TITLE: Thermodynamic Modeling of Magnetic Hysteresis in AMRR Cycles

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ABSTRACT

Current models of Active Magnetic Regenerative Refrigeration (AMRR) cycles are not able to capture the effect of magnetic hysteresis and are therefore strictly limited to second order magnetic transition (SOMT) materials. The discovery of the giant magnetocaloric effect (GMCE) in first order magnetic transition (FOMT) materials has generated substantial interest. FOMTs yield large adiabatic temperature changes but also exhibit significant magnetic hysteresis. This work quantifies the effects of magnetic hysteresis. Thermodynamically, hysteresis is treated as a source of entropy generation that is proportional to the area swept by the hysteresis loop experienced locally by the material during one refrigeration cycle. The 1-D numerical model presented by Engelbrecht (2008) is modified to include magnetic hysteresis. Hysteresis losses are shown to be directly proportional to regenerator volume. Therefore, at large refrigeration capacity to volume ratios, AMRR beds using layered FOMT materials significantly outperform the same cycle using layered SOMT refrigerants.

KEYWORDS

magnetocaloric, hysteresis, magnetic refrigeration, AMRR, thermodynamic

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NOMENCLATURE

A_c	cross-sectional area (m^2)
a_s	volume specific surface area ($\text{m}^2 \text{ m}^{-3}$)
AR	aspect ratio (-)
C_c	adiabatic temperature change scaling factor (-)
C_h	hysteresis scaling factor (-)
C_M	isothermal entropy change scaling factor (-)
COP	coefficient of performance (-)
$c_{\mu_0 H}$	constant field specific heat capacity ($\text{J kg}^{-1} \text{ K}^{-1}$)
d_h	hydraulic diameter (m)
d_p	particle diameter (m)
f	friction factor (-)
H	applied magnetic field (A m^{-1})
h	mass specific enthalpy (J kg^{-1})
J	total molecular angular momentum number (-)
k	thermal conductivity ($\text{W m}^{-1} \text{ K}^{-1}$)
L	regenerator length (m)
M	volume specific magnetic moment (A m^{-1})
M_{an}	anhysteretic magnetization (A m^{-1})
M_{irr}	irreversible magnetization (A m^{-1})
m	mass of magnetic refrigerant material (kg)
$\dot{m}$	mass flow rate (kg/s)
N_L	number of layers in regenerator bed
Nt	number of time steps
Nx	number of spatial steps
Nu	Nusselt number (-)
P	pressure (Pa)
Pr	Prandtl number (-)
Q	heat transfer (J)
q	mass specific heat transfer (J kg^{-1})
Re	Reynolds number (-)
S	entropy (J/K)
s	specific entropy ($\text{J kg}^{-1} \text{ K}^{-1}$)
T	temperature (K)
t	time (s)
U	internal energy (J)
u	mass specific internal energy (J kg^{-1})
V	volume (m^3)
v	mass specific volume ($\text{m}^3 \text{ kg}^{-1}$)
W	work (J)

w	mass specific work (J kg^{-1})
x	regenerator axial position (m)

Greek

ΔM	maximum irreversible magnetization (A m^{-1})
Δs_M	mass specific isothermal entropy change ($\text{J kg}^{-1} \text{K}^{-1}$)
ΔT_{ad}	adiabatic temperature change (K)
$\Delta \mu_0 H$	change in magnetic field (T)
ε	bed porosity (-)
η	efficiency (-)
μ	viscosity (N-s m^{-2}) or magnetic permeability (T m A^{-1})
$\mu_0 H$	applied field (Tesla)
ρ	density (kg m^{-3})
τ	cycle duration (s)

Subscripts

ad	adiabatic
amp	amplitude
an	anhysteretic
C	cold
$Curie$	Curie temperature
eff	effective
f	fluid or final
gen	generation
H	hot
i	spatial step index
irr	irreversible
j	temporal step index
mag	magnetic
max	maximum
min	minimum
$motor$	motor
r	regenerator material
ref	reference or refrigeration
s	saturation

INTRODUCTION

The lanthanum-iron-silicon hydrides ($\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$) are a promising material for use in active magnetic refrigerators since they exhibit a large magnetocaloric effect (MCE) near room temperature. These materials exhibit a first-order itinerant-electron metamagnetic phase transition between the ferromagnetic and paramagnetic phases, during which the material exhibits magnetic hysteresis (Fujita, Fujieda et al. 2003). The Curie temperature can be tuned between approximately 200 K to 336 K by adjusting the concentration of hydrogen in the compound (y), depending on the concentration of iron and silicon (x).

Fujieda et al. (2002) report an isothermal entropy change of $\Delta s_M = -24 \text{ J kg}^{-1} \text{ K}^{-1}$ and an adiabatic temperature change of $\Delta T_{ad} = 6.9 \text{ K}$ for $\text{La}(\text{Fe}_{0.89}\text{Si}_{0.11})_{13}\text{H}_{1.3}$ under an applied field change from 0 to 2 Tesla. The particular composition with $x = 0.11$ and $y = 1.3$ has a Curie temperature of approximately 291 K. These materials tend to be less expensive to manufacture than pure gadolinium, but require precise and tedious processing to modulate the silicon and hydrogen compositions (Gschneidner, Pecharsky et al. 2005). Zhao et al. (2009) demonstrated that hysteresis losses may be reduced with the partial substitution of praseodymium for lanthanum with only a slight decrease in the magnetocaloric effect. $\text{La}_{0.5}\text{Pr}_{0.5}(\text{Fe}_{0.88}\text{Si}_{0.12})_{13}\text{H}_{1.6}$ exhibited a magnetocaloric effect of $\Delta s_M = -17.7 \text{ J kg}^{-1} \text{ K}^{-1}$ at a Curie temperature of approximately $T_{Curie} = 320 \text{ K}$ with a hysteresis loss per cycle of 2.3 J kg^{-1} (Jin-liang, Jun et al.).

Lanthanum-iron-silicon hydride is just one of many first-order magnetic transition (FOMT) materials that are being developed. These materials all exhibit significant magnetic hysteresis as compared to the second order magnetic transition (SOMT) materials that have been considered for magnetic refrigeration. The relatively large magnetocaloric effect of a FOMT material as compared to an SOMT material, characterized either by the adiabatic temperature change or isothermal entropy change, will improve the performance of an active magnetic regenerative refrigeration (AMRR) system. However, FOMT materials also exhibit magnetic hysteresis whereas SOMT materials typically do not. Hysteresis will reduce the performance of an AMRR system. This work presents a thermodynamic model of magnetic hysteresis for applications in magnetic cooling systems and evaluates the effect of hysteresis on system performance. We are unaware of any other numerical models that can accurately capture the effects of magnetic hysteresis on the performance of an AMRR cycle. Hysteresis has previously been considered through the use of scaling factors that are applied to the efficiency of a system simulated without hysteresis (Nielsen, et al., 2011).

The objective of this work is to quantify the effects of magnetic hysteresis on the performance of AMRR cycles that use FOMT materials. Thermodynamically, magnetic hysteresis is treated as a source of entropy generation that is proportional to the area swept by a hysteresis loop for one refrigeration cycle. The effects of thermal hysteresis on the AMRR cycle are neglected in this analysis. The one-dimensional, transient numerical model presented by Engelbrecht (2008), is modified to include entropy generation due to magnetic hysteresis in the regenerator energy equations. This model accepts inputs such as the regenerator geometry, magnetic refrigerant material, and heat transfer fluid and calculates the steady-state temperature distribution for the complete AMRR cycle as a function of time and space through an iterative successive substitution technique that is subject to a specified convergence tolerance. The final fluid and regenerator material temperature distributions are used to quantify the performance of the cycle.

The modified model is used to optimize the performance of an AMRR cycle that uses FOMT materials and compare it to the performance of an AMRR cycle that uses SOMT materials. The effect of parameters such as regenerator volume, mass flow rate profile, regenerator aspect ratio, and bed layering geometry on the performance of the cycle is studied.

THERMODYNAMIC TREATMENT OF HYSTERESIS

Since hysteresis is an irreversible process, it is necessary to model the phenomenon as an internal entropy generation. Consider an adiabatic process undergone by a magnetocaloric material with hysteresis. Such a process will be irreversible and therefore an entropy balance on this system, carried out on a unit mass basis, requires that:

$$\frac{ds}{dt} = \frac{\dot{S}_{gen}}{m} \quad (1)$$

where s is the specific entropy of the magnetic material, m is the mass of the material and $\dot{S}_{gen}$ is the rate of entropy generation, which must have a positive value. The rate of entropy generation per unit mass is defined using the irreversible magnetization (M_{irr}) and it is assumed to be related to the area within a hysteresis curve. The rate at which the hysteretic curve area is being swept out per time is given by:

$$T \frac{\dot{S}_{gen}}{m} = v M_{irr} \left| \frac{d\mu_0 H}{dt} \right| \quad (2)$$

or, solving for the entropy generation rate per unit mass:

$$\frac{\dot{S}_{gen}}{m} = \frac{v M_{irr}}{T} \left| \frac{d\mu_0 H}{dt} \right| \quad (3)$$

where v is the specific volume of the magnetic material, M_{irr} is the irreversible magnetization per unit volume, T is the temperature of the magnetic material, and $\frac{d\mu_0 H}{dt}$ is the rate of change of the applied magnetic field. The irreversible magnetization is a function that must be unambiguously positive and it represents the deviation of the magnetization from the anhysteretic magnetization. The irreversible magnetization is a function of temperature and applied field as well as the minimum and maximum values of the applied field experienced by the material during each cycle. Assuming that the degree of hysteresis does not depend on the rate of the process (i.e., magnetic hysteresis is assumed to be independent of magnet sweep rate), the total magnetization induced in the material (M) is the sum of the anhysteretic (M_{an}) and hysteretic (M_{irr}) terms:

$$M\left(T, \mu_0 H, \mu_0 H_{min}, \mu_0 H_{max}, \frac{d\mu_0 H}{dt}\right) = M_{an}(T, \mu_0 H) - M_{irr}(T, \mu_0 H, \mu_0 H_{min}, \mu_0 H_{max}) \text{sign}\left(\frac{d\mu_0 H}{dt}\right) \quad (4)$$

The signum function appearing in Eq. (4) returns the sign of its argument and causes the irreversible magnetization to be subtracted from the anhysteretic magnetization when the applied field is increasing and added to it when it is decreasing; this leads to the typical magnetic hysteresis curve.

The value of M_{irr} must be equal to zero when the applied field reaches its minimum and maximum values:

$$\begin{aligned} M_{irr}(T, \mu_0 H = \mu_0 H_{min}, \mu_0 H_{min}, \mu_0 H_{max}) &= 0 \\ M_{irr}(T, \mu_0 H = \mu_0 H_{max}, \mu_0 H_{min}, \mu_0 H_{max}) &= 0 \end{aligned} \quad (5)$$

The absolute value of the rate of change of the applied field is used in Eq. (3) to ensure that the entropy generation rate is unambiguously positive. Defining the entropy generation rate with an irreversible magnetization term permits the calculation of the entropy generation rate at any point along the M - H curve. Graphically, the irreversible magnetization can be thought of as a method of “fattening out” the M - H curve by adding or subtracting (depending on the direction of the rate of applied field change) magnetization from the anhysteretic magnetization function.

The time rate of change of specific entropy in Eq. (1) is separated into its magnetic field and temperature driven components, which yields:

$$\frac{ds}{dt} = \frac{c_{\mu_0 H}}{T} \left(\frac{dT}{dt} \right) + \left(\frac{\partial v M_{an}}{\partial T} \right)_{\mu_0 H} \left(\frac{d\mu_0 H}{dt} \right) = \frac{\dot{S}_{gen}}{m} \quad (6)$$

By substituting Eq. (3) into Eq. (6):

$$\frac{c_{\mu_0 H}}{T} \left(\frac{dT}{dt} \right) + \left(\frac{\partial v M_{an}}{\partial T} \right)_{\mu_0 H} \left(\frac{d\mu_0 H}{dt} \right) = \frac{v M_{irr}}{T} \left| \frac{d\mu_0 H}{dt} \right| \quad (7)$$

which represents the final relation between temperature, applied field, irreversible magnetization, anhysteretic magnetization, and time for an adiabatic process.

ENTROPY DATA DERIVATION

Using raw data for magnetization as a function of applied field for a hysteretic material, an anhysteretic entropy function can be derived using Maxell's relations. The anhysteretic magnetization function (M_{an}) is the hypothetical magnetization curve of a material in the absence of magnetic hysteresis. The anhysteretic magnetization is calculated by taking the average of the positive (M_{pos}) and negative (M_{neg}) magnetization curves for a given applied field and temperature:

$$M_{an}(\mu_0 H, T) = \frac{M_{pos}(\mu_0 H, T) + M_{neg}(\mu_0 H, T)}{2} \quad (8)$$

where M_{pos} is the magnetization when the change in field with time is positive and M_{neg} is the magnetization when the change in field with time is negative.

The irreversible magnetization (M_{irr}) is defined in order to recover the area swept by a given hysteresis curve. The irreversible magnetization can be calculated according to:

$$M_{irr}(\mu_0 H, T) = \frac{|M_{neg}(\mu_0 H, T) - M_{pos}(\mu_0 H, T)|}{2} \quad (9)$$

The absolute value in Eq. (9) ensures a positive value for the irreversible magnetization which is necessary since it is directly related to the entropy generation.

With the anhysteretic and irreversible magnetization calculated, the specific entropy of $\text{La}(\text{Fe}_{0.885}\text{Si}_{0.115})_{13}\text{H}_{1.21}$ can be mapped as a function of both applied field and temperature. The total differential of the specific entropy (ds) of the material can be divided into its temperature-driven and applied magnetic field-driven components:

$$ds = \left(\frac{\partial s}{\partial T} \right)_{\mu_0 H} dT + \left(\frac{\partial s}{\partial \mu_0 H_{int}} \right)_T d\mu_0 H_{int} \quad (10)$$

where $\mu_0 H_{int}$ is the internal field. The Maxwell relation that relates the entropy to the anhysteretic magnetization can be utilized:

$$\left(\frac{\partial s}{\partial \mu_0 H} \right)_T = \left(\frac{\partial v M_{an}}{\partial T} \right)_{\mu_0 H} \quad (11)$$

as well as the definition of specific heat:

$$\left(\frac{\partial s}{\partial T} \right)_{\mu_0 H} = \frac{c_{\mu_0 H}}{T} \quad (12)$$

where $c_{\mu_0 H}$ is the constant field specific heat. Substituting Eqs. (11) and (12) into Eq. (10) yields:

$$ds = \frac{c_{\mu_0 H}}{T} dT + \left(\frac{\partial v M_{an}}{\partial T} \right)_{\mu_0 H} d\mu_0 H_{int} \quad (13)$$

Eq. (13) can be integrated over the temperature and internal magnetic field of the data sample to obtain an entropy function.

NUMERICAL MODEL INCORPORATION

The regenerator energy balance equation in the spatially one-dimensional active magnetic refrigerator model developed by Engelbrecht (2008) is modified to include an irreversible entropy generation source that can be used to represent the effect of magnetic hysteresis on the thermodynamic cycle. The model solves a pair of coupled partial differential equations in time and space for a cyclic steady state condition. One equation describes the energy balance for the fluid flow through the matrix and the other describes the energy balance for magnetocaloric material and entrained fluid in the matrix. The regenerator geometry is specified by a hydraulic diameter (d_h), porosity (ε), and specific surface area (a_s). These variables are sufficiently general such that a number of geometries may be specified (i.e., packed bed and parallel plate). Additionally, the numerical model requires user-defined applied magnetic field and coolant mass flow rate profiles as a function of time and space. This allows for modeling of various bed configurations such as rotary or reciprocating beds. A schematic of the energy balance for a differential segment of the regenerator is shown in Figure 1:

Figure 1 . Differential segment of the regenerator with energy balance terms included

The differential control volume has differential length dx and cross-sectional area A_c . In Figure 1, Nu_f is the Nusselt number for heat transfer between the fluid and magnetic material which is assumed to be a function of Reynolds number (Re_f) and Prandtl number (Pr_f). The parameter k_f is the thermal conductivity of the fluid and d_h is the hydraulic diameter or length scale used to define the Nusselt number. The temperatures T_r and T_f are the local temperatures of the regenerator and fluid, respectively. The parameter A_c is the cross-sectional area, a_s is the specific surface area of the matrix, ε is the porosity, k_{eff} is the effective thermal conductivity of the magnetic material in its fluid matrix, and u_r is the specific internal energy of the magnetocaloric material.

An energy balance on the differential segment in Figure 1 is written as:

$$\frac{Nu}{d_h} k_f a_s A_c (T_f - T_r) + A_c (1 - \varepsilon) \mu_o H \frac{\partial M}{\partial t} + k_{eff} A_c \frac{\partial^2 T_r}{\partial x^2} = \rho_r A_c (1 - \varepsilon) \frac{\partial u_r}{\partial t} \quad (14)$$

Eq. (14) can be expressed in terms of entropy. An entropy balance on a differential piece of magnetocaloric material is:

$$dS = \frac{\delta Q}{T_r} + \delta S_{gen} \quad (15)$$

where dS is the change in the entropy of the material, δS_{gen} is the internal entropy generation related to hysteresis and δQ is the heat transfer to the material. An energy balance on the same differential piece of material provides:

$$dU = \delta Q - \delta W \quad (16)$$

where dU is the change in the internal energy of the material and δW is the magnetic work out of the material which is equal to the product of applied field and the change in magnetization. By combining the entropy and the energy balances, Eqs. (16) and (15), we obtain:

$$dU = T_r dS - T_r dS_{gen} + \mu_0 H d(V M) \quad (17)$$

Dividing through by the mass of the material leads to:

$$du = T_r ds - T_r \left(\frac{dS_{gen}}{m} \right) + \mu_0 H d(v M) \quad (18)$$

where dS_{gen}/m is the mass-specific entropy generation. Eq. (18) can be rearranged:

$$\frac{\partial u}{\partial t} - \frac{\mu_0 H}{\rho_r} \frac{\partial M}{\partial t} = T_r \frac{\partial s}{\partial t} - T_r \left(\frac{\dot{S}_{gen}}{m} \right) \quad (19)$$

Substituting Eq. (19) into Eq. (14) provides the overall energy balance equation for the regenerator expressed in terms of the rate of entropy change and the rate of entropy generation:

$$\frac{Nuk_f}{d_h} a_s A_s (T_f - T_r) + k_{eff} A_c \frac{\partial^2 T_r}{\partial x^2} = \rho_r A_c (1 - \varepsilon) T_r \left(\frac{\partial s_r}{\partial t} - \frac{\dot{S}_{gen}}{m} \right) \quad (20)$$

Eq. (20) can be further simplified by expressing the change in the regenerator specific entropy in terms of its magnetic field and temperature driven components:

$$\frac{Nuk_f}{d_h} a_s A_s (T_f - T_r) + k_{eff} A_c \frac{\partial^2 T_r}{\partial x^2} = \rho_r A_c (1 - \varepsilon) \left[c_{\mu_0 H} \frac{\partial T_r}{\partial t} + T_r \left(\frac{\partial s_r}{\partial \mu_0 H} \right) \frac{\partial \mu_0 H}{\partial t} - \frac{\dot{S}_{gen}}{m} \right] \quad (21)$$

The last term in Eq. (21) is related to the rate of internal entropy generation due to hysteresis. The mass-specific entropy generation rate is expressed in terms of an irreversible magnetization according to Eq. (3):

$$\frac{Nuk_f}{d_h} a_s A_s (T_f - T_r) + k_{eff} A_c \frac{\partial^2 T_r}{\partial x^2} = \rho_r A_c (1 - \varepsilon) \left[c_{\mu_0 H} \frac{\partial T_r}{\partial t} + T_r \frac{\partial s_r}{\partial \mu_0 H} \right] \frac{\partial \mu_0 H}{\partial t} - v_r M_{irr} \left| \frac{\partial \mu_0 H}{\partial t} \right| \quad (22)$$

Eq. (22) is the final form of the regenerator energy balance equation that includes hysteresis effects expressed in terms of the irreversible mass specific magnetization ($v M_{irr}$).

The regenerator is discretized into Nx spatial nodes where the axial location of each node is given by:

$$x_i = \left(i - \frac{1}{2} \right) \frac{L}{Nx} \quad i = 1..Nx \quad (23)$$

In Eq. (16) i is the axial subscript and L is the length of the regenerator bed. The temporal grid is similarly separated into Nt nodes where the cycle time is given by:

$$t_j = j \frac{\tau}{Nt} \quad j = 0..Nt \quad (24)$$

where j is the temporal subscript.

Following Eq. (22), the regenerator energy balance equation for each control volume can be discretized as follows:

$$\begin{aligned} & \frac{Nu_{f,i,j} k_{f,i,j}}{d_h} a_s A_s \frac{L}{Nx} (T_{r,i,j+1} - T_{f,i,j+1}) + A_c \frac{L}{Nx} (1-\varepsilon) \rho_r c_{\mu_0 H,i,j} \frac{(T_{r,i,j+1} - T_{r,i,j}) Nt}{\tau} \\ & + \frac{Nx k_{eff,i,j} A_c}{L} [T_{r,i,j+1} - T_{r,i-1,j+1}] + \frac{Nx k_{eff,i,j} A_c}{L} [T_{r,i,j+1} - T_{r,i+1,j+1}] = \\ & -A_c (1-\varepsilon) \rho_r T_{r,i,j} \left(\frac{\partial s_r}{\partial \mu_0 H} \right)_{i,j} \frac{\partial \mu_0 H}{\partial t} \left(x_i, \frac{t_{j+1} + t_j}{2} \right) + \\ & A_c (1-\varepsilon) \rho_r (v_r M_{irr})_{i,j} \left| \frac{\partial \mu_0 H}{\partial t} \left(x_{Nx}, \frac{t_{j+1} + t_j}{2} \right) \right| \quad i = 2..Nx-1 \end{aligned} \quad (25)$$

Since the ends of the bed are assumed to be insulated, the conduction terms vanish at the hot end ($i=1$) and the cold end ($i=Nx$). The regenerator energy balance at the hot end is:

$$\begin{aligned} & \frac{Nu_{f,l,j} k_{f,l,j}}{d_h} a_s A_s \frac{L}{Nx} (T_{r,l,j+1} - T_{f,l,j+1}) + A_c \frac{L}{Nx} (1-\varepsilon) \rho_r c_{\mu_0 H,l,j} \frac{(T_{r,l,j+1} - T_{r,l,j}) Nt}{\tau} \\ & + \frac{Nx k_{eff,l,j} A_c}{L} [T_{r,l,j+1} - T_{r,2,j+1}] = -A_c (1-\varepsilon) \rho_r T_{r,l,j} \left(\frac{\partial s_r}{\partial \mu_0 H} \right)_{l,j} \frac{\partial \mu_0 H}{\partial t} \left(x_l, \frac{t_{j+1} + t_j}{2} \right) \\ & + A_c (1-\varepsilon) \rho_r (v_r M_{irr})_{l,j} \left| \frac{\partial \mu_0 H}{\partial t} \left(x_{Nx}, \frac{t_{j+1} + t_j}{2} \right) \right| \end{aligned} \quad (26)$$

Similarly, by neglecting conduction at the cold end of the bed, the regenerator energy balance at the cold end is:

$$\begin{aligned}
& \frac{Nu_{f,Nx,j} k_{f,Nx,j}}{d_h} a_s A_s \frac{L}{Nx} (T_{r,Nx,j+1} - T_{f,Nx,j+1}) + A_c \frac{L}{Nx} (1-\varepsilon) \rho_r c_{\mu_0 H,1,j} \frac{(T_{r,Nx,j+1} - T_{r,Nx,j}) Nt}{\tau} \\
& + \frac{Nx k_{eff,Nx,j} A_c}{L} [T_{r,Nx,j+1} - T_{r,Nx-1,j+1}] = -A_c (1-\varepsilon) \rho_r T_{r,Nx,j} \left(\frac{\partial s_r}{\partial \mu_0 H} \right)_{Nx,j} \frac{\partial \mu_0 H}{\partial t} \left(x_{Nx}, \frac{t_{j+1} + t_j}{2} \right) \quad (27) \\
& + A_c (1-\varepsilon) \rho_r (v_r M_{irr})_{i,j} \left| \frac{\partial \mu_0 H}{\partial t} \left(x_{Nx}, \frac{t_{j+1} + t_j}{2} \right) \right|
\end{aligned}$$

where $(v_r M_{irr})$ is the mass specific irreversible magnetization due to magnetic hysteresis. The final terms in Eqs. (25) - (27) represent the effect of the hysteretic entropy generation.

REFRIGERATION CAPACITY STUDY

The model is initially used to compare the performance and capacity of an AMRR cycle with a FOMT and a SOMT material in a generic refrigeration application. Figure 2 shows the COP and refrigeration capacity $(\dot{Q}_{ref})$ as a function of the heat transfer fluid mass flow rate amplitude for a properly configured AMRR system with the SOMT material Gd-Er as the magnetic refrigerant. The SOMT material does not exhibit hysteresis and provides a relatively modest magnetocaloric effect over a broad range of temperatures around the Curie temperature. Shown are curves depicting beds that consist of a single material with uniform properties and a layered bed in which the Curie temperature varies with axial position.

Table 1 presents the inputs to the AMRR model used to generate Figure 2 and Figure 3. The aspect ratio is defined as the length of a single regenerator bed to its diameter. The dwell ratio is defined as the ratio of the time in which the mass flow rate is zero to the total cycle time.

Table 1. Inputs to AMRR model for parametric studies

Parameter	Value	Parameter	Value
cycle time	0.2 s	dwel ratio	1/2
max magnetic field	1.5 T	regenerator type	packed sphere
heat transfer fluid	water	particle diameter	0.0002 m
porosity	0.36	motor efficiency	0.9
heat rejection temperature	310 K	pump efficiency	0.7
load temperature	299 K	number of beds	6
aspect ratio	0.2	regenerator volume	6 L
mass density	7170 kg/m ³	thermal conductivity	9.5 W/m-K

In the case of a continuously layered bed, the Curie temperature (T_{Curie}) is assumed to vary linearly between the hot and cold reservoirs according to:

$$T_{Curie}(x) = T_H - (T_H - T_C) \frac{x}{L} \quad (28)$$

where x/L is the dimensionless regenerator position, and T_H and T_C are the hot and cold reservoir temperatures, respectively. The Curie temperature of the material within each nodal control volume is computed using Eq. (28). The total entropy function for the refrigerant is shifted along the temperature axis such that the local Curie temperature matches that specified by Eq. (28). In the case of a bed composed of a single material (i.e., a single layer bed), the Curie temperature is assumed to be a constant and is evaluated as the arithmetic mean of the hot and cold reservoir temperatures:

$$T_{Curie} = \frac{(T_H + T_C)}{2} \quad (29)$$

The number of nodes used to produce the results that follow is 60; therefore, in the case of a continuously layered bed, the number of layers is 60. The effect of number of nodes and layers on the results has been investigated by Brey (2012) where it is shown that performance is negligibly affected by increasing the number of nodes/layers beyond 60.

Figure 2. *COP* and refrigeration capacity as a function of heat transfer fluid mass flow rate amplitude for Gd-Er. Curves for single layered and continuously layered beds are shown.

As Figure 2 illustrates, there exist two mass flow rate amplitudes that can achieve a given refrigeration capacity up to some maximum possible refrigeration capacity for the bed. The larger mass flow rate amplitude value corresponds to a condition where the regenerator matrix is flooded with heat transfer fluid. This condition is undesirable since the value of the *COP* is reduced dramatically due to the excessive pump work requirement and other losses that tend to increase with flow rate, as shown by Engelbrecht (2008). For a given refrigeration capacity, the operating condition that corresponds to the smaller value of mass flow rate should be chosen, since this condition yields a higher *COP*. Figure 2 also shows the effect of layering the regenerator bed. The *COP* at any operating condition is increased by layering the bed; the increase in performance is particularly dramatic at the low mass flow rate conditions which correspond to the optimal operating conditions.

Figure 3 illustrates the refrigeration capacity curve for the FOMT hysteretic material $\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$. The FOMT material exhibits significant hysteresis but provides a large magnetocaloric effect over a narrow range of temperatures around the Curie temperature. Near the Curie temperature for $\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$, the entropy generation due to hysteresis is approximately 15% of the isothermal entropy change. Figure 3 illustrates the curves for cases with and without magnetic hysteresis modeled. The bed is continuously layered (i.e., each node is assigned a separate Curie temperature) and operates under the conditions listed in Table 1. The effect of hysteresis on cycle performance becomes less significant at large mass flow rate amplitudes because hysteretic losses become small relative to the pump work. Additionally, by comparison of Figures 2 and 3 it becomes clear that the potential advantage of FOMT materials over SOMT materials is an increased refrigeration capacity for a given operating condition and bed size. For example, the refrigeration capacity for a continuously layered bed with hysteretic

$\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$ is approximately five times greater than that of the same system operating with a continuously layered bed of Gd-Er at an operating point where the $\text{COP} = 4$. Also, Figure 3 shows that FOMT type materials may be particularly advantageous for applications requiring large refrigeration capacities.

Figure 3. COP and refrigeration capacity as a function of heat transfer fluid mass flow rate amplitude for $\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$. Curves for layered beds with and without hysteresis are shown.

VOLUME, LOAD TEMPERATURE AND, BED LAYERING STUDY

In this study, a cooling capacity of $\dot{Q}_{ref,d} = 5.00$ kW is chosen as a design criteria for a typical space cooling application. A gadolinium-erbium compound ($Gd_{1-x}Er_x$) with a tunable Curie temperature is used to model a SOMT type material and a lanthanum-iron-silicon hydride compound ($La(Fe_{1-x}Si_x)_{13}Hy$) with a tunable Curie temperature is used to model a FOMT type material. The beds are layered continuously such that the Curie temperature of each layer varies linearly between load and heat rejection temperatures. The external heat exchangers are not explicitly modeled in this study. Additional detail is provided in Brey (2012).

Model Inputs

Table 1 lists the inputs for the AMRR model for the space conditioning parametric study. These parameters are held constant, while the total regenerator volume (V), aspect ratio (AR), and mass flow rate amplitude ($\dot{m}_{amp}$) are varied.

During the first quarter of the cycle, the magnetic field is linearly ramped from zero to its maximum value of 1.5 Tesla; it then remains at its maximum value for the next quarter, decreases linearly from its maximum value to zero during the third quarter, and finally is zero for the final quarter of cycle time. The mass flow rate and magnetic field profiles are assumed to be only functions of time, and do not vary spatially across the regenerator bed.

Design Strategy

The AMRR model requires inputs that include the magnetocaloric refrigerant material properties, working fluid properties, regenerator geometry (both overall size as well as the characteristics of the matrix), magnetic field profile, and mass flow profile. The model begins with an initial temperature distribution and numerically iterates the partial differential equations forward through time until it reaches a temperature distribution that satisfies the convergence criteria for cyclic steady state regenerator operation. The main outputs of the model include the regenerator and fluid temperature distributions, from which important performance metrics such as the coefficient of performance (COP) and refrigeration capacity ($\dot{Q}_{ref}$) can be calculated for a given magnetocaloric refrigerant. Additional details regarding the model are provided by Brey (2012). The model is written in MATLAB and can be downloaded from <http://sel.me.wisc.edu/>.

In subsequent studies, the cooling capacity is specified for a specific system (i.e., overall volume of regenerator and aspect ratio) and the mass flow rate is selected in order to maximize the COP . This process is an iterative process in which mass flow rate is varied systematically in order to identify the lower of the two values that will provide the required cooling capacity, as discussed previously in the context of Figure 2. For more details regarding this process please refer to Brey (2012).

For a given regenerator volume and cooling capacity, there exists an optimal aspect ratio that balances axial conduction and pressure drop losses and therefore yields a maximum coefficient of performance. Thus, for each volume, the aspect ratio is varied until an optimal value of the COP is found using the golden section search optimization technique.

Modeling Results

Regenerator Volume

For a fixed load temperature of $T_C = 299$ K, the regenerator volume is varied to examine its effect on COP ; the optimal aspect ratio is selected for each volume. All other model parameters are held constant at the values listed in Table 1. Figure 4 shows the coefficient of performance (COP) as a function of regenerator volume; each point on the curve is associated with a unique, optimal aspect ratio and assumes a continuously layered bed. The three materials simulated are the SOMT material $Gd_{1-x}Er_x$, and the FOMT material $La(Fe_{1-x}Si_x)_{13}H_y$; the FOMT material is modeled with and without hysteresis in order to show clearly the impact of this phenomenon on the performance.

By inspection of Figure 4, it is clear that the hysteretic losses become greater with an increasing volume. Consequently, for a given refrigeration capacity and temperature span, there exists an optimal regenerator volume where the COP is maximized. For a 5 kW application with $T_C = 299$ K and $T_H = 310$ K, Figure 4 demonstrates that a magnetic cooler using a continuously layered bed of $La(Fe_{1-x}Si_x)_{13}H_y$ performs better at this optimal volume (~ 4.3 L) than it would with a continuously layered bed of $Gd_{1-x}Er_x$. However at larger regenerator volumes, above approximately 7 liters, the gadolinium-erbium refrigerant outperforms the FOMT $La(Fe_{1-x}Si_x)_{13}H_y$ refrigerant because of the increasing losses related to hysteresis.

Figure 4. COP as a function of regenerator volume at its optimal aspect ratio and a load temperature of $T_C = 299$ K for a layered bed with each material indicated.

Load Temperature

In this study, the load temperature (T_C) is varied while the volume is held constant at 8 liters and the cooling capacity is held constant at 5 kW in order to examine the effect of load temperature on the cycle COP . The aspect ratio is optimized at each value of load temperature in order to yield the highest COP . The bed is assumed to be continuously layered and the other model inputs, including the heat rejection temperature, are maintained at the values listed in Table 1. Therefore, reducing the load temperature corresponds to increasing the temperature range that must be spanned by the cycle. Figure 5 illustrates the COP as a function of the load temperature with the bed set to its optimal aspect ratio for each material.

If hysteresis is neglected for $\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$ then it is more efficient than the gadolinium-erbium compound at all values of load temperatures. However, when hysteresis is accounted for, $\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$ is only more efficient than $\text{Gd}_{1-x}\text{Er}_x$ at load temperatures that are below approximately 297.5 K, for this particular application and bed volume. For a load temperature of 294 K, the COP resulting from $\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$ is approximately 25% greater than that of $\text{Gd}_{1-x}\text{Er}_x$.

Figure 5. COP as a function of load temperature (T_C) at the optimal aspect ratio for a layered regenerator bed with volume of 8 liters.

Figure 6 shows the *COP* as a function of load temperature (T_C) at the optimal aspect ratio for a regenerator bed volume of 2 liters. Figure 7 and Figure 8 show the *COP* as a function of load temperature (T_C) at the optimal aspect ratio for a regenerator bed volume of 4 and 6 liters, respectively. Comparing the figures for regenerator volumes between 2 and 8 liters, the performance of a continuously layered bed with lanthanum-iron-silicon hydride with hysteresis modeled is less sensitive to changes in regenerator volumes than the performance of a continuously layered bed with gadolinium-erbium. As the volume is increased, the load temperature at which the *COP* of an AMR cycle with $\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$ and Gd-Er is identical decreases. Hence for smaller volumes (e.g., 2 L), $\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$ will outperform Gd-Er for load temperatures below at least 304 K with a heat rejection temperature (T_H) of 310 K for a 5 kW cooling application.

Figure 6. *COP* as a function of load temperature (T_C) at the optimal aspect ratio for a layered regenerator bed with a volume of 2 liters ($\text{La(Fe}_{1-x}\text{Si}_x\text{)}_{13}\text{H}_y$ magnetic hysteresis modeled).

Figure 7. *COP* as a function of load temperature (T_C) at the optimal aspect ratio for a layered regenerator bed with a volume of 4 liters ($\text{La(Fe}_{1-x}\text{Si}_x\text{)}_{13}\text{H}_y$ magnetic hysteresis modeled).

Figure 8. COP as a function of load temperature (T_C) at the optimal aspect ratio for a layered regenerator bed with a volume of 6 liters ($\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$ magnetic hysteresis is modeled).

Bed Layering

In the studies presented in the previous sections, the beds were assumed to be continuously layered. That is, the Curie temperature varies linearly between the load and heat rejection temperatures. In this study, the effect of the number of layers on cycle performance is evaluated at a specified refrigeration capacity of 5 kW, a regenerator volume of 8 liters, and a heat rejection temperature of 310 K for various load temperatures. Other pertinent model inputs are listed in Table 1. The equation used to model the spatial distribution of the Curie temperature is:

$$T_{Curie,i} = T_H - (T_H - T_C) \frac{\left[(i - 0.5) \frac{N_L}{N_x} \right] - 0.5}{N_L} \quad (30)$$

where i is the spatial node number, N_x is the number of spatial nodes, and N_L is the number of layers. The argument of the $\lceil \cdot \rceil$ operator indicates rounding up to the nearest integer.

Figure 9 shows the coefficient of performance as a function of the number of layers for various values of load temperature (again, the optimal aspect ratio is used for each condition) with a $\text{Gd}_{1-x}\text{Er}_x$ bed. For all load temperatures, the COP is negligibly affected by an increase in the number of layers above approximately 3 layers. Thus, for applications in this range of load temperatures, a bed with 3 layers is sufficient. It is clear that a relatively small number of layers can be used for a SOMT material. This is due to the fact that the magnetocaloric effect of an SOMT material is a relatively broad peak centered near the Curie temperature, as shown in Figure 10.

Figure 9. COP as a function of the number of layers in each regenerator bed for various load temperatures at their respective optimal aspect ratios with $\text{Gd}_{1-x}\text{Er}_x$ as the refrigerant.

Figure 11 shows the *COP* as a function of the number of layers for various load temperatures using $\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$ (again, the optimal aspect ratio is selected for each condition). For each temperature span, there exists a number of layers below which the cycle cannot produce the specified refrigeration capacity of 5 kW. As the temperature span increases, the minimum number of layers required to achieve the desired refrigeration capacity also increases; larger temperature spans require more layers to operate. Figure 12 shows the minimum number of layers required by an AMRR cycle running with regenerator beds layered with Gd-Er and $\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$ capable of achieving a refrigeration capacity of 5 kW at the optimal aspect ratio. Figure 12 shows that in general, a FOMT refrigerant requires many more bed layers than a SOMT refrigerant to operate properly at the same refrigeration capacity. The reason more layers are required is because the magnetocaloric effect for an FOMT material exhibits a large, narrow peak around the Curie temperature, as shown in Figure 10.

Figure 10. Isothermal entropy change as a function of normalized temperature for pure gadolinium and $\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$ under an applied field change of 0 to 1.5 Tesla.

Figure 11. COP as a function of the number of layers in each regenerator bed for various load temperatures at their respective optimal aspect ratios with $\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$ as the refrigerant.

Figure 12. Minimum number of layers required each regenerator bed to achieve a refrigeration capacity of 5 kW for AMRR cycle running with Gd-Er and $\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$.

Hysteresis Scaling

In this study, the load temperature, the heat rejection temperature, volume and other model inputs listed in Table 1 are held constant, while the hysteresis scaling factor (C_h) is varied from 0 to 6 (i.e., 0% to 600%). $\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$ is used as the refrigerant in this study and the bed is assumed to be continuously layered. A hysteresis scaling factor of 0 indicates no hysteresis in the model while a factor of 1 indicates the amount of hysteresis that $\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$ would normally exhibit under these conditions. Figure 13 shows the COP at the optimal aspect ratio as a function of percent hysteresis for $\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$ for various regenerator volumes. Percent hysteresis is defined as the hysteresis scaling factor multiplied by 100%.

Figure 13. COP at the optimal aspect ratio as a function of percent hysteresis for $\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$ for various regenerator volumes. 100% hysteresis indicates normal hysteresis losses for $\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$ under specified conditions.

As shown by Figure 13, increasing the hysteresis is detrimental to the *COP* of the cycle for all regenerator volumes. However, for smaller volumes, increasing the percent hysteresis has less effect on cycle performance than it does for larger volumes. This behavior is related to the fact that hysteresis is a volumetric loss, which therefore becomes more important at larger regenerator volumes. Thus, a well-designed regenerator with an FOMT refrigerant should be small enough to negate hysteretic losses but large enough to provide the required cooling power at an acceptable *COP*.

MAGNETOCALORIC PROPERTY MODULATION

Two parameters that are often used in the literature to characterize magnetocaloric materials are the adiabatic temperature change (ΔT_{ad}) and the isothermal entropy change (Δs_M). These values are expressed for a given change in magnetic field ($\Delta \mu_0 H$), generally from 0 Tesla to a specified value. The specific isothermal entropy change is determined from a material's partial derivative of entropy with respect to magnetic field at constant temperature according to:

$$\Delta s_M(T, \mu_0 H_1, \mu_0 H_2) = \int_{\mu_0 H_1}^{\mu_0 H_2} \left(\frac{\partial s_r}{\partial \mu_0 H} \right)_T d\mu_0 H \quad (31)$$

which, by use of the Maxwell relations, can be rewritten as:

$$\Delta s_M(T, \mu_0 H_1, \mu_0 H_2) = \int_{\mu_0 H_1}^{\mu_0 H_2} \left(\frac{\partial v_r M_{an}}{\partial T} \right)_{\mu_0 H} d\mu_0 H \quad (32)$$

where $\left(\frac{\partial v_r M_{an}}{\partial T} \right)_{\mu_0 H}$ is the partial derivative of the mass specific anhysteretic magnetization with respect to temperature at constant applied field. The parameters $\mu_0 H_1$ and $\mu_0 H_2$ are the initial and final applied magnetic field.

The adiabatic temperature change can be written as:

$$\Delta T_{ad}(T, \mu_0 H_1, \mu_0 H_2) = \int_{\mu_0 H_1}^{\mu_0 H_2} \frac{T}{c_{\mu_0 H}} \left(\frac{\partial v_r M_{an}}{\partial T} \right)_{\mu_0 H} d\mu_0 H \quad (33)$$

where $c_{\mu_0 H}$ is the magnetic material mass specific heat capacity at a constant magnetic field. By inspection of Eq. (32), the isothermal entropy change is proportional to the partial derivative of the mass specific anhysteretic magnetization with respect to temperature at constant applied field. From Eq. (33), the adiabatic temperature change is also proportional to the partial derivative of the mass specific anhysteretic magnetization with respect to temperature at constant applied field and is inversely proportional to the constant applied field specific heat capacity.

Using a set of property data or an equation of state for the magnetic material, the adiabatic temperature and isothermal entropy changes can be computed. These values can then be artificially modulated using scaling factors in order to determine the relative effect of ΔT_{ad} and Δs_M on AMRR system performance. The scaling factor C_M is chosen to modify the partial derivative of the mass specific anhysteretic magnetization with respect to temperature at constant applied field and the scaling factor C_c is chosen to modify the mass specific heat at constant applied field. The entropy derivative is therefore written as:

$$ds = C_M \frac{c_{\mu_0 H}}{C_c} dT + C_M \left(\frac{\partial v_r M_{an}}{\partial T} \right)_{\mu_0 H} d\mu_0 H \quad (34)$$

In a similar fashion, Eqs. (32) and (33) can be written to include the scaling factors:

$$\Delta s_M (T, \mu_0 H_1, \mu_0 H_2, C_M) = C_M \int_{\mu_0 H_1}^{\mu_0 H_2} \left(\frac{\partial v_r M_{an}}{\partial T} \right)_{\mu_0 H} d\mu_0 H \quad (35)$$

$$\Delta T_{ad} (T, \mu_0 H_1, \mu_0 H_2, C_c) = C_c \int_{\mu_0 H_1}^{\mu_0 H_2} \frac{T}{c_{\mu_0 H}} \left(\frac{\partial v_r M_{an}}{\partial T} \right)_{\mu_0 H} d\mu_0 H \quad (36)$$

Note that now the isothermal entropy change and adiabatic temperature change are functions of C_M and C_c , respectively.

Utilizing the scaling factors, a parametric study is conducted to examine the effect of the adiabatic temperature and isothermal entropy changes on AMRR cycle performance. The inputs for the AMRR model are listed in Table 1. The mass flow and magnetic field profiles are the same as described in the Refrigeration Capacity Study.

Figure 14 shows the COP at the optimal aspect ratio of a continuously layered bed with a Gd-Er compound as a function of adiabatic temperature and (negative) isothermal entropy change. The field change used to define these parameters and also energize the cycle is from 0 to 1.5 Tesla. The base properties of the SOMT Gd-Er compound were modified according to Eqs. (34), (35), and (36); the unmodified Gd-Er compound is indicated in Figure 15. The volume-specific refrigeration capacity ($\dot{Q}_{ref}/V$) is held constant at 1.25 kW/L. By examination of Figure 14, the COP of this system is not a strong function of adiabatic temperature change for values above $\Delta T_{ad} = 4$ K, below this value of ΔT_{ad} the COP begins to decrease rapidly.

Figure 14. COP at the optimal aspect ratio as a function of negative isothermal entropy change and adiabatic entropy change for Gd-Er layered bed with a volume specific refrigeration capacity of 1.25 kW/L.

Figure 15 shows the regenerator volume specific refrigeration capacity as a function of adiabatic temperature change and isothermal entropy change for a continuously layered regenerator bed of Gd-Er with a specified $COP = 4$. As the figure shows, the volume specific refrigeration capacity is not a strong function of adiabatic temperature change above $\Delta T_{ad} = 4$ K for negative isothermal entropy changes below $\Delta s_M = 20$ J/kg-K.

Figure 15. Volume Specific refrigeration capacity at the optimal aspect ratios a function of negative isothermal entropy change and adiabatic entropy change for Gd-Er layered bed with a specified COP of

4.

Figure 16 is the COP at the optimal aspect ratio of a continuously layered bed using $La(Fe_{1-x}Si_x)_{13}H_y$ as the property basis. Again, the base properties are modulated using Eqs. (34), (35), and (36) in order to vary the adiabatic temperature and (negative) isothermal entropy change. The unmodified Lanthanum-Iron-Silicon Hydride compound is indicated in Figure 17. The field change is from 0 to 1.5 Tesla and the hysteresis is set to zero for this study. Adiabatic temperature changes below 4 K are shown. In this range, the COP becomes a strong function of ΔT_{ad} and only weakly depends on ΔS_M .

Figure 16. *COP* at the optimal aspect ratio as a function of negative isothermal entropy change and adiabatic entropy change for $\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$ layered bed with a volume specific refrigeration capacity of 1.25 kW/L. Hysteresis is set to zero.

Figure 17 is the *COP* at the optimal aspect ratio of a layered bed with modified $\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$ as a function of adiabatic temperature and (negative) isothermal entropy change with hysteresis set to its nominal value (i.e., the value expected for this material) for a regenerator volume of 4 liters. The field change is from 0 to 1.5. Adiabatic temperature changes below 4 K are shown. By comparison to Figure 16, the *COP* is slightly less due to the effect of hysteresis. The reduction in performance due to hysteresis is relatively small a relatively small bed (4 liters) is used. Again, because hysteresis is a volumetric loss, increasing the bed size increases the hysteretic losses.

Figure 17. COP at the optimal aspect ratio as a function of negative isothermal entropy change and adiabatic entropy change for $\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$ layered bed with a volume specific refrigeration capacity of 1.25 kW/L. Hysteresis is set to its nominal value for a regenerator with a volume of 4 L.

For a continuously layered regenerator bed in an AMRR cycle for a field change from 0 to 1.5 Tesla, an ideal material would have an adiabatic temperature change above 4 Kelvin and large isothermal entropy change near room temperature, with minimal hysteresis. However, hysteretic losses can be mitigated by designing for a minimal regenerator volume for a particular application.

CONCLUSIONS

Room temperature magnetic refrigeration is a promising alternative to vapor compression refrigeration because it has a high theoretical efficiency, uses non-ozone depleting refrigerants, and exhibits relatively quiet operation. The most encouraging thermodynamic cycle for magnetic refrigeration is the active magnetic regenerative refrigeration (AMRR) cycle using a layered magnetocaloric refrigerant configuration. Pure gadolinium is considered the baseline magnetic refrigerant for use in magnetic coolers because its magnetocaloric properties near room temperature have been measured and modeled most precisely. Gadolinium exhibits a modest magnetocaloric effect and has been successfully used in prototype magnetic coolers. However, materials having a much larger magnetocaloric effect have been reported. Due to the nature of their phase transition near room temperature, these compounds exhibit a large magnetocaloric effect but also exhibit magnetic hysteresis. This paper presents a thermodynamic model of AMRR systems with magnetic hysteresis that can be used to determine the performance of AMRR systems using magnetic materials that exhibit hysteresis. The model allows the magnetic material to be layered with varying Curie temperature.

The effect of magnetic hysteresis of AMRR systems was quantified through parametric studies with the model. For a specified refrigeration capacity and load temperature, a bed layered with $\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$ (a hysteretic material) showed a higher *COP* at relatively small volumes when compared to a bed layered with Gd-Er (a non-hysteretic material). Furthermore, for a specified refrigeration capacity and regenerator volume, a bed layered with $\text{La}(\text{Fe}_{1-x}\text{Si}_x)_{13}\text{H}_y$ was shown to outperform a bed layered with Gd-Er at large temperature spans. As regenerator volume increases, hysteretic losses outweigh the capacity gains associated with adding more refrigerant.

Parametric studies also implied that the adiabatic temperature change may not have a significant effect on the performance of a layered bed for $\Delta T_{ad} > 4$ K.

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