Wavelet-Galerkin analysis to study the coupled dynamic response of a tall building against transient wind loads

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Abstract

The wavelet-Galerkin analysis approach is explored for the solution of stochastic structural dynamic response of a tall building under transient nonstationary winds. The approach is obtained by combining the Galerkin expansion method with basis-functions selected from discrete orthonormal wavelets (namely, the compactly supported Daubechies wavelets). The expansion allows transforming the stochastic dynamic problem of the tall building, subject to time-dependent turbulent-induced forces and motion-induced self-excited forces, into a system of random algebraic equations in the domain of the wavelet coefficients. A reduced-order model of a benchmark tall building is employed as a numerical example. Nonstationary wind time histories, simulating the loading of a downburst, are artificially generated at discrete points along the vertical axis of the building by using the notions of evolutionary power spectral density of the turbulence and time-dependent amplitude modulation function. Important aspects such as the treatment of boundary conditions and wavelet resolution are examined. The paper also aims at investigating the influence of the order of the wavelets and the wavelet resolution on the numerical accuracy of the building response. **Even though the primary purpose of the study is to examine the feasibility of the WG analysis method for studying the transient stochastic response of the tall building, a “frozen” thunderstorm downburst model (a first approximation of slowly-varying time-dependent wind velocity profile with constant wind direction and negligible translational thunderstorm velocity) is also employed. Two time-independent synoptic wind velocity profiles (power-law model) and one non-synoptic downburst wind velocity profile (“Vicroy’s model”) are considered.**

Keywords: Wavelet-Galerkin method; Daubechies wavelet; coupled dynamics; transient response, nonstationary wind loading process; tall buildings; thunderstorm downburst.

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1 Nomenclature

$[A]$	WG approximation coefficient matrix (<i>sdof</i> system)
$\hat{A}_p(\omega, z_p, t)$	Deterministic modulation function
$[A_{11}], [A_{12}], [A_{21}], [A_{22}]$	WG approximation coefficient matrices in the x -, y - coordinates
a, b	Scale and translation parameters of the wavelet function
a_k	Scaling coefficient
$\{B_1\}, \{B_2\}$	WG approximation force coefficient matrices in the x -, y - coordinates
B	Width of the building cross section
b_{bxl}, b_{byl}	WG approximation force coefficient vectors in the x -, y - coordinates
$C_D, C'_D,$	Static along-wind force coefficient and its first derivative
C_L, C'_L	Static cross-wind force coefficient and its first derivative
$Coh_{u,pq}(\omega, z_p, z_q)$	Along-wind coherence function between two coordinates z_p, z_q of
c_{jk}	Detailed wavelet coefficients at small scale levels $j < j_0$
c_{j_0k}	Wavelet approximation coefficients at the j_0 -th scale
D	Depth of the building cross section
$F_{b,r}(z, t)$	Distributed buffeting force per unit height
$F_{a,r}(z, t, \dot{r}, \ddot{r})$	Distributed self-excited force per unit height
$F_{b,x}(z, t), F_{b,y}(z, t)$	Distributed buffeting forces per unit height in the x -, y - coordinates
$F_{a,x}(z, t, \dot{x}, \ddot{x}), F_{a,y}(z, t, \dot{y}, \ddot{y})$	Distributed self-excited forces per unit height in the x -, y - coordinates
f_l	WG approximation force vector (<i>sdof</i> system)
$H_u(\omega, z)$	Lower triangular matrix by Cholesky decomposition of stationary cross spectral matrix
H	Building height
x	Generic motion variable
I	Identity matrix
M	Number of nodes along the building height
M_r, C_r, K_r	Generalized mass, damping and stiffness coefficients of r -th coordinate
M_x, C_x, K_x	Generalized mass, damping and stiffness coefficients in the x coordinate
M_y, C_y, K_y	Generalized mass, damping and stiffness coefficients in the y coordinate
m, c, k	Mass, damping and stiffness coefficients, respectively (<i>sdof</i> system)
$m(z)$	Distributed mass of the building per unit height
N	Order or “genus” of the wavelet
N_n, N_x	Original and extended computational domain of the wavelet expansion
n_r, ζ_r	Natural frequencies and damping ratio
$Q_{b,r}(t)$	Generalized turbulent-induced buffeting force

$Q_{a,r}(t, r, \dot{r}, \ddot{r})$	Generalized motion-induced self-excited forces
$Q_{xx}, Q_{yx}, Q_{yy}, Q_{xy}$	Generalized motion-induced force terms in the x -, y - coordinates
$Q_{b,x}(t), Q_{b,y}(t)$	Generalized turbulent-induced forces in the x -, y - coordinates
q_{bxl}, q_{byl}	WG approximation buffeting force vectors in the x -, y - coordinates
r	Generalized coordinate index ($r=x$ or $r=y$)
$S_u^0(\omega, z)$	Stationary cross spectral matrix of the (x) along-wind turbulence
$S_u(\omega, z, t)$	Transient cross spectral matrix of the (x) along-wind turbulence
$S_{u,pp}^0(\omega, z_p)$	Stationary auto-power spectrum of the along-wind turbulence
$S_{u,pq}^0(\omega, z_p, z_q)$	Stationary cross-power spectrum of the along-wind turbulence
$S_{u,pp}(\omega, z_p, t)$	Evolutionary auto-power spectrum of the along-wind turbulence
$S_{u,pq}(\omega, z_p, z_q, t)$	Evolutionary cross-power spectrum of the along-wind turbulence
$U(z)$	Along-wind mean wind speed (stationary field)
$\bar{U}(z)$	Along-wind “frozen” wind velocity field (downburst)
$U_{tot,p}(z_p, t)$	Along-wind total wind velocity field (slowly varying, $U'_p(z_p, t)$, and turbulence fluctuations, $u'_p(z_p, t)$) at the building height z_p
$U'_p(z_p, t)$	Along-wind “slowly-varying” mean wind velocity (downburst) at the building height z_p
u_l	WG approximation displacement coefficient vector
$u(z, t)$	Along-wind zero-mean fluctuating wind field
$u_p(z_p, t), u'_p(z_p, t)$	Along-wind zero-mean stationary and transient wind fluctuation, respectively at the coordinate z_p
$v(z, t)$	Cross-wind zero-mean fluctuating wind field
$v_p(z_p, t), v'_p(z_p, t)$	Cross-wind zero-mean stationary and transient wind fluctuation, respectively at the coordinate z_p
$\{x_l\}, \{\dot{x}_l\}, \{\ddot{x}_l\}$	WG approximation coefficient vectors of the along-wind (x) displacement, velocity and acceleration, respectively
$\{y_l\}, \{\dot{y}_l\}, \{\ddot{y}_l\}$	WG approximation coefficient vectors of the cross-wind (y) displacement, velocity and acceleration, respectively
x, y	Along-wind (x) and cross-wind (y) coordinates of the building
x_l, y_l	WG approximation coefficients in the x -, y - coordinates
$x(t), \dot{x}(t), \ddot{x}(t)$	Along-wind (x) displacement, velocity and acceleration
$y(t), \dot{y}(t), \ddot{y}(t)$	Cross-wind (y) displacement, velocity and acceleration
z	Vertical coordinate of the building
z_p	Vertical coordinate of the generic p -th discrete node of the building

$\delta_{0,l-k}$	Kronecker delta
$\varphi(x)$	Mother scaling function
$\varphi_{j,k}(x)$	Scaling function at dilation j and translation k
ρ	Air density
$\Phi_r(z)$	Continuous mode shape of the building, r -th mode
ϕ_{ml}	Random phase angles, generated by Monte Carlo sampling
$\psi(x)$	Father wavelet function
$\psi_{a,b}(x)$	Wavelet function at dilation a and translation b
$[\Omega^{0,0}], [\Omega^{0,1}], [\Omega^{0,2}]$	2-term connection coefficient matrices, containing respectively $\Omega_{l-k}^{0,0}$, $\Omega_{l-k}^{0,1}$, $\Omega_{l-k}^{0,2}$
$\Omega_{l-k}^{0,0}, \Omega_{l-k}^{0,1}, \Omega_{l-k}^{0,2}$	2-term connection coefficient at the derivative order 0, at the derivative orders 0 and 1 and at the derivative orders 0 and 2
$\Omega_{l-k}^{d_1, d_2}$	2-term connection coefficients at the derivative orders d_1, d_2
$\Omega_{l_1 l_2 \dots l_n}^{d_1, d_2, \dots, d_n}$	Multiple-term connection coefficients at the multiple derivative orders $d_1, d_2, \dots, d_n$

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3 *Other symbols, subscript or superscript indices and operators:*

l, k	Translation parameters
j	Dilation parameter
p	Discrete nodal index ($p=1,2,\dots,M$)
$\langle , \rangle$	Inner product operator
$E[]$	Expectation operator
T	Transpose operator
$*$	Complex conjugate operator

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6 **1. Introduction**

7 Tall buildings are sensitive to wind excitation and often experience large wind-induced vibration
8 amplitudes due to small structural stiffness, small structural damping and low fundamental vibration
9 frequencies [1-3]. Large amplitude excitation and response of tall buildings result in many
10 engineering design issues, related to both structural serviceability, such as human discomfort (e.g.,
11 [4,5]), cumulative damage and ultimate failure. Wind-induced stochastic dynamics of a tall building
12 can potentially involve complex dynamic problems due to nonlinear structural behavior, motion
13 coupling and nonstationary winds (e.g., [6]). For the sake of simplification, however, assumptions on

the linear structural behavior and stationary wind loads are usually postulated. Also, the 3D-motion coupling is often neglected. Therefore, the stochastic response of a building can be adequately examined by means of linear elastic reduced-order models, which primarily describe the response features associated with the first fundamental vibration modes of the structure in terms of corresponding generalized coordinates [2,7-8]. A reduced-order model of a tall building usually includes coupled-motion differential equations due to the combination between time-dependent buffeting forces with motion-induced aeroelastic forces. Solution to the uncoupled stochastic dynamic response of the building, induced by stationary wind loads, can usually be found in the frequency domain using the Fourier transform, since the stochastic differential equations can be transformed into a simpler algebraic form [2,7]. In contrast, the solution of the motion equations in the time domain, needed in the case of coupled nonlinear building response is less often pursued since it may lead to complex and computationally demanding procedures [8,9].

Furthermore, recent investigations have explained that the fluctuating wind processes in extreme and local-convection wind events, such as thunderstorms and downbursts (e.g.,[10-12]) exhibit “non-synoptic” features, in which the wind speed is rapidly time-varying and fluctuations are no longer stationary but transient (e.g., [6,13]). The pressure loads in transient/nonstationary winds evolve with time both in amplitude and frequency. As a result, the stochastic response of a tall building due to transient winds is transient/nonstationary. Also, nonlinear effects may no longer be neglected [6].

Therefore, conventional assumptions on stationarity to evaluate the stochastic response of a tall building can be inadequate in the case of an extreme local-convection wind because of the time-varying amplitude and frequency properties of the transient wind speed fluctuations, and in relation to the use of efficient computational methods for the response. These are also the main reasons why numerical solutions for the wind-induced transient stochastic dynamics of a tall building might be extremely complicated [6]. Several studies have attempted to overcome such difficulties, for example by investigating the characteristics of extreme nonstationary wind events [10,14], by more accurately modelling the wind fluctuations [15], by estimating the evolutionary power spectra of transient/nonstationary wind velocities (e.g.,[16]), by digitally simulating the transient/nonstationary wind velocity field [17], by reformulating the problem of the along-wind transient response of tall

buildings in time domain [18] and in frequency domain [19], or by applying the concept of “response spectrum”, a technique extensively used in earthquake engineering, to thunderstorms [20].

Since the discovery of the wavelet transform in the late 80s, wavelets have emerged as a powerful computational tool for scientific analysis. Wavelets are calculated as continuously oscillatory functions and possess attractive analytical features: zero-mean, fast decay, short life, time-frequency representation, multi-resolution, etc. Therefore, wavelet transforms have been applied to various types of computational analyses in science and engineering, which include the study of nonstationary phenomena in the simultaneous time-frequency domain. For example, continuous wavelet transform has been used to generate artificial nonstationary seismic processes and nonstationary response of simplified systems [21,22]. In wind engineering, continuous wavelet transforms have been predominantly employed for signal processing, applied to nonstationary pressure analysis (e.g., [23]) and system identification (e.g., [24]).

Since Daubechies conceived the compactly supported (discrete) wavelets, known as Daubechies wavelets [25], wavelet-based computational tools have rapidly evolved in structural dynamics. The Daubechies wavelets possess the advantageous properties of being piecewise-defined functions, compact, orthogonal and of enabling multi-resolution analysis. Wavelets can be used to represent “computational solutions” at any pre-selected level of resolution. This property makes them particularly useful for developing an approximating solution to complex problems in structural dynamics, for example by Galerkin projection approach. The combined wavelet-Galerkin analysis method (WG) is a powerful approach for engineering computations, in which the Daubechies wavelets can be used as a basis of piecewise functions for the Galerkin projection. The advantageous properties of orthogonality and aptitude to multi-resolution analysis, typical of the Daubechies wavelets, can “empower” the Galerkin approximation.

The WG method has been successfully applied to several fields of engineering, such as the solution of partial and ordinary differential equations [26,27], the identification of linear and time-varying parameters of single-degree-of-freedom systems [28,29] and the analysis of simplified continuous mechanical systems [30]. One of the possible applications of the WG method involves the solution of stochastic structural dynamic problems of nonlinear structures subjected to

transient/nonstationary wind loading. Preliminary investigations on the use of the WG method for the wind-induced response analysis include the nonlinear stochastic dynamics of single-degree-of-freedom systems [31], the stationary response analysis of long-span bridges [32] and the stochastic dynamics of tall buildings [33]. Nevertheless, computational challenges of the WG method in stochastic dynamics have been observed. These include the accurate treatment of boundary (or initial) conditions, the estimation of the wavelet resolution, arbitrary time duration and computational complexity. These aspects have prevented the WG method from expanding to a wide spectrum of stochastic dynamic applications [30,34]. Fortunately, the treatment of boundary conditions and wavelet resolution along with an improved computation of the wavelet connection coefficients have been recently resolved [31,35].

In consideration of the recent advancements of the WG method to study stochastic structural dynamics problems, this paper proposes to use the WG method for the simulation of the coupled stochastic response of a tall building due to synthetically-generated transient winds by reduced-order dynamic models. The WG method is employed to transform the time-varying differential equations of motion, which couple the dynamics of the system with turbulent-induced buffeting forces and motion-induced aeroelastic forces, into a random algebraic system of equations in the wavelet domain; the unknown wavelet coefficients of this system can be solved very efficiently. The CAARC tall building [36] is used as a benchmark for the verification of the WG method. Multivariate evolutionary transient realizations of the wind turbulence field are artificially simulated along the vertical axis of the building, by utilizing the concept of amplitude modulation functions applied to a multivariate stationary wind field (e.g., [19]). Investigations are carried out to analyze the influence of the order of the Daubechies wavelet and wavelet resolution on the analysis of the transient building response. In order to approximately simulate the features of a transient wind, a “frozen” thunderstorm downburst model is employed, keeping in mind that the primary purpose of examining the feasibility of the WG analysis method. In this “frozen” thunderstorm downburst model time-independent wind velocity profile, constant wind directions as well as a fixed downburst center, which neglects the effect of the translational thunderstorm velocity on the downburst loading have been used. Apart from two time-

independent wind velocity profiles of a synoptic wind (power-law model), a non-synoptic wind profile (“Vicroy’s model”) is examined.

2. Wavelet-Galerkin analysis: background

The wavelets $\psi_{a,b}(x)$ are defined as a family of piecewise functions, generated from a “mother” wavelet by scaling (a) (as the inverse of a Fourier frequency) and translation (b) parameters, as

$$\psi_{a,b}(x) = \frac{1}{\sqrt{|a|}} \psi\left(\frac{x-b}{a}\right). \quad (1)$$

Wavelets possess very useful properties, which make them particularly attractive to represent transient nonstationary signals. Wavelets properties include “dilation” and “translation” and the concept of multi-resolution, which enables a signal to be observed on the simultaneous time-frequency plane. Dyadic and compact wavelets with $a = 2^{-j}$, $b = k2^{-j}$ are often employed along with the concept of discrete sampling (k, j are translation and dilation parameters, respectively).

Compactly supported wavelets are functions with non-zero values only within a finite interval and identically zero elsewhere. The family of compactly supported Daubechies wavelets (D) is very well suited for engineering computations. The “father” scaling functions $\varphi(x)$ of the Daubechies wavelet of order N , with support over the finite interval $\text{supp}(\varphi) = [0, N - 1]$, dilating and translating in the domain of a signal at any resolution; it can be found from a recursive expression [25]:

$$\varphi(x) = \sum_{k=0}^{N-1} a_k 2^{j/2} \varphi(2^j x - k), \quad (2)$$

where a_k : scaling coefficients, satisfying $\sum_{k=0}^{N-1} a_k = 2$; k, j : translation and dilation parameters; the scaling functions satisfy: $\int_{-\infty}^{+\infty} \varphi(x) dx = 1$. In the Daubechies wavelet family, “twin mother wavelet” functions are derived from the father scaling function as $\psi(x) = \sum_{k=-1}^{N-2} (-1)^k a_{k+1} \varphi(2x + k)$; they satisfy the property $\int_{-\infty}^{+\infty} \psi(x) x^k dx = 0$. The scaling functions and the wavelet functions are orthonormal, i.e., they satisfy $\int_{-\infty}^{+\infty} \varphi_l(x) \varphi_k(x) dx = \delta_{0,l-k}$, $\int_{-\infty}^{+\infty} \psi_l(x) \psi_k(x) dx = \delta_{0,l-k}$, $\sum_{k=0}^{N-1} a_k a_{k+l} = \delta_{0,l}$, in which l, k are both translation parameters and $\delta_{0,l}$ is the Kronecker delta. Figure 1 illustrates a few examples of Daubechies father scaling functions $\varphi(x)$ of various orders (N) along with their corresponding mother wavelet functions $\psi(x)$: $D2$ ($N=2$), $D6$ ($N=6$), $D8$ ($N=8$), $D10$

($N=10$) and $D20$ ($N=20$). The support interval of each pair of scaling function and wavelet function in the time domain widens, when the order of the Daubechies wavelet increases. The Wavelet expansion of an analytical signal $u(x)$, based on the pair of Daubechies father scaling function and mother wavelet function of order N at a pre-selected resolution level j is expressed in the form [25]:

$$u(x) = \sum_{l=0}^{N_x} c_{jl} \varphi_{jl}(x) + \sum_{a=0}^j \sum_{l=0}^{N_x} c_{al} \psi_{al}(x). \quad (3)$$

In the previous equation c_{jl} are “approximation” coefficients at the j -th resolution; $c_{jl} = \langle u(x), \varphi_{jl}(x) \rangle$, with the symbol $\langle . \rangle$ denoting inner product; $c_{al} = \langle u(x), \varphi_{al}(x) \rangle$ are “detailed” coefficients at very small scales $a < j$; $c_{al} = \langle u(x), \varphi_{al}(x) \rangle$ is the translation parameter; N_x is the computational domain. If the discrete wavelet decomposition in Eq. (3) is truncated at the j -th resolution level, the approximation of $u(x)$ at the j -th resolution is $u(x) \approx \sum_{l=0}^{N_x} c_{jl} \varphi_{jl}(x)$.

Three examples of $D4$ wavelets in their dilation and translation on a 100-second interval are indicated in Fig. 2a, while the concept of time-frequency resolution analysis of the $D4$ is illustrated in Fig. 2b. There is an apparent trade-off between time and frequency resolution in the wavelet analysis, i.e., the finer the time resolution the poorer the frequency resolution and conversely. Fortunately, low frequency resolution and high time resolution are usually needed for processing the fundamental (low) frequency components of “practical” signals; high frequency resolution and low time resolution are necessary for the higher frequency components.

The Galerkin method is a projection method that has been widely applied to the solution of differential equations in structural dynamics and engineering. This method seeks for an approximating solution through the projection of the exact solution onto a subspace spanned by a basis of functions. The Galerkin projection approximates an exact solution $u(x)$ to an equation $Au(x)=f$ by projecting it onto a subspace using a finite number of basis-functions $\varphi(x)$. If the approximating solution is defined as $u^*(x) = \sum_{l=1}^{N_n} u_l \varphi_l(x)$ on an inner-product space of N_n finite dimensions, it satisfies the conditions $u_l = \langle u^*, \varphi_l \rangle = \int_{-\infty}^{+\infty} u^*(x) \varphi_l(x) dx$ and $\langle Au^* - f, \psi \rangle = 0$, with u_l being unknown coefficients and $\psi(x)$ weight functions. If the inner-product operation in N_n finite dimensions is applied to the original equation in a general form as $\sum_{l=1}^{N_n} u_l \langle \psi_l, A\varphi_l \rangle = \langle \psi_l, f \rangle$, one obtains a matrix

equation $\overline{[A]}\{\overline{u}\} = \{\overline{f}\}$, where $\overline{[A]}$ is an N_n -by- N_n matrix with elements $\overline{A}_{l,k} = \langle \psi_l, A\varphi_k \rangle$, $\{\overline{u}\} = (u_1, u_2, \dots, u_{N_n})^T$, $\{\overline{f}\} = (\langle \psi_1, f_1 \rangle, \langle \psi_2, f_2 \rangle, \dots, \langle \psi_{N_n}, f_{N_n} \rangle)^T$. The basis function is often composed of a function, containing multiple piecewise sub-functions. Each sub-function is projected onto a given interval of the basis function's domain. The weight functions are chosen to be orthogonal to the basis functions; the unknown coefficients u_l can be estimated from the coefficient matrix.

In the WG method, the orthonormal and compactly supported Daubechies wavelets can be employed as the basis functions and weight functions in the Galerkin projection to find an approximate solution to a time-varying dynamic problem. If the time variable is denoted by t , a generic motion variable $u(t)$ can be expressed, at a preselected level of resolution j of the wavelet, as:

$$u(t) = \sum_{l=1}^{N_x} u_l \varphi_l(t). \quad (4)$$

In the previous equation u_l are approximation coefficients, derived from the inner product $u_l = \langle u(t), \varphi(t) \rangle = \int_{-\infty}^{\infty} u(t) \varphi(t) dt$. Similarly, the first and second derivatives $\dot{u}(t), \ddot{u}(t)$ can be approximated, in the Daubechies wavelet subspace, from the previous equation as:

$$\dot{u}(t) = \sum_{l=1}^{N_x} u_l \dot{\varphi}_l(t), \quad \ddot{u}(t) = \sum_{l=1}^{N_x} u_l \ddot{\varphi}_l(t). \quad (5)$$

The derivatives of the wavelets can be obtained correctly in the limit support, i.e., in the interval $[0, N-1]$. The inner products between approximating solutions of the displacement, velocity, acceleration in Eqs. (4-5) and each term of the expansion φ_k are required by the Galerkin expansion. Due to the orthogonality property of the Daubechies wavelets, these are:

$$\begin{aligned} \langle \varphi_k, \sum_{l=1}^{N_x} u_l \varphi_l \rangle &= \sum_{l=1}^{N_x} u_l \delta_{lk}, \\ \langle \varphi_k, \sum_{l=1}^{N_x} u_l \dot{\varphi}_l \rangle &= \sum_{l=1}^{N_x} u_l \Omega_{l-k}^{0,1}, \\ \langle \varphi_k, \sum_{l=1}^{N_x} u_l \ddot{\varphi}_l \rangle &= \sum_{l=1}^{N_x} u_l \Omega_{l-k}^{0,2}; \end{aligned} \quad (6a)$$

with

$$\begin{aligned} \delta_{0,l-k} &= \int_{-\infty}^{+\infty} \varphi_k(t) \varphi_l(t) dt, \\ \Omega_{l-k}^{0,1} &= \int_{-\infty}^{+\infty} \varphi_k(t) \dot{\varphi}_l(t) dt, \\ \Omega_{l-k}^{0,2} &= \int_{-\infty}^{+\infty} \varphi_k(t) \ddot{\varphi}_l(t) dt. \end{aligned} \quad (6b)$$

where $\delta_{0,l-k}$ is the Kronecker delta and $\Omega_{l-k}^{0,1}, \Omega_{l-k}^{0,2}$ are 2-term connection coefficients of the Daubechies wavelets (e.g., [26]); the “index of appearance” $l-k$ is dictated by the support (or the order) of the wavelet.

It is noteworthy that the connection coefficients exclusively depend on the wavelet resolution and the scaling functions within their limit support but not to depend on the analytical signal. The 2-term connection coefficients are only necessary for linear second-order dynamical systems. If higher-order derivatives, cross terms and nonlinear terms within the motion variables exist, 3-term connection coefficients or even higher multi-term connection coefficients may be needed to represent nonlinearity and coupling in the dynamical system (e.g., [30]).

3. Wavelet-Galerkin analysis: computation of connection coefficients

In a general application of the method, high-order multi-term connection coefficients can be defined as a function of the scaling function of order N at a pre-selected wavelet resolution j [35]:

$$\Omega_{l_1 l_2 \dots l_n}^{d_1, d_2, \dots, d_n}(N, j) = \int_{-\infty}^{+\infty} \varphi_{l_1}^{d_1}(t) \varphi_{l_2}^{d_2}(t) \varphi_{l_n}^{d_n}(t) dt = \int_{-\infty}^{+\infty} \prod_{i=1}^n \varphi_{l_i}^{d_i} dt. \quad (7)$$

In the previous equation, n is the index of the connected terms; the notation $d_1, d_2, \dots, d_n$ denotes the derivation order (e.g., $\varphi_{l_1}^d = \frac{d^d \varphi_{l_1}}{dt^d}$); $l_1, l_2, \dots, l_n$ are translation indices of the wavelets. In most applications and second-order dynamical systems, however, the first and second order of derivation are usually needed. Therefore, the connection coefficients of the form $\Omega_{l_1 l_2}^{d_1, d_2}$, with $d_1 = 0, d_2 = 0, 1, 2$ and $l_1, l_2 = 0, 1, \dots, N-1$ (up to the 2-term ones) need to be exclusively estimated. The 2-term connection coefficients can be estimated for an unbounded domain [34] as:

$$\Omega_{l-k}^{d_1, d_2} = \int_{-\infty}^{\infty} \varphi_l^{d_1}(t) \varphi_k^{d_2}(t) dx = 2^{d_1+d_2-1} \sum_{p,q} a_p a_{q-2(l-k)+p} \int_{-\infty}^{\infty} \varphi^{d_1}(t) \varphi_q^{d_2}(t) dx, \quad (8)$$

where $l-k$ is the index of the supported domain; a_p and $a_{q-2(l-k)+p}$ are the scaling coefficients of the scaling functions, defined in accordance with Eq. (2).

The wavelets are compactly supported, therefore the connection coefficients are also defined over a very limited range, depending on the number of supports (or the order of wavelet), indicated by the index ($l-k$). For instance, the Daubechies wavelet of the order N (DN) are compactly supported at (N -

1) discrete points, $0 \leq l, k \leq (N - 1)$, thus having a total of $(2N-3)$ pairs of connection coefficients; furthermore, the $(2N-3)$ 2-term connection coefficients can be determined with the indices $(l-k)$ on the support at the discrete “points” $[-N+2, -N+3, \dots, 0, \dots, N-3, N-2]$.

For facilitating the computations in the WG analysis, a sparse matrix form has been used for collecting the compactly supported connection coefficients. Computation of the connection coefficients of the Daubechies wavelets and accurate treatment of the boundary conditions are also essential to the implementation of the WG analysis. The 2-term connection coefficients applicable to an unbounded time interval, derived from the $D6$ Daubechies wavelet, with wavelet resolution $j=1$ only, were first computed by Latto et al. [34]. Romine and Peyton [35] extended the work by Latto et al. [34] to simulate the two ends of a bounded time interval and for resolutions other than $j=1$, providing an efficient method for implementation of arbitrary boundary conditions and arbitrary wavelet resolution. This study employs the approach proposed by Romine and Peyton [35], which is based on the expansion of the original computational domain of the signal (N_n discrete points in the wavelet domain) by adding $(N-1)$ points to the left of the original computational domain (before the initial time) and $(N-1)$ points to the right of original computational domain (beyond final time). The new computational domain has $N_x=(N_n+2N-1)$ wavelet expansion points. The WG analysis consequently uses N_x independent scaling functions in the computations.

The WG analysis expands a time-varying signal at a pre-selected resolution j . Therefore, the initial choice of wavelet resolution (j) is required for the computation of the connection coefficients. The resolution parameter of the Daubechies wavelets is 2^j at a scale j ; the resolution must be determined so that the scaling function is “centered”, given the number of discretization points, i.e., the sampling time of the original signal. The wavelet resolution (j) can be approximately found from the number of samples per unit time of the signal ($\bar{N}_x$) with $\bar{N}_x = 2^j$. Estimation of the 2-term connection coefficients for the arbitrary Daubechies wavelets at the arbitrary wavelet resolution can be numerically coded for a general application. Table 1 summarizes some examples of 2-term connection coefficients of the Daubechies wavelet at the orders $D4$, $D6$ and $D8$ and the wavelet resolution $j=6,8$ and for the derivative orders $d=0,1,2$. For example, the Daubechies scaling function

$D4$ establishes 5=(2N-3) 2-term connection coefficients in the support indices $(l-k)$ on $[-2,-1,0,1,2]$; the $D6$ has 8 connection coefficients according to the index $(l-k)$, supported on $[-4,-3,-2,-1,0,1,2,3,4]$; there are 13 connection coefficients with $(l-k)$, evaluated on $[-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6]$ for the $D8$.

4. Verification of the Wavelet-Galerkin analysis using a single-degree-of-freedom dynamical system

The WG analysis is employed to study the response of a single-degree-of-freedom (*s dof*) system due to random white-noise loading and verified against conventional solution methods, based on numerical integration. The equation of motion is:

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = f(t) \quad (9)$$

where m , c , k are respectively the mass, damping and stiffness coefficients; $f(t)$ is a random force; initial conditions at $t_0=0$ are assumed as $u(0) = 0, \dot{u}(0) = 0$. In the WG analysis, the system response $u(t)$, acceleration $\ddot{u}(t)$, velocity $\dot{u}(t)$ and the random force $f(t)$ can be projected into the wavelet domain. Firstly, the time-varying responses and the random force are approximated by using the Daubechies scaling function as in Eqs. (4-5). Secondly, the inner product operation as in Eq. (6) is applied to both sides of the motion equation in the wavelet domain. Finally, the *s dof* motion equation is obtained in the wavelet domain as:

$$m \sum_{l=1}^{N_x} \Omega_{l-k}^{0,2} u_l + c \sum_{l=1}^{N_x} \Omega_{l-k}^{0,1} u_l + k \sum_{l=1}^{N_x} \delta_{l,k} u_l = f_l \quad (10)$$

Eq. (10) must be solved for $l=1,...,N_x$. The resulting algebraic system can be written in a compact matrix form as:

$$[A]\{u_l\} = \{f_l\} \quad (11)$$

Each element of the matrix $[A]$ becomes $A_{l,k} = m\Omega_{l-k}^{0,2} + c\Omega_{l-k}^{0,1} + k\delta_{0,l-k}$; it depends on the connection coefficients and it is completely determined thorough the selected Daubechies scaling function D , the wavelet resolution and the boundary conditions; $\{f_l\}$ contains the wavelet coefficients of $f(t) = \sum_{l=1}^{N_x} f_l \varphi_l(t)$, which are random. It is noted that the second-order stochastic dynamic equation has been transformed into a first-order random coefficient algebraic equation, the solution of

which is much simpler and computationally advantageous. The resultant vector $\{u_l\}$ is actually the approximation of the exact solution in the compactly supported Daubechies wavelet-Galerkin analysis.

A realization of the random white noise force $f(t)$ is simulated artificially using the Monte Carlo method to illustrate the method. Daubechies scaling function $D6$ ($N=6$) is used. The wavelet resolution (j) can be estimated approximately by fitting $2^j = \bar{N}_n$, where $\bar{N}_n$ denotes number of samples per unit duration of the signal. In this example the wavelet resolution is fitted as $j=6.65$, since the time step is set to 0.01 seconds. The connection coefficient matrix $[A]$ is deterministic and can be pre-calculated.

Figure 3a shows, as an example, the dynamic displacement $u(t)$ of a given *s dof* system, subject to white-noise excitation force $f(t)$, with the following parameters: $m=1$ kg, $c=0.0628$ Ns/m and $k=39.4784$ N/m, corresponding to a natural frequency of 1Hz. The figure compares the results by WG analysis to the solution obtained by numerical integration, i.e., by Newmark- β method with $\alpha = 1/2, \beta = 1/4$. The error function of the displacement, between the WG analysis results and the “exact” solution by Newmark- β method (NM), is defined as $E(\%) = (x_{NM} - x_{WG})^2 / x_{NM}^2$, in which the variable x denotes resultant displacement. Error functions and power spectral density functions (PSD) of the response are illustrated in Fig.3b. Apparently, very good agreement in both the time-history solution and the PSD is observed between the WG analysis and the Newmark- β method.

5. Stochastic dynamic response of tall buildings: mathematical model

5.1. Reduced-order model and equations of motion

In this preliminary application of the WG analysis method, a reduced-order model of a tall building structure is briefly introduced and described in this section. The dynamic equations of motion are formulated under the hypothesis of linear structural response and modal superposition after decomposition into generalized coordinates, which only retain information on the fundamental modes of the structure, i.e., the first bending modes in the “ x ” and “ y ” directions of the building. The reader is referred to Fig. 4a for the designation of the “ x ” direction and loading plane corresponding to the

mean wind direction (later becoming time-invariant “frozen” direction in the case of the “frozen downburst” in Section 5.3). Figure 4b illustrates the two main lateral displacement variables and mean-wind and turbulence components at a generic elevation z along the vertical axis of the building. The generalized dynamic equation of the r -th mode can be written in the following form (e.g., [7]):

$$M_r \ddot{r}(t) + C_r \dot{r}(t) + K_r r(t) = \bar{Q}_f + Q_{bf,r}(t) + Q_{acf,r}(t, r, \dot{r}, \ddot{r}). \quad (12)$$

In the previous equation M_r, C_r, K_r are the generalized mass, damping and stiffness of the r -th mode. The variable $r = \{x, y\}$ is also used to designate the generalized coordinate of the fundamental modes: (x) “along-wind” lateral mode in the plane of the mean wind direction, (y) “cross-wind” transverse mode. The quantities $\bar{Q}_f, Q_{bf,r}(t)$ and $Q_{acf,r}(t, r, \dot{r}, \ddot{r})$ are, respectively, the generalized mean wind force, the time-dependent generalized buffeting and motion-dependent loads. The system and loading quantities, indicated in Eq. (12), are expressed in terms of generalized coordinates; these can be found as:

$$\begin{aligned} M_r &= \int_0^h \Phi_r^2(z) m(z) dz, \\ K_r &= 4\pi^2 n_r^2 M_r, \\ C_r &= 4\pi \zeta_r n_r M_r; \end{aligned} \quad (13a)$$

$$\begin{aligned} \bar{Q}_{f,r} &= \int_0^h \Phi_r(z) \bar{F}_r(z) dz, \\ Q_{bf,r}(t) &= \int_0^h \Phi_r(z) F_{b,r}(z, t) dz, \\ Q_{acf,r}(t, r, \dot{r}, \ddot{r}) &= \int_0^h \Phi_r(z) F_{ac,r}(z, t, r, \dot{r}, \ddot{r}) dz. \end{aligned} \quad (13b)$$

In the previous equations h is the total height of the building; z is the vertical coordinate along the building axis; $\Phi_r(z)$ is a continuous mode shape function; $m(z)$ is the distributed mass of the building per unit height; n_r, ζ_r are the fundamental natural frequencies and damping ratios. The lateral loading terms are denoted as $\bar{F}_r, F_{b,r}, F_{ac,r}$; these are, respectively, the distributed mean wind force, the buffeting forces and the self-excited forces per unit height, acting in the plane (or direction) of the “ r -th” mode. The global response (R) can be reconstructed from the generalized response (r) as $R = \Phi_r r$.

The distributed lateral wind forces $\bar{F}_r(z), F_{b,r}(z, t), F_{ac,r}(z, t, r, \dot{r}, \ddot{r})$ per unit height z for the two primary motion planes $r=x$ or $r=y$, compatible with the definition of generalized coordinates in Fig. 4, are derived as a first approximation by quasi-steady aerodynamic theory, applied to a rectangular line-like bluff body under turbulent wind (e.g., [8]), as:

$$\begin{aligned} \bar{F}_x(z) &= \left(\frac{1}{2}\right) \rho U(z)^2 D C_D, F_{b,x}(z, t) = \left(\frac{1}{2}\right) \rho(z) D [2C_D u + (C'_D - C_L) v], \\ F_{ac,x}(z, t, \dot{x}, \dot{y}, \theta) &= \left(\frac{1}{2}\right) \rho \bar{U}(z) D [-2C_D \dot{x} - (C'_D - C_L) \dot{y}] + \left(\frac{1}{2}\right) \rho \bar{U}(z)^2 D [C'_D \theta]; \end{aligned} \quad (14a)$$

$$\begin{aligned} \bar{F}_y(z) &= \left(\frac{1}{2}\right) \rho \bar{U}(z)^2 D C_L, \\ F_{b,y}(z, t) &= \left(\frac{1}{2}\right) \rho(z) D [2C_L u + (C'_L - C_D) v], \\ F_{ac,y}(z, t, \dot{x}, \dot{y}, \theta) &= \left(\frac{1}{2}\right) \rho(z) D [-2C_L \dot{x} - (C'_L - C_D) \dot{y}] + \left(\frac{1}{2}\right) \rho U(z)^2 D [C'_L \theta]. \end{aligned} \quad (14b)$$

In the previous equations $\rho, (z)$ and t are the air density and the time; **since the wind is stationary, the mean value $U(z)$ is independent of time (this hypothesis will later be relaxed in the case of downburst wind in Section 5.3).** The quantities B, D are the width and depth dimensions of the floor-plan; C_D, C_L, C'_D, C'_L are the static force coefficients, normalized with respect to D , and their derivatives. The mean wind load acts along the “ x ” direction; the fluctuating components of the wind velocity on the horizontal plans (Fig. 4b) are $u(z, t), v(z, t)$. The variables $\theta, \dot{x}$ and $\dot{y}$ designate torsional rotation, along-wind and cross-wind transverse velocities of the building at z . It is noted that the distributed wind forces in Eq. (14) are coupled due to a presence of the motion-dependent forces. Equations (14) do not include the effects of vortex shedding, in this first application of the method, even though it has been recognized that vortex shedding effects are of relevance to the estimation of the cross-wind response. Also, in the subsequent analysis of the response, torsional effects are not considered and the contribution of the angle θ is ignored. The extension of the numerical model to study the building dynamics in the time domain, accounting for such effects, can be readily incorporated in the future (e.g., [37] for vortex shedding). At the present time, however, this task is beyond the scope of the study, which is primarily an investigation on the feasibility of the WG analysis method.

It is generally agreed that the numerical solution of the coupled stochastic dynamic equations Eq.(14) in the time domain by numerical integration methods and other step-by-step methods is often extremely complex, not very accurate, and even impossible for large systems in many cases. For example, in many cases modal coupling, influenced by the motion-dependent forces, is neglected during the simulation of the building response in the time domain for the sake of simplicity.

5.2. Stochastic dynamic response of tall buildings in wavelet domain

In this section, the coupled motion equations in the generalized coordinates, described in Eq. (12), are combined with the system parameters and generalized forces in Eqs.(13-14) and transformed into the wavelet domain for subsequent WG analysis. The equation of motion of the building response in the global coordinates x (along-wind) and y (cross-wind) can be written in the following generalized form:

$$M_x \ddot{x}(t) + [C_x - Q_{xx}] \dot{x}(t) - Q_{yx} \dot{y}(t) + K_x x(t) = Q_{b,x}(t), \quad (15a)$$

$$M_y \ddot{y}(t) + [C_y - Q_{yy}] \dot{y}(t) - Q_{xy} \dot{x}(t) + K_y y(t) = Q_{b,y}(t). \quad (15b)$$

In the previous equations M_x, M_y, C_x, C_y, K_x and K_y are derived from Eq. (13); Q_{xx}, Q_{yx} are motion-induced generalized loading terms associated with the x -coordinate, linearly depending on the velocities $\dot{x}$ and $\dot{y}$; Q_{xx} and Q_{yx} are motion-induced loading terms related to the y -coordinate; $Q_{b,x}(t)$ is the turbulent-induced generalized force of the x -coordinate and $Q_{b,y}(t)$ is the turbulent-induced generalized force of the y -coordinate. These quantities can be determined as follows [33]:

$$Q_{xx} = - \int_0^h \phi_x [0.5 \rho U(z) D C_D] dz,$$

$$Q_{yx} = - \int_0^h \phi_x [0.5 \rho U(z) D (C'_D - C_L)] dz, \quad (16a)$$

$$Q_{xy} = - \int_0^h \phi_y [2(0.5) \rho U(z) D C_L] dz,$$

$$Q_{yy} = \int_0^h \phi_y [0.5 \rho U(z) D (C'_L - C_D)] dz. \quad (16b)$$

$$Q_{b,x} = - \int_0^h \phi_x (0.5) \rho U(z) D [2C_D u + (C'_D - C_L) v] dz,$$

$$Q_{b,y} = \int_0^h \phi_y \left(\frac{1}{2}\right) \rho U(z) D [2C_L u + (C'_L - C_D) v] dz. \quad (16c)$$

Similar to the WG analysis of the *sdo*f dynamical system, the following steps are repeated in the case of Eqs. (15-16): (1) time-dependent quantities are approximated by Galerkin projection as in Eqs. (3-4); (2) coefficients of the Galerkin-projected approximations are obtained by inner product, using the orthogonality of the wavelets, and assembled into connection coefficient matrices in Eq. (6); (3) resultant algebraic system of equations is numerically solved; (4) displacement, velocity and acceleration of the stochastic dynamic response of the structure are estimated. The coupled generalized equations of motion in the wavelet domain are (with $l = 1, \dots, N_x$):

$$\{M_x \sum_{l=1}^{N_x} \Omega_{lk}^{0,2} + (C_x - Q_{xx}) \sum_{l=1}^{N_x} \Omega_{lk}^{0,1} + K_x \sum_{l=1}^{N_x} \delta_{lk}\} x_l - \{Q_{yx} \sum_{l=1}^{N_x} \Omega_{lk}^{0,1}\} y_l = q_{bxl}, \quad (17a)$$

$$\{M_y \sum_{l=1}^{N_x} \Omega_{lk}^{0,2} + (C_y - Q_{yy}) \sum_{l=1}^{N_x} \Omega_{lk}^{0,1} + K_y \sum_{l=1}^{N_x} \delta_{lk}\} y_l - \{Q_{xy} \sum_{l=1}^{N_x} \Omega_{lk}^{0,1}\} x_l = q_{byl}. \quad (17b)$$

In the previous equations N_x is the computational domain; x_l , y_l are the WG-expansion coefficients of the approximate displacements in the “x” and “y” generalized coordinates, as in Eq. (4); q_{bxl} and q_{byl} are WG-based approximate generalized buffeting forces in the “x” and “y”, similar to Eq.(4); $\Omega_{lk}^{0,1}$, $\Omega_{lk}^{0,2}$ are 2-term connection coefficients obtained from Eqs. (6). The scalar equations (17a-17b) are extended to all $l = 1, \dots, N_x$ to form two coupled systems of algebraic matrix equations with random coefficients:

$$[A_{11}]\{x_l\} + [A_{12}]\{y_l\} = \{B_1\} \quad (18a)$$

$$[A_{21}]\{x_l\} + [A_{22}]\{y_l\} = \{B_2\} \quad (18b)$$

In the previous equations the quantities $\{B_1\} = \{q_{bxl}\}$, and $\{B_2\} = \{q_{byl}\}$ are column vectors regrouping the wavelet coefficients of the (known) random generalized buffeting forces; the vectors $\{x_l\}$ and $\{y_l\}$ regroup the WG-based approximating coefficients of the (unknown) dynamic displacements. Also, the following matrix terms are defined:

$$[A_{11}] = M_x[\Omega^{0,2}] + (C_x - Q_{xx})[\Omega^{0,1}] + K_x[I],$$

$$[A_{12}] = -Q_{yx}[\Omega^{0,1}], \quad (19a)$$

$$[A_{21}] = -Q_{xy}[\Omega^{0,1}],$$

$$[A_{22}] = M_y[\Omega^{0,2}] + (C_y - Q_{yy})[\Omega^{0,1}] + K_y[I]. \quad (19b)$$

in which $[\Omega^{0,2}]$, $[\Omega^{0,1}]$ denote to 2-term connection coefficient matrices and $[I]$ is the identity matrix.

Finally, the two algebraic matrix equations in Eq. (18) can be solved simultaneously and numerically to find the motion of the building. The WG-based approximating coefficients of the response, $\{x_l\}$ and $\{y_l\}$, are random since the generalized wavelet-domain loads, $\{B_1\}$ and $\{B_2\}$, are random. Moreover, the resultant random velocities and accelerations in the “x” and “y” generalized coordinates can be estimated as:

$$\{\dot{x}_l\} = [\Omega^{0,1}]\{x_l\}, \quad \{\dot{y}_l\} = [\Omega^{0,1}]\{y_l\} \quad (20a)$$

$$\{\ddot{x}_l\} = [\Omega^{0,2}]\{x_l\}, \quad \{\ddot{y}_l\} = [\Omega^{0,2}]\{y_l\} \quad (20b)$$

in which $\{\dot{x}_l\}$, $\{\dot{y}_l\}$, $\{\ddot{x}_l\}$ and $\{\ddot{y}_l\}$ are the WG-based approximation coefficients of the velocities and accelerations in x and y .

5.3. Simulation of transient wind loads on tall buildings (vertical line-like structures)

Multivariate transient wind fields must be digitally simulated at a series of discrete nodes, located along the vertical axis of the building (refer to Fig. 4a for the position of the nodes in the study example). The synthetically-generated realizations of the turbulence components u and v are later used to compute time-histories of the generalized wind forces, $Q_{b,x}$ and $Q_{b,y}$ in Eqs. (16c), from which the WG-expansion coefficients $\{B_1\} = \{q_{bxl}\}$, and $\{B_2\} = \{q_{byl}\}$ are calculated.

The digital simulation of a transient wind flow is based on the theory of evolutionary power spectral density functions (EPSD) (e.g., [38-40]), which exploits the property that partially-correlated nonstationary processes can be expressed by superposition of partially-correlated stationary processes, modulated by a slowly-varying deterministic time function (amplitude modulation). The spectrum of the stationary processes and the deterministic “modulation function” can be selected by matching a prescribed evolutionary spectrum. Therefore, multivariate time-histories of transient wind speed fluctuations can be reproduced by identifying a suitable deterministic time function to modulate a synthetically-generated sample of a multivariate stationary fluctuating wind process. In this study, a realization of the stationary wind speed process is digitally simulated using the spectral representation approach, either based on the Cholesky decomposition (e.g., [41]) or the proper orthogonal decomposition of the cross-power spectral density matrix of the turbulence (e.g., [42-43]). For

example, the total transient wind velocity fields in the two along-wind and cross-wind directions for the tall buildings can be expressed (e.g., [19]) as $U_{tot,p}(z_p, t) = U'_p(z_p, t) + u'_p(z_p, t)$, and $v'_p(z_p, t)$, in which p is nodal index ($p=1,2,\dots,M$), M is the number of nodes, z_p is the vertical coordinate of the discrete node, $U'_p(z_p, t)$ is now a time-varying “mean” wind velocity (slowly-varying fluctuations of the velocity), $u'_p(z_p, t)$ and $v'_p(z_p, t)$ are the random fluctuating nonstationary components of turbulence (high-frequency fluctuations). The terms $u'_p(z_p, t)$ and $v'_p(z_p, t)$ are found by combining two zero-mean stationary random processes with deterministic modulation function, $\hat{A}_p(\omega, z_p, t)$ as follows (e.g., [19]):

$$u'_p(z_p, t) = \hat{A}_p(\omega, z_p, t)u_p(z_p, t), \quad v'_p(z_p, t) = \hat{A}_p(\omega, z_p, t)v_p(z_p, t), \quad (21)$$

where $u_p(z_p, t), v_p(z_p, t)$ are spatially-correlated zero-mean stationary turbulence processes. In eq. (21) the function $\hat{A}_p$ is written in its most general form, which also depends on the circular frequency ω . It must be noted that, in a transient wind such a downburst, the time-varying “mean” velocity $U'(z_p, t)$ is often determined either from a phenomenological model (e.g., [15,44-46]) or from direct observations of the wind event (e.g.,[11]). In this study, the hypothesis of time-independent mean wind velocity profile $U'_p(z_p, t) \approx \bar{U}(z_p)$ was used, in which the variable $\bar{U}(z_p)$ is the average shape of a “frozen” downburst profile. For stationary winds the expression above simply becomes $\bar{U}(z_p) = U(z)$, i.e., the boundary layer profile and $\hat{A}_p$ is identically one. The assumption on the slowly-varying (i.e., mean) wind profile is compatible with the description of the generalized lateral load in Eqs. (14) and Eqs. (16) if the quantity $U(z)$ is substituted with $\bar{U}(z_p)$. This hypothesis is, however, a first approximation of an actual downburst wind, which was selected for the primary purpose of examining the feasibility of the WG analysis method.

The theory of evolutionary power spectra for the transient random fluctuating processes allows defining the elements of the evolutionary cross spectral matrix of the wind speed fluctuations as:

$$S_{pp}(\omega, z_p, t) = |\hat{A}_p(\omega, z_p, t)|^2 S_{pp}^0(\omega, z_p), \quad (22a)$$

$$S_{pq}(\omega, z_p, z_q, t) = \hat{A}_p(\omega, z_p, t)\hat{A}_q^{T*}(\omega, z_q, t)S_{pq}^0(\omega, z_p, z_q), \quad (22b)$$

where T and “*” denote the transpose and complex conjugate operators. The quantities $S_{pp}^0(\omega, z_p)$ and $S_{pq}^0(\omega, z_p, z_q)$ are the stationary auto- and cross-power turbulence spectra, respectively, whereas p and q are generic nodal indices. The cross-power spectrum of the stationary turbulence can be empirically estimated by Davenport spatial coherence function and Harris spectrum (e.g., [36,47]), as:

$$S_{pq}^0(\omega, z_p, z_q) = Coh_{pq}(\omega, z_p, z_q) \sqrt{S_{pp}^0(\omega, z_p) S_{qq}^0(\omega, z_q)}, \quad (23a)$$

$$Coh_{pq}(\omega, z_p, z_q) = \exp\left(-\frac{C\omega|z_p - z_q|}{\pi(\bar{U}(z_p) + \bar{U}(z_q))}\right), \quad (23b)$$

$$\frac{\omega S_{pp}^0(\omega, z_p)}{2\pi\sigma(z_p)^2} = \frac{0.6X(z_p)}{(2+X(z_p)^2)^{5/6}}. \quad (23c)$$

In Eqs. (23a-23b) $Coh_{pq}(\omega, z_p, z_q)$ is the spatial coherence function between node p and node q ; C is decay factor; in Eq. (23c) $\sigma(z_p) = I_p \bar{U}(z_p)$ is the standard deviation of the turbulence at z_p with I_p being the turbulence intensity, and $X(z_p) = \frac{1600\omega}{2\pi\bar{U}(z_p)}$. It is noted that the wind velocity $\bar{U}(z_p)$, used in the Harris spectrum and the Davenport coherence, are the time-invariant wind velocities at the building nodes with vertical coordinate z_p , i.e., compatible with the definition of “frozen” profile of the downburst in the case of nonstationary winds. The Cholesky decomposition is used to decompose the evolutionary cross spectral matrix of turbulence, written in a compact form as $S(\omega, z, t)$, which is real and symmetric by construction, as:

$$S(\omega, z, t) = |A(\omega, z, t)|^2 H(\omega, z) H^{T*}(\omega, z), \quad (24)$$

in which $H(\omega, t)$ denotes a lower triangular matrix obtained from the decomposition of the stationary-turbulence cross spectral matrix, which is assembled from Eqs. (23a-23c).

As a result, the multivariate transient process of the along-wind fluctuating wind velocity at the discrete nodes $p = 1, 2, \dots, M$, $u_p'(z_p, t)$, can be generated as (e.g., [41]):

$$u_p'(z_p, t) = 2\sqrt{\Delta\omega} \sum_{m=1}^M \sum_{l=1}^n |\hat{A}_p(\omega_l, z_p, t)| |H_{pm}(\omega_l, z_p)| \cos[\omega_l t - \vartheta_{pm}(\omega_l, z, t) + \phi_{ml}], \quad (25)$$

in which $\Delta\omega$ is a circular frequency interval or “step” $\Delta\omega = \omega_{up}/n$; ω_{up} is the upper cut-off circular frequency; n is the number of circular frequencies, used by the wave-superposition method; ω_l is a generic circular frequency ($\omega_l = l\Delta\omega$). The matrix decomposition of the wind spectrum leads to

$H_{pm}(\omega, z_p, t) = |H_{pm}(\omega, z_p)|e^{i\vartheta_{pm}(\omega, t)}$, with $\vartheta_{pm} = \tan^{-1}(Im[H_{pm}(\omega, t)]/Re[H_{pm}(\omega, t)])$ and ϕ_{ml} being independent phase angle distributed uniformly over interval $[0, 2\pi]$. Realizations of the random phase angles ϕ_{ml} can be synthetically generated by Monte Carlo sampling.

This study utilizes two simplified modulation functions: (i) a cosine-type modulation function, (ii) an exponential-type modulation function as a function of time t but independent of ω and the position (e.g., [19]), as:

$$\hat{A}_p(t) = ((1 - \cos(2\pi t/T_0))/2)^\eta \quad (26a)$$

$$\hat{A}_p(t) = \alpha_0 t^{\beta_0} e^{-\lambda t} \quad (26b)$$

In Eq. (26a), T_0 is a reference duration; η is a parameter controlling the width of the cosine window (with $\eta > 0$ an integer even number). In Eq. (26b), the following coefficients are needed: $\alpha_0 = \lambda^{\beta_0}/\beta_0^{\beta_0} e^{\beta_0}$, $\alpha_0 > 0$, $1 \geq \lambda \geq 0$, $\beta_0 \geq 0$, $t_{max} = \beta_0/\lambda$; t_{max} is time instant at which maximum amplitude of the modulation function is reached. Both the cosine modulation function (Eq. 26a) and the exponential modulation one (Eq. 26b) are strictly positive functions with maximum amplitudes equal to one. The width of the modulation windows is controlled by the parameters η , λ and the “location” of the maximum amplitude is influenced by T_0 and β_0/λ .

5.4. Discussion on the quasi-steady assumption, used to characterize lateral wind loads

The aerodynamic coefficients of the CAARC building (C_D , C_L , etc.) have been reproduced from the results of a previous study [48]. In this study, the aerodynamic coefficients were adapted from the stationary (synoptic-wind) pressure measurements along the building height in wind tunnel on a rigid model of the structure, as described in Melbourne (1980). Namely, the integration of the pressures at 2/3 of the height was considered as a suitable approximation for the equivalent strip-theory-based sectional force coefficients, later employed in the reduced-order model. This hypothesis was accepted also because previous investigations on the non-stationary response of tall buildings [15] have used the same approach since they have observed that most uncertainty in the loading, for such a tall structure, may be related to fluctuations in the wind velocity. Furthermore, since the dominant-mode vibration of the CAARC occurs at a slow frequency (0.2 Hz) and given the slowly-varying pattern of

the simulated non-stationary wind (e.g., through the modulating functions), interaction of wake effects with the dynamic response (mostly resonant) was considered as secondary. Therefore the quasi-steady load assumption was considered as adequate for the purposes of this study. The question on accurate load simulation is, however, still open in the wind engineering community as very few studies are available on the measurement of pressures (or forces) on tall buildings in non-stationary wind fields, simulating actual full-scale transient phenomena [49]. Ultimately, force coefficients should possibly be determined in a non-stationary wind flows but, currently, experimental difficulties are still present and prevent the systematic assessment of the loads.

5.5. Computational flowchart for WG analysis

In Fig. 5 a computational flowchart illustrates the various steps of the WG analysis for the coupled stochastic dynamics of a tall building. As can be seen in Fig. 5, the WG analysis employs the Galerkin projection and the compactly-supported Daubechies wavelet to approximate the dynamic motions, their derivatives and the stochastic wind loading in the wavelet domain. The wavelet connection coefficients, which exclusively depend on the selection of the wavelet and the designated resolution, are estimated. Subsequently, the connection coefficients and the dynamic parameters (mass, damping and stiffness) are assembled together in the connection coefficient matrices. Finally, the coupled motion equations of the tall building are transformed to algebraic equations in the wavelet domain, in which the unknown variables can be conveniently solved by linear algebra.

6. Numerical example: CAARC tall building

The system is modeled after the CAARC tall building [34], a benchmark structure of dimensions $B=30.5\text{m}$, $D=45.7\text{m}$ and $h=183\text{m}$ (Fig. 4a). Uniform mass per unit height is used, with $m(z)=m=220800\text{ kg/m}$. The structural system is treated as a shear-type building planar building with fundamental-mode linear mode shapes. The fundamental natural frequencies of the first along-wind mode (x lateral direction) and crosswind mode (y lateral direction) are $n_x=0.20\text{Hz}$ and $n_y=0.22\text{Hz}$. Modal damping ratios are $\zeta_x=\zeta_y=0.01$, according to the values prescribed in the benchmark problem

[34]. The normalized mode shape functions are $\Phi_x(z) = \Phi_y(z) = (z/h)$. The aerodynamic static coefficients and their first-order derivatives per unit height are taken as constant, independent of the height, as employed in a previous study [8,33]: $C_D = 1.1, C_L = -0.1, C'_D = 1.1, C'_L = 2.2$. The building model is approximated as a vertical cantilever, discretized into 41 nodes along the height, equally spaced at a distance of 4.575m (refer to Fig. 4a for the node position). In this study, the “mean” wind profile varies along the coordinate z . In the stationary analyses, as a first approximation, the synoptic wind profile of the stationary wind scenario is approximated by a boundary-layer power law model with exponential factor $\alpha = 0.25$ [36]; two examples of mean wind velocities at the rooftop node (41) are employed in the numerical simulations: $\bar{U}(z = 183\text{m}) = U_h = 20\text{m/s}$ and $U_h = 30\text{m/s}$ (stationary mean wind velocities at the rooftop node 41).

Initially, the same boundary layer profile is also used to simulate the loading of the downburst wind. In a second part of the study (Section 7.2), a time-independent wind profile, with time-independent fluctuations $U'_p(z_p, t) \approx \bar{U}_p(z_p)$, is used in the simplified “frozen” downburst model; this profile is determined by “Vicroy’s model” [44], with the maximum wind velocity U_{max} equal to 47m/s at the height $z_{max} = 67\text{m}$ [15]. The Vicroy’s model determines the time-independent wind profile at the height z in the downburst winds, as [44]:

$$U_{db}(z) = 1.22 \times [e^{-0.15z/z_{max}} - e^{-3.2175z/z_{max}}] \times U_{max} \quad (27)$$

where $U_{db}(z) = \bar{U}_p(z_p)$ is the downburst “frozen” mean wind velocity profile at the height z , U_{max} is the maximum mean velocity in the downburst wind profile, z_{max} is the elevation at which the maximum velocity occurs. The non-synoptic downburst wind profile is also employed to digitally simulate the transient wind fluctuations at various nodes along the building height, and for comparison with the synoptic wind profile, which relies on the power law. The same modulation functions have been employed for converting the digitally simulated turbulence realizations to the transient wind fluctuations at various nodes both for “power law” wind profile and non-synoptic “frozen” downburst wind profile.

7. Results and discussion

7.1. Investigation on feasibility of the WG method using simplified wind flow field for load analysis

Transient fluctuating wind velocities of the along-wind and cross-wind (turbulence) components in the x and y global structural coordinates, respectively have been artificially simulated at the building nodes using the modulation function method. The Harris spectrum and Davenport coherence function in Eq. (23) are used; a value of turbulence intensity equal to 0.15, constant along the height of the structure as a first approximation, is used to digitally generate the realizations of wind turbulence for both horizontal components. Both the cosine modulation function (Eq. 26a) and the exponential modulation one (Eq. 26b) are employed. Figure 5 investigates the effect of the modulation function parameters on the digital simulation of transient wind fields (along-wind turbulence fluctuations). The values $\eta = 0.5, 0.7, 1.0, 1.5, 2.5$ for the cosine modulation function and $\lambda = 0.05, 0.1, 0.25, 0.5, 0.7$ for the exponential modulation function have been selected to modulate the stationary fluctuating process. For comparison purposes, the same realization of the stationary stochastic process is used.

Fig. 6 shows that the width of modulating window reduces with an increase of the parameters η and λ . Additionally, a “short-life” transient gust can be obtained from the original realization of the stationary process by “widening” and “sharpening” the modulating windows through η and λ ; however, the dominant peaks of the original process are accordingly unaltered by the modulating windows. It seems that the cosine modulation function symmetrically sharpens the original signal on both ends, while the exponential modulation function works by better sharpening the left end as opposed to the right one (Fig. 6b). In the remainder of the study, $\eta = 1$ for the cosine modulation function and $\lambda = 0.1$ for the exponential modulation function have been used. The selection of these parameters creates almost similar effects, even though the effective duration of the transient turbulent record is wider with the cosine modulation function than with the exponential modulation function.

Digitally simulated time series of transient fluctuations of the wind speed in the (u) along-wind component at the building nodes 41 (rooftop), 30, 20, 10 and 5, with reference rooftop mean wind velocity $U_h=30$ m/s at $z=h=183$ m are depicted in Fig. 7. If Fig. 6a the reference stationary realizations are depicted, whereas Fig. 6b and Fig. 6c illustrate the corresponding transient fluctuating winds using

the cosine modulation function and the exponential modulation function, respectively, combined with power-law wind profile. The record has a duration of 300 seconds with 100 Hz sampling rate; the spectra of the stationary turbulence are limited to the 0-10 Hz frequency band. Transient fluctuating winds at other nodes and for the (v) cross-wind fluctuating component are not shown here for the sake of brevity.

The WG analysis is subsequently applied to approximate the global displacements at the discrete building nodes, induced by the simulated transient fluctuations of the (u) along-wind and (v) cross-wind turbulence fields. Daubechies wavelet $D6$ is employed. Fig. 8 illustrate a typical 300-second time history of the global displacements in the (x) along-wind and the (y) cross-wind directions at the rooftop node 41 with reference mean wind speed $U_h=30\text{m/s}$. The dynamic responses are dominated by the two fundamental lateral modes of the building. Stationary-turbulence dynamic displacement and two examples of transient-turbulence dynamic displacements (excluding the contribution of the boundary layer or frozen downburst wind profile) at the rooftop node 41, obtained by applying the two types of modulation function, are presented. The maximum dynamic global displacements at all building nodes can be estimated from the analysis of the time series of the generalized displacements and the mode shape functions ϕ_x and ϕ_y at all the building nodes. The envelope of the maximum dynamic displacements of the building along its height will be determined in the final step of this numerical investigation.

Power spectral densities of the global transient dynamic displacements in the (x) along-wind and (y) cross-wind directions at rooftop node 41 with mean wind speed $U_h=30\text{m/s}$ are also verified in Fig. 9. Spectral peaks are observed at 0.20 Hz for the (x) along-wind response and at 0.22 Hz for the (y) cross-wind response, corresponding to the frequencies of the x and y fundamental vibration modes of the building.

Coupling effect and influence of the motion-induced forces on the global transient displacement have also been investigated. Figure 10 illustrates the uncoupled (without the effect of motion-induced forces) and coupled (with the effect of motion-induced forces) dynamic displacements at the rooftop node 41 with $U_h=30\text{m/s}$. It is suggested that there is limited difference, observed in the transient dynamic displacements, between uncoupled-mode scenario and coupled one, except for a minor

difference at the left end of the realizations. The influence of the motion-induced forces appears to be of secondary importance for this case study and conditions. However, it is noted that, since the coupled dynamics can create larger dynamic vibration than the uncoupled-mode case, coupling effects should be included as they might be very important in the analysis of flexible vertical structures, when motion-induced forces are large. In any case, the numerical results confirm the validity and efficiency of the WG analysis method for the solution of coupled stochastic motion of the building.

Figure 11 investigates the influence of the order of the Daubechies wavelets on the numerical estimation of the transient response in the x along-wind and y cross-wind directions at the rooftop node 41 for $U_h=30\text{m/s}$. Various Daubechies wavelets $D2$, $D4$, $D6$, $D8$, $D10$ and $D20$ have been employed in this investigation. It can be seen from Fig. 11 that higher-order Daubechies wavelets, $D6$ to $D20$, produce qualitatively and quantitatively similar transient displacements. In contrast, lower-order wavelets $D2$, $D4$ create inaccurate solutions. Incorrect reconstruction of the response by $D2$ and $D4$ in the WG method is most likely due to the small number of support points. It is therefore recommended that the Daubechies wavelet $D6$ should “optimally” be employed in the WG analysis to accurately simulate the transient dynamics of the building model and to possibly reduce the computational time. Higher-order Daubechies wavelets with $N>6$ are clearly adequate, but they require longer computational time due to the processing of a larger number of connection coefficients.

The influence of the wavelet resolution on the transient response of the building is analyzed in Fig. 12. This figure illustrates the global transient displacements in the x along-wind direction and the y cross-wind direction at the rooftop node 41 for $U_h=30\text{m/s}$. The wavelet resolution has been fitted as $j=6.65$, which enables to create 100 moving wavelets on the unit time interval (equal to the digital sampling of the turbulence and the loads). Some slightly lower wavelet resolutions $j=6.45$ (87 wavelets per unit time interval), $j=6.55$ (94 wavelets per unit time interval) and slightly higher wavelet resolutions $j=6.75$ (108 wavelets per unit time interval), $j=6.85$ (115 wavelets per unit time interval) are also selected for examination. Figure 12 suggests that slightly different wavelet resolutions produce considerably dissimilar transient displacements. It seems from this investigation that lower wavelet resolutions generate higher transient dynamic displacement in the x along-wind direction, but lower transient displacement in the y cross-wind direction. The differences in the 2-term connection

coefficients of the Daubechies wavelet D_6 , at the investigated wavelet resolutions, are presented in Table 2. The 2-term connection coefficients considerably change with a slight difference in the resolution. It is observed that the selection of wavelet resolution is crucial for the WG analysis, since an incorrect choice may significantly overestimate or underestimate the dynamic building response.

7.2. Investigation on global response of the CAARC building under simulated thunderstorm winds

Maximum values of global displacements at the building nodes (Nos. 1-41) are estimated from the corresponding time series to construct envelopes of the global dynamic response. Figure 13 illustrates the envelope of the maximum vibration amplitudes at all the nodes along the building height for $U_h=20\text{m/s}$ and $U_h=30\text{m/s}$ in the (x) along-wind and (y) cross-wind directions, respectively. In the figure the maximum dynamic response, produced by the stationary wind realization, is compared to the ones induced by the simulated transient winds with cosine modulation function (“Transient 1”) and exponential modulation function (“Transient 2”). Even though this investigation is limited to a restricted selection of modulation-function parameters, the maximum global response to stationary winds at the rooftop node 41 for $U_h=20\text{m/s}$ exhibit higher displacements (about 17%) in both the x direction and the y direction (14%) than the ones due to “Transient 1”; the stationary-wind maximum displacements are larger by about 30% in the x 26% in the y direction when compared to “Transient 2” case. Similarly, if $U_h=30\text{m/s}$ is examined, the stationary-wind maximum displacements at node 41 are about 19% higher in the x direction and 17% higher in the y direction compared to the “Transient 1” case, about 30% higher in the x and 33% higher in the y compared to the “Transient 2” case. It is suggested that the modulation function, used to generate the transient-wind fluctuations, operates by reducing the overall input energy of the original stationary-wind fluctuations. Consequently, the shorter the length of the modulation window or the “sharper” the transient wind process is, the lower the dynamic building responses are obtained.

Figure 14 shows comparisons between the synoptic power-law profile model in the stationary winds and the non-synoptic downburst profile model in the simplified “frozen downburst”. The non-synoptic downburst profile is derived from the Vicroy’s model with a maximum mean wind velocity $U_{max}=47\text{m/s}$ at elevation $z_{max}=67\text{m}$. Two synoptic power-law profiles are constructed for

comparison with the non-synoptic wind. In the first one (“Synoptic 1”) the mean wind speed at $h=183$ m is taken as $U_{h,1}=47$ m/s= U_{max} (the same as the maximum velocity of the downburst); in the second one velocity the mean wind speed at $h=183$ m is $U_{h,2}=38$ m/s, which coincides with the “time-independent speed of the downburst profile at the same height. Time histories of maximum dynamic displacements due to all three cases, at the rooftop node 41, are illustrated in Figure 14a and 14b for the x -, y - directions respectively. It can be noticed that the non-synoptic downburst profile model produces extremely large amplitudes at the rooftop node in comparison with the synoptic profile 2. Concretely, the maximum dynamic displacement in the x along-wind direction at the rooftop node 41 reaches 0.72m (67% larger) in the case of non-synoptic downburst wind profile, compared to 0.43m due to power-law wind profile, $U_{h,2}=38$ m/s (“Synoptic 2”), see Figure 14a. In contrast, the y cross-wind maximum displacement at the rooftop node 41 in the non-synoptic profile is 0.108m (52% larger) in comparison with 0.071m at the same point for the synoptic profile with $U_{h,2}=38$ m/s (“Synoptic 2”), see Figure 14b. Maximum displacement envelopes in the x -, y - directions along the building height for the same three cases (two synoptic cases and one non-synoptic profile model) are presented in Figure 15a and Figure 15b. It is confirmed that the non-synoptic time-independent mean wind profile of the simplified “frozen” downburst winds induces extremely large vibration amplitudes in the x along-wind direction at all floors on the tall building to compare with the synoptic profile, $U_{h,2}=38$ m/s (“Synoptic 2”) in Figure 15a. Similarly, the displacement envelope of the “frozen” downburst wind in the y cross-wind direction is also obviously larger than that of the “Synoptic 2” profile, see Figure 15b. Finally, it is worth mentioning that in all the previous analyses the comparisons are based on the dynamic lateral displacements induced by high-frequency turbulence fluctuations; the effect of the time-independent wind load (or mean load in the case of synoptic winds) has not been included, i.e. $\bar{Q}_{f,r} \approx 0$ with $r = x$ in Eq. (13b), since the main purpose of this study was the feasibility of the WG analysis method. Future investigations will also consider the combination of this effect along with the dynamic loading. Future studies will also examine the results of the proposed model in comparison with wind tunnel aeroelastic analysis results, available in the literature for synoptic winds.

8. Conclusions

The WG analysis method was explored to estimate the coupled transient dynamic response of the 183m CAARC tall building subjected to digitally-simulated realizations of transient turbulent wind loads. A reduced-order model of the benchmark structure was constructed. The resultant response of the building was analyzed by comparing the stationary boundary-layer wind vibration against a sample of transient dynamic solutions, obtained by varying the modulation function, used to construct the time-frequency representation of nonstationary turbulent velocity fluctuations. Some investigations examined the influence of the coupling of the motion-induced forces, the order of the Daubechies wavelet and the estimation of wavelet resolution. The numerical results suggest that WG method is a very powerful analysis method for the solution of fully-coupled transient structural dynamics of tall buildings, even though motion-induced coupling did not considerably influence the building response in the specific case study. The Daubechies wavelet $D6$ is recommended in the WG analysis due to its ability to correctly replicate the dynamic response and its efficiency in terms of computational time. The accurate estimation of the wavelet resolution is essential to ensure the accuracy of the WG analysis.

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818 **List of Tables**

819	Table 1. 2-term connection coefficients of Daubechies wavelets $\Omega_{l-k}^{d_1 d_2}(N, j)$
820	Table 2. 2-term connection coefficients of $D6$ at different wavelet resolutions
821	

822 Table 1.

Dau.	$l-k$	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
$D4$	$\Omega_{l-k}^{0,1}(4,6)$					5.333	-42.666	0	42.666	-5.333				
$D4$	$\Omega_{l-k}^{1,2}(4,6)$					131070	262140	0	262140	131070				
$D6$	$\Omega_{l-k}^{0,1}(6,6)$			-0.021	-0.935	9.293	-47.693	0	47.693	-9.293	0.935	0.021		
$D6$	$\Omega_{l-k}^{0,2}(6,6)$			350	7490	-57420	222200	-345230	222200	-57420	7490	350		
$D8$	$\Omega_{l-k}^{0,1}(8,6)$	0	0.011	0.142	-2.149	12.287	-50.752	0	50.752	-12.287	2.149	-0.142	-0.011	0
$D8$	$\Omega_{l-k}^{0,2}(8,8)$	-5	-110	-690	9890	-45740	173150	-273020	173150	-45740	9890	-690	-110	5

823 Note: d_1, d_2 : derivative indices, l,k : support indices of wavelets, N : wavelet order, j : wavelet resolution

824

825 Table 2.

j	$l-k$	-4	-3	-2	-1	0	1	2	3	4
6.45	$\Omega_{l-k}^{0,1}(6,j)$	-0.029	-1.277	12.694	-65.150	0	65.150	-12.694	1.277	0.029
6.55	$\Omega_{l-k}^{0,1}(6,j)$	-0.032	-1.369	13.606	-69.826	0	69.826	-13.606	1.369	0.032
Fitted	$\Omega_{l-k}^{0,1}(6,j)$	-0.034	-1.457	14.481	-74.321	0	74.321	-14.481	1.457	0.034
6.75	$\Omega_{l-k}^{0,1}(6,j)$	-0.036	-1.572	15.629	-80.210	0	80.210	-15.629	1.572	0.036
6.85	$\Omega_{l-k}^{0,1}(6,j)$	-0.039	-1.685	16.750	-85.966	0	85.966	-16.750	1.685	0.039
6.45	$\Omega_{l-k}^{0,2}(6,j)$	71	874	-6697	25915	-40264	25915	-6697	874	71
6.55	$\Omega_{l-k}^{0,2}(6,j)$	47	1003	-7693	29768	-49252	29768	-7693	1003	47
Fitted	$\Omega_{l-k}^{0,2}(6,j)$	53	1137	-8715	33724	-52398	33724	-8715	1137	53
6.75	$\Omega_{l-k}^{0,2}(6,j)$	62	1324	-10151	39279	-61029	39279	-10151	1324	62
6.85	$\Omega_{l-k}^{0,2}(6,j)$	71	1420	-11660	45120	-70104	45120	-11600	1420	71

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