbf{D} | \boldsymbol{\theta}^k, M_j) p(\boldsymbol{\theta}^k | M_j)} \right)^{-1} \quad (13)$$

where $g(\boldsymbol{\theta})$ is the importance sampling density and can be taken as the prior, therefore:

$$p(\mathbf{D} | M_j) = N_{MC} \left(\sum_{k=1}^{N_{MC}} \frac{1}{p(\mathbf{D} | \boldsymbol{\theta}^k, M_j)} \right)^{-1} \quad (14)$$

3.2 Posterior probability estimation through asymptotic approximation

Generating MCMC samples for each model class is computationally expensive. The number of model classes needed in a specific application can become very large (e.g., in order of hundreds), and therefore making the sampling technique impractical for posterior estimation. This section reviews a simplified procedure for estimation of the posterior probability of a model class with very small computational efforts and reasonable accuracy. In the case of uniform prior probability, the most probable structural model parameters can be estimated by minimizing the natural log posterior PDF of Equation (9) with respect to $\boldsymbol{\theta}$:

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} \left(\sum_{m=1}^{N_m} (e_{\lambda_m})^2 + \frac{1}{wN_s} e_{\boldsymbol{\Phi}_m}^T e_{\boldsymbol{\Phi}_m} \right) \quad (15)$$

Equation (15) can be solved using any available global optimization method (multistart function of MATLAB [47] is used in this study). In the next step, the most probable error standard deviation (non-structural updating parameter) can be estimated using the estimated $\hat{\boldsymbol{\theta}}$ from Equation (15), by taking the derivative of natural log posterior PDF of Equation (1) with respect to σ :

$$\hat{\sigma}_{\lambda}^2 = \frac{\sum_{m=1}^{N_m} \hat{e}_{\lambda_m}^2 + \frac{1}{wN_s} \hat{\mathbf{e}}_{\boldsymbol{\Phi}_m}^T \hat{\mathbf{e}}_{\boldsymbol{\Phi}_m}}{N_m(N_s + 1)} \quad (16)$$

where $\hat{e}_{\lambda}$ and $\hat{\mathbf{e}}_{\boldsymbol{\Phi}}$ are the prediction errors at $\hat{\boldsymbol{\theta}}$. Finally, the evidence terms can be approximated by Laplace's method [28, 48]:

$$\operatorname{Ln}(p(\mathbf{D} | M_j)) \approx \operatorname{Ln}(p(\mathbf{D} | \hat{\boldsymbol{\theta}}, \hat{\sigma}_{\lambda}, M_j)) + \operatorname{Ln} \left(p(\hat{\boldsymbol{\theta}}, \hat{\sigma}_{\lambda} | M_j) (2\pi)^{N_p/2} |\mathbf{H}(\hat{\boldsymbol{\theta}}, \hat{\sigma}_{\lambda})|^{-1/2} \right) \quad (17)$$

where $\mathbf{H}$ is the Hessian matrix of $-Ln\left(p(\mathbf{D}|\boldsymbol{\theta},\sigma_\lambda,M_j)p(\boldsymbol{\theta},\sigma_\lambda|M_j)\right)$ and N_p is the size of model parameters. The asymptotic approximation (17) requires the evaluation of the Hessian $\mathbf{H}$ at the most probable value of the parameters. Beck and Yuen [28] showed that the evidence can be approximated as the following, which is known as the Bayesian Information Criterion [49, 50]:

$$Ln\left(p(\mathbf{D}|M_j)\right) \approx Ln\left(p(\mathbf{D}|\hat{\boldsymbol{\theta}},\hat{\sigma}_\lambda,M_j)\right) - \frac{N_p}{2} Ln(n) \quad (18)$$

where n is the number of data points used in the optimization.

4. Numerical Application

4.1. SAC 9-story building

The considered SAC 9-story steel moment frame is shown in Fig. 2. The building was designed for Los Angeles, California according to the ASCE-7 [51]. The floor masses are reported as 1,067 metric tons (73.10 kips-sec²/ft) for the roof, 991 tons (67.86 kips-sec²/ft) for floors 9 to 3, and 1,008 tons (69.04 kips-sec²/ft) for floor 2. For simplicity in this study, the floor masses are all considered as lumped masses and only in the horizontal direction. Steel elastic Young's modulus is reported as 200.02 GPa (29,000 ksi). To simulate the effects of fabrication and construction errors, the following changes are assumed between the original design and the constructed building.

The true/exact structural model is shown in Fig. 3 as model M0 and includes the following changes to the original design:

- a) 2nd floor is moved 2.54 cm (1 inch) downward.
- b) 4th floor is moved 5.08 cm (2 inch) upward.

- c) 6th floor is moved 5.08 cm (2 inch) downward.
- d) 9th floor is moved 5.08 cm (2 inch) upward.
- e) Axis B is moved 5.08 cm (2 inch) to the right, while axes A and C are fixed.
- f) Axis E is moved 5.08 cm (1 inch) to the left, while axes D and F are fixed.
- g) Elastic Young's modulus of W14×500 and W36×160 is set as 200.09 GPa (29,011 ksi).
- h) Elastic Young's modulus of W14×455 and W36×135 is set as 200.16 GPa (29,021 ksi).
- i) Elastic Young's modulus of W14×283 and W30×99 is set as 200.07 GPa (29,008 ksi).
- j) Elastic Young's modulus of W14×257, W14×233, W24×68, and W27×84 is set as 200.12 GPa (29,015 ksi).
- k) First floor's support is horizontally bounded with a spring of stiffness 50.8 GN/m (2.9×10^5 kips/in).
- l) The supports on the basement are modeled with rotational springs with the stiffness of 1.31 GN-m/rad (1.16×10^7 kips-in/rad).
- m) Mass of the roof is set as 1069 tons (73.25 kips-sec²/ft); floor 9 to 6 as 981 tons (67.20 kips-sec²/ft); floor 6 to 3 as 997 tons (68.30 kips-sec²/ft); and floor 2 as 1001 tons (68.54 kips-sec²/ft). Mass of first floor is also set as 1001 tons (68.54 kips-sec²/ft).

To evaluate the effects of modeling errors on model updating results, three initial FE models with three levels of modeling errors are considered.

M1. Minor modeling errors: all the changes to the original SAC model (points *a* to *m*) are considered. The modeling errors are in the column section properties. Locations of column splices are modified from the real model M0 (see Fig. 3b). Model M1 has very minor modeling error and therefore, it is expected to provide minimal estimation errors in the identification results.

M2. Moderate modeling errors: contains modeling errors of M1 and also neglects points g to j and point l . The basement supports are also modeled as pins (see Fig. 3c).

M3. Large modeling errors: contains all the errors of M1 plus those induced by neglecting points a to m . The bay lengths and story heights are modeled as the original SAC model shown in Fig. 2. The story masses in this model are assumed as 1067 tons (73.10) for the roof, 991 tons (67.86) for floors 9 to 3, and 1008 tons (69.04) for floor 2. The first floor's support is modeled as a pin in the horizontal direction (see Fig. 3d).

In Fig. 3, the modeling errors in each initial FE model are circled. Models M1 to M3 are the initial FE models to be used in the proposed damage identification process. Note that these FE models are not considered as different model classes in the Bayesian FE model updating, but are introduced to evaluate the performance of the proposed approach at different levels of modeling error. The MAC values between the mode shapes of the initial models M1 to M3 and those of the true model M0 are reported in Table 1. Since the initial model M3 has 9 DOFs with non-zero masses, a maximum of 9 modes are considered for all model updating cases. Modes 8 and 9 of the true model cannot both be formed separately in the M3 model. For the 8th mode of model M3, MAC values are calculated with both 8th and 9th modes of model M0. As it can be seen from Table 1, higher modes are more affected by modeling errors. The MAC values of the first 5 modes are all close to one, but they decrease for higher modes (0.59 for 8th mode of model M2). These effects are not systematic as the MAC values of the 10th mode are close to one, or the initial model M3 shows better MAC values than the initial model M2 although it has larger modeling errors.

Natural frequencies of the four initial models are shown as bar plots in Fig. 4. The maximum changes in the natural frequencies are observed for Model M3. Although model M2 has larger

modeling errors than model M1, its natural frequencies (except mode 8) are closer to the natural frequencies of the exact model M0. This underlines the complex effects of modeling errors on modal parameters.

4.2. Damage scenarios

Overall 10 model parameters are updated for each initial model, including one standard deviation (σ_λ), and 9 structural model parameters (θ) defined as the effective Young's modulus factor of all the beams and columns of a story (elements of the first and second stories are considered a one updating parameter). Three arbitrary damage scenarios of different levels are simulated in this study:

D0 (no damage)

D1 (small damage): $\theta_2 = 0.9$, i.e., 10% loss of stiffness in the beams and columns of the second story.

D2 (moderate damage): $[\theta_2 \ \theta_3 \ \theta_7] = [0.7 \ 0.8 \ 0.8]$

D3 (severe damage): $[\theta_1 \ \theta_2 \ \theta_3 \ \theta_4 \ \theta_7] = [0.7 \ 0.5 \ 0.6 \ 0.8 \ 0.7]$

The effective Young's moduli of the finite elements in the damaged structure are defined as $E_i = \theta_i E_i^0$, where θ_i is the i^{th} updating parameter, E_i^0 is the undamaged and E_i is the damaged effective Young's modulus of i^{th} parameter. Natural frequencies of the exact model M0 at all four damage states are also plotted in Fig. 4. These natural frequencies and their corresponding mode shapes will be considered as the “identified” modal parameters in the absence of noise or estimation error and are used for FE model updating. The changes in the natural frequencies of M0 at damage case D2 are in the same range as the changes in the natural frequencies of the initial model M3 with no damage.

4.3. *Effects of modeling errors on damage identification results*

In this section, effects of modeling errors and subsets of data used in the updating process are studied on the damage identification results. Twelve cases of FE model updating are performed using three different subsets of modes and the four initial FE models M0-M3. The three considered subsets of modes are the natural frequencies and mode shapes of (1) all vibration modes except mode 8, (2) the first 6 modes, and (3) the first 3 modes. As a representative of damage identification results, updated model parameters at the moderate damage level D2 are provided in Table 2. Note that these results are based on noise-free identified modal parameters and complete mode shapes (i.e., no missing mode shape components). The stiffness reduction factors with estimation errors larger than 0.05 are shown with bold-underlined font, representing a relatively poor identification.

From Table 2, it can be seen that the results are very accurate if no modeling errors are present (i.e., initial model M0 is used) regardless of the subset of modes used in the updating process. In the presence of modeling errors, the number of poor identification cases generally increases by using initial models with larger modeling errors. As expected, the updating results are sensitive to the subset of modes used. However, it is interesting to see that more poor identifications are observed when nine modes are used during the updating process. The model parameter estimates become more accurate when using the first six modes as compared to those using the first three modes, but adding more modes does not improve the accuracy of the updating results. The increase in the number of poor identification cases when using nine modes is due to the fact that modeling errors have larger effects on the higher vibration modes (see Table 1). Therefore, and as a starter, it is recommended to exclude the modes with low MAC

values between the mode shapes of the initial FE model and the corresponding identified mode shapes.

5. Damage identification results using the proposed approach

The proposed Bayesian FE model updating process is implemented for damage identification of the SAC building for the three considered damage scenarios D1, D2, and D3 and using the three initial models M1, M2, and M3. In this application, to simulate the effects of measurement noise and system identification errors, the “identified” natural frequencies and mode shape components are polluted with 0.5% and 2% white noise processes, respectively. In addition, the identified mode shapes are assumed to be incomplete, i.e., only half of the mode shape components (at the 2nd, 4th, 6th, 8th floor and the roof) are assumed to be available. The first nine vibration modes are assumed to be identifiable. Fourteen different subsets of modes are considered for model updating and are shown in Table 3. Each subset of modes defines a model class in the proposed identification process. Certain modes are not considered when the modeling errors are relatively high. For example, modes 7, 8, 9 are excluded when the initial model M2 is used for updating, and modes 8, 9 are excluded when the initial model M3 is used. This is due to the fact that the effect of modeling errors on these modes is very high (low MAC values) as it can be seen in Table 1.

5.1. Identified results from MCMC samples

In the first step of the proposed identification process, for each of the three initial models and the three damage scenarios (total of nine cases), 15000 MCMC samples are generated per model class using the adaptive Metropolis-Hastings algorithm of [52]. In this algorithm, the adaption is performed on the proposal probability distribution functions of the standard Metropolis-Hastings

algorithm [53, 54]. As an illustration, the MAP estimates of the model parameters for the case of M2 initial model and D3 damage level are provided in Table 4 for each model class of Table 3. The probability of each model class is calculated using Equations (10) and (14), and is shown in the last column. Based on these results, the best subset of modes is the first 6 modes or model class iv although the identified MAP values have some bias, especially for θ_5 , θ_6 , and θ_8 . The results of Table 4 are obtained for a fixed weight factor of one.

In the second step of the proposed updating process, 15 model classes with different weight factors ranging from 0.01 to 10 are defined and the FE model updating is performed for each model class in order to obtain the most probable updating parameters through model averaging (Equation 12). Larger weight factors give more importance to the eigenvalue errors in the likelihood function compared to the mode shape errors (see Equation 8). The optimal subset of modes selected in the previous step is used for all the 15 model classes of this step. Table 5 shows the MAP values of updating parameters for damage scenario D3 and the initial model M2 when different weight factors are used. This table also provides the error function standard deviation and posterior probability of the 15 model classes with different weight factors. It can be seen that the accuracy of the MAP estimates is sensitive to the selected weight factors, for example weight factors 1 and 10 provide inaccurate updating parameters and have zero posterior probabilities. Unlike the first step, the posterior probability of more than one model can be significant and therefore the updating parameters are estimated using the Bayesian model averaging.

The final MAP estimates of the updating parameters for each of the nine identification cases (M1-M3 and D1-D3) are provided in Table 6. The MAP estimates are computed using the optimal subset of the modes and averaging over different weight factors. The third column of

Table 6 shows the selected model class number listed in Table 3 while the fourth column of the table provides the most probable weight factor and its posterior probability in the parentheses. Except the case M3-D3, the case with largest modeling errors and most severe structural damage, updating parameters are estimated with relatively good accuracy.

5.2. *Identified results using asymptotic approximation*

In this section, the proposed updating process is implemented using the asymptotic approximation described previously. Table 7 reports the estimated MAP values of the updating parameters using the asymptotic approximation procedure. The results can be compared to their counterparts using MCMC sampling method of Table 6. It can be observed that the asymptotic approximation provides reasonably accurate identification results in a fraction of the computational cost. The asymptotic approximation method is especially useful when the number of considered model classes gets large. More details about the accuracy of the asymptotic approximation method can be found in [48].

6. **Conclusions**

The effects of modeling errors on damage identification results obtained from FE model updating are systematically studied using the numerically simulated modal data of a 9-story steel moment frame. The study is performed using three initial FE models with different levels of modeling errors and at three levels of damage severity. It has been observed that identified damage (stiffness reduction factors in this study) is very sensitive to (1) the subset of modes used, and (2) the weight factors assigned to different error functions (residuals). It is also shown that including a larger number of modes in the updating process does not necessarily improve the damage identification results. To further mitigate the effects of modeling errors in the updating

process, a two-step damage identification process is proposed. In the first step, multiple model classes are defined using different subsets of modes. The optimum subset is selected using a Bayesian model class selection technique. In the second step, different weight factors are defined for the residuals and FE model updating is performed for each of the considered weight factors. The optimal updating parameters are estimated from model averaging (weighted by model evidences) over the updating cases with different weight factors. A new likelihood function is also proposed to be used in the damage identification process, including the modal data at both damaged and undamaged states of the structure. Use of the proposed likelihood function eliminates the need to update a reference FE model and therefore the initial FE model can be used directly for identification purposes.

The proposed model updating process provided accurate damage identification results for the considered structure, except for the case of the largest modeling errors M3 when combined with the most severe structural damage D3. It is also observed that the results obtained from an asymptotic approximation are in good agreement with those obtained from MCMC samples. Therefore, the asymptotic approximation is recommended as an efficient and relatively accurate alternative to MCMC in the real-world applications where the number of model classes is large. Overall, the combination of using the proposed likelihood function, the optimal subset of modes, and the optimal weight factors provides drastic improvement to the damage identification results in the presence of modeling errors. Thus, the use of this framework is recommended in applications to civil structures when a large number of vibration modes are available to be used for model updating. It should be noted that the considered modeling errors of this study, cannot fully represent all types of modeling uncertainties that are experienced in real-world applications.

Therefore, the proposed damage identification process needs to be further examined on different types of operational, full-scale structures using in-situ data.

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APPENDIX

This section provides a discussion and more details about derivation of the proposed error functions in Equations (2) and (3). Following the classical damage identification approach of performing two sets of FE model updating based on the data at damaged and undamaged conditions, the eigenvalue error function (shown by $\mathcal{E}$ here) of mode m in damaged condition can be written as:

$$\mathcal{E}_{\lambda_m}^d = \frac{\tilde{\lambda}_m^d - \lambda_m^d(\boldsymbol{\theta})}{\tilde{\lambda}_m^d} = 1 - \frac{\lambda_m^d(\boldsymbol{\theta})}{\tilde{\lambda}_m^d} \quad (\text{A-1})$$

where $\mathcal{E}_{\lambda_m}^d$ is due to measurement error $\mathcal{E}_{\lambda_m, meas}^d$ and model error $\mathcal{E}_{\lambda_m, model}^d$, i.e.,

$$\mathcal{E}_{\lambda_m}^d = \mathcal{E}_{\lambda_m, meas}^d + \mathcal{E}_{\lambda_m, model}^d \quad (\text{A-2})$$

The measurement error is best represented by a zero mean Gaussian distribution,

$\mathcal{E}_{\lambda_m, meas}^d \sim N\left(0, \left(\tilde{\sigma}_{\lambda_m}^d\right)^2\right)$. The model error can be considered as a Gaussian distribution with non-

zero mean (to account for bias), $\varepsilon_{\lambda_m, model}^d \sim N\left(\bar{\mu}_{\lambda_m}^d, \left(\bar{\sigma}_{\lambda_m}^d\right)^2\right)$. Thus, the total model error is also

Gaussian with non-zero mean to account for the bias, $\varepsilon_{\lambda_m}^d \sim N\left(\bar{\mu}_{\lambda_m}^d, \left(\tilde{\sigma}_{\lambda_m}^d\right)^2 + \left(\bar{\sigma}_{\lambda_m}^d\right)^2\right)$. The error

bias should be considered for predicting a response from the calibrated models [24, 55].

A similar analysis applies for the error function between the identified eigenvalues at the undamaged state and the initial model.

$$\varepsilon_{\lambda_m}^h = \frac{\tilde{\lambda}_m^h - \lambda_m^{initial}}{\tilde{\lambda}_m^h} = 1 - \frac{\lambda_m^{initial}}{\tilde{\lambda}_m^h} \quad (\text{A-3})$$

where $\varepsilon_{\lambda_m}^h$ is due to measurement error $\varepsilon_{\lambda_m, meas}^h$ and model error $\varepsilon_{\lambda_m, model}^{initial}$, i.e.,

$$\varepsilon_{\lambda_m}^h = \varepsilon_{\lambda_m, meas}^h + \varepsilon_{\lambda_m, model}^{initial} \quad (\text{A-4})$$

The measurement error here is also assumed to have a zero mean Gaussian distribution,

$\varepsilon_{\lambda_m, meas}^h \sim N\left(0, \left(\tilde{\sigma}_{\lambda_m}^h\right)^2\right)$. Since the initial model is not updated, the model error $\varepsilon_{\lambda_m, model}^{initial}$ is

deterministic and has a value $\bar{\mu}_{\lambda_m}^{initial}$ that accounts for the bias. Thus, the error function is also

Gaussian with non-zero mean to account for the bias, $\varepsilon_{\lambda_m}^h \sim N\left(\bar{\mu}_{\lambda_m}^{initial}, \left(\tilde{\sigma}_{\lambda_m}^h\right)^2\right)$.

The error in Equation (2) is defined as the difference

$e_{\lambda_m} = \varepsilon_{\lambda_m}^d - \varepsilon_{\lambda_m}^h \sim N\left(\bar{\mu}_{\lambda_m}^d - \bar{\mu}_{\lambda_m}^{initial}, \left(\tilde{\sigma}_{\lambda_m}^h\right)^2 + \left(\tilde{\sigma}_{\lambda_m}^d\right)^2 + \left(\bar{\sigma}_{\lambda_m}^d\right)^2\right)$ with mean $\mu_{\lambda_m} = \bar{\mu}_{\lambda_m}^d - \bar{\mu}_{\lambda_m}^{initial}$ and

variance $\sigma_{\lambda_m}^2 = \left(\tilde{\sigma}_{\lambda_m}^h\right)^2 + \left(\tilde{\sigma}_{\lambda_m}^d\right)^2 + \left(\bar{\sigma}_{\lambda_m}^d\right)^2$. The mean accounts for the difference of the bias in the

undamaged/initial and damaged states. Assuming same/similar bias in the undamaged and

damaged states yields $\mu_{\lambda_m} = \bar{\mu}_{\lambda_m}^d - \bar{\mu}_{\lambda_m}^{initial} = 0$. This assumption gets violated as the level of structural damage increases.

Following a similar analysis for the mode shapes, the error function in Equation (3) can be derived as:

$$\mathbf{e}_{\Phi_m} = \boldsymbol{\varepsilon}_{\Phi_m}^d - \boldsymbol{\varepsilon}_{\Phi_m}^h \sim N\left(\bar{\boldsymbol{\mu}}_{\Phi_m}^d - \bar{\boldsymbol{\mu}}_{\Phi_m}^{initial}, \tilde{\boldsymbol{\Sigma}}_{\Phi_m}^h + \tilde{\boldsymbol{\Sigma}}_{\Phi_m}^d + \bar{\boldsymbol{\Sigma}}_{\Phi_m}^d\right) \sim N\left(\boldsymbol{\mu}_{\Phi_m}, \boldsymbol{\Sigma}_{\Phi_m}\right) \quad (\text{A-5})$$

where $\tilde{\boldsymbol{\Sigma}}_{\Phi_m}^h + \tilde{\boldsymbol{\Sigma}}_{\Phi_m}^d + \bar{\boldsymbol{\Sigma}}_{\Phi_m}^d$ includes covariance from both measurement and model errors. The

measurement error covariance matrices are usually assumed to be diagonal, i.e., $\tilde{\boldsymbol{\Sigma}}_{\Phi_m}^h = \left(\tilde{\sigma}_{\Phi_m}^h\right)^2 \mathbf{I}$

and $\tilde{\boldsymbol{\Sigma}}_{\Phi_m}^d = \left(\tilde{\sigma}_{\Phi_m}^d\right)^2 \mathbf{I}$, while the model error covariance matrix $\bar{\boldsymbol{\Sigma}}_{\Phi_m}^d$ is usually a full matrix. In

this paper, the latter covariance matrix is also assumed to be diagonal. If the variances for all mode shape components and different modes are identical, the covariance matrix of mode shape errors can be written as: $\boldsymbol{\Sigma}_{\Phi_m} = \sigma_{\Phi_m}^2 \mathbf{I} = \sigma_{\Phi}^2 \mathbf{I}$. Similar to eigenvalue error function, the mean of

mode shape error function in Equation (A-5) is assumed to be zero, i.e., $\boldsymbol{\mu}_{\Phi_m} = \bar{\boldsymbol{\mu}}_{\Phi_m}^d - \bar{\boldsymbol{\mu}}_{\Phi_m}^{initial} = \mathbf{0}$.

This assumption becomes less realistic with increasing level of structural damage.

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Table 1. MAC values between mode shapes of initial models M1 to M3 to those of model M0

Mode # of M0	1	2	3	4	5	6	7	8	9	10
M1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98	0.98	1.00
M2	1.00	1.00	1.00	1.00	0.99	0.96	0.76	0.59	0.73	0.98
M3	1.00	1.00	0.99	0.99	0.99	0.98	0.95	0.83*	0.81*	0.99*

* MAC is calculated for M3 mode shapes of modes 8, 8, 9 respectively.

Table 2. Effects of modeling errors on model updating results of case D2

Identified modes	FE Model	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9
Exact values	-	1.00	0.70	0.80	1.00	1.00	1.00	0.80	1.00	1.00
Nine modes (all except 8)	M0	1.00	0.70	0.80	1.00	1.00	1.00	0.80	1.00	1.00
	M1	1.02	0.72	0.79	<u>0.94</u>*	1.03	1.00	0.81	0.99	1.01
	M2	0.97	<u>0.81</u>	0.76	0.95	0.97	1.01	0.84	1.01	0.99
	M3	<u>0.92</u>	<u>0.87</u>	0.82	<u>0.89</u>	1.05	0.97	0.80	1.03	1.00
First six modes	M0	1.00	0.70	0.80	1.00	1.00	1.00	0.80	1.00	1.00
	M1	1.01	0.69	0.82	0.96	1.03	0.99	0.80	0.99	1.01
	M2	1.00	0.69	0.83	<u>0.94</u>	1.04	0.98	0.80	0.99	1.02
	M3	1.00	0.68	0.82	0.96	1.02	1.00	0.81	1.00	0.99
First three modes	M0	1.00	0.70	0.80	1.00	1.00	1.00	0.80	1.00	1.00
	M1	1.00	0.70	0.81	0.99	1.00	1.01	0.80	1.00	1.00
	M2	1.00	0.67	0.85	0.97	1.01	1.00	0.79	1.00	1.00
	M3	0.99	<u>0.78</u>	<u>0.72</u>	1.03	0.99	1.02	0.81	0.99	1.00

* Bold, underlined values represent identification results with more than 0.05 error.

Table 3. Modes used in the updating process for each model class

Model Class	i	ii	iii	iv	v	vi	vii	viii	ix	x*	xi ⁺	xii ⁺	xiii ⁺	xiv ⁺
Mode 1	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓
Mode 2	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Mode 3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Mode 4		✓	✓	✓				✓	✓	✓	✓	✓	✓	✓
Mode 5			✓	✓	✓	✓			✓	✓	✓	✓	✓	✓
Mode 6				✓		✓	✓	✓	✓	✓	✓			✓
Mode 7										✓		✓		
Mode 8											✓		✓	
Mode 9														✓

* This model class is considered when the initial models M1 or M3 is used during FE model updating process

+ These model classes are considered when the initial model M1 is used during FE model updating process

Table 4. Identified parameters for different model classes using model M2 at damage case D3

Model class	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	$Ln(\sigma_i)$	$p(M_j \mathbf{D})$
Exact	0.70	0.50	0.60	0.80	1.00	1.00	0.70	1.00	1.00	-	-
i	0.68	0.48	0.66	0.77	0.97	1.07	0.67	1.02	1.00	-5.29	0.00
ii	0.69	0.50	0.59	0.81	0.98	1.00	0.68	1.01	1.01	-5.26	0.00
iii	0.67	0.53	0.57	0.81	1.02	0.98	0.69	1.00	1.03	-4.97	0.00
iv	0.69	0.52	0.57	0.76	1.06	0.92	0.74	0.94	1.04	-4.90	1.00
v	0.67	0.53	0.57	0.82	1.00	1.07	0.67	1.00	1.04	-4.94	0.00
vi	0.70	0.52	0.56	0.78	1.07	0.90	0.74	0.94	1.03	-4.88	0.00
vii	0.65	0.52	0.68	0.65	1.32	0.87	0.73	1.00	0.96	-5.18	0.00
viii	0.67	0.52	0.60	0.73	1.15	0.89	0.75	0.94	1.02	-5.13	0.00
ix	0.69	0.52	0.57	0.77	1.06	0.92	0.74	0.94	1.04	-4.85	0.00

Table 5. Updating parameters for different weight factors using model M2
at damage scenario D3

Weight Factor	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	$Ln(\sigma_i)$	$p(M_j \mathbf{D})$
Exact	0.70	0.50	0.60	0.80	1.00	1.00	0.70	1.00	1.00	-	-
0.01	0.69	0.54	0.55	0.81	0.96	0.99	0.70	1.03	0.99	-2.97	0.00
0.05	0.69	0.54	0.55	0.79	0.98	0.98	0.70	1.01	0.99	-3.74	0.00
0.10	0.69	0.54	0.55	0.78	0.99	0.97	0.71	0.99	1.00	-4.05	0.00
0.11	0.69	0.54	0.55	0.79	0.99	0.97	0.71	1.00	1.00	-4.09	0.07
0.12	0.69	0.54	0.56	0.78	1.00	0.97	0.71	0.99	1.00	-4.11	0.11
0.13	0.69	0.54	0.56	0.78	1.00	0.97	0.71	0.99	1.01	-4.16	0.00
0.14	0.69	0.54	0.56	0.78	1.00	0.96	0.72	0.99	1.00	-4.20	0.01
0.15	0.69	0.54	0.56	0.78	1.00	0.96	0.71	0.99	1.01	-4.22	0.60
0.20	0.69	0.54	0.56	0.78	1.01	0.96	0.72	0.99	1.00	-4.34	0.21
1.00	0.69	0.52	0.57	0.76	<u>1.07</u>	<u>0.91</u>	0.74	<u>0.94</u>	1.04	-4.91	0.00
10.00	0.71	0.49	0.60	0.76	<u>1.09</u>	<u>0.89</u>	0.74	<u>0.90</u>	<u>1.08</u>	-5.39	0.00

Table 6. Identified updating parameters using MCMC simulations

FE Model	Damage Case	Selected Model Class	Weight factor (Probability)	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	$Ln(\sigma_i)$
exact	D1	-	-	1.00	0.90	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-
M1	D1	x	0.10 (0.30)	1.00	0.90	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-4.62
M2	D1	iv	0.10 (0.33)	1.00	0.90	1.00	0.99	1.00	1.00	1.00	1.01	0.99	-4.62
M3	D1	x	0.10 (0.42)	1.00	0.89	1.01	1.00	1.01	0.99	1.00	1.00	1.00	-4.53
Exact	D2	-	-	1.00	0.70	0.80	1.00	1.00	1.00	0.80	1.00	1.00	-
M1	D2	x	0.11 (0.28)	0.99	0.71	0.80	0.98	1.02	0.99	0.80	1.02	0.99	-4.57
M2	D2	iv	0.09 (0.46)	1.02	0.72	0.77	1.00	0.98	1.00	0.80	1.03	0.98	-4.40
M3	D2	x	0.09 (0.86)	0.97	0.68	0.85	0.99	1.05	0.97	0.80	1.00	1.01	-4.24
Exact	D3	-	-	0.70	0.50	0.60	0.80	1.00	1.00	0.70	1.00	1.00	-
M1	D3	x	0.10 (0.27)	0.70	0.51	0.60	0.76	1.05	0.98	0.70	1.00	1.01	-4.38
M2	D3	iv	0.15 (0.60)	0.69	0.54	0.56	0.78	1.00	0.96	0.71	0.99	1.00	-4.23
M3	D3	iv	0.12 (0.53)	0.71	<u>0.44</u>	<u>0.71</u>	<u>0.71</u>	<u>1.24</u>	0.95	0.72	0.96	1.03	-4.21

Table 7. Identified model parameters through the proposed procedure

FE Model	Damage Case	Selected Model Class	Selected weight factor	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	$Ln(\sigma_i)$
Exact	D1	-	-	1.00	0.90	1.00	1.00	1.00	1.00	1.00	1.00	1.00	-
M1	D1	x	0.10 (0.29)	1.00	0.90	1.00	0.99	1.00	1.00	1.00	1.01	1.00	-5.09
M2	D1	iv	0.11 (0.26)	1.00	0.90	1.00	0.99	1.00	1.00	1.00	1.01	0.99	-5.18
M3	D1	x	0.11 (0.25)	1.00	0.89	1.01	1.00	1.01	0.99	1.00	1.00	1.00	-5.04
Exact	D2	-	-	1.00	0.70	0.80	1.00	1.00	1.00	0.80	1.00	1.00	-
M1	D2	x	0.10 (0.27)	0.99	0.71	0.80	0.98	1.02	0.99	0.80	1.02	0.99	-5.05
M2	D2	iv	0.09 (0.29)	1.02	0.72	0.77	1.00	0.98	1.00	0.80	1.03	0.98	-4.95
M3	D2	x	0.09 (0.87)	0.97	0.68	0.85	0.99	1.05	0.97	0.80	1.00	1.01	-4.72
Exact	D3	-	-	0.70	0.50	0.60	0.80	1.00	1.00	0.70	1.00	1.00	-
M1	D3	x	0.11 (0.25)	0.70	0.51	0.60	0.76	1.05	0.98	0.70	1.00	1.00	-4.86
M2	D3	iv	0.15 (0.28)	0.69	0.54	0.56	0.78	1.00	0.96	0.72	0.99	1.00	-4.74
M3	D3	iv	0.11 (0.23)	0.71	0.44	0.71	0.70	1.24	0.95	0.72	0.95	1.03	-4.76

Figure 1. Two-step damage identification process using FE model updating technique

— Moment Resisting Connection — Simple (hinged) Connection

Figure 2. Original SAC 9-story steel moment frame

Figure 3. Models M0 to M3 (modeling errors circled)

Figure 4. Variation of modal frequencies at different levels of modeling error (top) and damage severity (bottom)