Multi-Hazard System-Level Logit Fragility Functions

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Abstract

Fragility functions are used to represent the probability of failure of a structure or lifeline system conditional upon a hazard or set of hazards and are essential in the performance-based design process. Continuous lognormal damage fragilities are traditional, but recent formulations have implemented logit transformations from the family of generalized linear models for categorical data with a binary outcome (e.g., failure, no failure). In wind engineering, single hazard parameters derived from correlated variables (e.g., integrated kinetic energy, IKE) have been employed to indirectly include the effect of more than one hazard variable; however, even with more general hazard metrics, the lognormal formulation is still unduly restrictive for realistic fragility modeling. Using a statistical approach based on a logit formulation, a shift towards more robust fragility functions can be achieved. Because

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of its simplicity and ability to represent multiple predictor variables to improve the fitted model, this paper proposes use of the logit formulation of the fragility function at the system level for two or more simultaneous weather hazards. Successful applications of the model to characterize lifeline system-level fragility functions for electric power delivery during Hurricane Isaac in Louisiana and Hurricane Sandy in New York City using in-situ damage and hazard data are shown. While the results here are empirically derived, the modeling approach may be expanded for other structural systems subject to multiple loadings or demand variables.

Keywords: Structural engineering, Wind engineering, Structural fragility, Hurricanes, Electric Power Delivery, Logistic regression

1. Introduction

Fragility functions are a cornerstone of the performance-based design process in civil engineering. A fragility function is defined here as the probability of failure of a structure or structural system conditional upon a hazard or set of hazards. The estimation of building or bridge fragility for single hazard parameters such as peak ground acceleration for seismic loadings or 3-second wind gusts for wind loadings, is considered mature, e.g.[1]. Whether the fragility is formulated as a probability of damage, or the exceedance of a given capacity, derived empirically or analytically, the representation is based upon a lognormal formulation as discussed in detail by Shinozuka et al. [1]. That is, the fragility function F is modeled as a lognormal distribution with the two parameters c and ζ as follows:

$$F(a) = \Phi \left[\frac{\ln \left(\frac{a}{c} \right)}{\zeta} \right] \quad (1)$$

where a represents the peak demand parameter; $\Phi(\cdot)$ is the standardized normal distribution function; c and ζ are the two parameters of the lognormal distribution.

Many representations of structural fragilities of the form in equation (1) exist for different types of building constructions, e.g. [2, 3, 4, 5]. One of the disadvantages of using the lognormal approach for modeling fragilities is the inability to capture the influence of more than one hazard simultaneously. Some investigators have tried to circumvent this problem by implementing hazard parameters that incorporate more than one variable. For example, for seismic fragilities, the Japanese Meteorological Agency (JMA) intensity scale or the instrumental Modified Mercalli Index [I_{mm}] have been used [6]. The integrated kinetic energy [IKE] metric created by Powell and Reinhold [7] includes a combination of different aspects of the wind and storm surge potential, but it is challenging to use in predictions. For example, two IKE data points are presently available from HWind [8] on a daily basis for storms; however, the time scale is too long for use in estimating the level of structural damage required for fragility models. Due to the limitations of the lognormal approach described in equation (1), research in the use of logit transformations for seismic and wind hazards has been undertaken, e.g. [6, 9, 10, 11]. These research investigations were focused on individual hazards or damaged structures, such as bridges, and the use of empirical data was limited. In the next section, the modeling of fragilities through logit transformations is presented before application to post-event in-situ analysis for lifeline system is

presented.

2. Logit transformation and regression models for fragilities

2.1. Structural damage due to a single hazard

The fragility function F represents a conditional probability here in the general form of $E(\mathfrak{S}|H)$ where “ E ” represents the expectation, “ $\mathfrak{S}$ ” is a binary event and “ H ” is a hazard measure such as wind speed or peak ground acceleration. The logit transformation undertaken to evaluate $F = E(\mathfrak{S}|H)$ is as follows.

Define a binary event, $\mathfrak{S}$ as below:

$$\mathfrak{S} = \begin{cases} 1 & \text{system is damaged} \\ 0 & \text{system is not damaged} \end{cases} \quad (2)$$

The probability distribution of $\mathfrak{S}$ is as follows:

$$\mathfrak{S} = \begin{cases} 1 & P(\mathfrak{S} = 1) = \kappa(H) \\ 0 & P(\mathfrak{S} = 0) = 1 - \kappa(H) \end{cases} \quad (3)$$

where $P(*)$ = probability of $*$; $\kappa(H)$ denotes a number on $[0, 1]$. For a single hazard H , the logistic response function F has the form, e.g. [13]:

$$F = E(\mathfrak{S}|H) = P(\mathfrak{S} = 1|H) = \frac{\exp(\beta_0 + \beta_1 H)}{1 + \exp(\beta_0 + \beta_1 H)} \quad (4)$$

where $E(\cdot)$ = expected value; β_0 and β_1 are parameters of the logistic regression model.

The logit transformation of F is given by

$$y(H) = \ln \left[\frac{\kappa(H)}{1 - \kappa(H)} \right] = \beta_0 + \beta_1 H \quad (5)$$

Equation (5) is for one hazard variable H only. When using this format, typically, the parameter β_0 is referred to as the “intercept” and β_1 as the “coefficient”. The ratio of $\kappa(H)$ to $(1 - \kappa(H))$ is sometimes referred to as the “odds ratio”. The advantage to using the logit model is that the function F is transformed to a linear function y , which is easier to fit to actual data. It is noted that for the two parameter F , the β_0 parameter dictates the placement along the hazard variable axis, whereas the β_1 parameter controls the slope of the so-called “S” curve. Logistic transformation and regression models for fragilities have been used successfully in the earthquake community for evaluating fragilities of buildings, bridges and lifeline systems for single hazard metrics, e.g. [6, 11]. Lallemand et al. [12] note that lognormal and logit transformation models result in very similar curves but that logistic regression has the advantage of being easy to understand in that the estimated model coefficients represent the log of the odds ratio of damage due to a unit increase in the independent variable.

2.2. Structural damage due to multiple hazards

Distinct hazards generated simultaneously from a single event (e.g. hurricane) cause single and joint hazard damage and failure to structures and infrastructure. Modeling the effects of multiple hazards requires representation of hazard and hazard-structure interactions, as well as the cascading nature of hazard-induced damage [14, 15]. The interaction of multiple hazards is widely believed to cause damage conditions that differ from the aggregation of single-hazard damage [16, 17]. Therefore, statistical methods, which

74 represent system-level response, are appropriate to represent infrastructure
 75 response to a range of multiple hazard conditions.

76 Equation (5) can be modified to include the influence of more than one
 77 hazard. Hazards may be considered separately as given in equation (6)

$$\text{Linear model : } y = \beta_0 + \beta_1 H_1 + \dots + \beta_m H_m \quad (6)$$

78 Here “ y ” represents the dependent variable of fragility; H_i represents the
 79 hazard variables; and β_i are the fitted coefficients.

80 The linear model shown in equation (6) has been applied by Koutsourelakis [10] for geotechnical systems subject to earthquake hazards, where the
 81 hazard variables were the peak ground acceleration, and the root-mean-square intensity and the Arias intensity, which were calculated based upon
 82 the ground acceleration time history. The structure was modeled simplis-
 83 tically as located on a rigid foundation atop a deep sand deposit. It was
 84 concluded that including all three seismic hazard parameters provided a su-
 85 perior representation of the fragility function.

86 Gehl et al. [18] developed fragility functions with uncorrelated and corre-
 87 lated seismic intensity metrics for buildings. A simple unreinforced masonry
 88 structure was used in numerical simulations to provide examples of fragility
 89 functions. In each formulation, two intensity metrics were used for the two
 90 damage states of “yield” and “collapse” respectively. The use of two hazard
 91 parameters was found to be superior to a single-value hazard based fragility
 92 function.

93 In the companion papers [19, 20], fragility functions for aging highway
 94 bridge networks were modeled using both seismic loading variables and ma-

terial deterioration effects as hazards. The material deterioration parameters were based on data derived from instrumented bridges and included surface chloride concentration, chloride diffusion coefficient and corrosion rate. The highway bridge network was modeled with bridges represented as nodes and roads represented as the connecting links. In [20], a fully parameterized fragility function with seven independent variables was derived. Only one of these variables was a seismic hazard intensity measure. Monte Carlo simulations were used to derive the fragilities employed for the bridges and highway networks.

Kameshwar and Padgett [19] derived fragility functions for bridges located in Charleston, South Carolina, subject to earthquake and hurricane hazards, respectively. Comparisons between earthquake and hurricane fragilities were made on the basis of numerical simulations. The models provided insight into the relative influence of each hazard parameter upon bridge risk.

Mardfekri and Gardoni [21] developed a probabilistic framework for the structural performance of offshore wind turbines subject to both wind and seismic loadings. The hazard data were modeled using extreme value seismic and wind load probability distributions at specific site locations. Annual probabilities of being in any specified damage state for the wind turbine at any particular location using the total probability rule were evaluated.

In all cases, fragilities were derived from numerical simulations. For network fragilities, the component fragilities of networks were determined and combined according to a network modeling approach. The ability of logistic models to characterize system response has been demonstrated, but the application of logistic models has been mostly for seismic hazards and the

122 influence of material aging and deterioration of concrete bridges. Although
 123 correlations between seismic parameters were discussed by Koutsourelakis
 124 [10], interactions among the hazard variables were not directly modeled lo-
 125 gistically; that is, products of hazard variables were not directly employed in
 126 the models.

127 Equation (7) provides the so-called interaction terms; it contains the lin-
 128 ear model of equation (6) combined with a weighted sum of hazard product
 129 terms such as “ $H_j H_k$ ”:

$$\text{Interaction model : } y = \beta_0 + \beta_1 H_1 + \beta_2 H_2 + \dots + \beta_m H_m + \sum_{1 \leq j \neq k \leq m}^m \beta_{jk} H_j H_k \quad (7)$$

130 It is noted that the more hazard parameters considered, the more data
 131 are required for appropriate fitting; however, it is also anticipated that as
 132 data collections amass online each year, the data required for model fitting
 133 will be readily available in the near future. In the next section, interaction
 134 models will be presented for the hurricane hazards of storm surge, wind speed
 135 and rainfall using post-event in-situ hazard and damage data.

136 **3. Application to electric power lifeline systems for multiple weather** 137 **hazards**

138 *3.1. Background on fragility models for electric power delivery lifelines*

139 Electric power is generated, transmitted and then fed through distribution
 140 network systems to buildings and other structural systems. Voltage drops
 141 occur at substations. Industrial or commercial facilities may have their own

142 substations. Distribution lines may be located underground or overhead. For
143 seismic loadings, substations are the most vulnerable structural components
144 of the system [22]. For hurricanes, the towers and poles that comprise the
145 lines feeding power to facilities are more vulnerable than the substations,
146 especially in heavily wooded regions. If substations fail in hurricanes or
147 other storms, it is due to flooding caused by excessive rainfall or storm surge,
148 or through damaging wind-borne debris. Design guides and standards are
149 available for the various structural components of the power grid, e.g. [23,
150 24, 25, 26]. These guides may be implemented at a component level to assess
151 the structural fragility as a “stand-alone” element of the network itself.

152 Seismic-based fragilities for the electric power delivery system have been
153 investigated in greater detail than for wind-induced failures. In particular,
154 the FEMA-based Multi-hazard Loss Estimation Methodology HAZUS-MH
155 program [27] provides users with loss estimates based upon five possible
156 structural damage states for generation facilities, substations and distribu-
157 tion circuits subject to seismic hazards. The hazards are characterized by
158 peak ground acceleration. It is noted that HAZUS runs on ArcGIS software
159 platforms, so geo-coding of the hazard data has been undertaken.

160 Park et al. [6] investigated the failure of an urban power delivery system
161 through the use of the geographical information system [GIS] based power
162 distribution grid overlaid with actual seismic intensities for the Nisqually
163 earthquake of 2001. A thorough analysis of damage due to earthquake haz-
164 ards in the region, yielded a lifeline fragility based upon the damage index,
165 defined as the “length of a line damaged relative to the total length of a power
166 line”. It is noted that substation damage was not significant for the Nisqually

167 earthquake. It was found that the damage index was equivalent to the total
168 number of customer outages divided by the total number of customers at
169 the neighborhood or county level. The importance of this result is that the
170 number of outages is associated with the distribution system, which affects
171 customers directly, and usually power companies supply these data for every
172 earthquake or storm. In the US, the power grid is more vulnerable to storm
173 events, whether these are winter storms or hurricanes, than earthquakes. In-
174 deed, almost every hurricane season in the US produces remarkable outages
175 in the Gulf of Mexico region. Recent Atlantic storms such as Sandy have
176 wreaked havoc on the power grid in the northeastern US [28].

177 Formulations of the lifeline fragilities have been modified in recent years
178 to characterize the loss of service through a fragility function, as opposed to
179 a structural variable. In the case of electric power delivery, the “damage”
180 variable of interest as used in this investigation was the inoperability $X(t)$,
181 or inability to provide service. The maximum fraction of customers without
182 power at time t , is denoted by X_{max} , where $X_{max} = \max\{X(t), \text{for all } t\}$. In
183 this section, the results of logit models described in equations (5), (6) and
184 (7) fit to data collected for Hurricane Isaac and Sandy will be provided as a
185 proof of concept in the approach. First, the data employed in the analysis
186 are described and the limitations and caveats discussed. Then the model
187 results are presented.

188 *3.2. Data used for the hurricane fragility analysis*

189 *3.2.1. Isaac*

190 Hurricane Isaac made first landfall at Southwest Pass in Louisiana on
191 August 29, 2012 at 00:00 UTC with maximum sustained winds of 36 m/s (70

192 kt) [29]. According to Berg [29], the storm center moved back over water and
 193 then made second landfall west of Port Fourchon, Louisiana, later on August
 194 29, 2012 at 08:00 UTC. The meteorological data of wind speed, inundation
 195 levels due to storm surge, and rainfall were cataloged for locations within
 196 the State of Louisiana. The description and sources of data are presented in
 197 Table 1.

198 There are limitations regarding the collected data. First, all of the hazards
 199 are uncertain random variables. Second, the geo-coded power grid was not
 200 available for overlay due to security reasons, so the damage data consisted
 201 of power delivery data per parish (county) obtained in spreadsheet format
 202 from the Louisiana Public Service Commission (LPSC) [30]. These data
 203 are not exact representations of damage, but close approximations, as noted
 204 previously. The LPSC provided outage time series data per parish as well
 205 as per utility or company at approximately half hour intervals from August
 206 28 to September 7, 2012. The $X(t)$ function per parish, from which the
 207 peak X_{max} values were derived, has been studied in detail, e.g. [31, 32].
 208 The X_{max} data were geo-coded and then overlaid with hazard data using
 209 ArcGIS software [33]. Figure 1 illustrates the ratio of peak customer outages
 210 to the total number of customers X_{max} per parish (county) in the State of
 211 Louisiana for Isaac. It can be seen that the percentages vary widely over the
 212 range of zero to one hundred. Figure 2 shows a map of the peak outages per
 213 parish. Although outage, wind speed and rainfall data were available for 64
 214 parishes, only 20 were affected by storm surge. Therefore, the data set for
 215 the fragilities focused on the 20 parishes affected by all three hazards.

Figure 1: Peak outages X_{max} in percent per parish (county) in the State of Louisiana for Isaac (parishes with X_{max} less than 10% are not displayed)

3.2.2. Sandy

Hurricane Sandy made landfall near Brigantine, New Jersey, to the north-east of Atlantic City, on October 29, 2012 at approximately 23:30 UTC [34]. It was categorized as a post-tropical cyclone with an estimated intensity of 36 m/s [70 kt] and a minimum pressure of 945 mb [34]. The specifics of the hazard data are given in Table 2.

The limitations regarding the Isaac data also apply for Hurricane Sandy: all hazards are uncertain random variables and outage data were collected by Consolidated Edison (ConEd) employees and reported to the New York

Figure 2: Map of peak power outages per parish in the State of Louisiana for Isaac overlaid with Hurricane Isaac Best Track

(Source: Parish boundary data provided by PennWell MAPSearch; Isaac Best Track and Intensities data provided by National Hurricane Center of National Oceanic and Atmospheric Administration (NOAA))

225 State Public Service Commission (NYSPSC). Specifically, NYSPSC provided
 226 electric power outage time series data per locality at approximately half hour
 227 intervals from October 29 to November 29, 2012 (some localities had longer
 228 or shorter periods) in spreadsheet format. The term “locality” is similar
 229 to “neighborhood” and its boundaries are defined by ConEd, who supplies
 230 electric power to Manhattan. It is noted that the spatial unit of “locality”
 231 is smaller than “parish” or “county” and the majority of the power lines

232 in Manhattan are located underground. Figure 3 illustrates the ratio of
 233 peak customer outages to the total number of customers X_{max} per locality
 234 in Manhattan of the City of New York. The outage data were mostly zero
 235 or 100% for this data set. As with the Isaac data, the X_{max} data were
 236 geo-coded and then overlaid with hazard data using the ArcGIS software.
 237 Although outage data, rainfall and wind speed data were available for 32
 238 localities in Manhattan, only 24 were affected by storm surge. Therefore,
 239 the data set for the fragilities focused on the 24 localities affected by all
 240 hazards. Figure 4 shows the localities of lower Manhattan that lost power
 241 during Hurricane Sandy. Also shown for reference is the location of ConEd's
 242 East 13th Street complex (including two transmission substations) near the
 243 intersection of East 13th Street and Avenue C.

244 3.3. Model Analysis

245 3.3.1. Fitting methods

246 Classical fitting techniques for logistic regression are based on an odds
 247 ratio [35]; that is, for this case, the variable X_{max} would only take on values of
 248 zero and unity, rather than the fractional values derived from the outage data.
 249 The model fitting investigation showed that the fractional values provided a
 250 superior fit to those for the binary X_{max} variable. The software environment
 251 R [36] and 'arm' package [37] were used to fit the models.

252 The Maximum Likelihood Estimation (MLE) approach was used to es-
 253 timate fragility model parameters by maximizing the likelihood function in
 254 equation (8), where Θ^{OPT} are the optimal estimated parameters that maxi-
 255 mize the likelihood function, $f(i)$ is the probability density function of vari-
 256 ables with parameters θ , and $\prod_{i=1}^n f(y_i, X_i; \theta)$ is the likelihood function. The

Figure 3: Peak outages X_{max} in percent per locality in Manhattan of the City of New York for Hurricane Sandy

257 Akaike Information Criterion (AIC) was used to select the best fitted model.
 258 AIC provides an indication of the goodness of fit for the statistical model
 259 to the observed data. The AIC (equation (9)) adjusts the log-likelihood
 260 function (equation (10)) for the affected number of parameters (K), where
 261 $\prod_{i=1}^n f(y_i, X_i; \theta)$ is the likelihood function, and $f(i)$ is the probability density
 262 function of variables with parameters θ .

$$\Theta^{\text{OPT}} = \arg \max \left[\prod_{i=1}^n f(y_i, X_i; \theta) \right] \quad (8)$$

Figure 4: Locality map for lower Manhattan of the City of New York (Source of Imagery Basemap: Esri, DigitalGlobe, GeoEye, Earthstar Geographics, CNES/Airbus DS, USDA, USGS, AEX, Getmapping, Aerogrid, IGN, IGP, swisstopo, and the GIS User Community)

$$\text{AIC} = -2(\mathcal{L}(\theta|x)) + 2K \quad (9)$$

$$\mathcal{L}(\theta|x) = \log \left(\prod_{i=1}^n f(y_i, X_i; \theta) \right) \quad (10)$$

263 The best fitted model of a group of models fit to the same data set should
 264 have the smallest AIC value. The symbol Δ_i is defined as “the difference of
 265 AIC value between the i th model and the best fitted model” [38]. The strategy
 266 developed by Burnham and Anderson [38] was used to compare the goodness
 267 of fit of multiple models of a given sample set (Table 3). The larger the Δ_i ,
 268 the less plausible it is that the fitted model is the best model. As seen in
 269 Table 3, models with $\Delta_i > 10$ are considered inappropriate.

270 3.3.2. Results

271 The logistic regression results for Hurricane Isaac using the peak outage
 272 X_{max} as a direct response variable are summarized in Table 4 for the sample
 273 size of $N = 20$ data points as described in section 3.2.1.

274 Table 4 contains the fitted model parameters for the single hazard linear
 275 models, two hazard linear models, a three hazard linear model, two hazard
 276 models with interactions and finally, a three hazard model with multiple in-
 277 teractions. The significance of the fitted coefficients is provided through the
 278 p-values. The probability value (p-value) of a hypothesis test is the probabil-
 279 ity of obtaining a value of the test statistic as extreme, or more extreme, than
 280 the one observed, if the null hypothesis is true. The models are ranked in
 281 each category by the smallest to largest AIC values. Although the three haz-
 282 ard model with multiple interactions provides the overall lowest AIC value

283 of 374.91, not all of the coefficients are significant. The two hazard linear
 284 model with rainfall and wind speed (AIC = 398.40) is superior to models
 285 for either rainfall or wind speed alone (AIC = 428.49, 462.68, respectively).
 286 Figure 5 shows the 3-D surface for this particular model. The equation of
 287 the 3-D surface has the form as in equation (11):

$$F(X_{max}|H_1, H_2) = \frac{\exp(-5.024 + 0.005H_1 + 0.158H_2)}{1 + \exp(-5.024 + 0.005H_1 + 0.158H_2)} \quad (11)$$

288 where H_1 and H_2 denote the rainfall [mm] and wind speed [m/s] hazard
 289 variables respectively.

Figure 5: 3D surface for two hazard linear model with rainfall and wind speed for Isaac

290 Adding an interaction term to the two hazard linear model with rainfall

and wind speed does not significantly decrease the AIC value (398.40 versus 398.54). The three hazard linear model with AIC of 400.71, which is close to 398.40, does not have a storm surge inundation coefficient value that is significant. In summary, the results suggest that the multiple hazard fragilities are better than the single hazard fragility representation, which confirms anecdotal observations regarding combined wind plus rain and surge damage, versus each alone. The lack of significance for the coefficients of the three hazard models with interactions may be due in part to the small data set.

In order to test the logistic model for an electric power delivery fragility in another region of the country, the Sandy data for the localities of Manhattan in the City of New York were used. As noted previously, the spatial resolution for this storm was much smaller than for Isaac. Because the majority of the power infrastructure was located underground, the three hazard model including wind speed was not a good fit.

The maximum storm surge and rainfall variables are reported here for Sandy, as shown in Table 5. The two hazard model results were superior to the single hazard models, which confirms the importance of multiple hazards on lifeline fragility. The interaction term did not appear to add much to the model fit and the coefficients were not as significant. In summary, the fragility based on combined storm surge inundation and rainfall hazards was the best model for Manhattan. Figure 6 illustrates this fragility model. The equation of the 3-D surface has the form as in equation (12):

$$F(X_{max}|H_1, H_3) = \frac{\exp(-51.372 + 1.399H_1 + 1.394H_3)}{1 + \exp(-51.372 + 1.399H_1 + 1.394H_3)} \quad (12)$$

where H_1 and H_3 denote the rainfall [mm] and storm surge inundation [m]

314 hazard variables respectively.

315 This result confirms observations that the bulk of the power grid in lower
316 Manhattan is underground, with the exception of two power substations
317 near the East River at East 13th Street. Post event surveys showed that
318 wind speed effects were significant in Westchester County where overhead
319 lines were more prominent.

Figure 6: 3D surface for two hazard linear model with rainfall and storm surge for Sandy

320 4. Discussion

321 The characterization of lifeline fragilities using multiple hazards, includ-
322 ing hazard interactions, as shown in section 3, is an attractive alternative to

323 lognormal single hazard models. The degree to which this additional charac-
 324 terization is useful may only be apparent after the damage due to multiple
 325 storms has been analyzed. However, the fragility function is only one facet of
 326 the performance-based design process [4, 5]. Ultimately, in order to evaluate
 327 the performance of the delivery system for storm hazards, the multivariate
 328 probability density of the hazards is required. The theory of total probabil-
 329 ity indicates that an integration of the product of the conditional probability
 330 $F(X_{max}|H = h)$ and marginal $f(h)$ or multivariate densities $f(h_i, h_j, \dots)$ will
 331 yield a probability function. That is, the cumulative distribution function
 332 $F(X_{max})$ may be determined from the following integration if a multivariate
 333 model for n hazards is used,

$$F(X_{max}) = \iiint_{h_1, \dots, h_n} F(X_{max}|h_1, \dots, h_n) f(h_1, \dots, h_n) dh_1 \dots dh_n. \quad (13)$$

334 The representation of joint probability distributions for wind speed and
 335 storm surge data has been investigated recently, e.g. [39, 40, 41]. Pei et
 336 al. [39, 40] considered the joint occurrence of both hazards in a loss es-
 337 timation scheme for Charleston County, South Carolina. They created a
 338 simulation approach that utilized over 50,000 years of synthetic hurricane
 339 data to model joint distributions numerically. Phan et al. [41] implemented
 340 a proof of concept investigation using bivariate Normal distributions. Ac-
 341 cording to Johnson and Kotz [42] even if two variables are not Normal, their
 342 joint distribution may be Normal. However, one of the challenges in fitting
 343 the bivariate distribution of any type is the restriction of satisfying stabil-
 344 ity conditions for the functional form even if the marginal distributions do
 345 not completely specify the joint distribution, e.g.[42]. For a two hazard logit

346 fragility model, the joint hazard distribution would be sufficient. For more
 347 complex three hazard fragilities, the multivariate hazard distribution would
 348 be required. For hurricane hazards, a form of a logistic hazard distribution
 349 $P_{H_1 H_2 H_3}$ is suggested as follows:

$$P_{H_1 H_2 H_3}(h_1, h_2, h_3) = \frac{1}{1 + e^{-\lambda_1 h_1} + e^{-\lambda_2 h_2} + e^{-\lambda_3 h_3} + \sum_{i,j,i \neq j}^3 e^{-\lambda_{ij} h_i h_j}} \quad (14)$$

350 where H_1 , H_2 , and H_3 are defined as three random hazard variables; λ_1 , λ_2 ,
 351 and λ_3 are coefficients.

352 It is noted that correlations among the hazard variables would need to be
 353 characterized for these distributions. However, a multivariate approach has
 354 yet to be fully explored for storm hazards. Whether the complexity of this
 355 model will provide the appropriate characterization remains to be seen.

356 In addition, the collection of data sets for modeling hazard and related
 357 damage will be essential. It is anticipated that predictive models of col-
 358 lected weather hazards will become available in the future from atmospheric
 359 scientists at the space and time scales necessary for engineering models.

360 5. Summary and conclusions

361 The methodology developed in this paper is useful to researchers, sys-
 362 tem owners, government agencies, insurance companies, and others who
 363 have access to large data sets describing system functionality during ex-
 364 treme events to facilitate generation of multi-hazard system-level fragility
 365 functions. The results of the case studies will be useful to practitioners
 366 evaluating system-level performance in Louisiana and New York. Most im-
 367 portantly, the methodology and results generated will be useful for multiple

368 users who rely on loss functions for downstream analysis, including compar-
369 ative risk assessment, overall system reliability and catastrophe models (e.g.
370 used in the insurance industry).

371 In summary, logistic regression models have been used successfully to
372 characterize fragilities for electric power delivery subject to hurricane haz-
373 ards, where fragility is defined here as a conditional probability function.
374 The system-level fragility models, while representing only single event sce-
375 narios, are based on geo-coded post-event in-situ hurricane weather hazard
376 and power delivery system damage data rather than numerical simulations.
377 Multiple simultaneous hazards of wind speed, rainfall and storm surge were
378 used for the fragility model fits. This multi-hazard characterization property
379 of the logit models is a distinct advantage over lognormal models. However,
380 extensive GIS data are required for fitting the models. It is anticipated that
381 continued investigations into the models presented here for other storms in
382 the same geographical region will yield not only predictive capabilities, but
383 also needed insight into the hurricane damage process. The characterization
384 of the multivariate distributions of the simultaneous weather hazards will
385 also improve damage predictions for hurricane prone regions.

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Table 1: Hazard Data Employed in the Isaac Analysis

Data	Description	Source
Wind Speed [m/s]	H*Wind: Maximum sustained wind speed swaths for Hurricane Isaac for the State of Louisiana.	HWind Scientific [†] [8]
Inundation due to Storm Surge [m]	Depth of water above local land elevation as determined for storm surge using hindcasting.	Coastal Emergency Risks Assessment (CERA) [43]
Rainfall [mm]	Observed daily precipitation for the duration of the storm.	National Weather Service, Advanced Hydrologic Predictive Service [44]
[†] Note: The Hurricane Research Division of the National Oceanic & Atmospheric Administration produced surface wind analysis of tropical cyclones from 1993 - 2013 as part of the H*Wind Project. In 2014 this activity was transferred to HWind Scientific, a US private sector firm. The images and data sets formerly available on the Hurricane Research Division website are now publicly available on the HWind Scientific website.		

Table 2: Hazard Data Employed in the Sandy Analysis

Data	Description	Source
Wind Speed [m/s]	H*Wind: Maximum sustained wind speed swaths for Hurricane Sandy for the New York City.	HWind Scientific [†] [8]
Storm Surge Inundation [m]	Water surface elevation above local ground interpolated using High Water Marks (HWMs) and Storm Surge Sensor data from the United States Geological Survey (USGS)	Federal Emergency Management Agency (FEMA) Modeling Task Force (MOTF) [45]
Rainfall [mm]	Observed daily precipitation for the duration of the storm.	National Weather Service, Advanced Hydrologic Predictive Service [44]

[†] Note: The Hurricane Research Division of the National Oceanic & Atmospheric Administration produced surface wind analysis of tropical cyclones from 1993 - 2013 as part of the H*Wind Project. In 2014 this activity was transferred to HWind Scientific, a US private sector firm. The images and data sets formerly available on the Hurricane Research Division website are now publicly available on the HWind Scientific website.

Table 3: Interpretation of AIC difference Δ_i [38]

Δ_i	Level of Empirical Support of Model i
$0 \sim 2$	Substantial
$4 \sim 7$	Considerably less
> 10	Essentially none

Table 4: Hurricane Isaac Fragility Fit Results

‡ Significance code for the p-value Column: between 0 ~ 0.01 ***; 0.01 ~ 0.05 **; 0.05 ~ 0.1 *					
Model Variables	Coefficients	Estimate	Standard Error	‡p-value	AIC
Single Hazard Linear Model Results <i>listed in order of lowest AIC</i>					
$X_{max} \sim$ Wind [m/s]	Intercept β_0	-5.748	0.267	<2e-16***	428.49
	Wind [m/s]	0.248	0.011	<2e-16***	
$X_{max} \sim$ Rainfall [mm]	Intercept β_0	-3.206	0.159	<2e-16***	462.68
	Rainfall [mm]	0.011	0.0004	<2e-16***	
$X_{max} \sim$ Inundation [m]	Intercept β_0	-2.165	0.121	<2e-16***	477.38
	Inundation [m]	1.119	0.049	<2e-16***	
Two Hazard Linear Model Results <i>listed in order of lowest AIC</i>					
$X_{max} \sim$ Rainfall [mm] + Wind [m/s]	Intercept β_0	-5.024	0.291	<2e-16***	398.40
	Rainfall [mm]	0.005	0.001	1.69e-07***	
	Wind [m/s]	0.158	0.019	3.31e-16***	
$X_{max} \sim$ Inundation [m] + Wind [m/s]	Intercept β_0	-4.681	0.356	<2e-16***	413.06
	Inundation [m]	0.399	0.096	3.07e-15***	

	Wind [m/s]	0.168	0.021	3.05e-15***	
$X_{max} \sim$ Inundation [m] + Rainfall [mm]	Intercept β_0	-2.796	0.186	<2e-16***	452.99
	Inundation [m]	0.520	0.133	9.26e-05***	
	Rainfall [mm]	0.006	0.001	3.11e-06***	
Three Hazard Linear Model Result					
$X_{max} \sim$ Inundation [m]+Rainfall [mm]+Wind [m/s]	Intercept β_0	-4.956	0.371	<2e-16***	400.71
	Inundation [m]	0.043	0.146	0.767	
	Rainfall [mm]	0.004	0.001	0.001***	
	Wind [m/s]	0.156	0.022	6.31e-13***	
Two Hazard Model with Interactions Results <i>listed in order of lowest AIC</i>					
$X_{max} \sim$ Rainfall [mm] + Wind [m/s] + (Rainfall $\times$ Wind)	Intercept β_0	-5.526	0.707	5.40e-15***	398.54
	Rainfall [mm]	0.007	0.003	0.0093***	
	Wind [m/s]	0.179	0.033	9.33e-08***	
	Rainfall $\times$ Wind	-7.75e-05	9.42e-05	0.4111	
$X_{max} \sim$ Inundation [m] + Wind [m/s] + (Inundation $\times$ Wind)	Intercept β_0	-5.018	0.520	<2e-16***	412.78
	Inundation [m]	0.682	0.284	0.0162***	

	Wind [m/s]	0.179	0.026	2.38e-12***	
	Inundation $\times$ Wind	-0.009	0.009	0.310	
$X_{max} \sim$ Inundation [m] + Rainfall [mm] + (Inundation $\times$ Rainfall)	Intercept β_0	-3.359	0.296	<2e-16***	447.82
	Inundation [m]	0.976	0.216	6.33e-06***	
	Rainfall [mm]	0.008	0.001	2.04e-07***	
	Inundation $\times$ Rainfall	-0.001	0.000	0.00926***	
Three Hazard Model with Multiple Interactions Result					
$X_{max} \sim$ Inundation [m] + Rainfall [mm] + Wind [m/s] + (Inundation $\times$ Rainfall) + (Inundation $\times$ Wind) + (Rainfall $\times$ Wind) + (Inundation $\times$ Rainfall $\times$ Wind)	Intercept β_0	-11.140	1.430	6.84e-15***	374.91
	Inundation [m]	0.040	0.423	0.925	
	Rainfall [mm]	0.042	0.009	2.55e-06***	
	Wind [m/s]	0.413	0.065	2.32e-10***	
	Inundation $\times$ Rainfall	-0.006	0.002	0.0061***	
	Inundation $\times$ Wind	-0.004	0.014	0.7942	
	Rainfall $\times$ Wind	-0.001	3.16e-04	3.85e-06***	
	Inundation $\times$ Rainfall $\times$ Wind	2.44e-04	6.81e-05	0.0003***	

Table 5: Hurricane Sandy Fragility Fit Results for Manhattan Localities Only

‡ Significance code for the p-value Column: between 0 ~ 0.01 ***; 0.01 ~ 0.05 **; 0.05 ~ 0.1 *					
Model Variables	Coefficients	Estimate	Standard Error	‡p-value	AIC
Single Hazard Linear Model Results <i>listed in order of lowest AIC</i>					
$X_{max} \sim$ Rainfall [mm]	Intercept β_0	-45.346	2.504	<2e-16***	1693.30
	Rainfall [mm]	1.410	0.0770	<2e-16***	
$X_{max} \sim$ Inundation [m]	Intercept β_0	-8.770	0.417	<2e-16***	2438.40
	Inundation [m]	1.880	0.088	<2e-16***	
Two Hazard Linear Model Results: With and Without Interaction					
$X_{max} \sim$ Inundation [m] + Rainfall [mm]	Intercept β_0	-51.372	2.667	<2e-16***	1353.10
	Inundation [m]	1.394	0.096	<2e-16***	
	Rainfall [mm]	1.399	0.079	<2e-16***	
$X_{max} \sim$ Inundation [m] + Rainfall [mm] + Inundation $\times$ Rainfall	Intercept β_0	-42.982	7.244	<2.96e-09***	1357.30
	Inundation [m]	-0.251	1.378	0.855	
	Rainfall [mm]	1.138	0.224	3.84e-07***	
	Inundation $\times$ Rainfall	0.051	0.043	0.232	