Modeling floor systems for collapse analysis

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ABSTRACT

Gravity load redistribution and collapse resistance of structures following loss of load bearing elements due to natural and manmade hazards significantly depend on the floor response. Effects of floor system modeling on collapse resistance of damaged structures are studied in this paper. In particular, axial and flexural constraints imposed by the floor slab on joists and beams of RC structures is evaluated and characterized. The axial constraint imposed by the slab leads to additional compressive force, enhancing flexural response and in turn load carrying capacity of beams and joists. The flexural constraint can be captured by proper modeling approaches and can help improve the response of floor joists and beams. It is shown that the method by which the floor system is modeled has considerable effects on the floor response following loss of load bearing elements. The effects of modeling techniques on the structural response are also evaluated from an energy point of view. The effects of flexural-axial, as well as torsional cracking of the floor slab are evaluated and characterized.

Keywords: Modeling techniques, Collapse resistance, Floor system, Compressive membrane action, Energy balance.

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1. INTRODUCTION

Progressive collapse is defined as the spread of an initial local failure from element to element, eventually resulting in collapse of the entire structure, or a portion of the structure disproportionate to the initial damage (ASCE 7, 2010). Allen and Schriever (1972) defined progressive collapse as a situation where local failure of a primary structural component(s) leads to the collapse of adjoining members which in turn leads to additional collapse. Thus, the extent of collapse is disproportionate to the initial damage. One approach often used to evaluate collapse resistance and integrity of structures (direct design, ASCE 7, 2010) is to suddenly remove a column of the structure and study the building response. This study utilizes the aforementioned approach.

Compressive membrane action (particularly in slabs), also known as arch action, has been studied by several researchers (Westergaard and Slater, 1921, Ockleston, 1958, Christiansen, 1963, Park, 1965, Vecchio and Tang, 1990). Park (1965) and Park and Gamble (2000) proposed different formulations to capture arch action. Bazan (2008) and Sasani and Kropelnicki (2008) have demonstrated this action can be captured through Bernoulli beam theory, making its utilization more effective. That is, in order to account for arch action, the special formulations mentioned above would not be needed and the same beam theory used to model axial-flexural interaction of beam elements can capture arch action as well. The compressive membrane action develops as a result of constrained tendency of RC beams to grow in length as their sections crack and yield in flexure (Sasani and Kropelnicki, 2008). The constraint is primarily provided by the floor system, which leads to the development of axial compressive force in beams as they deform following structural damage. In this paper, the effects of modeling floor systems on such response is evaluated. In particular, the effects of relative elevation of the slab with respect to the beam centerline is studied.

Sagiroglu and Sasani (2014) studied the effects of including the elevation difference between the center lines of beams and floor slabs in analytical modeling. It was reported axial compressive force and flexural resistance increased at the supports of beams bridging over the lost columns. Kazemi and Sasani (2015) studied a structure with deep beams and observed similar effects. The main objective of this paper is to identify and characterize the underlying effects of the slab elevation (with respect to beam and joist centerlines) on building response in

damaged reinforced concrete (RC) structures. The response of two finite element modeling techniques that simulate progressive collapse resistance of two RC frame structures following initial local failure is evaluated. In model A, the floor is modeled conventionally, i.e. all structural nodes are in the same plane. In model B (with raised slab), however, using rigid elements, the vertical elevation of the slab is increased, such that the slab is placed at its actual height relative to the beams and joists. Behavior of each model following sudden loss of an exterior first floor column is evaluated.

2. CHARACTERISTICS OF STRUCTURES

Each structure is a seven-story building designed as an ordinary moment frame structure with a longitudinal span of 26' (7.93 m) and transverse span of 30' (9.14 m). The height of the first floor is 12'-8" (3.86 m) and that of floors above is 11'-6" (3.51 m). A floor live load of 50 lb/ft² (2.39 kN/m²) and 100 lb/ft (1.46 kN/m) wall weight on spandrel beams are applied. Note the floor dead load of each building is different because they are dependent on the dimensions of floor system elements. Each building is assumed to be located at a site class C with a 1-s spectral acceleration of 0.1 g and seismic response coefficient of 0.0534 for ordinary frame structures (ASCE 7, 2010). The concrete is assumed to have a nominal compressive strength of 5 ksi (34 MPa) and unit weight of 150 lb/ft³ (24 kN/m³). The reinforcing steel bars have a yield strength of 60 ksi (413 MPa) and modulus of elasticity of 29,000 ksi (20×10^5 MPa).

The floor is a one-way joist system in the transverse direction and has a 4 in (102 mm) thick slab. Figure 1 shows the building floor plan where sizes of the beam, column, and joist sections are given. Element dimensions are kept the same over the height of the structure. However, the reinforcement details change 3 times over the height of the structure. A floor dead load (excluding beam and column weights) of 100 lb/ft² (4.78 kN/m²) is considered. In the design of spandrel beams and joists, ACI 318 (2014) integrity requirements are satisfied. Figure 2 (a) and (b) show the longitudinal reinforcement in the spandrel beams and joists at the 2nd floor. The reinforcement of the 4 in (102 mm) thick slab is #3@12 in (D10@305 mm).

The main structural integrity requirements include: (1) for spandrel beams, at least 1/6th of top bars required for negative moment at support, but not less than two bars and at least 1/4th of bottom reinforcement to be continuous; and (2) for joists require to have at least one bottom bar to be continuous or spliced with a Class A tension splice or a mechanical or welded splice

satisfying and at a noncontinuous end of the beam or joist, the bottom bars must be terminated with a standard hook (ACI 318, 2014).

3. ANALYTICAL MODEL

Using the Open System for Earthquake Engineering Simulation (OpenSees, 2012), two models for each structural system are developed. Both material and geometric nonlinearities are accounted for. Distributed plasticity in beam-column elements with force formation is used to account for material nonlinearity. The co-rotational coordinate transformation is used to account for geometric nonlinearity. Reinforcing steel bar buckling and fracture are accounted for. In order to study the resistance of progressive collapse, the first floor middle exterior column D3 is suddenly removed (Sasani and Kropelnicki, 2008 and Sagioglu and Sasani, 2014).

Although portions of the slab are accounted for as beam flanges and are modeled at the correct height in analytical model A (see Figure 3a), the frame elements modeling the beams, joists, and remaining slab lie in the same horizontal plane (i.e. centerlines of beams and slabs are at the same height). There is an 8 in (203 mm) height difference between the centerlines of the beams and the slabs. Therefore, in model B (see Figure 3b), the slab elements lie on a plane that is 8 in (203 mm) higher than beam and joist centerlines and the connection between the slab and beam elements is achieved by using rigid vertical link elements (Sagioglu and Sasani, 2014).

A force-based nonlinear beam-column element is used to model the structure in OpenSees (2012). One important advantage of this choice is that the accuracy of the solution can be improved for such an element by simply increasing the number of integration points, while for displacement based elements the number of elements and in turn the number of degrees of freedom of the structure need to increase (OpenSees, 2012). The beams and joists are modeled with L and T shaped sections to account for the contribution of slabs, with an effective flange width of 16 in (406 mm) on each side (ACI 318, 2014). Because the nonlinear shell elements in OpenSees are not completely verified, the floor is modeled using a grid of nonlinear beam elements. The slab in the transverse direction is modeled as the flanges of the joists. In the longitudinal direction, the slab is modeled using four parallel lines of beam elements with rectangular sections between adjacent longitudinal frames. The cross sections of nonlinear beams, joists, and slabs are discretized into fibers.

A fiber section does not have torsional stiffness. In this study, elastic torsional stiffness is aggregated with the fiber section in OpenSees. In a preliminary analysis, the result of column removal analyses showed that sections of joists crack in torsion. Therefore, given that the torsional stiffness of the joist after cracking is only a small fraction of the initial stiffness, it is reasonable and conservative to ignore the torsional stiffness of the joists. Therefore, the joists in this study are modeled with zero torsional stiffness. The analytical modeling approach discussed above is extensively verified against laboratory tests and field studies on response of actual RC structures following initial damage (see for example Sasani and Sagioglu, 2008).

4. OVERVIEW OF RESULTS: JOIST FLOOR SYSTEMS

Figure 4 compares the vertical displacement histories at the top of removed column D3 for models A and B of the joist floor building system. The peak displacements for models A and B are -4.6 in (117 mm) and -3.8 in (97 mm) while the permanent displacements are -4.3 in (109 mm) and -3.3 in (84 mm), respectively. That is, the peak and permanent displacements of model A are about 21% and 29% larger than those for model B, respectively. The following sections investigate how the position of the floor slab with respect to the spandrel beam and joists in structural models affects the peak and permanent displacement.

4.1 Equilibrium Condition of Center Joist at Permanent Displacement

The equilibrium condition of the second floor joist located between the face of column D2 and removed column D3 (referred to as the center joist) is identified as the primary factor in explaining why the vertical displacements of model A are larger than those in model B. Note that the response of the center joists on floors above is almost identical to that of the second floor, because the building moves downwards almost equally on all floors. Figure 5 shows the equilibrium condition of the free-body diagram of the center joists for models A and B at permanent displacement. In order to better describe the effects of different parts of the floor structural system, Figure 6 shows a plan view of the second floor area within C2-C3-D2-D3. The center joist is intersected by two longitudinal beams and four slab elements labeled f through k, creating six nodes and five segments. In Figure 5, each joist is cut at the faces of the columns and the internal forces at the cuts are shown at the joist ends. The four downward forces applied on

each joist (top of the figures) represent the gravity loads. The vertical and horizontal forces and the moments applied to the joist by the slab elements (see also Figure 6) are also shown.

As can be seen in Figure 5, the shear forces at the face of removed column D3 for models A and B (on the right ends of the joists) are 5.0 k (22.2 kN) and -0.6 k (2.7 kN). In other words, at the permanent displacement, the center joist imposes a 5.0 k (22.2 kN) downward force on the spandrel beam in model A, and a 0.6 k (2.7 kN) upward force on the spandrel beam in model B. This difference in joist shear forces transferred to the spandrel beam is primarily caused by three factors discussed in the following paragraphs.

First, after the sudden removal of column D3, and as a result of the center joist displacing downwards, the joist sections crack and rebars yield developing a tendency to grow in length at mid height throughout its entire length (Bazan, 2008 and Sasani and Kropelnicki, 2008). Furthermore, as the displacement at the top of removed column D3 causes additional negative moment to develop at the face of column D2 (center joist at point k), additional axial tensile strain develops at the top of the section of the center joist in the vicinity of point k which leads to concrete cracking, rebar yielding, and tendency to grow at this location. For both models A and B, the center joist is longitudinally constrained by its adjoining slab elements so it cannot elongate to the full extent of its tendency to grow. This phenomenon causes the center joist to develop axial compressive force. The effects of axial compressive force developed in floor elements and in turn on the system response will be further discussed in section 4.2.

The position of the adjoining slab relative to the center joist affects the level of constraint and therefore axial compressive force. For model A, the mid height of the slab and joists are located at the same elevation. For model B, however, the mid height of the slab is almost located at the top of the joists. The adjoining slab constrains the center joist's elongation near point k (due to the negative moment) more effectively in model B than in model A. Figure 5 shows that the axial compressive force and moment capacity at this location for model B are larger compared to those for model A, being 94.3 k (419.5 kN) and 1962 k-in (222 kN-m) compared to 68.4 k (304.3 kN) and 1708 k-in (193 kN-m), respectively. According to the P-M interaction, larger axial compressive force is correlated with larger moment capacity (below the balance point). In other words, the flexural moment in the center joist for model A at the face of column D2 is smaller

than that in model B. This leads to the joist in model A relying more on the vertical support provided by the spandrel beam than in model B to reach equilibrium.

Second, as the joist deflects downwards and its slope increases, torsion develops in the slab elements, which are perpendicular to the joist (moments shown in Figure 5 at g, h, i, and j). These torsions help the joist resist gravity loads, i.e. transferring more gravity load away from removed column D3. Note that while it has been verified that the maximum torsion developed in the slab elements are smaller than their cracking torsion (based on ACI 318, 2014), the effects of possible torsional cracking of the slab elements are discussed later in this paper (Section 4.5). Figure 5 shows that the center joist at the intersections with the slab elements (points g, h, i, and j) has one component of moment for model A but two for model B. The moment in model A is transferred to the center joist from torsion in the adjoining slab elements. This same moment exists in model B, however an additional moment develops due to the eccentricity of the slab with respect to the joist centerline. This additional moment is the resisting moment produced by the combination of horizontal shear force in the adjoining slab elements and eccentricity of these elements with respect to the centerline of the center joist, which is 8 in (203 mm) and helps the joist resist gravity loads. For example, the resisting moment at point j of 251 k-in (28 kN-m) equals the horizontal shear force of 31.4 k (139.7 kN) multiplied by 8 in (203 mm). The sum of these moments due to the eccentricity of the slab horizontal shear on the center joist in model B is 631 k-in (71 kN-m), which considerably helps the joist in transferring more gravity loads away from the location of the removed column D3.

Third, Figure 5 shows the axial compressive force and moment in the center joist at the face of column D3 for models A and B are 41.0 k (182.4 kN) and 239 k-in (27 kN-m) compared to 15.4 k (68.5 kN) and 414 k-in (47 kN-m), respectively. These values are not consistent with what is expected based on P-M interaction, as discussed earlier. The reason for this discrepancy is that models A and B yield at different locations under positive moments. Figure 7 (a) and (b) show the axial compressive force and bending moment diagrams of the center joist. The center joist for models A and B yield at points g and f, respectively, which are shown by solid circles in Figure 7. This can be explained by examining the response of the f-g segment of the joist in the two models. After column D3 is removed in model A and the positive moment in joist segment f-g increases, yielding occurs at point g where the bottom bar cut-off is located. This yielding limits the positive moment developed at the joist end of model A at the face of column D3 to 239 k-in

(27 kN-m). In other words, the moment for model A at the face of column D3 is rather small (compared to model B) because the demand is smaller following joist yielding at point g. Figure 8 shows the equilibrium condition at permanent displacement of the joist segment at point g. The joist of model B does not yield at this location primarily because of its additional moment resistance produced by the eccentricity of the adjoining slab, which is $48.6 \times 8 = 389$ k-in ($216 \times 0.203 = 44$ kN-m). Note that at the right section of node g in model B, where there is one bottom bar, the moment demand is only 355 k-in (40 kN-m) compared to a much larger value of 643 k-in (73 kN-m) for model A. Instead, model B yields at the joist front end, i.e., at the face of column D3. This leads to a larger positive moment in the center joist at the face of column D3 of model B compared to that for model A (see Figures 5 and 7), which in turn leads to the center joist in model B to transfer more gravity loads away from the removed column location compared to that of model A.

In summary, these three factors affect the moment equilibrium of the center joist for models A and B, and in turn determine the shear force at the face of removed column D3.

4.2 Axial Forces in Spandrel Beam and Slab

As discussed in section 4.1, the center joist develops compressive axial force when its tendency to grow is constrained by the slab, enhancing the flexural resistance of the joist. The spandrel beam also experiences this phenomenon. The effects of axial compressive force in the spandrel beam at the element and system response is discussed in the following sections. Refer back to Figure 6 in order to analyze the spandrel beam and slab. The spandrel beam and four slab elements are intersected by five joist elements labeled a through e. The slab element closest to the spandrel beam is referred to as the first row slab element.

4.2.1 Model A

The sudden removal of column D3 causes the spandrel beam to displace downwards. Its section develops flexural cracking and yielding resulting in a tendency to grow. Because it is constrained, the spandrel beam develops axial compressive force (Sasani and Kropelnicki, 2008), and thus, the slab (through horizontal shear in the joists) develops axial tensile force.

Figure 9 shows the axial force in each segment of the spandrel beam bridging over the lost column and the first row slab elements (see Figure 6) versus vertical displacement at the top of

removed column D3. Upon the removal of column D3, the axial compressive and tensile force in the spandrel beam and slab, respectively, are larger at segment d-e (closer to the removed column) compared to other segments. At a vertical displacement of -2.8 in (-71 mm), the axial tensile forces in the first row slab element begin to drop due to segment d-e flexural-axial cracking. This in turn limits further development of compressive axial force in the spandrel beam and the axial compressive force in the spandrel beam starts to drop. The first row slab segments in closer proximity to cracked segment d-e experience larger drops in axial tensile force compared to those farther away. This results in the axial compressive and tensile forces in the spandrel beam and slab being larger at segment a-b compared to d-e at permanent displacement.

After the first row slab segment d-e cracks, axial tensile force transfers into its reinforcing bars and the remainder into the next rows of slab elements. At permanent displacement, the axial tensile force in the first row slab segment d-e is about 40 k (177.9 kN). This is expected considering the slab segment has 0.66 in^2 (426 mm^2) of reinforcing bars with a yield strength of 60 ksi (413 MPa).

At permanent displacement, the axial compressive forces in the spandrel beam at the faces of columns C3 and D3 are 157.8 k (702.8 kN) and 120.1 k (533.8 kN), respectively. To determine how the structural system provides constraints to develop the axial compressive forces, the equilibrium condition is represented in a free body diagram. By taking transverse cut Q-Q through segment a-b as shown on Figure 6 as well as cutting the columns directly above and below the second floor slab, the longitudinal forces of all elements on the left side of the second floor were investigated. This included axial forces in the spandrel beam and slab elements as well as longitudinal shear forces in columns directly above and below the second floor slab. It was apparent that the axial compressive forces in the spandrel beam are almost entirely resisted by the axial tensile forces of slab elements between axis 2 and 3. The contribution of longitudinal column shear to resist the spandrel beam axial compressive force was negligible.

4.2.2 Model B (with elevated slab)

Similar to the center joist, the sudden removal of column D3 causes the spandrel beam to develop positive moment near the removed column and additional negative moment at the face of column C3. Because the centerline of the slab is approximately located at the top of the spandrel beam, the combination of spandrel beam growth and axial tensile strain at the top

section (due to negative moment) cause the slab to develop larger axial tensile forces at the face of column C3 compared to D3 (a-b compared to d-e). Figure 10 shows a diagram of the axial force in each segment of the spandrel beam and first row slab element versus vertical displacement at the top of removed column D3. At a vertical displacement of -2.9 in (74 mm), the first row slab element axial tensile forces drop due to segment a-b cracking, which again limits the development of axial compressive force in the spandrel beam.

4.3 Equilibrium Condition of Spandrel Beam at Permanent Displacement

Figure 11 shows the equilibrium conditions of the free-body diagrams of the second floor spandrel beam between the faces of column C3 and removed column D3 at permanent displacement, which is discussed below to determine why the displacements of models A and B differ. Table 1 summarizes different components of moments with respect to the left end of the spandrel beam that affect the beam equilibrium, which are discussed below. Positive values represent resisting components. As can be seen, the sum of all the moments for each case, which is shown on the last column, is almost equal to zero, verifying the movement equilibrium. Throughout this section, the effects of different demand and capacity (resisting) components on the moment equilibrium of the beam with respect to its left end (face of column C3) are discussed. On the demand side, the effects of larger forces transferred by the center joist to the spandrel beam bridging over the lost column in each model is evaluated. This larger demand would require the spandrel beam in model A to develop larger end moments that go hand in hand with larger axial compressive force, both requiring larger displacements and deformations. It will be discussed later in this paper that while the axial compressive force enhances the flexural capacity of the beam, it also imposes more demand on the beam through the P- Δ effect.

Let us consider the demanding downward forces transferred to the spandrel beam at the face of removed column D3 for models A and B (8.6 k (38.3 kN) and 6.3 k (28.0 kN), respectively). It should be noted that the difference between these forces is primarily due to the response of the center joist. The static moment demands of these two forces with respect to the left end of the spandrel beam (i.e. their contribution to the moment equilibrium) for models A and B are 2520 k-in (285 kN-m) and 1846 k-in (209 kN-m), respectively, having a 674 k-in (76kN-m) difference.

In order for the spandrel beam in model A to resist this additional demand, larger end moments are required. This in turn requires larger vertical displacement and section curvature that leads to development of larger axial compressive force. According to the P-M interaction, at the face of column C3 and removed column D3, both the axial compressive force and moment capacity for model A are larger compared to those for model B. The sum of the spandrel beam end moments for models A and B are 6646 k-in (751 kN-m) and 5825 k-in (658 kN-m), respectively, an 821 k-in (93 kN-m) difference.

On the other hand, and as was discussed before, the larger axial compressive force imposes larger demand on the beam through P- Δ effect. According to the P- Δ effect, an axial compressive force in conjunction with vertical displacement produces a moment demand. The sum of these moments about the face of column C3 for models A and B are 580 k-in (66 kN-m) and 322 k-in (36 kN-m), a 258 k-in (30 kN-m) difference. Both the axial compressive forces and associated vertical displacement throughout the length of the spandrel beam for model A are larger compared to those for model B. For example, the axial compressive force and vertical displacement at removed column D3 for models A and B are 120.1 k (534.2 kN) at -4.3 (109 mm) in compared to 80.4 k (357.6 kN) at -3.3 in (84 mm).

There are some other less significant differences between the components affecting the moment equilibrium of the beam in models A and B. For instance, the moment transferred to the spandrel beam from the torsion in each joist is negligible. This is because the joists cracked upon column removal and their cracked torsional stiffness is negligible. Table 1 shows the sum of these moments as well as moments due to the vertical forces (shear in joists) transferred to the spandrel beam. Note that the change in internal forces from Model A to B may have adverse effects on the joints, which needs to be accounted for.

4.4 Effects of building characteristics

In order to evaluate the effects of building characteristics on the system response, a parametric study is conducted. The results are presented in terms of the ratio of building maximum vertical displacement using model A over that of model B; $R = \Delta_{\max}(\text{Model A}) / \Delta_{\max}(\text{Model B})$. Increasing the aspect ratio of the floor panels by increasing the span of the longitudinal beams by 15% increases R from 1.21 to 1.53, while decreasing the length of the longitudinal span by 15% reduces this ratio from 1.21 to 1.14. Increasing the steel yield stress by 15% reduces R to from 1.21 to 1.16 and

decreasing the yield stress by 15% results in $R=1.49$. Similarly, increasing and decreasing the concrete compressive strength by 1 ksi (6.9 MPa) lead to R values of 1.19 and 1.31, respectively.

The effects of the location of column removal on the vertical displacement following column removal is extensively studied by Sagioglu and Sasani (2014). They reported that column removal on floors above, particularly on the top floor leads to a larger vertical displacement. The maximum displacement was observed under removal of a corner column at the top floor, which led to an R value of 1.56 at the same time.

In all the cases described above, the larger the vertical displacement of the structure, the larger is the value of R . Therefore, if the soil type and/or the building location leads to a smaller lateral load in design, the structure would have less longitudinal reinforcement and in turn R will increase. In particular, if the seismic coefficient drops from 0.0534 to 0.03, R will increase from 1.21 to 1.38.

4.5 Energy Balance

The vertical shear forces in the center joist at the face of removed column D3 for models A and B have different effects on the spandrel beam, and in turn on the building response after column removal. As discussed earlier, at the permanent displacement, the vertical force applied by the center joist pushes down and slightly holds up the spandrel beam for models A and B, respectively. Figure 12 (a) shows the vertical shear force applied at the face of column D3 as a function of vertical displacement. From an energy point of view, the external work due to this vertical force contributed differently on the spandrel beam's energy balance. For model B, Figure 12 (b) shows the external work due to the vertical load almost continually decreases. Because the external work is negative, the vertical force helps the spandrel beam resist other external work. For model A, however, the external work due to the vertical load remains approximately zero until reaching a vertical displacement of -2.7 in (69 mm). This is because the vertical force in the spandrel beam oscillates about zero until reaching a vertical displacement of -2.7 in (69 mm), as shown in Figure 12 (a). After this displacement, the vertical force and external work significantly increase. The increasing vertical force also affects the vertical velocity of the main node at the top of removed column D3. As shown in Figure 12 (c), after vertical displacement of -2.7 in (69 mm), the velocity of the node above the removed column in model A is not consistent with that in model B and requires more time to decelerate to zero. A related plot is shown in Figure 12 (d), which is the total kinetic energy of spandrel beam. As the

external work suddenly increases for model A, the kinetic energy of the beam also increases requiring more time (or more vertical displacement) for model A reach equilibrium.

4.6 Slab Cracking in Torsion

The cracking torsion for slab elements g and j is 154 k-in (17 kN-m) while that of h and i is 134 k-in (15 kN-m). Figure 5 shows the maximum torsion in the center joist's adjoining slab elements, which are less than the cracking torsions. The response of the building is evaluated assuming that the slab elements have no torsional stiffness. Compared to original models A and B discussed earlier, a larger peak and permanent displacement develops at the top of removed column D3 when slab elements crack. The increase in peak and permanent displacement for model B is less compared to model A, being 3.9% compared to 8.9% for model B and 5.7% compared to 10.9% for model A. This is due to the slab elements in model B providing moment resistance to the joists (despite being assumed to have no torsional stiffness) as a result of eccentricity between the slab and joists, as discussed previously.

5. CONCLUSIONS

The performance of a seven story RC structure with a joist floor system is studied and it is demonstrated that the modeling technique has a significant effect on the building response following sudden loss of a first floor exterior column. Model A uses a conventional modeling method in which all the structural nodes of a floor are placed on the same plane. Model B is more representative of actual construction, using rigid vertical elements such that the slab centerline is elevated relative to the beam and joist centerlines. Note that the analytical modeling technique discussed in this paper does not account for element shear, torsional or bond failure, nor for joint failure. Also the analysis is conducted using Euler–Bernoulli beam theory that plane sections remain plane.

Following the column removal, the peak and permanent displacements are about 21% and 29% larger in model A than model B, respectively. It is demonstrated that from an equilibrium point of view, the response of the joist previously supported by the removed column is the primary cause of this difference. This is due to two important factors:

- At the back of the joist, where the section yields under negative moment and the top of the beam is under tensile strain, the slab in model B can impose a more

significant constraint on the beam's tendency to grow, leading to a higher axial compressive force and in turn a higher moment capacity.

- Given that the slab elements in model B are connected to the joist 8 in (203 mm) above the centerline of the joist, due to this eccentricity they a) impose moments to the joist that enhance its load carrying capacity; and b) help avoid formation of a positive hinge in the middle of the joist, which also improves the load resistance of the joist.

These factors lead to the fact that while at the permanent displacement, the joist in model B supports the spandrel beam bridging over the lost column with a small vertical force of 0.6 k (2.7 kN). However, this same joist in model A pushes the spandrel beam downwards by 5.0 k (22.2 kN).

The effects of the joists on the spandrel beam are also evaluated from an energy point of view. While at the peak and permanent displacements, the joist shear in model A *imposes* an external work of approximately 8 k-in (0.90 kN-m) and 7.6 k-in (0.86 kN-m) on the spandrel beam, respectively, while the joist shear in model B *absorbs* about 7.9 k-in (0.89 kN-m) and 7.6 k-in (0.86 kN-m), respectively. In order to recognize the importance of the energy absorbed by the joist, note that the total internal energy of the spandrel beam in model B at the peak and permanent displacements are about 100 k-in (11 kN-m) and 90 k-in (10 kN-m), respectively.

The response of the spandrel beam is also evaluated. It is shown that in model A, the spandrel beam needs to develop larger beam end moments to resist the additional demand from the joist, which is achieved by a larger vertical displacement and larger axial compressive forces due to a higher tendency of the beam to grow (larger flexural cracks and yielding). It is also demonstrated that while the axial compressive force helps enhance the flexural capacity, it also imposes larger moment demand due to P- Δ effects, with the former effect being more significant.

It is shown that the constraint to develop the axial compressive force in the spandrel beam is primarily provided by the slab elements. As a result, the axial-flexural cracking of the slab elements limits the maximum axial compressive force that can develop in the spandrel beam. After cracking, the slab reinforcement determines how much tension can develop in the slab, and in turn how much compression in the spandrel beam.

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	Moment due to Shear at Face of Removed Column D3	Sum of end moments	P-Δ Effect	Nodal Loads	V and T (Shear and Torsion) of Joists	Sum
Model A	-2520 (-285)	6646 (751)	-580 (-66)	-2355 (-266)	-1193 (-135)	-2 (0)
Model B	-1846 (-209)	5825 (658)	-322 (-36)	-2355 (-266)	-1302 (-147)	0
Difference	-674 (-76)	821 (93)	-258 (-29)	0	109 (12)	-2 (0)

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