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5 **ESTIMATING PERMEABILITY FROM THIN SECTIONS WITHOUT**
6 **RECONSTRUCTION: DIGITAL ROCK STUDY OF 3D PROPERTIES FROM 2D**
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14 Right Running Head: Permeability from thin sections
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ABSTRACT

We present a new approach for predicting permeability of natural rocks using thin sections. Our approach involves two steps: (1) computing permeability of the thin sections for flow normal to the face, and (2) application of new robust 2D-3D transforms that relate thin section permeability to 3D rock permeability using calibration parameters. We perform step 1 using Lattice-Boltzmann and finite difference schemes, which are memory efficient. We discuss two models to perform step 2. Our two-step approach is fast and efficient, since it does not require reconstruction of the unknown 3D rock using 2D thin section information. We establish the applicability of this new approach using a dataset comprised of LBM-computed permeability of rock samples from various geologic formations, including Fontainebleau sandstone, Berea sandstone, Bituminous sand, and Grosmont carbonate. We find that for sandstones our approach predicts fairly accurate permeability with little calibration. Predicting permeability of carbonates from thin sections is more challenging due to microstructural complexity thus model parameters require more calibration. For general workflow, we propose to first calibrate the proposed models using the available 3D information on the rock microstructure (from microCT, SEM, etc.) and then predict the permeability for rocks from the same geological formation for which only 2D thin sections are available.

Keywords: digital rock, image segmentation, permeability, petrology, sandstone, carbonate, thin sections

1. INTRODUCTION

Digital Rock Physics (DRP) offers significant potential for reducing turnaround times for laboratory-based rock analysis and real time rock property prediction near the drill sites. Most commonly, DRP computations are carried out on a three dimensional (3D) microstructure obtained using modern x-ray tomographic imaging techniques (Arns et al. 2001, 2002, 2004, 2005; Keehm, 2003; Knackstedt et al. 2009; Dvorkin et al. 2011; Madonna et al., 2012; Andrä et al. 2013a,b). However, acquiring 3D microstructures can be both expensive and time consuming; to date, having 3D micro-tomographic images for field studies has been more the exception, than the rule. In contrast, two dimensional rock thin sections are relatively widely and routinely available. Figure 1 shows 2D thin sections for four sandstones along with the corresponding binary images that were segmented using the Otsu algorithm (Otsu, 1979). It is of considerable interest to address whether it is possible to use thin sections to gain information on the corresponding properties of the 3D media. If successful, quantitative rock property information can be obtained from conventional thin sections or images of cuttings taken at the well site with a minimum of preparation.

Several authors have previously addressed estimating single-phase permeability of rocks upon statistically reconstructing a 3D rock from a 2D image (e.g., Adler et al. 1990; Yeong and Torquato, 1998; Hilfer and Manwart, 2001; Øren and Bakke, 2002; Keehm et al., 2004). Reconstructing 3D images presents several challenges, including inadequate reproduction of pore connectivity, unrealistic reconstructed pore shapes, and non-uniqueness of the solution, which depends on the choice of the reconstruction method.

Berryman and Blair (1986) presented a method for estimating permeability from 2D SEM images. In their approach, porosity and specific surface area were estimated from the image-

derived two-point spatial correction function. Combining these with known values of electrical formation factor and a form of the Kozeny-Carman relation gave reasonable agreement with measured permeabilities.

Recently, Saxena and Mavko (2015) proposed a simple approach for estimating elastic properties or sonic velocities of rocks using 2D thin sections. This method is based on computing elastic properties directly on the 2D microstructure, and then applying a power-law transform to estimate 3D rock moduli. This approach does not require reconstruction of the 3D porous media from 2D images. An additional advantage is that, for the same available computer memory, the 3D rock stiffness can be estimated from a larger 2D field of view, compared to the limited field of view of a 3D image.

In this paper, we use a similar approach to investigate the link between permeability of natural rocks in 3D and the permeability computed directly from its 2D slices (thin sections from the same 3D rock), without resorting to 3D reconstruction. Permeability is computed on a 2D microstructure that has the exact pore geometry observed in the thin section. A calibratable heuristic model then transforms the exact 2D result to an estimate of the 3D permeability. To our knowledge, no previous study has performed a detailed analysis of possible 2D-3D correlations for permeability of natural rocks. Since permeability is a dimensional quantity, simulating flow using 2D images directly accounts for the spatial scale.

We analyze a dataset that includes permeability computed using the Lattice-Boltzmann method for 12 digital samples of natural rocks in 3D, along with the permeability of the corresponding 2D thin sections. For the rocks considered in this study, we note that simple models, upon calibration, can provide reasonably accurate estimates of rock permeability (in 3D) if only 2D information is available of the rock microstructure.

The main advantage of computing 2D permeability from the thin section is that it automatically captures effects of realistic pore shapes and scales. This approach still shares the problem of non-uniqueness of estimating effects of the third dimension, though no worse than with reconstruction techniques. The advantages of using 2D thin sections for DRP analysis must be ultimately weighed against the ability to estimate accurate 3D properties using thin sections

2. DIGITAL ROCKS

We consider the same twelve 3D digital rocks as Saxena and Mavko (2015): four binary digital samples which were generated from segmented digital images of natural rocks – a Fontainebleau sandstone, a Berea sandstone, a Bituminous sand sample (referred as Bitumen Rock), and a Grosmont carbonate. Another eight samples were created by digitally reducing the porosity of the original four samples by assuming uniform overgrowths on grain surfaces.

2.1 Fontainebleau sandstone

The Fontainebleau sandstone sample image was originally reported in Andrä et al. (2013a). This image has a voxel scale of 7.5 μm . Figure 2a shows the segmented binary digital rock (288×288×300) with grain and pore voxels. For this sample, the porosity is 14.7%, and permeability is between 1100 - 2000 mDarcy (Keehm, 2003; Andrä et al., 2013b). Figure 3a shows the distribution of 2D thin section porosity as estimated from multiple slices of the original cube. Grains of Fontainebleau sandstone are typically composed of quartz (bulk modulus: 36 GPa; shear modulus: 45 GPa). In this paper, we refer to the original sample reported in Andrä et al. (2013a) as sample C. We generated two additional samples (A and B) by digitally expanding the grains of the original Fontainebleau sandstone sample C (shown in Figure 4). Porosity of sample B is 11%, and the porosity of sample A is 5%.

Multiple thin sections, defined as lying parallel to the x-y plane, are sampled from each of the 3D cubes, each of dimensions $288 \times 288 \times 1$. These new microstructures are permeated with pipes that are parallel to the z direction. We generate such microstructures using a total of 100 thin sections drawn randomly from the 3D cubes. The properties (porosity and permeability) computed for these microstructures are referred in this paper as the 2D properties.

2.2 Berea sandstone

The Berea sandstone dataset was also taken from Andrä et al. (2013a). Figure 2b shows the segmented sample of size $800 \times 800 \times 800$ with a voxel edge length of $0.74 \mu\text{m}$. Laboratory measured connected porosity of this sample is approximately 20%. Permeability as provided by the company Berea Sandstone™ Petroleum Cores (Ohio, USA) is between 200 and 500 mDarcy. The reported porosity of the segmented digital sample is 18% (Andrä et al., 2013a). Figure 3b shows the distribution of 2D thin section porosity. We refer to the original sample as sample C. Similar to the Fontainebleau samples, we also generated samples A and B by digitally expanding the grains of the original Berea sandstone sample C (shown in Figure 4). Porosity of sample B is 13%, and the porosity of sample A is 5%. We once again generate microstructures with dimensions $800 \times 800 \times 1$ using only the 2D information for 100 thin sections drawn from each 3D Berea rock.

2.3 Bitumen Rock

A digital bituminous sand rock sample was imaged, processed, and later segmented using commercially available Avizo software. The segmented digital sample of size $400 \times 400 \times 400$ is shown in Figure 2c. The spatial resolution of this digital rock is $4 \mu\text{m}$. Imaging revealed that the sample has quartz grains (volume fraction 0.56), bitumen-filled pores (volume fraction 0.37) and

air-filled pores (volume fraction 0.07). To avoid partial saturation we digitally replaced all air-filled pores with quartz. This sample is referred to as Bitumen Rock sample C. Figure 3c shows the distribution of 2D thin section porosity. Similar to our analysis of the Fontainebleau and Berea sandstone samples, we digitally alter the grains of Bitumen Rock sample C to generate samples B and A of porosity 17% and 5%, respectively (shown in Figure 5). We generate microstructures of dimensions $400 \times 400 \times 1$ using only the 2D information for 100 thin sections drawn from each 3D rock.

2.4 Grosmont carbonate

The carbonate sample was also taken from Andrä et al. (2013a). The sample was extracted from the Grosmont formation, Alberta, Canada. Figure 2d shows the digital rock sample of size $800 \times 800 \times 800$. The voxel edge length is $2.02 \mu\text{m}$. The sample porosity is 24%, and laboratory measured permeability ranges from 150 mDarcy to 470 mDarcy. Figure 3d shows the distribution of 2D thin section porosity. Altering grains of this original Grosmont carbonate sample C we generate samples B and A of porosity 13% and 5%, respectively (shown in Figure 5). Similar to the analysis above, we also generate microstructures of dimensions $800 \times 800 \times 1$ using only the 2D information for 100 thin sections drawn from each 3D rock.

3. ABSOLUTE PERMEABILITY

Darcy's law states that the volume flux Q of a viscous fluid per unit time, through a sample of porous material is linearly proportional to the pressure difference ΔP applied to the opposite faces of the porous sample and the cross-section area A , and inversely proportional to the fluid viscosity η and sample length L :

$$Q = -k \frac{A \Delta P}{\eta L} . \quad (1)$$

The proportionality constant k is a fundamental property of the porous medium referred to as the absolute permeability, which has units of length squared. The most common permeability unit is Darcy or mDarcy (mDarcy), such that 1 Darcy = 10^{-12} m² and 1 mDarcy = 10^{-15} m². Permeability depends on porosity but in a non-unique manner. A commonly used approximate permeability-porosity relation is the Kozeny-Carman equation (Kozeny 1927; Carman 1937):

$$k = \frac{1}{72} \frac{\phi^3}{(1-\phi)^2} \frac{1}{\tau^2} D^2 , \quad (2)$$

where, ϕ is porosity, D is grain diameter, and τ is tortuosity. This model assumes that a porous medium can be represented as a solid block permeated by parallel cylindrical pipes. The proportionality constant 1/72 is sometimes replaced with a fitting parameter.

4. PERMEABILITY COMPUTATIONS

To estimate permeability of digital rocks (3D composites) and thin sections (2D composites permeated with pipes of arbitrary cross section that are parallel to the z direction) we use the Lattice-Boltzmann method (LBM) (Ladd, 1994a,b; Bosl et al., 1998; Keehm et al., 2004; Keehm, 2003). The LBM method describes fluid motion with the interaction of a large number of particles following simple local rules that are based on cellular automata theory. The LBM method recovers the Navier-Stokes equation at the macroscopic scale (Rothman and Zaleski, 1997; Ladd, 1994a,b). The primary advantage of using LBM is its speed and usability for very complex microstructures of rocks (Cancelliere et al., 1990, Ladd, 1994, Martys and Chen, 1996, Keehm, 2003). This method does not idealize the microstructure and hence provides a rigorous

estimate of flow properties. Andrä et al. (2013b) recently compared permeability calculated using the LBM with other numerical schemes and found reasonable agreement between various numerical approaches. In this paper, we use an implementation of LBM to simulate single-phase flow with a “fixed flux” scheme (Keehm, 2003). We add 15 buffer layers at the inlet and the outlet (Andrä et al., 2013b). The absolute permeability is estimated using Darcy’s law in the z direction.

The following sample sizes were used in the property computations: 3D Fontainebleau sandstone sample of size $288 \times 288 \times 300$ and thin sections of size $288 \times 288 \times 1$, 3D Berea sandstone sample of size $800 \times 800 \times 800$ and thin sections of size $800 \times 800 \times 1$; 3D Bitumen Rock sand sample of size $400 \times 400 \times 400$ and thin section of size $400 \times 400 \times 1$; 3D Grosmont carbonate sample of size $800 \times 800 \times 800$ and thin section of size $800 \times 800 \times 1$. For each 3D rock sample, we calculate permeability of 100 thin sections that were drawn randomly. We also compute 2D permeability using a finite difference (FD) scheme that also operates directly on 2D microstructures.

We compare the LBM-computed velocity fields for 3D digital rocks and the corresponding 2D thin sections (Figures 6 and 7; for Sample C). For each lithology, the same color scale is used to display the LBM-computed velocity fields for the 3D microstructure and the corresponding 2D thin section so that a direct comparison can be made. The computed velocity fields are quite distinct in 2D and 3D. Furthermore, the computed permeability for the 3D rocks and the corresponding 2D thin sections (100 thin sections for each 3D sample) show quite distinct values for all four lithologies (Figures 8 – 11). The computed permeability (3D permeability, large blue symbols) for all rocks is generally much smaller than the permeability values calculated from thin sections (2D permeability, small red symbols for LBM and green symbols for FD). Tables 1

– 4 list the calculated permeability values for all rocks. We also report the arithmetic mean, the harmonic mean, and the average of 2D permeabilities, calculated for all thin sections drawn from the same 3D rock, using

$$k = \langle k \rangle \text{ (arithmetic mean) ,} \quad (3)$$

$$k = \langle k^{-1} \rangle^{-1} \text{ (harmonic mean) ,} \quad (4)$$

$$k = \frac{\langle k \rangle + \langle k^{-1} \rangle^{-1}}{2} \text{ (average of arithmetic and harmonic means) .} \quad (5)$$

The computed permeabilities of 3D rocks show values consistent with those exhibited by laboratory measurements (open grey symbols), even though the laboratory measurements were neither taken on the same samples nor at the same scale as the digital computations. For comparison we also show predictions of the Kozeny-Carman relation (equation 2) for a variety of grain sizes (dashed gray curves).

5. 2D-3D TRANSFORMATIONS FOR PERMEABILITY

We now present heuristic models for estimating 3D permeability from the LBM-computed 2D permeabilities for the Fontainebleau sandstone, the Berea sandstone, the Bitumen Rock, and the Grosmont carbonate. It is clear that no single approach can yield perfect 2D-to-3D transforms, since this problem is fundamentally non-unique given that the same 2D microstructure can be cut from many different 3D composites. Therefore, in this section, we discuss models that capture possible deviations from 2D to 3D that are relevant for natural rocks.

5.1 Kozeny-Carman approach (Model 1)

The Kozeny-Carman expression for permeability can also be expressed as

$$k \approx \frac{1}{2} \frac{\phi^3}{\tau^2 S_0^2} . \quad (6)$$

where ϕ is the porosity, S_0 is the specific surface area and τ is the tortuosity. Using equation 6 we can write

$$\frac{k_{3D}}{k_{2D}} \approx \frac{S_{2D}^2}{\tau_{3D}^2 S_{3D}^2} . \quad (7)$$

For an arbitrary packing of identical spherical grains in 3D, the specific surface area is $S_{3D} = 3(1-\phi)/R$, where R is the sphere radius. Similarly, the specific surface area for an arbitrary packing of parallel circular cylinders is $S_{2D} = 2(1-\phi)/R$, where R is the cylinder radius. We use a model for porosity-dependent tortuosity given by (Comiti and Renaud, 1989): $\tau_{3D} = 1 - p \log \phi$, where p is an empirical parameter ($p = 0.41$ for sphere packs; $p = 0.63$ for cubes). Therefore, the estimated 3D permeability is given by

$$k_{3D} \approx \frac{4}{9} \frac{k_{2D}}{(1 - p \log \phi)^2} . \quad (8)$$

5.2 Flow path deviation approach (Model 2)

Model 1 does not incorporate the effects of change in flow field from pipes of fixed diameter in 2D to variable diameter in 3D. Thus, we consider another approach that accounts for sinusoidally changing pore diameter in 3D along with distortion of pipe diameter upon slicing. Details of derivation for the model are presented separately in Appendix A. The final expression for model 2 is given by

$$k_{3D} \approx 0.24 \left(\frac{1}{1+\bar{\delta}} \right)^4 \left(\frac{2(1-\bar{\delta}^2)^{7/5}}{2+3\bar{\delta}^2} \right) \left(1 + \frac{16\pi^2}{3} \frac{\bar{\delta}^2}{\bar{\lambda}^2} \frac{(1-\bar{\delta}^2)}{2+3\bar{\delta}^2} \right)^{-1} k_{2D} . \quad (9)$$

In equation 9, $\bar{\delta}$ is the normalized magnitude of variation in radius, and $\bar{\lambda}$ is the “wavelength” of variation. The factor 0.24 in equation 9 accounts for distortion of tube diameter upon slicing in all possible directions. For our analysis, we fix $\bar{\lambda} = 2$ and vary $\bar{\delta}$ as a calibration parameter.

Varying the calibration parameters, we compare the model-calculated 3D permeabilities (using the “average” 2D LBM-computed permeabilities as inputs into the models) with those computed using the LBM. The model-calculated values, using model 1, for the Fontainebleau sandstone samples (A, B, and C) show a good agreement with the LBM-computed 3D permeability when the parameter p is varied between 1 and 2 (Figure 12). Similarly, we note good agreement between the predictions of model 2 and the LBM-computed permeability for model parameter $\bar{\delta}$ between 0.3 and 0.5. The required model parameters for Berea sandstone are very similar to those required for the Fontainebleau sandstone (Figure 13), even though Berea sandstone is known to have a slightly more complex microstructure when compared to Fontainebleau sandstone. Bitumen Rock samples have considerably larger porosity than Fontainebleau and Berea sandstones. The calibration required to get an agreement between model-calculated and LBM-computed 3D permeabilities is similar to that required for the other two sandstone rocks, but more calibration is required at larger porosities (Figure 14). In summary, we note that parameter range $p \sim 1-3$ (model 1) and $\bar{\delta} \sim 0.3-0.5$ (model 2) contains most of the sandstone data.

Model parameters p and $\bar{\delta}$ depend on the details of rock microstructure. We investigate their dependence on porosity. Using the LBM-computed permeabilities (using “average” 2D and 3D) in equations 8 and 9 we invert for the parameters for all digital sandstone samples (Figure 15).

We find that parameters p and $\bar{\delta}$ sharply decline with increase in porosity, and are well described by the following empirical relations (Figure 15):

$$p = 3e^{-7\phi}, \quad (10)$$

$$\bar{\delta} = e^{-9\phi}. \quad (11)$$

We refer to models 1 and 2 with parameters in equations 10 and 11 as the calibrated models. We apply the two models to calculate permeability for the sandstone thin sections shown in Figure 1. The predictions of the calibrated models agree well with the laboratory-measured permeabilities of the core samples (from which the thin sections were extracted). We note that the laboratory measured permeabilities are also well contained by transforming the LBM-computed 2D permeability with the range of model parameters discussed above (Figure 16). Model 2 performs slightly better than model 1.

Carbonate microstructure is often significantly more complicated than sandstones and the models discussed in this paper may be too simplistic to perform 2D-3D transformations. This is evident when we compare the distribution of 2D porosities in sandstone samples (Figures 8 to 10) with the corresponding distribution in Grosmont carbonate samples (Figure 11) which are more heterogenous. Moreover, the current imaging methods (e.g., micro-CT, thin sections) do not resolve details of microporosity in carbonates (Andrä et al., 2013a). Yet, for Grosmont carbonate samples, the transformed 2D trends using all models agree with the LBM-computed 3D permeability for the following parameter ranges: $p \sim 3-6$ (model 1) and $\bar{\delta} \sim 0.5-0.7$ (model 2) (Figure 17). This subject requires further study using a sufficiently large dataset of thin sections and laboratory-measured permeability.

6 CONCLUSIONS

We conclude that fairly accurate estimates of permeability of natural rocks can be obtained from 2D thin sections without reconstruction of the 3D rock, especially for sandstones. This assertion is based on the numerical dataset presented in this paper, which includes permeability calculations of 12 digital rocks and over 1200 thin sections from various geologic formations, including Fontainebleau sandstone, Berea sandstone, Bituminous sand, and Grosmont carbonate. We also demonstrated the validity of the proposed method by comparing the numerically predicted 3D permeability (computed using 2D thin sections) with laboratory measured permeability of four sandstone samples.

The proposed approach is fast and efficient, since it does not require reconstruction of the unknown 3D rock using 2D thin section information. Since thin sections are often more readily available when compared to full 3D images, we propose to first calibrate the model parameters using the available 3D information on the rock microstructure (from microCT, SEM, etc.) and then predict the permeability for rocks from the same geological formation for which only 2D thin sections are available. In the absence of any information on 3D microstructure, we suggest using the model parameters discussed in this paper.

It is important to note that the proposed approach also highlights the uncertainties and challenges with regards to estimating permeability from 2D thin sections. Firstly, there is no unique solution to this problem since infinitely many 3D samples can have identical 2D profile. The proposed 2D-to-3D transforms discussed here attempt to capture the impact of changes in flow profile from 2D to 3D for natural rocks and these transforms are not universal. In addition to these considerations, acquisition of both 3D samples and 2D thin sections have many

additional challenges, including sample bias, damage, and acquisition type (core, rotary, percussion, and cuttings).

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8 APPENDIX A: DEVIATIONS IN FLOW FROM 2D TO 3D

The 2D flow calculation assumes that each pore is a straight tube of arbitrary cross section extending perpendicular to the 2D image (i.e. along the z direction). We consider two deviations from this geometry that are expected in a three-dimensional rock.

8.1 Variations in pore diameter in the direction of flow

Flow in a pipe varies as R^4 , where R is some representative scale (in the simple case of a round pipe, $2R$ is the diameter; for a square pipe, $2R$ is the side; for an elliptical pipe $2R$ is the minor axis length). Therefore the largest pipes dominate the overall flow. Reducing the pipe scale by 50% reduces the flow by 94%.

We assume that in 3D, each flow path observed in the thin section encounters widening and narrowing, in the direction of flow – the natural result of the distribution of pores and throats. If the thin section is extensive enough, and if the rock is isotropic, then the distribution of tube sizes observed in the thin section is the same as in 3D (ergodic assumption). The largest pipe in 2D is representative of the upper end of the size distribution. Therefore, it is more likely that “behind” the thin section, the larger pipes narrow. Since the largest pipes in the thin section

dominate the flow, we expect that the narrowing of the largest pipes will cause 3D flow to most often be smaller than the computed 2D flow.

We use expressions adapted from Sisavath et al. (2001) for flow in a pipe with sinusoidally varying radius (Figure A-1) to estimate the effect of the narrowing:

$$k = k_0 \left(\frac{1}{1 + \bar{\delta}} \right)^4 \left(\frac{2(1 - \bar{\delta}^2)^{7/5}}{2 + 3\bar{\delta}^2} \right) \left(1 + \frac{16\pi^2}{3} \frac{\bar{\delta}^2}{\bar{\lambda}^2} \frac{(1 - \bar{\delta}^2)}{2 + 3\bar{\delta}^2} \right)^{-1}. \quad (\text{A-1})$$

where k_0 is the permeability if the pipe has radius $R_{\max}$, $\bar{\delta} = \Delta R / R_{\text{avg}}$ is the normalized magnitude of variation in radius, and $\bar{\lambda}$ is the “wavelength” of variation, normalized by R_{avg} . If $\bar{\delta} = 0.3$, $\bar{\lambda} = 2$, then the permeability is reduced by 0.15, relative to the 2D permeability. An example is presented in Figure A-2; we show various sinusoidal pipes of varying radius. Each pipe has the same 2D porosity (area highlighted with yellow) but distinct absolute permeability, specific surface area, and flux-averaged tortuosity.

8.2 Distortion of tube diameter upon slicing

In the 2D problem, a circular pore of radius R might represent a circular tube of radius R with the pipe perpendicular to the thin section. In 3D, if the pipe deviates from the perpendicular by angle q , then the circular pore corresponds to a pipe with elliptical cross section with semi-major axis R , and semi-minor axis $R \cos q$ (Figure A-3). The permeability of a pipe with elliptical cross section is

$$k = k_{\text{cir}} \frac{2 \cos^3 \theta}{1 + \cos^2 \theta}. \quad (\text{A-2})$$

345 where k_{cir} is the permeability of the circular pipe. The path length of the deviated pipe is also
 346 increased by $1/\cos \theta$, a form of tortuosity. Assuming that the rock is isotropic, then averaging
 347 over all angles yields

$$348 \quad k = k_{cir} \frac{1}{4\pi} \int_{\varphi=0}^{2\pi} 2 \int_{\theta=0}^{\pi/2} \frac{2\cos^4 \theta}{1 + \cos^2 \theta} \sin \theta d\theta d\varphi . \quad (A-3)$$

$$= 0.24 k_{cir}$$

349 Therefore, the 3D permeability is diminished from the 2D permeability by a factor ~ 0.24 .
 350 Combining results in equations A-2 and A-3 we get

$$351 \quad k_{3D} = 0.24 \left(\frac{1}{1 + \bar{\delta}} \right)^4 \left(\frac{2(1 - \bar{\delta}^2)^{7/5}}{2 + 3\bar{\delta}^2} \right) \left(1 + \frac{16\pi^2}{3} \frac{\bar{\delta}^2}{\lambda^2} \frac{(1 - \bar{\delta}^2)}{2 + 3\bar{\delta}^2} \right)^{-1} k_{2D} . \quad (A-4)$$

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11. FIGURES

Figure 1a. Thin sections of two sandstones: sandstones X1 and X2. All thin sections have the same resolution ($0.327\ \mu\text{m}$) and edge length (3.3 mm). Laboratory measured core porosities of these rocks are 0.35 and 0.36, respectively. Measured permeabilities are 866 and 1754 mDarcy, respectively.

Figure 1b. Thin sections of two sandstones: sandstones Y1 and Y2. All thin sections have the same resolution ($0.327\ \mu\text{m}$) and edge length ($3.3\ \text{mm}$). Laboratory measured core porosities of these rocks are 0.21 and 0.20, respectively. Measured permeabilities are 110 and 561 mDarcy, respectively.

Figure 2. Digital rocks: (a) Fontainebleau sandstone sample of size $288 \times 288 \times 300$, (b) Berea sandstone sample of size $800 \times 800 \times 800$, (c) Bitumen Rock sand sample of size $400 \times 400 \times 400$, (d) Grosmont carbonate sample of size $800 \times 800 \times 800$. Pores are shown in black and mineral in gray. Voxel edge length for Fontainebleau, Berea, Bitumen Rock, and Grosmont samples are $7.5 \mu\text{m}$, $0.74 \mu\text{m}$, $4 \mu\text{m}$, and $2.02 \mu\text{m}$, respectively.

Figure 3. Histograms of porosity estimated from thin sections for rock samples shown in Figure 2. Average porosity from thin sections is shown with the red pentagon symbol.

Figure 4. Subcubes of the Fontainebleau and Berea sandstone samples that are shown in Figure 2; original images are referred as “sample C”. For each rock type, samples A and B are derived by expanding the grains in sample C, resulting in reduced porosity. Each subcube is of size 200×200×200. Pores are shown in black and mineral in gray. For these subcubes, the voxel edge length for the Fontainebleau and Berea samples are 7.5 μm and 1.48 μm, respectively.

Figure 5. Subcubes of Bitumen Rock and Grosmont carbonate samples that are shown in Figure 2; original images are referred as “sample C”. For each rock type, samples A and B are derived by expanding the grains in sample C, resulting in reduced porosity. Each subcube is of size 200×200×200. For these subcubes, the voxel edge length for the Bitumen rock and Grosmont samples are 4 μm and 4.04 μm , respectively.

Figure 6. Warmer colors indicate high velocity fields for 3D (left) and 2D thin sections (right) for rocks shown in legend. Red and white arrows indicate volume and planar velocity fields, respectively. For each lithology, same color scale is used for 3D and 2D. Calculations are shown only for sample C.

Figure 7. Warmer colors indicate high velocity fields for 3D (left) and 2D thin sections (right) for rocks shown in legend. Red and white arrows indicate volume and planar velocity fields, respectively. For each lithology, same color scale is used for 3D and 2D. Calculations are shown only for sample C.

Figure 8. LBM-computed permeability versus porosity for different Fontainebleau sandstone samples. Calculations are shown for the 3D rock samples (large blue symbol) and the corresponding thin sections (100 for each rock sample; smaller red symbols). Also shown are permeability trends calculated using the Kozeny-Carman relation ($\tau = 1$ and $\phi_c = 0$) for various grain sizes versus porosity (dashed gray curves with corresponding grain sizes). Laboratory data for sandstones, shown in open gray symbols, are taken from Han (1986) and Gomez (2009).

Figure 9. LBM-computed permeability versus porosity for different Berea sandstone samples. Calculations are shown for the 3D rock samples (large blue symbol) and the corresponding thin sections (100 for each rock sample; smaller red symbols). Also shown are permeability trends calculated using the Kozeny-Carman relation ($\tau = 1$ and $\phi_c = 0$) for various grain sizes versus porosity (dashed gray curves with corresponding grain sizes). Laboratory data for sandstones, shown in open gray symbols, are taken from Han (1986) and Gomez (2009).

Figure 10. LBM-computed permeability versus porosity for different Bitumen Rock samples. Calculations are shown for the 3D rock samples (large blue symbol) and the corresponding thin sections (100 for each rock sample; smaller red symbols). Also shown are permeability trends calculated using the Kozeny-Carman relation ($\tau = 1$ and $\phi_c = 0$) for various grain sizes versus porosity (dashed gray curves with corresponding grain sizes). Laboratory data for sandstones, shown in open gray symbols, are taken from Han (1986) and Gomez (2009).

Figure 11. LBM-computed permeability versus porosity for different Grosmont carbonate samples. Calculations are shown for the 3D rock samples (large blue symbol) and the corresponding thin sections (100 for each rock sample; smaller red symbols). Also shown are permeability trends calculated using the Kozeny-Carman relation ($\tau = 1$ and $\phi_c = 0$) for various grain sizes versus porosity (dashed gray curves with corresponding grain sizes). Laboratory data for sandstones, shown in open gray symbols, are taken from Gomez (2009).

Figure 12. Model-calculated permeabilities for a range of model parameters as shown in the legend. Rest of the symbols are the same as in Figure 8.

Figure 13. Model-calculated permeabilities for a range of model parameters as shown in the legend. Rest of the symbols are the same as in Figure 9.

Figure 14. Model-calculated permeabilities for a range of model parameters as shown in the legend. Rest of the symbols are the same as in Figure 10.

Figure 15. Inverted parameters p (Figure a; Model 1) and $\bar{\delta}$ (Figure b; Model 2) using both 2D and 3D permeabilities for all digital sandstones (Fontainebleau, Berea, and Bitumen Rock).

Figure 16. Laboratory measured (blue symbols) versus model-calculated permeabilities for the thin sections shown in Figure 1. LBM-computed 2D permeabilities shown in open red small symbols. Transformed 2D-to-3D using the calibrated models 1 and 2 are shown in open larger symbols.

Figure 17. Model-calculated permeabilities for a range of model parameters as shown in the legend. Rest of the symbols are the same as in Figure 11.

Figure A-1. Flow in a pipe with sinusoidally varying radius.

Figure A-2. All sinusoidal pipes have the same 2D porosity (circular area highlighted in yellow), however, their 3D shapes are different. The degree of sinusoidal nature, i.e., variation in radius, increases from top to bottom. Numerically-calculated permeability (in m^2), tortuosity, and specific surface area (in m^{-1}), are shown in the legend. 3D microstructure is of size 200 x 100 x 100 with voxel size 0.002 m.

Figure A-3. A circular pore of radius R in 2D represents a circular tube of radius R (left). An ellipsoidal pore in 2D may also correspond to a circular tube that deviates from the perpendicular by angle θ (right).

12. TABLES

Property/Rock	Font A	Font B	Font C
Sample size	288×288×300	288×288×300	288×288×300
Pixel size	7.5 μm	7.5 μm	7.5 μm
3D porosity	0.05	0.1	0.15
2D porosity	0.05 ± 0.01	0.1 ± 0.01	0.15 ± 0.02
LBM 3D permeability	119	813	2028
LBM 2D permeability			
Arithmetic mean (AM)	8174	23299	37221
Geometric mean (GM)	7521	22094	35711
Average (AM + GM)/2	7848	22697	36466
Standard deviation	3479	7874	10907
FD 2D permeability			
Arithmetic mean (AM)	7907	22435	35860
Geometric mean (GM)	6722	20096	32827
Average (AM + GM)/2	7315	21266	34344
Standard deviation	3400	7912	11006

Table 1: Results of Fontainebleau sandstone computations. All permeabilities in mDarcy.

Property/Rock	Berea A	Berea B	Berea C
Sample size	800×800×800	800×800×800	800×800×800
Pixel size	0.74 μm	0.74 μm	0.74 μm
3D porosity	0.04	0.1	0.18
2D porosity	0.04 ± 0.02	0.1 ± 0.03	0.19 ± 0.04
LBM 3D permeability	3	27	114
LBM 2D permeability			
Arithmetic mean (AM)	817	1909	3286
Geometric mean (GM)	431	1404	2691
Average (AM + GM)/2	624	1656	2988
Standard deviation	1298	1880	2486
FD 2D permeability			
Arithmetic mean (AM)	799	1876	3272
Geometric mean (GM)	268	1042	2228
Average (AM + GM)/2	533	1459	2750
Standard deviation	1348	2026	2727

Table 2: Results of Berea sandstone computations. All permeabilities in mDarcy.

Property/Rock	Bit. Rock A	Bit. Rock B	Bit. Rock C
Sample size	400×400×400	400×400×400	400×400×400
Pixel size	4.0 μm	4.0 μm	4.0 μm
3D porosity	0.05	0.17	0.37
2D porosity	0.05 ± 0.01	0.17 ± 0.02	0.37 ± 0.03
LBM 3D permeability	18	794	7476
LBM 2D permeability			
Arithmetic mean (AM)	1546	9326	40614
Geometric mean (GM)	1439	9083	40110
Average (AM + GM)/2	1425	9204	40362
Standard deviation	601	2146	6305
FD 2D permeability			
Arithmetic mean (AM)	1520	9154	38267
Geometric mean (GM)	1339	8708	37292
Average (AM + GM)/2	1429	8931	37780
Standard deviation	568	2064	6032

Table 3: Results of Bitumen rock computations. All permeabilities in mDarcy.

Property/Rock	Grosmont A	Grosmont B	Grosmont C
Sample size	800×800×800	800×800×800	800×800×800
Pixel size	2.02 μm	2.02 μm	2.02 μm
3D porosity	0.05	0.13	0.24
2D porosity	0.05 ± 0.03	0.13 ± 0.04	0.24 ± 0.04
LBM 3D permeability	2.8	64	360
LBM 2D permeability			
Arithmetic mean (AM)	17719	19912	26197
Geometric mean (GM)	3790	9279	17423
Average (AM + GM)/2	10755	14595	21810
Standard deviation	25905	24151	25016
FD 2D permeability			
Arithmetic mean (AM)	19686	28389	38942
Geometric mean (GM)	1047	5458	13597
Average (AM + GM)/2	10366	16924	26270
Standard deviation	30149	39448	46714

Table 4: Results of Grosmont computations. All permeabilities in mDarcy.