Investigation of ejector-equipped Joule-Thomson refrigerator operating below 77 K

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Abstract

The lowest attainable refrigeration temperature of a nitrogen based Joule-Thomson refrigerator is generally limited to 77 K since the compressor suction pressure is usually higher than atmospheric pressure. The Joule-Thomson process with an ejector is proposed to achieve a refrigeration temperature as low as 68 K by adjusting the evaporation pressure down to 28 kPa and boosting the return stream pressure up to 147 kPa. A one-dimensional numerical model is developed to predict the performance of the ejector at cryogenic temperature, and its accuracy is compared with experimental data. The analysis results show that the addition of the ejector in the Joule-Thomson refrigeration cycle increases up to 5 times the overall efficiency, where the maximum achievable COP and exergy efficiency are 0.0195 and 6.65%, respectively. Other featured advantages of the proposed Joule-Thomson refrigeration cycle with ejector are the simplicity of cycle, minimization of mechanical moving components, cost effectiveness, and high reliability compared to other cryogenic refrigeration methods using pumps or cold compressors in Joule-Thomson cycles.

Key words: Cryogenic; Ejector; Joule-Thomson; Refrigeration; Sub-atmospheric;

Nomenclature

A	area (m^2)
C	velocity of sound (m s^{-1})
EOS	equation of state
h	enthalpy (J kg^{-1})
JT	Joule-Thomson
LNG	liquefied natural gas
M	Mach number
$\dot{m}$	mass flow rate (kg s^{-1})
P	pressure (kPa)
s	entropy ($\text{J kg}^{-1} \text{K}^{-1}$)
T	temperature (K)
V	velocity (m s^{-1})
x	Quality

Greek symbols

η	isentropic efficiency
ρ	density (kg m^{-3})
ψ	exergy (kJ s^{-1})
μ	entrainment ratio

1. Introduction

The cooling mechanism of the Joule-Thomson (JT) refrigerator arises from isenthalpic expansion of the high pressure refrigerant to a low pressure state. This isenthalpic expansion is often easily achieved by non-moving devices such as valves, orifices or capillary tubes. JT refrigerators, therefore, generally demonstrate high reliability since the expansion device is free from mechanical wear and vibration.

The configuration of an ideal JT refrigeration cycle is described in Figure 1 with its temperature-entropy (T-s) diagram for better understanding the operating mechanism in low temperature applications. First, the refrigerant is compressed to high pressure, then the heat of compression is rejected by an after-cooler (1→2). Next, the high pressure, room temperature refrigerant is precooled by the returning cold stream as it passes through the heat exchanger or

recuperator (2→3). Then, the temperature of the refrigerant is further lowered by isenthalpic expansion through the JT valve (3→4). Heat is then absorbed at the evaporator as the refrigerant evaporates (4→5). The low pressure refrigerant then returns to the compressor to complete the cycle (5→1). Detailed processes of the compression and aftercooling (1-2) are simplified as isothermal line, and the temperature of warm side at the heat exchanger is assumed as identical in Figure 1 (b).

The refrigeration temperature of the single pure refrigerant JT process is determined by the saturation temperature of the refrigerant and the pressure after the JT expansion. The suction pressure of commercial compressors is usually higher than atmospheric pressure. Therefore, the pressure after the JT expansion will also be higher than atmospheric pressure. Accordingly, a refrigeration temperature below 77 K cannot be attained with pure nitrogen, as its saturation temperature at atmospheric pressure is approximately 77 K.

There are several methods to achieve a refrigeration temperature below 77 K. Firstly, it is possible to mix a lower boiling point refrigerant such as neon (Lee et al., 2014) or helium with nitrogen. However, these fluids decrease the thermodynamic efficiency of the refrigerator because the JT coefficients of neon and helium are negative at ambient temperature (Barron, 1985). Secondly, pumps or cold compressors can be used to lower the pressure after the JT expansion below 100 kPa. Thereby achieving a temperature lower than 77 K. This configuration is depicted in Figure 2 (a) and (b), respectively. The pumps act as room temperature sub-atmospheric compressors when installed in such a refrigeration cycle. The term of ‘pumps’ will be used in this paper to distinguish from cold compressors.

Room temperature pumps (or sub-atmospheric compressors) are readily available, less expensive, and the compression heat from them can be easily dissipated to the atmosphere since the compression is achieved outside of the cold box. On the other hand, the use of large heat exchanger

and stream tubes to mitigate the pressure loss in the sub-atmospheric pressure return stream is disadvantage.

If a cold compressor is used after the JT expansion instead of a room temperature vacuum pump, several issues related to the sub-atmospheric pressure condition can be eliminated. The low pressure after the JT expansion can be recovered by employing a cold compressor to boost the return stream pressure above atmospheric pressure. Conversely, the advantages of using a cold compressor lead to disadvantages as they are limited in availability, are expensive, and the compression heat, which needs to be rejected at low temperature, increases the entropy generation of the overall refrigeration cycle.

The problems related to the use of pumps or cold compressors can be alleviated by using an ejector, a hydraulic compressor operating without moving components. The pressure during the evaporation process can be lowered below atmospheric pressure by the pumping effect of the ejector, which can lower the refrigeration temperature. Although the compression efficiency of an ejector is lower than mechanical compressors, the major advantage is high reliability even at extremely low temperature due to its simple structure and the elimination of mechanical moving components.

The operating mechanism of the ejector is briefly introduced with the structure of the ejector in Figure 3. Two nozzles are incorporated in the primary and the secondary flow inlets. The fluid pressure decreases while the local velocity increases as fluid flows through a nozzle. Usually, the primary flow is choked at the nozzle throat, and then the velocity continuously increases to supersonic state in the diverging nozzle. At the exit of the primary nozzle, the static pressure of the primary flow is lower than the secondary flow inlet pressure. Secondary flow, therefore, is induced by the pressure difference. The primary flow and the induced secondary flow meet each other at the mixing section, and the secondary fluid is accelerated by the high velocity of the primary flow. The

1 velocity decreases and the pressure increases while the mixed stream enters through the constant
2 area section and the diffuser. Generally, performance of the ejector is expressed by the entrainment
3 ratio and the compression ratio. The entrainment ratio is defined by the ratio between the secondary
4 mass flow rate and the primary mass flow rate (m_s/m_p). The compression ratio is defined by the
5 ratio between the ejector exit pressure and the secondary flow inlet pressure ($P_{exit}/P_{s,in}$). Normally,
6 higher entrainment and compression ratios are preferred because the primary flow is used as the
7 driving force of the ejector.
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9 In cryogenic refrigeration, Rietdijk first proposed the use of ejector in the helium refrigeration
10 cycle for lowering the helium bath temperature (Rietdijk, 1966). The saturation temperature of
11 helium is 4.2 K at atmospheric pressure, thus, the ejector was used for lowering the saturation
12 pressure of the helium bath to achieve a refrigeration temperature below 4.2 K. The purpose of
13 lowering the helium bath temperature and pressure was successful even with the room temperature
14 ejector operation (P. Rudolf von Rohr, 1985). In this case, a part of compressed flow at ambient
15 temperature is bypassed for the ejector primary flow, and the remained refrigerant is expanded in
16 the JT expansion valve. Following reasons were the major motivations to improve the process,
17 firstly the structure of ejector was simple, and secondly, the ejector produces no additional
18 pollution of the main helium stream such as oil particles.
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20 The idea of using ejector to increase the thermodynamic efficiency of cryogenic refrigeration
21 systems (Daney et al., 1973; Johnson and Daggett, 1987; Pirtle and Kerney, 1984, 1986; Pirtle et
22 al., 1982) have been proposed, however, most of the previous results were qualitative analysis with
23 the preliminary test results of ejector itself. Recently, Yu et al. suggested a modified JT
24 refrigeration cycle using ejector. In this case, ejector-JT cycle shows 10% to 16% higher efficiency
25 compared to the JT cycle without ejector (Yu, 2008). Meanwhile, a statistical entropy analysis
26 carried out for low capacity LNG liquefaction plants (Arkharov et al., 2010, 2012) reported
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1 interesting results. The plant in Petergof, St. Petersburg Oblast is operated with a simple JT
2 expansion cycle, and the plant in Razvilka Moscow Oblast is equipped with one ejector. Another
3 plant in Balashikha Moscow Oblast is operated with two ejectors. One ejector equipped in JT cycle
4 showed the best efficiency (+1%), and the two ejectors equipped in JT cycle showed the lowest
5 efficiency (-27%) compared to a simple JT cycle without ejector. These examples cannot be
6 considered as a typical result of using ejector in JT cycle since the number of cases are too small,
7 and each cycle configuration is different for adopting the ejector. Certainly, the adoption of ejector
8 does not always guarantee the increase of thermodynamic efficiency, therefore, JT cycle combined
9 with ejector should be carefully analyzed.
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21 In this research paper, the refrigeration cycle operating with nitrogen as the refrigerant is
22 proposed with an ejector instead of a cold compressor to achieve a refrigeration temperature down
23 to 68 K. Figure 4 describes the process of the ejector-JT refrigeration cycle with T-s diagram. The
24 high pressure refrigerant is expanded through the ejector prior to the JT expansion to induce the
25 secondary flow, which is connected to the evaporator. After passing the ejector, the refrigerant
26 enters the liquid-vapor phase separator. The saturated liquid in the phase separator is further
27 expanded to a lower pressure by the JT valve. The vapor flow at the ejector exit returns to the
28 compressor, and the cold sensible heat is regenerated to precool the high pressure refrigerant at the
29 heat exchanger. To evaluate the proposed ejector-JT refrigeration cycle, one-dimensional ejector
30 modeling is established to predict the thermodynamic efficiency of the proposed refrigeration cycle.
31 Moreover, the usefulness of the ejector in cryogenic refrigeration is also discussed in this paper.
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50 **2. Modeling of ejector**

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55 Keenan published the first comprehensive theoretical one-dimensional (1-D) model for the
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ejector (Keenan, 1950). The 1-D model assumes that the flow velocity only changes along the flow direction. The losses in the ejector were not taken into account in Keenan's initial model, however, this 1-D model became the basis of the ejector design and analysis. Since then numerous models have been published to simulate the actual phenomenon inside the ejector.

In the models, calculations may not proceed employing the ideal gas correlation in the cryogenic ejector because the low temperature gas and two-phase fluid do not follow ideal gas behavior. Therefore, real fluid properties should be used in the cryogenic ejector model. In this paper, the basic calculation process follows the models of the helium ejector (Daney et al., 1973; Johnson and Daggett, 1987; Pirtle and Kerney, 1984, 1986). The proposed model in this paper is not focused on the accuracy but concentrated on the simplicity such that the model can be applied not only for the single-phase operation but also for the two-phase ejector operation. The notations used in 1-D ejector model are described in Figure 3.

The following assumptions have been made for the development of the ejector model.

- Flow of streams at the ejector inlets and outlet are stationary and at steady state.
- The velocity distribution is uniform along the radial direction, and it only varies with the axial direction.
- Friction loss at the wall is negligibly small.
- Gravitational effect is neglected.
- Losses through the nozzle and diffuser are considered with isentropic efficiency.

2.1. Primary nozzle (converging part)

The stationary inlet primary flow is accelerated while the fluid passes through the nozzle, and the velocity reaches sonic velocity ($M=1$) at the nozzle throat. From the known temperature (T_{p1})

and pressure (P_{p1}) at the inlet conditions, the enthalpy (h_{p1}) and the entropy (s_{p1}) are calculated by the equation of state (EOS) using REFPROP (Lemmon EW, 2010), as equations (1) and (2), respectively.

$$h_{p1} = EOS(T_{p1}, P_{p1}) \quad (1)$$

$$s_{p1} = EOS(T_{p1}, P_{p1}) \quad (2)$$

Initially, the pressure (P_{pt}) at the nozzle throat is assumed, and the isentropic enthalpy ($h_{pt,s}$) at the nozzle throat is calculated by EOS (eqn (3)). Then, the enthalpy (h_{pt}) at the nozzle throat is calculated from the definition of nozzle isentropic efficiency (eqn. (4)). The velocity (V_{pt}) at the nozzle throat is calculated from the energy balance (eqn. (5)). If the fluid is single-phase, the velocity of sound (C_{pt}) is calculated by EOS (eqn. (6)).

$$h_{pt,s} = EOS(P_{pt}, s_{p1}) \quad (3)$$

$$h_{pt} = h_{p1} - \eta_n (h_{p1} - h_{pt,s}) \quad (4)$$

$$V_{pt} = \sqrt{2(h_{p1} - h_{pt})} \quad (5)$$

$$C_{pt} = EOS(P_{pt}, h_{pt}) \quad (6)$$

The fluid condition at the nozzle exit may be supersaturated because of the sudden expansion of the fluid near the saturation limit. However, the supersaturation is usually neglected in ejector modelling. Because this phenomenon does not influence on significant mass flow rate change or the

state of the expanded two-phase flow (Ahammed et al., 2014; Elbel and Hrnjak, 2008), a simple thermodynamic equilibrium state is assumed. If the fluid state is in two-phase at the throat, the velocity of sound is not directly calculated by REFPROP. Thus, the sound velocity is calculated by the differential ratio between the pressure and the density. (Cengel et al., 2011) The density difference was calculated at +2% increased pressure and -2% decreased pressure from the original pressure value.

$$C_{pt} = \left(\frac{\partial P}{\partial \rho} \right)_s^{1/2} \cong \left[\frac{\Delta P}{\Delta \rho} \right]_s^{1/2} \quad (7)$$

In summary, the pressure (P_{pt}) at the nozzle throat is assumed and iterated until the nozzle throat velocity reaches sonic velocity ($M=1$). A flow chart in Figure 5 shows the iterative procedure. Pressure, temperature, and velocity at the primary nozzle throat are determined when the calculation is converged. This calculation method was also applied by Liu et al. (Liu and Groll, 2013).

The primary flow velocity continuously increases through the diverging nozzle. Munday and Bagster assumed that the primary flow fans out without mixing with the secondary flow for some distance after exiting the primary nozzle exhibited by a significant difference in velocity between the primary and the secondary flows (Munday and Bagster, 1977). This theory was validated by recent CFD calculations and supersonic ejector flow visualizations (Little et al., 2012). Thus it is assumed that there is a virtual extension of the diverging nozzle as depicted in Figure 3, which accelerates the primary flow above the sonic velocity, and the secondary flow up to the sonic velocity. In essence, this virtual duct extension acts as a diverging nozzle for the primary flow and as a converging nozzle for the secondary flow.

The primary flow and the secondary flow start to mix when the secondary flow is choked at the

end of the virtual duct. Thus, the ejector operates in the critical mode such that both primary and secondary flows are choked. Thereby resulting in the maximum entrainment ratio (Huang and Chang, 1999). Mixing takes place from the virtual nozzle exit to the constant area inlet under constant pressure due to choking pressure of the secondary flow. Thus, the choke flow condition of the secondary flow needs to be calculated.

2.2. Secondary nozzle

The calculation procedure for the secondary nozzle is the same as that of the primary nozzle. The pressure at the nozzle throat is initially assumed and iterated until the velocity at the nozzle throat reaches sonic velocity ($M=1$). From the known temperature (T_{s1}) and pressure (P_{s1}) at the inlet conditions, the enthalpy (h_{s1}) and the entropy (s_{s1}) are calculated by EOS (eqns. (8) and (9)).

$$h_{s1} = EOS(T_{s1}, P_{s1}) \quad (8)$$

$$s_{s1} = EOS(T_{s1}, P_{s1}) \quad (9)$$

The pressure (P_{st}) at the nozzle throat is initially assumed, and the isentropic enthalpy ($h_{st,s}$) at the nozzle throat is calculated by EOS (eqn. (10)). Then, the enthalpy (h_{st}) at the nozzle throat is calculated from the definition of nozzle isentropic efficiency (eqn. (11)). The velocity (V_{st}) at the nozzle throat is calculated by an energy balance (eqn. (12)). If the fluid is single-phase at the throat, the velocity of sound (C_{st}) is calculated by EOS (eqn. (13)). If the fluid is two-phase at the throat, the velocity of sound is calculated by eqn. (7).

$$h_{st,s} = EOS(P_{st}, s_{s1}) \quad (10)$$

$$h_{st} = h_{s1} - \eta_n (h_{s1} - h_{st,s}) \quad (11)$$

$$V_{st} = \sqrt{2(h_{s1} - h_{st})} \quad (12)$$

$$C_{st} = EOS(P_{st}, h_{st}) \quad (13)$$

2.3. Primary nozzle (diverging section)

The real and the virtual segments of the primary diverging nozzle are considered as one diverging nozzle in the calculation. The pressure (P_{p2}) at the exit of the primary nozzle is assumed to be the same as the secondary flow choke pressure (eqn. (14)). Thus the enthalpy and velocity of the primary flow at the exit of the nozzle are calculated by the following steps.

The isentropic enthalpy ($h_{p2,s}$) at the exit of the nozzle is calculated by EOS (eqn. (15)). Then the enthalpy (h_{p2}) at the exit of the nozzle is calculated from the definition of the isentropic efficiency of the nozzle. The velocity (V_{p2}) at the nozzle exit is calculated by energy balance (eqn. (17)). If the fluid is single-phase at the throat, the velocity of sound (C_{p2}) is calculated by EOS (eqn. (18)). If the fluid is two-phase at the throat, the velocity of sound is calculated by eqn. (7).

$$P_{p2} = P_{st} \quad (14)$$

$$h_{p2,s} = EOS(P_{p2}, s_{pt}) \quad (15)$$

$$h_{p2} = h_{pt} - \eta_n (h_{pt} - h_{p2,s}) \quad (16)$$

$$V_{p2} = \sqrt{2 \left(h_{pt} + \frac{V_{pt}^2}{2} - h_{p2} \right)} \quad (17)$$

$$C_{p2} = EOS(P_{p2}, h_{p2}) \quad (18)$$

2.4. Constant pressure mixing section

Conservation of mass, momentum, and energy equations are used to calculate the mixing process of the primary and secondary flows. The flow area at each location (throat of the secondary nozzle and exit of the primary nozzle) can be calculated with eqn. (19) with known velocity, density, and mass flow rates. Independent variables are four: V_3 , A_3 , h_3 and ρ_3 . The usable equations are four: mass, momentum, energy, and equation of states. REFPROP is used as equations of states (EOS), which means the relation of thermodynamic properties are not provided by explicit mathematical formula. Therefore this problem is calculated using iterative method to find the solution. In this particular model, we assumed h_3 as the initial value to calculate other variables with the iterative process.

The initial value of h_3 is assumed which is lower than the enthalpy at the two inlets of the mixing section (h_{p2} and h_{st}). Then, the velocity at the exit of the mixing section (V_3) is calculated from eqn. (20). The cross-sectional area at the exit of the mixing section (A_3) is calculated from eqn. (21), and the density at the exit of the mixing section (ρ_3) is calculated from eqn. (22). Finally, the initially assumed value of the enthalpy at the exit of the mixing section (h_3) is compared with the enthalpy (h_{31}) calculated by EOS in eqn. (23) until the deviation ($\frac{h_3 - h_{31}}{h_3}$) is less than 0.1%.

The state at the exit of the mixing section is determined when the calculation converges. Figure 6 describes the iteration procedure.

In addition, there is also another method to define the state of the mixing section by assuming the density first instead of the enthalpy. At first, we can assume ρ_3 , and then enthalpy h_3 (eqn.(23)), velocity V_3 (eqn.(20)), and area A_3 (eqn.(21)) can be calculated. The ρ_{31} can be calculated by the mass balance equation (eqn.(22)), and this value can be compared with the assumed initial value of ρ_3 until the calculation converges.

Two calculation procedures showed identical thermodynamic state of the mixing section. Therefore, either procedures could be applied to the model.

$$\dot{m} = \rho VA \quad (19)$$

$$\dot{m}_p \left(h_{p2} + \frac{V_{p2}^2}{2} \right) + \dot{m}_s \left(h_{st} + \frac{V_{st}^2}{2} \right) = \dot{m}_3 \left(h_3 + \frac{V_3^2}{2} \right) \quad (20)$$

$$P_{p2}A_{p2} + \dot{m}_p V_{p2} + P_{st}A_{st} + \dot{m}_s V_{st} = P_3A_3 + \dot{m}_3 V_3 \quad (21)$$

$$\dot{m}_p + \dot{m}_s = \dot{m}_3 = \rho_3 V_3 A_3 \quad (22)$$

$$h_{31} = EOS(P_3, \rho_3) \quad (23)$$

2.5. Constant area section

The velocity and pressure change due to wall friction are neglected while the mixed flow passes through the constant area section. However, shock phenomenon is taken into account when the flow velocity is higher than the sonic velocity ($M > 1$). The pressure increases and the velocity decreases

after the shock, thus, the flow velocity changes to subsonic. The practical shock phenomenon in the supersonic ejector occurs as a result of several oblique shock waves (Bartosiewicz et al., 2005; Zhu et al., 2007). In addition, the shock appears not only at the end of the constant area section but also in the diffuser (Weiss et al., 2010). In the 1-D calculation model, the complex shock phenomenon is simply assumed as a normal shock at the end of the constant area section to simulate the increased pressure and the decreased velocity. Instead of the normal shock equations combined with the ideal gas relation, an iterative method is employed using the real gas property from REFPROP. This calculation method was adopted from the 1-D model of the helium two-phase ejector by Pirtle and Kerney (Pirtle and Kerney, 1984). Details of the calculation procedure are described as follows.

The density after the shock is higher than the density before the shock, thus, a slightly increased density is assumed as the initial density after the shock. The solution for the state after the shock can be calculated from conservation of mass, momentum, and energy. By assuming a value for the density (ρ_4) after the shock, the velocity after the shock (V_4) is calculated by eqn. (24). Then, the pressure (P_4) and the enthalpy (h_4) after the shock are calculated by eqs. (25) and (26), respectively. Finally, the density (ρ_{41}) is calculated by EOS from pressure (P_4) and enthalpy (h_4) in eqn. (27). The iterations are repeated until the deviation ($\frac{\rho_4 - \rho_{41}}{\rho_4}$) is less than 0.1%. The state after the shock is confirmed when the calculation converges. The iteration process before and after the shock is depicted in Figure 7.

$$\rho_3 V_3 A_3 = \rho_4 V_4 A_4, (A_3 = A_4) \quad (24)$$

$$P_3 A_3 + \dot{m}_3 V_3 = P_4 A_4 + \dot{m}_4 V_4 \quad (25)$$

$$h_3 + \frac{V_3^2}{2} = h_4 + \frac{V_4^2}{2} \quad (26)$$

$$\rho_{41} = EOS(P_4, h_4) \quad (27)$$

2.6. Diffuser

The velocity of the mixed flow after the shock gradually decreases, and the pressure increases by passing through the diffuser. The isentropic enthalpy at the exit of the diffuser ($h_{5,s}$) is calculated (eqn. (29)) from the conservation of energy and the definition of diffuser isentropic efficiency. Then, the pressure at the exit of the diffuser (P_5) is calculated from $h_{5,s}$ and s_4 by EOS. If P_5 and h_5 are calculated, other state properties at the exit of the diffuser are determined by EOS.

$$h_4 + \frac{V_4^2}{2} = h_5 + \frac{V_5^2}{2}, V_5 \approx 0 \quad (28)$$

$$h_{5,s} = h_4 + \eta_d(h_5 - h_4) \quad (29)$$

$$P_{5s} = EOS(h_{5,s}, s_4) \quad (30)$$

If the state at the exit of the diffuser is two-phase, a phase separator is installed at the exit. Then the liquid phase is further expanded through the JT valve, and it returns to the ejector as secondary flow. The vapor phase returns to the compressor, where becomes the primary flow in the ejector. Therefore, the entrainment ratio (μ) is expressed as vapor quality (x_5) at the exit of the diffuser as in eqn. (31). In this case, the initial entrainment ratio is updated by the calculated entrainment ratio

from eqn. (31) until the deviation is less than 0.1%.

$$\mu = \frac{\dot{m}_s}{\dot{m}_p} = \frac{1 - x_5}{x_5} \quad (31)$$

3. Validation of modeling

3.1 Single-phase operation of the ejector

We validate the 1-D ejector performance prediction model from existing experimental results from previous literatures. The main performance parameters, the entrainment ratio and the compression ratio are compared. Inlet conditions for the primary and secondary flows are input as boundary conditions. The mass flow rate at primary flow and the entrainment ratio are also inputs to the model that calculates the ejector exit pressure.

Results from the ejector refrigeration system presented by Huang et al (Huang et al., 1999) are compared with the model presented in this paper. The vapor-phase of the R141b (ranged from 400 kPa to 600 kPa) enters the primary nozzle. The saturated vapor-phase of the R141b (at 40 kPa) is induced as secondary flow into the ejector, from which the mixed flow then discharges at the outlet of the diffuser. The suction pressure is now lower than atmospheric pressure, thereby obtaining a refrigeration temperature of 8 °C which is the saturation temperature of R141b at 40 kPa. Generally, this sub-atmospheric low pressure cannot be achieved with conventional compressors. Thus, the ejector serves as the vacuum pump for the sub-atmospheric condition.

The primary and secondary inlet conditions are used as input values to the model, and the exit pressure of the ejector is calculated for a given entrainment ratio. The isentropic efficiency of the

nozzles and the diffuser are assumed to be 85%. The mass flow rate of the primary flow is not presented in the paper, thus it is estimated with the information of primary nozzle throat diameter and choking condition. Calculated values for the exit pressure of the ejector are compared with experimental results of Huang et al as shown in Figure 8. The calculated values correspond to the experimental data to within $\pm 10\%$ error range.

Another ejector comparison employing experimental data from Hemidi et al (Hemidi et al., 2009) with air as the working fluid is used to validate the model. The compressed air ranged from 400 kPa to 600 kPa for the primary flow, and the secondary flow is induced from ambient air. The exit pressure of the ejector is controlled by an adjustable valve at the exit of the ejector. In this case, the isentropic efficiency of the nozzles and the diffuser are assumed to be 80%. As before, the mass flow rate of the primary flow is not shown in the reference, thus it is estimated with the primary flow inlet pressure and the primary nozzle throat diameter. The exit pressure at the ejector is calculated at the critical entrainment ratio from the experimental data. Calculation results are compared with the critical back pressure data in Figure 9. These calculations agree well for three cases from the experiments within $\pm 5\%$ accuracy.

3.2 Two-phase operation of the ejector

For two-phase flow at the exit of the ejector, the initial entrainment ratio is changed through iteration to match the calculated vapor quality at the exit of the ejector. Therefore, both the entrainment and compression ratios are compared for two-phase ejector operation. Experimental results for an ejector employed in the transcritical CO_2 refrigeration cycle (Elbel and Hrnjak, 2008) are selected to validate the model for two-phase operation. Due to operation in the transcritical region, the primary and the secondary flow pressures are very high, 10 MPa and 3.5 MPa,

respectively. The primary and the secondary flow inlet conditions of the experiment are applied, and the ejector performance is calculated assuming an isentropic efficiency of 80% for the nozzle and diffuser. The calculated values for the exit pressure of the ejector and the entrainment ratio are presented with the experimental data in Figure 10 and Figure 11. Calculated values for the exit pressure compare well with experiment values to within $\pm 5\%$. Predicted entrainment ratios tend to be higher than experiment values up to +25%.

4. Analysis of N₂ ejector-JT refrigeration cycle

Nitrogen was selected as the refrigerant for the refrigeration cycle analysis. The isentropic efficiency for ejector nozzles and diffuser are assumed to be 80%, since most of ejector component efficiencies are ranged from 70% to 90% at the on-design operating condition (Besagni et al., 2015b; Liu and Groll, 2013; Varga et al., 2009). In fact, ejector component efficiency varies with the operating condition, geometry, and manufacturing technique (Besagni et al., 2015a; Chen et al., 2014). However, our study focuses on the general aspect of the total system which is the refrigeration cycle combined with the ejector. Therefore, the ejector geometry is assumed to be optimized for the specific operating condition. According to this assumption, nozzles and diffusers in the ejector have constant isentropic efficiencies.

The configuration of the cycle is shown in Figure 12. The refrigeration temperature is set as 68 K to avoid nitrogen freezing when the secondary flow expands through the secondary nozzle. Therefore, the secondary flow inlet condition is fixed by the saturated vapor at 68 K (saturation pressure: 28.86 kPa). From the given inlet conditions for primary and secondary flow, the ejector exit pressure and vapor quality are calculated by the proposed 1-D ejector model.

Several assumptions have been made to estimate the overall refrigeration efficiency. For

example, the compression process is assumed to be a four-stage compression system. Each compression ratio is set to 2:1 to regulate the discharge temperature of the compressor below 400 K, which is the generally accepted maximum temperature at the compressor discharge to avoid overheating. Isentropic efficiency of each compressor is assumed to be 80%, and the refrigerant temperature after the heat rejection (stream 2) is assumed to be 300 K. The performance of the heat exchanger is assumed as a minimum temperature difference of 3 K between the hot and cold side refrigerants, where the heat exchanger size is determined by the process simulator. The pressure drops in heat exchangers, compressors, pumps, and evaporators are neglected. The pressure drop of the refrigerant flow is taken into account only at the expansion valve. Finally, the efficiency of the ejector-JT refrigeration cycle is calculated from the definition of COP (coefficient of performance: cooling load / compression work) and the exergy efficiency (COP/COP_{ideal}) where COP_{ideal} is the Carnot efficiency (Ersoy and Bilir, 2012).

For the primary flow inlet pressure below 1900 kPa, vapor-phase is maintained at the exit of the ejector so that phase separation does not occur at the ejector exit. According to this reason, the maximum compressor discharge pressure is varied from 1900 kPa to 2600 kPa. Calculation results of each case are summarized in Table 1, and the details of each stream information is listed in Table 2 when the primary inlet pressure is 2400 kPa. The mass flow rate between the primary flow and the secondary flow are determined by the entrainment ratio, and the entrainment ratio does not change when the primary flow rate is increased. Therefore, the analysis is carried out for 1 g/s of refrigerant flow rate (secondary flow rate) which is convenient to extend for other mass flow rate condition.

The exit pressure of the ejector exhibits similar levels from 128 kPa to 136 kPa regardless of varying discharge pressure at the compressor. Therefore, the compression ratio of the exit pressure of the ejector to the inlet pressure of the secondary flow is almost constant, as 4.5:1. Since the

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entrainment ratio, i.e. ratio of the secondary flow rate to the primary flow rate, can be expressed by the vapor quality at the exit of the ejector from eqn. (33), high vapor quality at the exit of the ejector results in very small entrainment ratio from 0.032 to 0.037. COP and exergy efficiency are slightly changed according to the primary inlet pressure, however, the deviation is not significant.

An ejector usually shows the best performance at the critical mode of operation, as the maximum entrainment ratio is achieved. Then the entrainment ratio at the critical mode of operation is determined from operating conditions such as pressure and temperature at the ejector inlet and outlet. From calculation, the optimum operating pressure is found to be 2400 kPa for the ejector-JT refrigeration cycle (ejector component: 80%) yielding the highest entrainment ratio and COP. At optimum primary flow inlet pressure 2400 kPa, the COP and exergy efficiency of the ejector-JT cycle are 0.0195 and 6.65%, respectively.

5. Discussion

The JT refrigeration cycle configured with room temperature pumps or cold compressors are analyzed for the same refrigeration conditions. For the equitable comparison, the cycles are configured as a single-stage operation without aid of a precooling stage, nitrogen as a single refrigerant, a refrigeration temperature of 68 K, 80% efficiency for the compressor and vacuum pump, and the heat exchanger performance having minimum temperature difference between streams of 3 K.

First, the JT refrigeration cycle with vacuum pump is analyzed. Figure 13 displays the process diagram for the JT refrigeration cycle employing vacuum pumps and compressors with after-coolers. The pressure after the JT expansion needs to be maintained at 28 kPa, and the refrigeration supply pressure should be higher than 1300 kPa to achieve a refrigeration temperature of 68 K. Therefore,

several compressors in series are necessary to boost the sub-atmospheric pressure to 1300 kPa. For this purpose, at least four compressor stages are necessary to limit the maximum temperature to 400 K at the compressor discharges. Therefore, the maximum compressor discharge pressure is varied from 1300 kPa to 2600 kPa, and the calculation results of each case are summarized in Figure 14. The cooling capacity, COP, and exergy efficiency almost linearly increase with the compressor discharge pressure. Thus, refrigeration performance calculation for the compressor discharge pressure higher than 2600 kPa is omitted. Details of each stream information are listed in Table 3, when the compressor discharge pressure is 2400 kPa. The high pressure nitrogen is precooled to 119 K by the return stream at the heat exchanger. Then the precooled nitrogen expands through JT valve to low pressure (28.86 kPa), and the lowest temperature of 68 K is achieved. Since the refrigerant flow rate is assumed as 1 g/s, the maximum cooling capacity is small as 3.54 W. COP and exergy efficiency gradually increase with the increase of compressor discharge pressure, and the maximum COP and exergy efficiency are 0.00628 and 2.14% at 2600 kPa.

Secondly, another JT refrigeration cycle method using a cold compressor is analyzed. Figure 15 displays the process diagram. In this case, the sub-atmospheric return-stream (28 kPa) after the JT expansion process is compressed to 100 kPa by the cold compressor with the cold compressor inlet temperature set to 68 K (saturated vapor state). The simulation indicates that the nitrogen temperature should be lower than 100 K before the expansion (stream location 3 in Figure 15) to achieve 68 K after the expansion (stream location 4 in Figure 15). To demonstrate this process, the return-stream inlet temperature (stream location 6 in Figure 15) may have a lower temperature than that in the stream 3. However, a temperature lower than 100 K at stream location 6 is not achievable, because the heat generated by the cold compressor feeds into the return-stream (location 6). The cold compressor discharge temperature increases to 106 K when the compressor isentropic efficiency of 80% is assumed. The temperature of the high pressure refrigerant at the inlet of JT

expansion process is determined from the recuperation precooling with the cold stream at the discharge of the compressor. Therefore, the inlet temperature to the JT expansion process is always higher than 106 K. Thus, it is impossible to reach 68 K with the JT refrigeration cycle employing a cold compressor.

The exergy efficiencies of the ejector-JT cycle and the JT cycle with vacuum pumps are compared with respect to the compressor discharge pressure in Figure 16. For this analysis, the isenthalpic efficiency of the primary nozzle, the secondary nozzle, and the diffuser were also changed from 70% to 90%. As the compressor discharge pressure increases, the exergy efficiency steadily increases for low outlet pressure through 2600 kPa for the JT cycle with vacuum pumps. On the other hand, the exergy efficiency of the ejector-JT cycle peaks at certain compressor outlet pressure, and the optimum compressor outlet pressure is slightly different to the ejector component efficiency. The efficiency of ejector-JT cycle shows always higher than that of JT cycle more than three to five times. If the compressor outlet pressure is extremely high, JT cycle may show higher efficiency. While this is possible, this cycle configuration becomes too complicated. The complicated setup of the pumps, compressors, after-coolers and low-pressure pipes are challenges to the realization of this process. One should be pointed out that the calculation results studied in this paper have errors for the entrainment ratio ($\pm 25\%$) and ejector exit pressure ($\pm 5\%$). These errors attribute to the uncertainty of total cycle efficiency by $\pm 26\%$.

For the comparison of thermodynamic characteristic between the JT cycle and the ejector-JT cycle, exergy analysis has been carried out. Exergy of each stream and exergy destruction of each component are calculated by eqn. (32). (Cengel et al., 2011; Venkatarathnam, 2008). The performance of actual system can be different from the calculation results due to the unavoidable heat transfer and component inefficiencies, however, the exergy destruction rate is still an important measure to analyze the losses and entropy generation. In this analysis, exergy losses of heat

exchangers, an ejector and JT valves are calculated based upon the exergy balance between the incoming streams and outgoing streams.

$$\psi = (h - h_0) - T_0(s - s_0) \quad (32)$$

The results are summarized in Table 4 and Table 5. Most of losses are found in the JT valve and the heat exchanger in JT cycle. Exergy destruction in the JT valve and the heat exchanger are mainly caused by the isenthalpic expansion and the heat transfer, respectively. In ejector-JT cycle, the maximum loss is found in the ejector and the heat exchanger. Losses of ejector represented by isentropic efficiency of nozzles and diffuser account for the exergy destruction of ejector itself. It is notable that exergy destruction in the JT valve is very small in ejector-JT cycle because the refrigerant is expanded at saturated liquid state. The absolute value of total exergy destruction in ejector-JT cycle is larger than that of JT cycle since the refrigerant flow rate is large in ejector-JT cycle. Therefore, exergy destruction is divided by the cooling capacity of each refrigeration cycle for the fair comparison. The exergy destruction per cooling capacity shows that the considerable loss in JT valve is reduced by adopting ejector. Reduction of loss in the heat exchanger is additional benefit by decreasing the temperature difference between the cold stream and the hot stream.

The advantage of the ejector-JT cycle is that it requires only four compressors as the pumps are substituted with one ejector. As a result, the compression process is simplified, and the cycle operates from above atmospheric pressure which is advantageous for reduced maintenance. In addition, thermodynamic efficiency of refrigeration cycle can be improved considerably.

6. Conclusion

1 The JT refrigeration cycle equipped with an ejector has been designed to achieve 68 K with
2 nitrogen as the refrigerant. The performance of the ejector is analyzed with a 1-D ejector prediction
3 model that has been developed for cryogenic applications. The important feature of this 1-D model
4 is that the appropriate fluid property is used for each calculation step. As a result, the proposed
5 model can be applied for the prediction of ejector performance operating in either single- ($\pm 10\%$
6 uncertainty) or two-phase conditions ($\pm 25\%$ uncertainty). The analysis shows that the addition of an
7 ejector in JT refrigeration cycle enables the JT refrigeration cycle to work below 77 K with pure
8 nitrogen. The cryogenic ejector refrigeration cycle is considered to be more attractive for sub-
9 atmospheric operation due to higher efficiency (3 to 5 times), compactness, lower cost, and higher
10 reliability than the JT-cycle with vacuum pumps.
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Table list

Table 1 Calculation result of N₂ ejector- JT refrigeration cycle at constant mass flow rate of 1 g s⁻¹ for the secondary flow.

Table 2 Stream information of N₂ ejector- JT refrigeration cycle when the primary inlet pressure is set as 2400 kPa.

Table 3 Stream information of N₂ JT refrigeration cycle when the compressor discharge pressure is set as 2400 kPa.

Table 4 Exergy of each stream in N₂ JT and ejector-JT refrigeration cycle when the compressor discharge pressure is set as 2400 kPa.

Table 5 Exergy destruction at the components of refrigeration cycle

Figure list

Figure 1 (a) Cycle configuration and (b) operating mechanism of JT refrigerator in T-s diagram.

Figure 2 Cycle configuration of JT refrigerator with (a) vacuum pump at room temperature and (b) cold compressor at low temperature.

Figure 3 Configuration and internal structure of the ejector.

Figure 4 (a) Cycle configuration of ejector equipped JT refrigeration cycle and (b) operating mechanism in T-s diagram.

Figure 5 A flow chart for the description of iteration process from the primary nozzle inlet to the primary nozzle throat.

Figure 6 A flow chart for the description of iteration process in the mixing section.

Figure 7 A flow chart for the description of iteration process before and after the shock.

Figure 8 Comparison of ejector exit pressure from the present model and the experimental data (Huang et al., 1999).

Figure 9 Comparison of ejector exit pressure from the present model and the experimental data (Hemidi et al., 2009).

Figure 10 Comparison of the ejector exit pressure from the present model and the experimental data (Elbel and Hrnjak, 2008).

Figure 11 Comparison of the ejector entrainment ratio from the present model and the experimental data (Elbel and Hrnjak, 2008).

Figure 12 The process diagram of the ejector-JT refrigeration cycle.

Figure 13 The process diagram of the JT cycle with vacuum pumps that draws the low pressure return to a sub-atmospheric pressure.

Figure 14 Calculation result of N₂ JT refrigeration cycle at constant mass flow rate of 1 g s⁻¹.

Figure 15 The process diagram of the JT cycle employing a the cold compressor.

Figure 16 The exergy efficiency comparison: Ejector-JT cycle and the JT cycle with the vacuum pump.

Table 1 Calculation result of N₂ ejector- JT refrigeration cycle at constant mass flow rate of 1 g s⁻¹ for the secondary flow.

Primary inlet pressure	1900 kPa	2000 kPa	2100 kPa	2200 kPa	2300 kPa	2400 kPa	2500 kPa	2600 kPa
Primary inlet temperature	114.6 K x=0.9595	115.5 K x=0.9595	116.4 K x=0.9737	117.3 K x=0.9833	118.2 K x=0.9948	119.3 K	120.5 K	121.6 K
Secondary inlet pressure	Fixed by 28.86 kPa (saturation pressure at 68 K)							
Entrainment ratio	0.0327	0.0338	0.0348	0.0357	0.0363	0.0373	0.0371	0.0362
Ejector exit pressure	128.6 kPa	129.8 kPa	131.0 kPa	132.1 kPa	133.3 kPa	134.2 kPa	135.3 kPa	136.5 kPa
Cooling capacity	186.6 W	186.4 W	186.3 W	186.1 W	185.9 W	185.8 W	185.7 W	185.5 W
COP	0.0186	0.0188	0.0190	0.0192	0.0193	0.0195	0.0192	0.0184
Exergy efficiency	6.33%	6.42%	6.49%	6.57%	6.57%	6.65%	6.54%	6.29%

(x: vapor quality)

Table 2 Stream information of N₂ ejector- JT refrigeration cycle when the primary inlet pressure is set as 2400 kPa.

stream	1	2a	2b	2c	2d	2e	2f	2g
Vapor fraction	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Temperature (K)	297.0	381.2	300	385.1	300	384.9	300	385.2
Pressure (kPa)	134.2	276.0	276.0	568.0	568.0	1167	1167	2400
Mass flow (g s ⁻¹)	26.78	26.78	26.78	26.78	26.78	26.78	26.78	26.78
stream	2	3	Eo	V	L	4'	5	
Vapor fraction	1.0000	1.0000	0.9640	1.0000	0.0000	0.1068	1.0000	
Temperature (K)	300	119.3	79.83	79.83	79.83	68.0	68.0	
Pressure (kPa)	2400	2400	134.2	134.2	134.2	28.86	28.86	
Mass flow (g s ⁻¹)	26.78	26.78	27.78	26.78	1.00	1.00	1.00	

Table 3 Stream information of N₂ JT refrigeration cycle when the compressor discharge pressure is set as 2400 kPa.

stream	1	2a	2b	2c	2d	2e	2f	2g
Vapor Fraction	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Temperature (K)	297.0	382.5	300	387.7	300	392.6	300	387.2
Pressure (kPa)	28.86	60.0	60.0	126	126	263	263	550
Mass Flow (g s ⁻¹)	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
stream	2h	2i	2j	2k	2	3	4	5
Vapor Fraction	1.0000	1.0000	0.9640	1.0000	1.0000	0.9018	0.9854	1.000
Temperature (K)	300	387.1	300	387.2	300	119	68.0	68.0
Pressure (kPa)	550	1149	1149	2400	2400	2400	28.86	28.86
Mass Flow (g s ⁻¹)	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Table 4 Exergy of each stream in N₂ JT and ejector-JT refrigeration cycle when the compressor discharge pressure is set as 2400 kPa.

JT								
Stream	1	2a	2b	2c	2d	2e	2f	2g
Exergy (kJ s ⁻¹)	-0.1111	-0.0357	-0.0464	0.0311	0.0193	0.0974	0.0844	0.1613
Stream	2h	2i	2j	2k	2	3	4	5
Exergy (kJ s ⁻¹)	0.1495	0.2263	0.2145	0.2913	0.2793	0.4273	0.1124	0.1021
Ejector-JT								
stream	1	2a	2b	2c	2d	2e	2f	2g
Exergy (kJ s ⁻¹)	0.6656	2.649	2.373	4.382	4.0800	6.083	5.78	7.787
Stream	2	3	Eo	V	L	4'	5	
Exergy (kJ s ⁻¹)	7.478	11.04	6.222	5.483	0.7392	0.7311	0.1021	

Table 5 Exergy destruction at the components of refrigeration cycle

JT	Exergy destruction	Portion	Exergy destruction per cooling capacity
JT valve	0.3149 kJ s ⁻¹	80.7%	103.4
Evaporator	0.0103 kJ s ⁻¹	2.6%	3.38
Heat exchanger	0.0652 kJ s ⁻¹	16.7%	21.4
Total	0.3904 kJ s ⁻¹	100%	128.2
Ejector-JT			
Ejector	4.9201 kJ s ⁻¹	72.2%	26.5
JT valve	0.0081 kJ s ⁻¹	0.1%	0.044
Evaporator	0.629 kJ s ⁻¹	9.2%	3.39
Heat exchanger	1.2554 kJ s ⁻¹	18.4%	6.76
Total	6.8124 kJ s ⁻¹	100%	36.7

(a)

(b)

Figure 1 (a) Cycle configuration and (b) operating mechanism of JT refrigerator in T-s diagram.

(a)

(b)

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(a)

(b)

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Figure 14 Calculation result of N₂ JT refrigeration cycle at constant mass flow rate of 1 g s⁻¹.

Figure 15 The process diagram of the JT cycle employing a the cold compressor.

Figure 16 The exergy efficiency comparison: Ejector-JT cycle and the JT cycle with the vacuum pump.