Auction-based Resource Allocation for Cooperative Cognitive Radio Networks

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Abstract

Cooperative cognitive radio network (CCRN) is a promising paradigm to increase spectrum utilization and exploit spatial diversity. The allocation of two coupled resources, *i.e.* spectrum and secondary relay nodes, plays a fundamental role in the performance of CCRNs. However, previous studies either lack of incentives for both primary users (PUs) and relay nodes to participate in or consider spectrum auction and relay auction separately. In this paper, we consider a static cooperative cognitive radio network scenario with several PUs and multiple secondary user coteries, each of which consists of a set of secondary users who are interested in sharing the same secondary relay node. We model the problem of joint spectrum allocation and relay allocation as a hierarchical auction and propose two auction schemes, *i.e.* TERA and UERA. TERA is the first Truthful auction mechanism for Efficient Resource Allocation in CCRNs which satisfies critical economic properties. UERA is an approximate truthful scheme which adopts a uniform price for fairness in the same secondary user coteries. Furthermore, we theoretically prove TERA and UERA can achieve near-optimal revenue. Finally, extensive simulation results show that TERA and UERA are efficient and able to improve the utility of PUs and relay nodes significantly.

Keywords: Cooperative cognitive radio networks, auction, resource allocation, relay, spectrum allocation

1. Introduction

The proliferation of wireless devices and the rapid development of wireless communication applications have caused two critical challenges: one is the huge starvation for the valuable spectrum resources, and the other is the guarantee of Quality of Service (QoS) for wireless users. Recently, cooperative cognitive radio network (CCRN) [1] which leverages both the technology of cognitive radio and cooperative communication is shown to be a promising paradigm to solve those two challenges. By utilizing the cognitive radio technology [2], CCRN allows unlicensed users (secondary users, SUs) to dynamically access the licensed spectrum occupied by spectrum holders (primary users, PUs) to improve spectrum utilization, and with the technique of cooperative communication [3], wireless transmitters in CCRN can improve their transmission capacity dramatically by exploiting spatial diversity of relay nodes.

According to who serve as relay nodes and who are the benefited transmitters, researchers have proposed three kinds of CCRN paradigms. The first CCRN paradigm [44, 47] is that primary users may select some of secondary users to be the cooperative relays, and in return grant transmission opportunities to those SUs. Noticing that primary users are generally considered to be able to support their target throughput alone and thus the cooperation from secondary users may have less attraction for PUs, Zou *et al.* [45] present a new CCRN paradigm,

in which primary users will serve as relays for SUs to earn revenue. However, to ensure cooperation between PUs and SUs, complicated cross-layer designs [46] are usually required in those two paradigms. So, in this paper, we will consider the CCRN paradigm [37] that a secondary user chooses some other secondary users to relay its transmission data.

In our considered CCRN paradigm, secondary users who hunger for high transmission rate and high quality channels may contend for both secondary relays' access time and primary users' spectrum. Furthermore, a secondary relay node can cooperate with a secondary user only if they share a common spectrum. Therefore, the allocation of spectrum and secondary relay nodes are coupled and should be addressed jointly. Although numerous previous researches [1, 4, 5, 6] have been conducted on this topic, those works assume that PUs and secondary relay nodes are willing to share their resources without reward which is not always practical. In reality, however, PUs and secondary relays are usually selfish and reluctant to share their resources for free, thus it is necessary to provide to participate in. A natural solution for this problem is to utilize the auction-based approach [7] for resource allocation in CCRNs.

The challenges for designing a practical and effective auction in our considered CCRN are two-fold. Firstly, the character that spectrum allocation and relay allocation are coupled makes the auction design for resource allocation in our considered CCRN paradigm special but challenging. Even though a large volume of researches have considered either spectrum auction [8, 9, 10] or relay auction [11, 12, 13] in the literature, those works can not be applied here because they do not consider spectrum auction and relay auction jointly. To our best knowledge, Han *et al.*

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¹This is an extended and enhanced version of [36].

[44] is the first to investigate auction-based resource allocation in CCRNs. However, they considered the paradigm that primary users choose secondary users as relays which is different with this paper. Secondly, a relay node usually can be shared by multiple secondary users in various situations [14, 15, 16], e.g. cellular networks or WLAN. Thus, a relay node may sell its access time to multiple SUs to improve its utility. However, it is a great challenge to allocate the access time efficiently while providing sufficient incentives for relay nodes.

To deal with the above challenges, we attempt to design an appropriate auction framework for allocating spectrum and secondary relays between SUs fairly and effectively. To be specific, we first assume secondary users are independent so that they can choose their desired secondary relays freely according to their preferences, and we call those secondary users who are interested in the same secondary relay as a secondary user coterie (SUC). Then, we let each secondary relay node play as a middle agent who takes charge of collecting bids from its corresponding SUC. According to SUC's bids, secondary relay node will determine its bid for spectrum. Afterwards, all secondary relay nodes contend for spectrum from PUs and winning relays will allocate both its gained spectrum and its access time to SUC. In the above procedure, PUs can gain payment from secondary relay nodes, and secondary relays can also gain financial compensation due to the surplus between its collected fees from SUC and its payment to PUs. So, our design can provide ample incentive for both PUs and secondary relays to participate in.

Under the aforementioned guideline, without loss of generality, we consider a static cooperative cognitive radio networks with several PUs and multiple secondary user coterie, each of which consists of a set of secondary users who are interested in sharing the same secondary relay node. To depict the character that a relay node can be accessed by multiple SUs in the same secondary user coterie, we assume that each SU has a valuation on that relay node and a budget constraint on the maximum payment it willing to pay. Specifically, the valuation represents the benefit that the SU gets when it accesses the relay node alone, while the budget determines the total access time that a SU can get from relay node which will be illustrated later. We model this problem as a hierarchical auction: First, each relay node collects bids from the corresponding SUC and conducts a virtual auction to allocate its access time to the SUs in that SUC. Next, a double auction is conducted between relay nodes and PUs to determine the allocation of spectrum. Finally, the winning relay node will perform the auction result in the first step to allocate the spectrum and access time to the winning SUs.

Based on the hierarchical auction framework, we first propose TERA, a Truthful auction mechanism for Efficient Resource Allocation in CCRNs. In TERA, each secondary relay will randomly separate its corresponding SUC into two subsets and charge different unit price for those two subsets. Even though TERA satisfies truthfulness, we notice that it may be unfair to charge different unit prices in TERA inasmuch as winners in those two subsets just get the identical relay node and spectrum. Therefore, to ensure the fairness among SUs in the same SUC, we further propose UERA which will charge a u-

niform unit price for the SUs who get the identical relay node and spectrum. UERA is not a truthful scheme indeed, but we prove that it achieves another appealing property known as approximate truthful [17] which means that each participant has no more than a small additive incentive to reply non-truthfully. Furthermore, we prove that both TERA and UERA satisfy other critical economic properties, such as individual rationality, budget balance, supply limits and computational efficiency, which are defined in Section 3.4.

By theoretical analysis, we show that TERA can achieve near-optimal revenue with high probability and UERA can achieve near-optimal expected revenue. Moreover, Our simulations verify the performance of our proposed schemes. Especially, TERA can improve the utility of PUs and relay nodes significantly up to 125% and 151% compared with a random method respectively. UERA can also improve the utility of PUs and relay nodes significantly up to 108% and 134% compared with a random method while ensure fairness among SUs.

The remainder of this paper is organized as follows. In Section 2, a brief review of related work is given. We provide some preliminaries and give the problem formulation in Section 3. Next, we propose and analyze our TERA scheme in Section 4. UERA is presented in Section 5. Simulation results are given in Section 6. Finally, we conclude our work in Section 7.

2. Related Work

In this section, we briefly review related works for relay allocation and spectrum allocation in wireless networks.

2.1. Non-Auction Work

Relay allocation has been extensively studied in cooperative networks in the past decade. Extensive related studies concentrate on the performance of transmission capacity. The authors in [3] studied the relay node assignment problem in a network environment, where multiple source-destination pairs compete for the same pool of relay nodes in the network. The authors in [18] gave a relay assignment algorithm with interference mitigation for cooperative communication. However, in [3, 18], a relay node can be assigned to at most one source node which is impractical. In contrast, the authors in [14, 15] studied the problems that multiple source nodes share the same relay nodes, which are similar to our work. The other line of previous work aims at improving energy efficiency. For example, Huang *et al.* proposed a series of power allocation schemes in different scenarios, such as energy harvesting enabled relay networks [39] and relay-enhanced OFDM systems [40]. Nevertheless, those works cannot be applied in CCRNs because they did not consider the spectrum allocation.

Besides, numerous previous works have addressed the spectrum allocation problem in cognitive radio networks. Sun and Zhu [48] proposed an efficient distributed algorithm in multi-hop cognitive radio networks to maximize spectrum utilization. Huang *et al.* [42] gave a comprehensive survey on the energy-efficient resource allocation techniques in green-energy-powered cognitive radio networks. To maximize the network

capacity of cognitive radio networks, Yousefvand *et al.* [43] presented a novel interference constraint capacity-aware spectrum allocation model. However, the character that relay and spectrum allocation is coupled makes the resource allocation in CCRN unique and challenging.

The joint allocations of spectrum and relay in CCRNs have attracted lots of attention recently. The authors in [6] studied the resource allocation in Orthogonal Frequency Division Multiplexing (OFDM)-based CR networks with cooperative relays. In [4], the authors investigated the problem of energy-efficient in cooperative cognitive radio networks by taking power control, relay assignment and channel allocation into consideration. Afterwards, they considered the relay allocation and spectrum allocation jointly to max-min the transmission rate in [5]. Huang *et al.* [41] tried to minimize the overall power consumption while satisfying throughput requirements for a cooperative downlink multi-user system by considering spectrum and power allocation jointly. Some other energy-efficient schemes are also reviewed in [42]. In [1], the authors explored the utilization of multiple-input-multiple-output (MIMO) techniques to improve capacity. Even though those works can achieve considerable performance, they lack the incentives for PUs and relay nodes to participate in.

2.2. Auction-based Work

Auction approaches for resource allocation can provide incentives for agents in nature, and lots of auction-based works have been done on resource allocation in wireless networks [7]. We will give a brief review on related auction scheme.

Auction designs for relay allocation in cooperative communication have been extensively studied in the literature [11, 12, 13, 19, 20]. In [12], the authors were the first to design a truthful auction scheme for cooperative communications. In [13], the authors proposed SAIL, which is a Strategy-proof and Approximately efficient combinatorial auction for relay assignment in cooperative communication. In [11], the authors utilized auction to study the power allocation problem. The authors [20] devised a truthful repeated double auction that is able to bound the collusion probability of each entity. However, most of the existing auction mechanisms for relay allocation assume that one relay can only be allocated to one buyer while in our work a relay node can be shared by multiple source nodes. Even though the authors [19] considered the scenario that a relay shared by multiple nodes and proposed two auction mechanisms to coordinate the relay power allocation among users, they did not consider truthful which is critical to auction design.

Extensive works on spectrum auction has been done in the last decade. The reusability makes spectrum fundamentally different from conventional goods, thus a large portion of previous works concentrate on auction mechanisms which are spectrum reusable, such as [2, 8, 10, 21]. In addition to spectrum reusable, many works are drawing their attention to social efficiency, e.g. [9, 22, 23, 24]. Besides, there are also some other works bring new perspective in spectrum auction. For example, the authors [25, 26] novelly introduced group-buying into spectrum auction. In [27], Tang and Jain considered the hierarchical auction mechanisms. Nearly all those works considered

truthfulness as the primary property to guarantee. Even though truthfulness is an attractive property, it appears to be too strong an assumption [17], which will prevent other desirable properties, such as privacy and efficiency. Therefore, many other works [17, 28, 29, 30] also consider a slight relaxation known as approximate truthful [17] in mechanism design.

To our best knowledge, Han *et al.* [44] is the first to investigate auction-based resource allocation in CCRNs. However, they considered a different CCRN paradigm from our work. That is, they considered the paradigm that primary users choose secondary users as relays, but no existing works have discussed the CCRN paradigm that a secondary user employs some other secondary users as relays.

3. Preliminaries and Problem Formulation

3.1. Cooperative Communications

Cooperative communication(CC) is a two-slot transmission mode. In the first time slot, source node s transmits data to the destination d . This transmission can be overheard by relay r due to the broadcast nature of wireless communication. In the second time slot, node r forwards the data received in the first time slot to d . There are two categories of CC, *i.e.* Amplify-and-Forward(AF) and Decode-and-Forward(DF). We denote $c_R(s, r, d)$ the capacity under CC and $c_D(s, d)$ the capacity under direct transmission. For the expressions of $c_R(s, r, d)$ and $c_D(s, d)$, readers can refer to [3] for more details and we omit it for saving space.

3.2. System Architecture and Design Rationale

Similar to [37], we consider the system architecture of a cooperative cognitive radio network (CCRN) as illustrated in Fig. 1, where multiple individual secondary users (SUs) and primary users (PUs) are resided in this network. Each primary user has a licensed channel which is under-utilized. While, on the other hand, secondary users have a voracious appetite for channels. Furthermore, secondary users are mainly classified into two categories: secondary relays and secondary transmitters (Note that we do not describe secondary transmitter's corresponding secondary receiver in Fig. 1 for conciseness). In reality, secondary transmitters usually hunger for high transmission rate and high quality channels, so they will contend for both secondary relays and primary users' channels. This makes the allocation of secondary relays and channels coupled and challenging.

To tackle this challenge, we strive to handle this problem in a natural way. Specifically, we first assume secondary transmitters are independent so that they can choose their desired secondary relays freely according to their preferences. Note that the authors in [38] have shown that a single relay is sufficient to enhance a transmitter's transmit rate, so we only consider the situation that a secondary transmitter chooses only one secondary relay. Besides, for those secondary transmitters who only need channels, we can think that they choose to relay themselves. Once secondary transmitters' choices are done, we will call those secondary transmitters who are interested in sharing the same secondary relay as a secondary user

Figure 1: System Architecture

coterie (SUC). Then, according to their demands, all the secondary transmitters in the same SUC may be classified into two categories: 1) those who only want to acquire access time from secondary relay, 2) those who want to acquire both access time from secondary relay and channels from PUs. In real scenarios, secondary relay can first split its access time between those two categories based on its own benefit. Subsequently, for the former category, secondary relay can resort to numerous literatures [11, 12, 13, 19, 20]. While how to allocate resource in the second category is the primary concern of this paper.

Considering that secondary relays and PUs may be selfish and reluctant to share their access time and channels respectively, we thus resort to the classic and efficient auction method. Our primal rationale is let secondary relay play as a middle agent. Specifically, secondary relay first collects bids from its corresponding SUC. Then, it determines its bid for channels according to SUC's bids. Finally, all secondary relays contend for channels from PUs and winning relays will allocate both its gained channel and its access time to SUC. In the whole procedure, secondary transmitters will pay money to resource providers (*i.e.* secondary relays and PUs). Secondary relays can gain financial compensation due to the surplus between its collected fees from SUC and its payment to PUs. So, our work can provide adequate incentive for both secondary relays and PUs to participate in.

In next subsection, we will describe our auction model in details. Besides, we may use secondary transmitter and secondary user, secondary relay and relay interchangeably when there is no confusion in the following content.

3.3. Auction Model

Based on previous discussion, we consider a static cooperative cognitive radio network consisting of m primary users and n relay nodes. We use $\mathcal{H} = \{h_1, h_2, \dots, h_m\}$ to denote the set of PUs and $\mathcal{R} = \{r_1, r_2, \dots, r_n\}$ to denote the set of relay nodes. Similar to [10], we assume that each PU contributes one distinct channel and the channels are homogenous to buyers so that their requests are not channel-specific. Besides, each PU h_k has a minimum required payment $\bar{A}_k$ to sell its channel. On the other hand, each relay node r_i can be accessed by a set

$\mathcal{S}_i = \{s_i^1, s_i^2, \dots, s_i^{n_i}\}$ of secondary users. Each SU s_i^j has a valuation $\bar{v}_i^j$ for accessing relay r_i alone and a budget $\bar{b}_i^j$ which represents the financial constrain of s_i^j . Denote d_i^j as the destination of $s_i^j \in \mathcal{S}_i$, then $\bar{v}_i^j$ is calculated as $\bar{v}_i^j = \max\{\frac{c_R(s_i^j, r_i, d_i^j)}{c_D(s_i^j, d_i^j)} - 1, 0\}$, obviously, we have $\bar{v}_i^j > 0$ if $c_R(s_i^j, r_i, d_i^j) > c_D(s_i^j, d_i^j)$ and $\bar{v}_i^j = 0$ otherwise. We further assume that the total access time of r_i is one unit and each SU in $\mathcal{S}_i$ shares r_i in a Time Division Multiple Access (TDMA) manner. Note that our motivation of introducing a budget $\bar{b}_i^j$ on each SU s_i^j is two-fold: 1) The budget represents the willingness of s_i^j to use r_i , *i.e.* s_i^j may put on more budget if s_i^j hungers for larger bandwidth and vice versa. 2) The budget is used to determine the access time allocated to s_i^j if it wins the auction which will be illustrated in Section 4.

Figure 2: Auction Model

We design the budget constrained spectrum auction as a hierarchical auction. As showed in Fig. 2, our auction framework consists of two single-round sealed-bid auctions, which are called *tier 1 auction* and *tier 2 auction* respectively. The tier 2 auction is between each relay node r_i and the corresponding $\mathcal{S}_i$. The relay node r_i acts as seller while the SUs in $\mathcal{S}_i$ are buyers. In this auction, r_i sells its access time to $\mathcal{S}_i$. The tier 1 auction is between the PUs and the relay nodes. The PUs sell their idle spectrum to relay nodes.

The tier 2 auction is executed first between r_i and $\mathcal{S}_i$. Each SU s_i^j submits a bid $B_i^j = \{v_i^j, b_i^j\}$, which is his reported valuation and budget. Note that s_i^j may not report its true bid $\bar{B}_i^j = \{\bar{v}_i^j, \bar{b}_i^j\}$ if doing so can improve its utility. After collecting all the SUs' bids, r_i conducts the tier 2 auction to compute the access time allocated to each SU in $\mathcal{S}_i$ and the potential revenue collected from $\mathcal{S}_i$, denoted as $\bar{F}_i$. Next, The tier 1 auction is executed between the PUs and the relay nodes. In this procedure, each relay node r_i acts as a buyer and submits a bid F_i , while each PU h_k acts as a seller and submits an ask A_k . Relay r_i may not report its true bid $\bar{F}_i$ as well if doing so can improve its utility. Let $\mathbf{F} = \{F_1, F_2, \dots, F_n\}$ denote the set of bids reported by all relays. Similar, PU h_k may not report its true valuation $\bar{A}_k$ as well if doing so can improve its utility. Let $\mathbf{A} = \{A_1, A_2, \dots, A_m\}$ denote the set of asks reported by all PUs. Upon receiving the bids $\mathbf{F}$ and asks $\mathbf{A}$, an auctioneer conducts a double auction to decide how to allocate channels to relay

Table 1: Primary Notations

notation	annotation
h, r	PU, relay node
$\mathcal{H}, \mathcal{R}$	set of PUs, relays nodes
$\mathcal{H}_w, \mathcal{R}_w$	winning set of PUs and relays nodes
$\mathcal{S}_i$	the SU set may access relay r_i
m, n, n_i	the number of elements in $\mathcal{H}, \mathcal{R}$ and $\mathcal{S}_i$
$s_i^j, \mathcal{S}_i^w$	the j -th SU in $\mathcal{S}_i$, winning SU set of $\mathcal{S}_i$
$\mathcal{S}_i^1, \mathcal{S}_i^2$	two disjoint subset of $\mathcal{S}_i$ satisfies $\mathcal{S}_i^1 \cup \mathcal{S}_i^2 = \mathcal{S}_i$
$\bar{A}_k, \bar{v}_i^j, \bar{b}_i^j$	true ask of h_k , true valuation and budget of s_i^j
A_k, v_i^j, b_i^j	reported ask of h_k , valuation and budget of s_i^j
$\bar{F}_i, F_i$	revenue r_i collected from $\mathcal{S}_i$ and its reported value
U_{h_k}, U_{r_i}, u_i^j	utility of PU h_k , relay r_i and SU s_i^j
P_{h_k}, P_{r_i}, p_i^j	payment to PU h_k , prices for relay r_i and SU s_i^j
t_i^j	the access time allocated to s_i^j
$opt(\tilde{\mathcal{S}}_i)$	the optimal unit price which maximizes the revenue of selling at most $\frac{1}{2}$ access time in the SU set $\tilde{\mathcal{S}}_i$

nodes and the corresponding payments for winners.

Given the aforementioned settings, we denote $\mathcal{H}_w \subseteq \mathcal{H}$ as the set of winning PUs and $\mathcal{R}_w \subseteq \mathcal{R}$ as the set of winning relay nodes in the tier 1 auction. Let P_{r_i} be the price that winning relay r_i need to pay. Let P_{h_k} be the payment the auctioneer pays the winning PU h_k . Then the *utility* of PU h_k is defined as

$$U_{h_k} = \begin{cases} P_{h_k} - \bar{A}_k, & \text{if } h_k \in \mathcal{H}_w \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

Accordingly, the *utility* of relay node r_i is defined as

$$U_{r_i} = \begin{cases} \bar{F}_i - P_{r_i}, & \text{if } r_i \in \mathcal{R}_w \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

Furthermore, for each winning relay node r_i , let $\mathcal{S}_i^w \subseteq \mathcal{S}_i$ be the set of winning SUs in the tier 2 auction. Let t_i^j be the fraction of time allocated to SU s_i^j and p_i^j be the price that s_i^j needs to pay for r_i , then the *utility* of SU s_i^j is defined as

$$u_i^j = \begin{cases} t_i^j \cdot \bar{v}_i^j - p_i^j, & \text{if } s_i^j \in \mathcal{S}_i^w \text{ and } p_i^j \leq \bar{b}_i^j \\ -\infty, & \text{if } s_i^j \in \mathcal{S}_i^w \text{ and } p_i^j > \bar{b}_i^j \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

Note that the first branch of equation (3) represents the utility of s_i^j when it wins the auction and it can afford the auction. $t_i^j \cdot \bar{v}_i^j$ is the benefit that s_i^j access r_i for a fraction of time t_i^j and p_i^j is the price it needs to pay. The value $-\infty$ in the second branch means that the SU receiving nothing and paying nothing when the price goes over its budget.

3.4. Economic Properties

Many properties must be satisfied when designing an auction scheme. In the following, we introduce the critical properties we would like to achieve.

- *Truthful*: an auction is truthful if any agent's utility is maximized by announcing his true private valuation to the auction regardless of the strategies of other agents.

- *Individual Rationality*: an agent's utility from participating in the auction is non-negative. That is, U_{h_k} , U_{r_i} and u_i^j are non-negative in our model.
- *Budget Balance*: an auction is budget-balanced if the total payment from buyers are no less than the total amount of price paid to the sellers. This property is desired in the tier 1 auction to ensure the auctioneer will not receive a negative utility, *i.e.* $\sum_{r_i \in \mathcal{R}_w} P_{r_i} \leq \sum_{h_k \in \mathcal{H}_w} P_{h_k}$.
- *Supply Limits*: the auction never allocates more resources than are available. Specifically, it means $\sum_{s_i^j \in \mathcal{S}_i^w} t_i^j \leq 1$ for each relay node r_i in our work.
- *Computational Efficiency*: the run time of the auction is a polynomial function of the scale of the input.

The challenges are two-fold. First, the key difficulty lies in that the utility of a SU is $-\infty$ if the price exceeds its budget. As a consequence, the well-known mechanisms such as the VCG mechanism [31] are no longer truthful in our setting as pointed in [32]. Second, it is a challenge to allocate two different sources, *i.e.* the spectrum of PUs and the access time of relay nodes.

In the Section 4, we first present TERA which satisfies all the aforementioned properties. Notice that TERA may be unfair for SUs, we thus relax the truthfulness property slightly and propose UERA to ensure the fairness among SUs in Section 5.

4. Truthful Auction Scheme

In this section, we present TERA, a truthful spectrum auction mechanism for cooperative cognitive radio networks with budget constrained SUs. TERA consists of three procedures: In procedure 1, the tier 2 auction is conducted virtually by each relay node $r_i \in \mathcal{R}$ to calculate its potential revenue collected from SUs. Note that this auction will be put in practice only if r_i wins in the following procedure. In procedure 2, the tier 1 auction is performed to sell the PUs' spectrum to relay nodes. Finally, if a relay node wins in procedure 2, it will execute the auction results in procedure 1.

4.1. Procedure 1: Tier 2 Auction

In this procedure, every relay node $r_i \in \mathcal{R}$ conducts the tier 2 auction virtually to calculate the potential revenue $\bar{F}_i$ collected from SUs in $\mathcal{S}_i$. The details are as follows:

First, relay node r_i collects the bid vector $\{B_i^1, B_i^2, \dots, B_i^{n_i}\}$ from SUs in $\mathcal{S}_i$, where $B_i^j = \{v_i^j, b_i^j\}$ as previous mentioned. Then, inspired by [33, 34], we partition the SU set $\mathcal{S}_i$ randomly into two sets $\mathcal{S}_i^1$ and $\mathcal{S}_i^2$. After this partition, we calculate the optimal unit price for each subset. To make sure our auction satisfies the *supply limits* property, we sell at most half of the access time to each subset. More specifically, we define the optimal unit price $opt(\tilde{\mathcal{S}}_i)$ for a subset $\tilde{\mathcal{S}}_i \subseteq \mathcal{S}_i$ as the price which maximizes the revenue of selling at most $\frac{1}{2}$ time fraction in $\tilde{\mathcal{S}}_i$, *i.e.* if $\{v_i^j : s_i^j \in \tilde{\mathcal{S}}_i\}$ are sorted in decreasing order, and the optimal unit price is defined as

Algorithm 1 : The Tier 2 Auction for Relay r_i

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1: Partition  $\mathcal{S}_i$  randomly into two subsets  $\mathcal{S}_i^1$  and  $\mathcal{S}_i^2$ ;
2: Sort  $\mathcal{S}_i^1$  and  $\mathcal{S}_i^2$  by  $v_i^j$  in decreasing order;
3: Compute  $opt(\mathcal{S}_i^1)$  and  $opt(\mathcal{S}_i^2)$  based on equation (4);
4: The access time can be allocated to  $\mathcal{S}_i^1$  is  $t_1 = \frac{1}{2}$ ;
5: while  $t_1 \geq 0$  do
6:   take  $s_i^j \in \mathcal{S}_i^1$  that are not considered before randomly;
7:   if  $v_i^j \geq opt(\mathcal{S}_i^2)$  then
8:      $\mathcal{S}_i^w = \mathcal{S}_i^w \cup \{s_i^j\}$  and  $t_i^j = \min\{\frac{b_i^j}{opt(\mathcal{S}_i^2)}, t_1\}$ ;
9:      $t_1 = t_1 - t_i^j$ ;
10:  end if
11: end while
12: The access time can be allocated to  $\mathcal{S}_i^2$  is  $t_2 = \frac{1}{2}$ ;
13: while  $t_2 \geq 0$  do
14:   take  $s_i^j \in \mathcal{S}_i^2$  that are not considered before randomly;
15:   if  $v_i^j \geq opt(\mathcal{S}_i^1)$  then
16:      $\mathcal{S}_i^w = \mathcal{S}_i^w \cup \{s_i^j\}$  and  $t_i^j = \min\{\frac{b_i^j}{opt(\mathcal{S}_i^1)}, t_2\}$ ;
17:      $t_2 = t_2 - t_i^j$ ;
18:   end if
19: end while
20: Compute  $\bar{F}_i$  by equation (5)

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$$opt(\tilde{\mathcal{S}}_i) = v_i^{j^*} \quad (4)$$

where $j^* = \min\{j : \sum_{k=1}^j b_i^k \geq \frac{v_i^{j+1}}{2}\}$. Based on equation (4), we can get the optimal unit prices for subsets $\mathcal{S}_i^1$ and $\mathcal{S}_i^2$, denoted as $opt(\mathcal{S}_i^1)$ and $opt(\mathcal{S}_i^2)$ respectively. To make the auction *truthful* which will be proved later, we charge the unit price $opt(\mathcal{S}_i^2)$ for SUs in $\mathcal{S}_i^1$, and $opt(\mathcal{S}_i^1)$ for SUs in $\mathcal{S}_i^2$.

Finally, we need to allocate the access time of relay r_i to $\mathcal{S}_i^1$ and $\mathcal{S}_i^2$. Taking $\mathcal{S}_i^1$ for example, to allocate at most $\frac{1}{2}$ access time fraction to $\mathcal{S}_i^1$, we consider the SUs in it in a random order. If the valuation of SU $s_i^j \in \mathcal{S}_i^1$ satisfies $v_i^j \geq opt(\mathcal{S}_i^2)$, then SU s_i^j is a winner, *i.e.* $s_i^j \in \mathcal{S}_i^w$, and we allocate $t_i^j = b_i^j / opt(\mathcal{S}_i^2)$ access time to it, or all the remaining access time if less than $b_i^j / opt(\mathcal{S}_i^2)$ access time remains. Same method can be used to allocate access time to SUs in $\mathcal{S}_i^2$ using the unit price $opt(\mathcal{S}_i^1)$. Then, relay r_i can calculate the potential revenue collected from $\mathcal{S}_i^1$ as

$$\bar{F}_i = \sum_{s_i^j \in \mathcal{S}_i^w \cap \mathcal{S}_i^1} t_i^j \cdot opt(\mathcal{S}_i^2) + \sum_{s_i^j \in \mathcal{S}_i^w \cap \mathcal{S}_i^2} t_i^j \cdot opt(\mathcal{S}_i^1) \quad (5)$$

where the first item represents the revenue collected from subset $\mathcal{S}_i^1$, and the second item represents the revenue collected from $\mathcal{S}_i^2$. The whole procedure is showed in Algorithm 1.

4.2. Procedure 2: Tier 1 Auction

After the previous procedure, the tier 1 auction which is executed between the PUs and the relay nodes is reduced to the settings as those in McAfee's mechanism. Thus, the procedure 2 goes as follows:

Algorithm 2 : The Tier 1 Auction

```

1: Sort all relay nodes such that their bids are in non-increasing order, i.e.  $F_1 \geq F_2 \geq \dots \geq F_n$ ;
2: Sort all PUs such that their asks are in non-decreasing order, i.e.  $A_1 \leq A_2 \leq \dots \leq A_m$ ;
3:  $k = \arg \max_{l \leq (m,n)} F_l \geq A_l$ ;
4: Determine the winning relay nodes and PUs as  $\mathcal{R}_w = \{r_1, r_2, \dots, r_{k-1}\}$  and  $\mathcal{H}_w = \{h_1, h_2, \dots, h_{k-1}\}$ ;
5: for all  $r_i \in \mathcal{R}_w$ ,  $P_{r_i} = F_k$ ;
6: for all  $h_j \in \mathcal{H}_w$ ,  $P_{h_j} = A_k$ ;

```

First, TERA sorts the relays' bids in non-increasing order, while the PUs' asks in non-decreasing order, *i.e.*

$$F_1 \geq F_2 \geq \dots \geq F_n$$

$$A_1 \leq A_2 \leq \dots \leq A_m$$

Then, the auctioneer finds the largest k such that $F_k \geq A_k$ and claims that the first $k-1$ PUs and $k-1$ relays are winners. Finally, TERA charges each winning relay node r_i by the price $P_{r_i} = F_k$ and pays each winning PU h_j the payment $P_{h_j} = A_k$. The detailed algorithm is shown in Algorithm 2.

4.3. Procedure 3: Charges for SUs

As mentioned before, the auction in procedure 1 is conducted virtually by each relay node and the auction results of relay r_i will not be put in practice unless it wins in procedure 2. Thus, in this procedure, each winning relay announces the price each SU needs to pay and the access time allocated to each SU.

For each winning relay node $r_i \in \mathcal{R}_w$, the access time allocated to each winning SU $s_i^j \in \mathcal{S}_i^w$ is t_i^j which has been computed in procedure 1. In addition, if s_i^j falls into subset $\mathcal{S}_i^1$, its payment is $p_i^j = t_i^j \cdot opt(\mathcal{S}_i^2)$. Otherwise, its payment is $p_i^j = t_i^j \cdot opt(\mathcal{S}_i^1)$ if s_i^j is in subset $\mathcal{S}_i^2$.

4.4. Proof of Economic Properties for TERA

In this section, we only focus on proving the economic properties of the tier 2 auction because the tier 1 auction takes the McAfee's mechanism [35] which satisfies the economic properties mentioned in Section 3.4. In addition, The property of budget balance is also not proved for the tier 2 auction because it is self-evident.

Theorem 1. *The tier 2 auction is individual rationality.*

PROOF. We only need to prove the case that SU s_i^j wins in the tier 2 auction and the corresponding relay node r_i also wins in the tier 1 auction because in other cases the utility of s_i^j is always 0.

We assume that s_i^j is in subset $\mathcal{S}_i^1$, and the proof for s_i^j in $\mathcal{S}_i^2$ is analogous. From line 8 in Algorithm 1, we have $t_i^j = \min\{\frac{b_i^j}{opt(\mathcal{S}_i^2)}, t_1\} \leq \frac{\bar{b}_i^j}{opt(\mathcal{S}_i^2)}$. Thus, the price s_i^j pays to r_i is $p_i^j = t_i^j \cdot opt(\mathcal{S}_i^2) \leq \bar{b}_i^j$, and the utility of s_i^j is computed by the first branch of equation (3). From line 7 in Algorithm 1, we have $\bar{v}_i^j \geq opt(\mathcal{S}_i^2)$ and therefore $u_i^j = t_i^j \cdot \bar{v}_i^j - p_i^j = t_i^j \cdot (\bar{v}_i^j - opt(\mathcal{S}_i^2)) \geq 0$.

Theorem 2. *The tier 2 auction satisfies supply limits.*

PROOF. For each relay node r_i in our scheme, we divide S_i into two subsets and allocate at most half access time to each subset. Hence, this theorem stands naturally.

Theorem 3. *The tier 2 auction is computational efficiency.*

PROOF. For Algorithm 1, sorting takes $O(n_i \log n_i)$ time where n_i is the number of SUs in S_i . Besides, partition S_i and allocate access time to SUs need $O(n_i)$ time. Thus, the time complexity of Algorithm 1 is $O(n_i \log n_i)$.

Theorem 4. *The tier 2 auction is truthful.*

PROOF. To prove this theorem, we need to show that reporting the correct valuation and budget is a dominant strategy for every SU in S_i .

We assume that s_i^j is in subset S_i^1 , and the proof for s_i^j in S_i^2 is analogous. From the first branch of equation (3), we can see that s_i^j can improve its utility by manipulating its reported valuation v_i^j and b_i^j to decreasing $\text{opt}(S_i^2)$ or increasing t_i^j .

However, on the one hand, we can see that $\text{opt}(S_i^2)$ is determined independently of s_i^j 's valuation v_i^j and budget b_i^j in Algorithm 1. On the other hand, we can also prove that s_i^j can not improve its utility by manipulating t_i^j for two reasons: Firstly, when the access time is not sufficient to support all the SUs in S_i^w , s_i^j can not increasing t_i^j by misreporting because the time allocation is done in a random order, i.e. independent of v_i^j and b_i^j as well. Secondly, when the access time is sufficient, s_i^j may increase t_i^j by reporting a budget $b_i^j \geq \bar{b}_i^j$. Then its allocated time is $t_i^j = b_i^j / \text{opt}(S_i^2)$ and its payment is $p_i^j = t_i^j \cdot \text{opt}(S_i^2) = b_i^j > \bar{b}_i^j$ which leads to a utility of $-\infty$. Based on previous discussion, we have proved that s_i^j can not improve its utility by bidding untruthfully and this theorem holds.

4.5. Revenue Analysis of TERA

In this section, we will prove that TERA can achieve near-optimal revenue with high probability in Algorithm 1.

Without loss of generality, we assume $v_i^1 > v_i^2 > \dots > v_i^{n_i}$. We denote $F(S, p, t)$ as the revenue collected by allocating at most t access time to SUs set S at unit price p . Apparently,

$$F(S, p, t) = \min(tp, \sum_{j \in S, v_i^j \geq p} b_i^j) \quad (6)$$

which means that the revenue is the minimum of the total selling price (i.e. tp) and the total budgets of those SUs whose valuations are higher than the unit price p (i.e. $\sum_{j \in S, v_i^j \geq p} b_i^j$).

Besides, we denote $F(S, p) = \sum_{j \in S, v_i^j \geq p} b_i^j$ for convenience, then $F(S, p, t) = \min(tp, F(S, p))$.

Given the bids of all SUs in S_i , we denote $p^* = v_i^l$ as the optimal unit price at which $F(S_i, v_i^j, 1)$ is maximized, and denote $\text{OPT} = F(S_i, p^*, 1)$. Disregarding supply limits, we first prove that the revenues collected from S_i^1 and S_i^2 are nearly equal with high probability in the following lemma.

Lemma 5. *Let $\delta > 0$ and $b_{\max} = \max_j b_i^j$. The probability that*

$$|F(S_i^1, v_i^j) - F(S_i^2, v_i^j)| < \delta \text{OPT for all } v_i^j \geq p^*$$

is at least $1 - 2e^{-\delta^2/4\beta}$, where $\beta = b_{\max}/\text{OPT} \leq 1$.

PROOF. Let x_j be a variable representing whether SU s_i^j is in S_i^1 . $x_j = 1$ when $s_i^j \in S_i^1$ and $x_j = -1$ when $s_i^j \in S_i^2$. Then $|F(S_i^1, v_i^j) - F(S_i^2, v_i^j)| = |\theta_j|$ where $\theta_j = \sum_{k \leq j} x_k b_i^k$.

We define the following martingale:

$$\Theta_i = \begin{cases} \theta_i, & \text{if } i \leq \pi(\delta) \\ \theta_{\pi(\delta)}, & \text{otherwise.} \end{cases}$$

where $\pi(\delta) = \min\{i : |\theta_i| \geq \delta \text{OPT}\}$. Previously, we denote $p^* = v_i^l$, and now we have

$$\begin{aligned} & \Pr(|F(S_i^1, v_i^j) - F(S_i^2, v_i^j)| < \delta \text{OPT}, \forall j \leq \lambda) \\ &= \Pr(|\theta_j| < \delta \text{OPT}, \forall j \leq \lambda) \\ &= 1 - \Pr(|\theta_j| \geq \delta \text{OPT}, \forall j \leq \lambda) \\ &= 1 - \Pr(\pi(\delta) \leq \lambda) \\ &= 1 - \Pr(|\Theta_\lambda| \geq \delta \text{OPT}) \end{aligned}$$

By the Azuma-Hoeffding inequality, we have

$$\Pr(|\Theta_\lambda| \geq \delta \text{OPT}) \leq 2e^{\frac{-\delta^2 \text{OPT}^2}{2 \sum_{j \leq \lambda} b_i^j \cdot b_i^j}}$$

Since $\sum_{j \leq \lambda} b_i^j \cdot b_i^j \leq \sum_{j \leq \lambda} b_i^j \cdot b_{\max} = \beta \cdot \text{OPT} \cdot F(S_i, p^*)$, and according to Eq. (6), $F(S_i, p^*) \leq \text{OPT} + b_{\max} = \text{OPT}(1 + \beta) \leq 2\text{OPT}$. Thus, this lemma is proved.

Hereafter, when saying that an event happens with high probability, we means its probability is at least $1 - 2e^{-\delta^2/4\beta}$. From lemma 5, we can see that, with high probability, the revenue of each subset at the optimum price is almost the same. In the following lemma, we can prove that the revenue of one subset is close to either the revenue of the other subset or half of OPT with high probability.

Lemma 6. *With high probability, the inequality holds*

$$F(S_i^2, v_i^j) \geq \min\{F(S_i^1, v_i^j) - \delta \text{OPT}, \frac{1 - \delta}{2} \text{OPT}\}, \text{ for all } j.$$

PROOF. we analyze this lemma in two cases:

- CASE 1: ($v_i^j \geq p^*$) With Lemma 5, it follows that with high probability $F(S_i^1, v_i^j) - F(S_i^2, v_i^j) < \delta \text{OPT}$, i.e.

$$F(S_i^2, v_i^j) > F(S_i^1, v_i^j) - \delta \text{OPT}, \text{ for all } v_i^j \geq p^* \quad (7)$$

which proves this case.

- CASE 2: ($v_i^j < p^*$) Since $F(S_i^2, p^*) + F(S_i^1, p^*) = F(S_i, p^*) \geq \text{OPT}$. Combined with Eq. (7), we get with high probability

$$F(S_i^2, p^*) \geq \frac{1 - \delta}{2} \text{OPT} \quad (8)$$

By monotonicity, we have

$$F(S_i^2, v_i^j) \geq F(S_i^2, p^*), \text{ for all } v_i^j < p^*$$

which implies that $F(S_i^2, v_i^j) \geq \frac{1-\delta}{2}OPT$ when $v_i^j < p^*$.

Thus, we can obtain this lemma.

With the above discussion, we can give the final theorem.

Theorem 7. *Given $0 < \delta < 1$ and $\beta = b_{\max}/OPT$. The Algorithm 1 is truthful and can also achieve revenue at least $(1 - \delta)OPT$ with probability of $1 - O(e^{-G\delta^2/4\beta})$ for some constant G .*

PROOF. According to Eq. (5) and Eq. (6), the revenue collected by Algorithm 1 is

$$\bar{F}_i = F(S_i^1, \text{opt}(S_i^2), \frac{1}{2}) + F(S_i^2, \text{opt}(S_i^1), \frac{1}{2}) \quad (9)$$

By the definition, $\text{opt}(S_i^1)$ is the optimal unit price which maximizes the revenue of selling at most $\frac{1}{2}$ access time in subset S_i^1 . Thus, we can conclude that

$$F(S_i^1, \text{opt}(S_i^1), \frac{1}{2}) \geq F(S_i^1, p^*, \frac{1}{2}) = \min\{\frac{p^*}{2}, F(S_i^1, p^*)\} \quad (10)$$

Since $OPT = F(S_i, p^*, 1) = \min\{p^*, F(S_i, p^*)\}$, we can get $p^* \geq OPT$, implying that $\frac{p^*}{2} \geq \frac{1-\delta}{2}OPT$. Besides, take the same proof procedure as that of Eq. (8), we can obtain that with high probability $F(S_i^1, p^*) \geq \frac{1-\delta}{2}OPT$. Combined with Eq. (10), it follows that with high probability $F(S_i^1, \text{opt}(S_i^1), \frac{1}{2}) \geq \frac{1-\delta}{2}OPT$, which implies that

$$\frac{\text{opt}(S_i^1)}{2} \geq \frac{1-\delta}{2}OPT \quad (11)$$

and

$$F(S_i^1, \text{opt}(S_i^1)) \geq \frac{1-\delta}{2}OPT \quad (12)$$

By lemma 6, Eq. (12) implies with high probability

$$F(S_i^2, \text{opt}(S_i^1)) \geq \frac{1-3\delta}{2}OPT \quad (13)$$

To conclude, Eq. (11) and Eq. (13) can deduce that with high probability

$$F(S_i^2, \text{opt}(S_i^1), \frac{1}{2}) \geq \frac{1-3\delta}{2}OPT \quad (14)$$

Taking the same proof, we can get the same conclusion for $F(S_i^1, \text{opt}(S_i^2), \frac{1}{2})$. By substituting the results to Eq. (9), this theorem is obtained.

5. Uniform Price Auction Scheme

Notice that, the tier 2 auction in preceding scheme will charge different unit prices for winners in subset S_i^1 and S_i^2 . However, it is unfair inasmuch as they just get the identical relay node and spectrum. To ensure the fairness among SUs in the same SUC, we further propose UERA which will charge a uniform unit price for the SUs who get the identical relay node and spectrum. It is worth mentioning that truthful appears to be too strong an assumption [28], [17], which will prevent other desirable properties. Therefore, we consider a slight relaxation known as approximate truthful [17], a good substitute for the property of truthful.

Approximate truthful, or ϵ -truthful is proposed by Schummer [17], in which each participant has no more than an ϵ additive incentive to reply non-truthfully. Formally, the definition is as follows:

Definition 1. (ϵ -truthful) Let ϵ as a small constant and denote $\bar{s}_i$ as the strategy when agent i plays truthfully. An auction is ϵ -truthful if for any agent i , for any strategy $s_i \neq \bar{s}_i$ and any other agents' strategy profile s_{-i} , its expected utility has,

$$E[u_i(\bar{s}_i, s_{-i})] \geq E[u_i(s_i, s_{-i})] - \epsilon.$$

Based on this definition, we will present the details of UERA and prove that UERA ensures fairness among SUs in the same SUC while achieving $(\exp(2\epsilon) - 1) \cdot \max_{s_i^j \in S_i} E[u_i^j(\bar{v}_i^j, \bar{b}_i^j)]$ -truthful. Moreover, we will show that UERA can achieve a high expected revenue in algorithm 3.

5.1. The Tier 2 Auction of UERA

UERA only differs with TERA on the tier 2 auction which is showed in algorithm 3. Algorithm 3 consists of two steps: price determination and winner selection respectively.

In the price determination step, relay node r_i will choose a unit price p with probability which is correlated positively with the revenue of this unit price. More specifically, we denote $\mathbb{P}$ as the set of candidates for the final unit price. Based on the discussion in section 4.5, we can see that the revenue collected from S_i with the unit price $p_j \in \mathbb{P}$ is $F_i^j = F(S_i, p_j, 1) = \min(p_j, \sum_{j \in S_i, v_i^j \geq p_j} b_i^j)$. So, we will choose p_j as the final unit price with the probability

$$\Pr(p_j) = \frac{\exp(\epsilon * F_i^j / B)}{\sum_{p_j \in \mathbb{P}} \exp(\epsilon * F_i^j / B)} \quad (15)$$

where ϵ is a small constant and B is an upper bound for each SU s_i^j 's budget. Note that this is an additional assumption required in UERA, but it is very reasonable since every agent in reality generally has a financial constraint and the auctioneer can estimate the value of B from historical data in previous auctions.

Denote p_{end} as the final selected unit price in the price determination step, then relay node r_i chooses the winners in a random manner as follows: Consider the SUs in a random order, if SU s_i^j 's valuation is not smaller than p_{end} , then it is a winner, i.e. $s_i^j \in S_i^w$, and we allocate $t_i^j = b_i^j / p_{\text{end}}$ access time to it, or

all the remaining access time if less than b_i^j / p_{end} access time remains. Finally, r_i can calculate the potential revenue collected from $\mathcal{S}_i$ as $\bar{F}_i = \sum_{s_i^j \in \mathcal{S}_i^w} t_i^j \cdot p_{end}$.

Algorithm 3 : The Tier 2 Auction for Relay r_i

```

1: Step 1: Price Determination
2: Sort  $\mathcal{S}_i$  by  $v_i^j$  in decreasing order;
3: for all  $p_j \in \mathbb{P}$  do
4:    $F_i^j = F(\mathcal{S}_i, p_j, 1) = \min(p_j, \sum_{k \in \mathcal{S}_i, v_i^k \geq p_j} b_i^k)$ ;
5: end for
6: for all  $p_j \in \mathbb{P}$  do
7:    $\Pr(p_j) = \frac{\exp(\epsilon * F_i^j / B)}{\sum_{p_j \in \mathbb{P}} \exp(\epsilon * F_i^j / B)}$ ;
8: end for
9: Choose  $p_{end} \in \mathbb{P}$  with probability distribution  $\Pr(p_j)$ ;
10: Step 2: Winner Selection
11: set  $t = 1$ ;
12: while  $t \geq 0$  do
13:   take  $s_i^j \in \mathcal{S}_i$  that are not considered before randomly;
14:   if  $v_i^j \geq p_{end}$  then
15:      $\mathcal{S}_i^w = \mathcal{S}_i^w \cup \{s_i^j\}$  and  $t_i^j = \min\{\frac{b_i^j}{p_{end}}, t\}$ ;
16:      $t = t - t_i^j$ ;
17:   end if
18: end while
19: Compute the revenue  $\bar{F}_i = \sum_{s_i^j \in \mathcal{S}_i^w} t_i^j \cdot p_{end}$ 

```

5.2. Performance Analysis of UERA

In this section, we mainly prove that the tier 2 auction of UERA satisfies the economic properties as individual rationality, supply limits and computational efficiency. Even though algorithm 3 does not satisfy truthfulness, we will prove that UERA ensures fairness among SUs in the same SUC while achieving approximate truthful which is a good substitute of truthfulness.

Theorem 8. *Algorithm 3 satisfies supply limits, individual rationality and has a time complexity of $O(n_i \log n_i)$*

PROOF. The proof is similar to that of TERA's.

Firstly, in the winner selection step of algorithm 3, r_i sells at most one unit access time. Thus, algorithm 3 satisfies supply limits.

Secondly, if a SU s_i^j wins in algorithm 3, then from line 15 we can see that $p_i^j = t_i^j \cdot p_{end} = \min\{\frac{b_i^j}{p_{end}}, t\} \cdot p_{end} \leq \bar{b}_i^j$. So, its utility is $u_i^j = t_i^j \cdot \bar{v}_i^j - p_i^j = t_i^j \cdot (\bar{v}_i^j - p_{end})$. According to line 14, we have $\bar{v}_i^j \geq p_{end}$ and it follows that $u_i^j \geq 0$, i.e. algorithm 3 satisfies individual rationality.

Thirdly, for algorithm 3, sorting takes $O(n_i \log n_i)$ time and the other operations need $O(n_i + |\mathbb{P}|)$ time. Thus, the time complexity is $O(n_i \log n_i + |\mathbb{P}|)$.

Next, we will prove that algorithm 3 is a $(\exp(2\epsilon) - 1) \cdot \max_{s_i^j \in \mathcal{S}_i} E[u_i^j(\bar{v}_i^j, \bar{b}_i^j)]$ -truthful scheme in theorem 9.

Theorem 9. *Algorithm 3 is $(\exp(2\epsilon) - 1) \cdot \max_{s_i^j \in \mathcal{S}_i} E[u_i^j(\bar{v}_i^j, \bar{b}_i^j)]$ -truthful.*

PROOF. Let ϵ be a small constant. When a SU s_i^j bids $B_i^j = \{v_i^j, b_i^j\}$, denote $E[u_i^j(v_i^j, b_i^j)]$ as its expected utility. We now examine the following three cases one by one.

- **CASE 1:** ($b_i^j > \bar{b}_i^j$) If s_i^j wins in this case, since the winner selection step of algorithm 3 selects winners in a random order, and s_i^j may get $t_i^j = \frac{b_i^j}{p_{end}}$ access time. Thus, its payment is $p_i^j = t_i^j \cdot p_{end} = b_i^j > \bar{b}_i^j$. From Eq. (2), its utility will be $-\infty$ and it follows that $E[u_i^j(v_i^j, b_i^j)] = -\infty$. From theorem 8, we know that v_i^j has a non-negative expected utility when bids truthfully, i.e. $E[u_i^j(\bar{v}_i^j, \bar{b}_i^j)] \geq 0$. So, it is obvious that $E[u_i^j(\bar{v}_i^j, \bar{b}_i^j)] \geq E[u_i^j(v_i^j, b_i^j)]$.
- **CASE 2:** ($b_i^j \leq \bar{b}_i^j$ and $v_i^j \geq \bar{v}_i^j$) Compared with bidding truthfully, if s_i^j bids in this case, then F_i^j may increase for all $\bar{v}_i^j \leq p_j \leq v_i^j$ and F_i^j remains the same for other cases. Denote $\Pr(p_j)$ and $\Pr'(p_j)$ as the probability distribution when s_i^j bids truthful and un-truthful respectively. Then, we have:

$$\begin{cases} \Pr'(p_j) \leq \Pr(p_j), & p_j < \bar{v}_i^j \\ \Pr'(p_j) \geq \Pr(p_j), & \bar{v}_i^j \leq p_j \leq v_i^j \\ \Pr'(p_j) \leq \Pr(p_j), & p_j > v_i^j. \end{cases} \quad (16)$$

On the other hand, when $p_j < \bar{v}_i^j$ or $p_j > v_i^j$, s_i^j 's utilities remain the same. However, if $\bar{v}_i^j \leq p_j \leq v_i^j$, SU s_i^j will get a non-positive utility if it bids untruthful due to $u_i^j = t_i^j \cdot (\bar{v}_i^j - p_j) \leq 0$. Thus, we can conclude that s_i^j will not increase its expected utility by bids $b_i^j \leq \bar{b}_i^j$ and $v_i^j > \bar{v}_i^j$, i.e. $E[u_i^j(\bar{v}_i^j, \bar{b}_i^j)] \geq E[u_i^j(v_i^j, b_i^j)]$.

- **CASE 3:** ($b_i^j \leq \bar{b}_i^j$ and $v_i^j < \bar{v}_i^j$) This case is more complex and some theorems are proved in the appendix section for ease of organization. Theorem 13 in the appendix shows that

$$E[u_i^j(\bar{v}_i^j, \bar{b}_i^j)] \geq E[u_i^j(v_i^j, b_i^j)] - (\exp(2\epsilon) - 1) \cdot E[u_i^j(\bar{v}_i^j, \bar{b}_i^j)] \quad (17)$$

i.e. a SU s_i^j has no more incentive than $(\exp(2\epsilon) - 1) \cdot E[u_i^j(\bar{v}_i^j, \bar{b}_i^j)]$ to report un-truthfully. Therefore, no SU can improve its expected utility than $(\exp(2\epsilon) - 1) \cdot \max_{s_i^j \in \mathcal{S}_i} E[u_i^j(\bar{v}_i^j, \bar{b}_i^j)]$

Base on the above discussion, this theorem is proved.

At last, we would like to analysis the revenue r_i collected in Algorithm 3. As in previous discussion, let $OPT = \max_{p_j \in \mathbb{P}} F(\mathcal{S}_i, p_j, 1)$ be the optimal revenue r_i can collect. We can prove that Algorithm 3 also can achieve near-optimal expected revenue. We first proof that, with high probability, Algorithm 3 chooses price which can bring high revenue.

Lemma 10. Denote $\mathbb{P}_\gamma = \{p_j : F_i^j \geq OPT - \gamma B\}$ as the set of price whose revenues are bounded by $OPT - \gamma B$, and its complementary set is $\bar{\mathbb{P}}_\gamma = \{p_j : F_i^j < OPT - \gamma B\}$, then we have

$$\Pr(\mathbb{P}_{2\gamma}) \geq 1 - \exp(-\epsilon\gamma) \cdot \frac{|\bar{\mathbb{P}}_{2\gamma}|}{|\mathbb{P}_\gamma|} \quad (18)$$

where $|\mathbb{P}_\gamma|$ is the number of elements in $\mathbb{P}_\gamma$.

PROOF. Apparently, $\Pr(\mathbb{P}_{2\gamma}) + \Pr(\bar{\mathbb{P}}_{2\gamma}) = 1$, so

$$\Pr(\mathbb{P}_{2\gamma}) = 1 - \Pr(\bar{\mathbb{P}}_{2\gamma}) \quad (19a)$$

$$\geq 1 - \frac{\Pr(\bar{\mathbb{P}}_{2\gamma})}{\Pr(\mathbb{P}_\gamma)} \quad (\text{Due to } \Pr(\mathbb{P}_\gamma) \leq 1) \quad (19b)$$

$$= 1 - \frac{\sum_{p_j \in \bar{\mathbb{P}}_{2\gamma}} \exp(\epsilon * F_i^j / B)}{\sum_{p_j \in \mathbb{P}_\gamma} \exp(\epsilon * F_i^j / B)} \quad (19c)$$

$$\geq 1 - \frac{\sum_{p_j \in \bar{\mathbb{P}}_{2\gamma}} \exp(\epsilon * (OPT - 2\gamma B) / B)}{\sum_{p_j \in \mathbb{P}_\gamma} \exp(\epsilon * (OPT - \gamma B) / B)} \quad (19d)$$

$$= 1 - \exp(-\epsilon\gamma) \cdot \frac{|\bar{\mathbb{P}}_{2\gamma}|}{|\mathbb{P}_\gamma|} \quad (19e)$$

Based on lemma 10, we can deduce the following theorem which shows that Algorithm 3 can achieve high expected revenue.

Theorem 11. Algorithm 3 has expected revenue at least $OPT - \frac{3B}{\epsilon} \ln(e + \frac{\epsilon OPT |\mathbb{P}|}{B})$.

PROOF. From lemma 10, we can see that if $\gamma \geq \frac{1}{\epsilon} \ln(\frac{OPT |\bar{\mathbb{P}}_{2\gamma}|}{\gamma B |\mathbb{P}_\gamma|})$, then $\Pr(\mathbb{P}_{2\gamma}) \geq 1 - \frac{\gamma B}{OPT}$, which means that

$$E[F_i^j] \geq \Pr(\mathbb{P}_{2\gamma}) \cdot (OPT - 2\gamma B) \geq OPT - 3B\gamma \quad (20)$$

Thus, take $\gamma = \frac{1}{\epsilon} \ln(e + \frac{\epsilon OPT |\mathbb{P}|}{B})$, if $\frac{1}{\epsilon} \ln(e + \frac{\epsilon OPT |\mathbb{P}|}{B}) \geq \frac{1}{\epsilon} \ln(\frac{OPT |\bar{\mathbb{P}}_{2\gamma}|}{\gamma B |\mathbb{P}_\gamma|})$, this theorem will be obtained.

Obviously, $\gamma \geq 1/\epsilon$, which implies

$$\frac{1}{\epsilon} \ln(e + \frac{\epsilon OPT |\mathbb{P}|}{B}) > \frac{1}{\epsilon} \ln(\frac{OPT |\mathbb{P}|}{\gamma B}) \quad (21a)$$

$$\geq \frac{1}{\epsilon} \ln(\frac{OPT |\bar{\mathbb{P}}_{2\gamma}|}{\gamma B}) \quad (\text{Due to } |\mathbb{P}| \geq |\bar{\mathbb{P}}_{2\gamma}|) \quad (21b)$$

$$\geq \frac{1}{\epsilon} \ln(\frac{OPT |\bar{\mathbb{P}}_{2\gamma}|}{\gamma B |\mathbb{P}_\gamma|}) \quad (21c)$$

Therefore, this theorem holds.

6. Numerical Results

In this section, we conduct extensive simulations to evaluate the performance of TERA and UERA.

6.1. Simulation Setup

We consider a CCRN structured as Fig. 2 whose nodes are randomly deployed in a 1000×1000 square. The parameters for cooperative communications are followed from [3]. Specifically, we use AF mode for CC, and let the bandwidth be 22 MHz for all channels and the transmit power for each node is 1 W. We further assume that the path loss exponent is 4 and the variance of noise is 10^{-10} W.

By default, we set $n = 10$, $m = 10$, and varied n_i from 20 to 120 with an increment of 20 for all $i = 1, \dots, n$. For the tier 2 auction, $\bar{v}_i^j$ is calculated as described in Section 3.3, and we further assume that SU s_i^j 's budget $\bar{b}_i^j$ is randomly distributed over $(0, B]$ where $B = 0.1$ is an upper bound for all SUs' budget. Besides, for UERA, we set $\epsilon = 0.4$ by default and assume that each relay r_i chooses the final price from 50 possible price in Algorithm 3, i.e. $|\mathbb{P}| = 50$. In the tier 1 auction, A_k is randomly distributed in $(0, 1]$. All results are averaged on 100 random repetitions.

To our best knowledge, this is the first work considering the fundamental resource allocation problem in CCRNs and there are no existing auction schemes which can be compared with ours. Thus, in this paper, we just compare our work with the following two schemes:

- **Upper:** This scheme is similar to TERA and UERA, except that it calculates $OPT = F(S_i, p^*, 1)$ in the tier 2 auction for relay r_i . Note that Upper is not a truthful scheme, but it provides an upper bound for the performance of TERA and UERA.
- **RND:** It replaces the Algorithm 1 of TERA with a random method: r_i randomly selects a SU s_i^j and then considers all SUs in a random order. If a SU s_i^j 's valuation is larger than s_i^j 's, r_i will allocate access time to s_i^j . The truthful property of RND is trivial and we do not prove here for saving space.

6.2. Running Time

We exam the running time of TERA under different network size in Fig. 3(a). In all the simulations, the number of relay nodes is just set as the same value of PUs' number, i.e. $n = m$. From Fig. 3(a), we can see that the running time is no more than 150ms even when the network size becomes large. The running time of UERA has a similar trend as TERA's, and we compare the running time of TERA and UERA in Fig. 3(b). We varied the value of n_i from 20 to 120 with an increment of 20 and find that UERA costs slightly more time than TERA when n_i is small. However, as the increase of n_i , TERA and UERA nearly take the same running time. The reason for those relationship is because we set $|\mathbb{P}| = 50$ in our simulations and UERA may take more time on choosing the final price p_{end} when n_i is small. But as n_i becomes larger, the additional time on choosing the final price is negligible. In conclude, TERA and UERA satisfy the property of computational efficiency.

(a) TERA's Running time as a function of n_i and m

(b) Running time: TERA vs. UERA

Figure 3: Running time

Figure 4: Tradeoff between revenue and approximate truthful

6.3. Tradeoff between revenue and approximate truthful

In this section, we analyse the tradeoff between revenue and approximate truthful for UERA. Theoretically, it is more likely to distinguish those price which can generate high revenue from those generate low revenue by setting ϵ larger. While, on the other hand, we can see that UERA is more vulnerable to un-truthful bids because malicious bidder may increase its utility by about $\Gamma = (\exp(2\epsilon) - 1) \cdot \max_{s_i^j \in S_i} E[u_i^j(\bar{v}_i^j, \bar{b}_i^j)]$ according to theorem 9. Thus, it is necessary to make a good tradeoff between revenue and approximate truthful by setting ϵ elabo-

ately. To this end, we inspect the following variable carefully

$$\Psi = Ave \left(\frac{\bar{F}_i}{(\exp(2\epsilon) - 1) \cdot \max_{s_i^j \in S_i} E[u_i^j(\bar{v}_i^j, \bar{b}_i^j)]} \right) \quad (22)$$

which is the average ratio between relay's revenue and the maximum additional utility introduced by bids un-truthfully. Intuitively, UERA is more robust to un-truthful bids when Ψ is larger.

Fig. 4 shows how ϵ and n_i affect the variable Ψ . We vary ϵ from 0.1 to 1 with an increment of 0.1 and show that Ψ increases significantly as the increase of ϵ at the beginning and achieve the maximum value when ϵ is about 0.4. However, when ϵ is larger than 0.4, Ψ will decrease. This phenomenon is caused by the different increase rates of $\bar{F}_i$ and Ψ respect to ϵ . At first, $\bar{F}_i$ will increase greatly with the increasing of ϵ because UERA is more likely to distinguish those price which can generate high revenue from those generate low revenue by setting ϵ larger. However, when ϵ is enough large, $\bar{F}_i$ will increase slowly while Ψ still increases fast because Ψ is proportional to $\exp(2\epsilon)$. Besides, we can also see that there is a positive correlation between Ψ and n_i due to more SUs will generate more revenue.

From Fig. 4, we can conclude that the parameter ϵ impacts the performance of UERA significantly. In practical implementation of UERA, the auctioneer can try different value for ϵ after collecting bids from bidders and find the appropriate ϵ which maximises some metrics (e.g. Ψ in this paper).

6.4. Profit Comparison for TERA and UERA

We analyse the profits of all participants in this section.

Fig. 5(a) shows the capacity gain of SUs. This metric captures the welfare that SUs can get from relays and is calculated as $\sum_{i=1}^n \sum_{j=1}^{n_i} t_i^j * c_R(s_i^j, r_i, d_i^j)$. From Fig. 5(a), we can see that the capacity gains of RND, UERA, TERA and Upper increase as the number of SUs n_i increases at first. The primary reason is that the access time is sufficient and the competition among SUs is not serious when n_i is small. So more SUs lead to higher capacity. But when the number of SUs exceeds a threshold (in Fig. 5(a) is 80), the capacity gains of TERA, UERA and Upper do not increase anymore. This is because the relays have reached their load capacity and they can not improve the performance of the network any longer. The capacity gains of SUs' in UERA is overall similar to that of TERA's, but UERA fluctuates stronger because it is more random an algorithm than TERA. Surprisingly enough, the capacity gain of RND may decrease as n_i increases which is because RND is more likely to select inefficient SUs as n_i increases. In conclusion, TERA and UERA outperforms RND significantly by about 53% to 130%. Additionally, to ensure the property of truthful (or approximate truthful), the degradation of TERA (or UERA) is only 11% compare with Upper which prove that TERA (or UERA) can greatly improve the capacity of the network while guaranteeing truthful (or approximate truthful).

Fig. 5(b) shows the average utility of relay nodes, which captures the expected profit of relays. Similar to Fig. 5(a), the average utility of relay nodes increase as n_i increases. However, the average utility of relay nodes still increases when

(a) The capacity gain of SUs

(b) The average utility of relays

(c) The average utility of PUs

Figure 5: Performance evaluation of TERA and UERA. Upper is an un-truthful auction scheme which provides an upper bound. RND is a truthful scheme which chooses winner randomly.

$n_i \geq 80$ in TERA, UERA and Upper, which is different from Fig. 5(a). Note that a SU s_i^j 's valuation is defined as $\bar{v}_i^j = \max\{\frac{c_R(s_i^j, r_i, d_i^j)}{c_D(s_i^j, d_i^j)} - 1, 0\}$ in this paper. Thus, even though the capacity gains of TERA, UERA and Upper do not increase anymore when $n_i \geq 80$, relays can select those SUs who have larger valuation but smaller capacity under direct transmission. In that

way, relays can improve their utilities as well. Besides, the average utility of relays in RND has the same trend as in Fig. 5(a) for the same reason. As a conclusion, the average utility of relays in TERA outperforms RND by 60% to 151% while the degradation is only about 10.6% compared to Upper. Moreover, the average utility of relays in UERA is about 20% smaller than TERA which is the expense paid for guaranteeing fairness among SUs (as showed in the dashed circle in Fig. 5(b)). Even though UERA generates smaller utility than TERA, it still outperforms RND by 37% to 134%. Therefore, UERA is also a profitable auction scheme which can ensure fairness as well.

Fig. 5(c) shows the average utility of PUs, which captures the expected profit of PUs. Fig. 5(c) has the similar tendency as Fig. 5(b), which shows that both PUs and relays can get more profits as n_i increases in TERA and UERA. To ensure truthful, the degradation of TERA is only about 9.5% compared to Upper. To ensure fairness, the degradation of UERA is only about 18% compared to TERA. On the other hand, the average utility of PUs in TERA outperforms RND by 71% to 125% and UERA outperforms RND by 50% to 108%. Fig. 5(b) and Fig. 5(c) verify that TERA can motivate the participation of relay nodes and PUs significantly, while UERA both can ensure fairness among SUs and achieves rather good profit for relay nodes and PUs.

7. Conclusion

In this paper, we investigate the auction mechanism design which considers the efficient allocation of spectrum and the access time of relay nodes jointly in cooperative cognitive radio networks. The framework of our mechanism is as follows: First, each relay node collects bids from the corresponding SUC and conducts a virtual auction to allocate its access time to the SUs in that SUC. Next, a double auction is conducted between relay nodes and PUs to determine the allocation of spectrum. Finally, the winning relay node will perform the auction result in the first step to allocate the spectrum and access time to the winning SUs. Based on this framework, we propose two schemes, *i.e.* TERA and UERA. TERA is a truthful scheme, while UERA is an approximate truthful scheme which ensures fairness among SUs. Both TERA and UERA satisfy other critical economic properties such as individual rationality, budget balance, supply limits and computational efficiency. We also prove that TERA can achieve near-optimal revenue with high probability and UERA can achieve near-optimal expected revenue. Finally, simulation results show the efficiency of TERA and UERA which demonstrate that TERA and UERA can provide significant incentive for both the PUs and relay nodes to participate in.

Appendix A. Proof of the third case of theorem 9

We first give a lemma which shows that SU s_i^j 's changes on its bids have only a little influence on the final probability distribution.

Lemma 12. Let $B_i^j = \{v_i^j, b_i^j\}$ as any bid of SU s_i^j and $\bar{B}_i^j = \{\bar{v}_i^j, \bar{b}_i^j\}$ as its true bid. When s_i^j bids with B_i^j and $\bar{B}_i^j$ respectively while other SUs' bids keep the same, the final probability distributions $\Pr(p_k)$ and $\bar{\Pr}(p_k)$ satisfy

$$\Pr(p_k) \leq \exp(2\epsilon) \cdot \bar{\Pr}(p_k) \quad (\text{A.1})$$

PROOF. Denote F_i^k and $\bar{F}_i^k$ as the revenues when s_i^j bids with B_i^j and $\bar{B}_i^j$ respectively. Without loss of generality, assume that $F_i^k = \bar{F}_i^k + f_k$ for all $p_k \in \mathbb{P}$, then

$$\frac{\Pr(p_k)}{\bar{\Pr}(p_k)} = \frac{\exp(\epsilon * F_i^k/B)}{\exp(\epsilon * \bar{F}_i^k/B)} \cdot \frac{\sum_{p_k \in \mathbb{P}} \exp(\epsilon * \bar{F}_i^k/B)}{\sum_{p_k \in \mathbb{P}} \exp(\epsilon * F_i^k/B)} \quad (\text{A.2a})$$

$$\leq \exp(\epsilon * f_k/B) \cdot \frac{\sum_{p_k \in \mathbb{P}} \exp(\epsilon * (F_i^k + |f_k|)/B)}{\sum_{p_k \in \mathbb{P}} \exp(\epsilon * F_i^k/B)} \quad (\text{A.2b})$$

$$\leq \exp(\epsilon * f_k/B) \cdot \exp(\epsilon * \max_{\tau} |f_{\tau}|/B) \quad (\text{A.2c})$$

$$\leq \exp(2\epsilon * \max_{\tau} |f_{\tau}|/B) \quad (\text{A.2d})$$

Furthermore, for all $p_k \in \mathbb{P}$

$$F_i^k = \min(p_k, \sum_{v_i^j \geq p_k} b_i^j) \quad (\text{A.3a})$$

$$= \min(p_k, \sum_{\tau \neq j, v_i^{\tau} \geq p_k} b_i^{\tau} + \sum_{v_i^j \geq p_k} b_i^j) \quad (\text{A.3b})$$

$$\leq \min(p_k, \sum_{\tau \neq j, v_i^{\tau} \geq p_k} b_i^{\tau} + \sum_{\bar{v}_i^j \geq p_k} \bar{b}_i^j + |b_i^j - \bar{b}_i^j|) \quad (\text{A.3c})$$

$$\leq \min(p_k, \sum_{\tau \neq j, v_i^{\tau} \geq p_k} b_i^{\tau} + \sum_{\bar{v}_i^j \geq p_k} \bar{b}_i^j) + |b_i^j - \bar{b}_i^j| \quad (\text{A.3d})$$

$$= \bar{F}_i^k + |b_i^j - \bar{b}_i^j| \quad (\text{A.3e})$$

which means that $|f_k| \leq |b_i^j - \bar{b}_i^j| \leq B$. Therefore, we have

$$\frac{\Pr(p_k)}{\bar{\Pr}(p_k)} \leq \exp(2\epsilon * \max_{\tau} |f_{\tau}|/B) \leq \exp(2\epsilon) \quad (\text{A.4})$$

and this lemma is obtained.

Now, we can also show that SU s_i^j 's changes on its bids do not impact his expected utility too much.

Theorem 13. When s_i^j 's bid $B_i^j = \{v_i^j, b_i^j\}$ satisfies that $b_i^j \leq \bar{b}_i^j$ and $v_i^j < \bar{v}_i^j$, its expected utility satisfies

$$E[u_i^j(v_i^j, b_i^j)] \leq \exp(2\epsilon) \cdot E[u_i^j(\bar{v}_i^j, \bar{b}_i^j)] \quad (\text{A.5})$$

where $\bar{B}_i^j = \{\bar{v}_i^j, \bar{b}_i^j\}$ is its true bid.

PROOF. Let $t_{i,j}^k$ be the expected time allocated to s_i^j when with

unit price $p_k \in \mathbb{P}$, we have

$$E[u_i^j(\bar{v}_i^j, \bar{b}_i^j)] \quad (\text{A.6a})$$

$$= \bar{\Pr}(p_k) * \sum_{p_k \in \mathbb{P}, p_k \leq \bar{v}_i^j} t_{i,j}^k \cdot (\bar{v}_i^j - p_k) \quad (\text{A.6b})$$

$$\geq \exp(-2\epsilon) \Pr(p_k) * \sum_{p_k \leq \bar{v}_i^j} t_{i,j}^k \cdot (\bar{v}_i^j - p_k) \quad (\text{A.6c})$$

$$\geq \exp(-2\epsilon) \Pr(p_k) * \sum_{p_k \leq v_i^j} t_{i,j}^k \cdot (\bar{v}_i^j - p_k) \quad (\text{A.6d})$$

$$= \exp(-2\epsilon) * E[u_i^j(v_i^j, b_i^j)] \quad (\text{A.6e})$$

Note that, the inequality (A.6c) is deduced from lemma 12. The inequality (A.6d) is because when $p_k \leq v_i^j$, s_i^j can get the same expected access time, however s_i^j will win more benefit, i.e. $\sum_{v_i^j \leq p_k \leq \bar{v}_i^j} t_{i,j}^k \cdot (\bar{v}_i^j - p_k)$, when reports his true valuation.

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