**A Matlab-Based Frequency-Domain Electromagnetic Inversion Code (FEMIC)
with Graphical User Interface**

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Abstract

We present a new Matlab-based¹ frequency-domain electromagnetic (EM) inversion code (FEMIC) for analysis of datasets collected using multi-frequency EM-induction instruments. The code includes routines for data filtering and calibration, forward modeling, inverse modeling, image appraisal (i.e., calculation of the depth of investigation), and visualization. A one-dimensional forward model is assumed, but two or three dimensional lateral regularization constraints can be applied during the inversion. The code can take advantage of the parallel-processing capabilities of multi-processor computers, thus facilitating efficient inversion of large datasets. Synthetic and field examples demonstrate the operation of the FEMIC code and showcase its capabilities. Source code is provided as material supplementary to this paper to allow modifications and extensions by others.

KeyWords: EM induction, Inversion, data filtering

¹ Any use of trade, product, or firm names is for descriptive purposes only and does not imply endorsement by the U.S. Government.

18 [1] **Introduction:**

19 Multi-frequency electromagnetic (EM) induction tools are increasingly used for
20 hydrogeophysics applications (e.g., Christensen and Sorensen, 1994; Fitterman and
21 Deszcz-Pan, 1998; Ong et al., 2010; Brosten et al., 2011; Binley et al., 2015). In
22 contrast to the galvanic resistivity imaging method, EM tools are less labor intensive
23 to operate, offer large-scale coverage, and can be used over almost any terrain (except
24 for urban areas). These advantages have led to [1] the development of small-loop
25 multi-frequency EM systems that can record high density data (usually 3 to 5
26 soundings per linear meter) that are tagged with location information (x, y and z),
27 typically using a global-positioning system (GPS), that can provide very high
28 resolution (centimeter-level) (e.g., Duncan et al., 1998; Bell et al., 2001) ; and [2]
29 increased interest in airborne EM surveys that can cover large areas (10's to 1000's of
30 km²) and image to ~100 m depth (e.g., Cook and Kilty, 1992; Macnae and Bishop,
31 2001) . However, inverting multi-frequency domain EM data to obtain meaningful
32 models is challenging because [1] it is time consuming especially when dealing with
33 three-dimensional (3D) problems, [2] data commonly are contaminated with both
34 systematic instrument errors and random noise (Lavoué et al., 2010; Minsley et al.,
35 2012a), and [3] the depth of investigation (DOI) varies with conductivity structure
36 and measurement quality, and must be estimated across the survey site to ensure
37 meaningful interpretation of the images in terms of subsurface structure (Brosten et
38 al., 2011). Whereas there are numerous commercially available and public-domain
39 software packages for resistivity inversion (e.g., Binley and Kemna, 2005; Gunther et
40 al., 2006; and Santos et al., 2010), there are few similar such software packages for
41 multi-frequency EM inversion. These challenges may limit the inexperienced user to a
42 qualitative interpretation of multi-frequency EM data, e.g., by directly plotting the
43 data for each frequency to obtain the spatial distribution of subsurface apparent

conductivity over an approximate depth interval (e.g., McNeill, 1980) or by using analytical solutions assuming 2-3 layered model (e.g., Everett, 2012). Qualitative interpretation of the data may provide useful information on the rough lateral extent of subsurface features but is very limited for characterizing the distribution of such features over depth. The development of user-friendly inversion codes can expand the availability of tools for estimating the spatial location and thicknesses of subsurface layers using multi-frequency EM data.

To the best of our knowledge, free open source code programs with graphical user interfaces (GUIs) for modeling and inversion of multi-frequency EM data are currently unavailable. The few commercially available software packages are not user-friendly in terms of raw data entry (e.g., Hydro Geophysics Group, 2007b), do not have 2D and 3D lateral constraints options, and do not take advantage of parallel-processing capabilities of modern multi-processor computers (e.g., UBC-GIF, 2000). New tools with open source codes are needed for the near surface geophysics community to fully capitalize on multi-frequency EM data.

The inversion of multi-frequency EM data commonly utilizes a one-dimensional (1D) forward solver, given the large size of most datasets and associated computational constraints (e.g., Auken et al., 2014). Furthermore, 1D inversion without lateral constraints can produce results with unrealistic blocky models characterized by sharp lateral variations as a result of either 3D structural effects or measurement noise (Martinelli and Duplaa, 2008; Minsley et al., 2011). Santos (2004) proposed a laterally constrained inversion approach, where a two-dimensional (2D) smoothness constraint is applied between adjacent 1D models. Auken et al (2005) describe a similar approach for inverting resistivity data, which is generalized to three-dimensional (3D) constraints for airborne EM data by Viezzoli et al (2008). Martinelli

and Duplaa (2008) proposed applying 1D inversion along with two horizontal smoothing filters; one applied to the raw data prior to inversion and the other applied post-inversion to the resultant 1D models.

We have developed a Matlab-based, Multi-Frequency Domain EM (FDEM) inversion program that offers [1] a user-friendly data pre-processing module, [2] parallel 1D laterally constrained inversion capabilities, and [3] a graphical user interface (GUI). The code is pseudo-3D, i.e., the forward modeling is 1D, but the regularization applied for the inversion is implemented in 2D or 3D. Although we present example data from a ground-based GEM-2 instrument (Geophex, Ltd.) in this paper, the code can also be applied to other multi-frequency EM data collected from land-based, water-based or airborne platforms (e.g., Minsley et al., 2012b). It could also be used for inversion of fixed-loop single-frequency data.

Here we summarize the theory behind the EM forward and inverse problems, describe the capabilities and operation of FEMIC, and present 2D synthetic and 3D field examples that demonstrate the operation of the FEMIC code. As supplementary material to this paper, we provide the Matlab public-domain source code for both the analysis routines and GUI to allow third-party code modifications and extensions.

[2] EM Theory overview

The basic principle of the EM method is that a transmitter coil is excited with alternating current at a specific frequency, which generates a changing primary magnetic field that in turn induces alternating electrical eddy currents in the subsurface [Ward and Hohmann, 1987; Huang and Won, 2000]. The induced currents produce secondary magnetic fields in the subsurface, which are detected by a receiver coil along with the primary field. The out-of-phase component of the ratio of the secondary field created by the eddy currents to the primary field is proportional to the

conductivity of the subsurface, whereas the in-phase component of the ratio is proportional to subsurface magnetic susceptibility (e.g., Won et al., 1998; and Farquharson et al. 2003). FEMIC only utilizes the out-of-phase data sets during the inversion process given its high signal-to-noise ratio compared with the in-phase data sets (e.g., Huang and Won, 2000). Figure 1 shows the data acquisition setup and a parameterization of the subsurface model. A description of EM theory is provided in the following sections.

2.1- 1D forward problem and governing equations

The equations for the magnetic field generated by an oscillating magnetic dipole above a half space are derived from the quasi-static forms of Ampere's and Faraday's laws of induction. For a layered earth model, the magnetic field propagates into the subsurface as described by Faraday's law:

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \quad (1)$$

where $\nabla \times$ is the curl operator, $\mathbf{E}$ is electrical field strength (V m⁻¹), μ is the magnetic permeability (Hm⁻¹), $\frac{\partial \mathbf{H}}{\partial t}$ is the alternating magnetic field strength (Am⁻¹) with time (t).

After neglecting displacement currents, the vertical component of the magnetic field for the vertical magnetic dipole ($\mathbf{H}_{VMD}$) and the horizontal component of the magnetic field for the horizontal magnetic dipole ($\mathbf{H}_{HMD}$) sources become

$$\mathbf{H}_{VMD} = \frac{m}{4\pi} \int_0^\infty [e^{-\mu_0(z+h^{tx})} + r_{TE} e^{\mu_0(z-h^{tx})}] \frac{\lambda^3}{\mu_0} J_0(\lambda r) d\lambda \quad (2)$$

$$\mathbf{H}_{HMD} = -\frac{m}{4\pi} \frac{1}{r} - \frac{m}{4\pi} \frac{2x^2}{r^3} \int_0^\infty [e^{-\lambda(z+h^{tx})} + r_{TE} e^{\lambda(z-h^{tx})}] \lambda J_1(\lambda r) d\lambda$$

$$-\frac{m}{4\pi r^2} \int_0^\infty [e^{-\lambda(z+h^{tx})} + r_{TE} e^{\lambda(z-h^{tx})}] \lambda^2 J_0(\lambda r) d\lambda \quad (3)$$

where m is the transmitter moment, r is the transmitter – receiver coil separation, h^{tx} is the positive transmitter coil elevation above the ground, λ is the horizontal wavenumber, $u_i = (\lambda^2 - k_i^2)^{1/2}$ where $k_i = (\omega^2 \mu_i \epsilon_i - j \omega \mu_i \sigma_i)^{1/2}$ is the wavenumber of the i th layer, ω is the angular frequency, σ_i is the conductivity, μ_i is the magnetic permeability (Hm^{-1}), ϵ_i is the permittivity of the i th layer and $j = \sqrt{-1}$, J_0 and J_1 are the zeroth and first order Bessel functions of the first kind, and r_{TE} is a reflection coefficient that is calculated using the following recursive form (Ward and Hohmann, 1987; Minsley, 2011):

$$r_{TE} = \frac{Y_0 - \hat{Y}_1}{Y_0 + \hat{Y}_1} \quad (4)$$

Subscripts in eq. 4 refer to layer numbers, where 0 refers to air. $\hat{Y}_1$ is determined recursively using the following expression (Ward and Hohmann, 1987; Minsley, 2011):

$$\hat{Y}_i = Y_i \frac{\hat{Y}_{i+1} + Y_i \tanh(u_i h_i)}{Y_i + \hat{Y}_{i+1} \tanh(u_i h_i)} \quad (5)$$

where h_i is the i th layer thickness. The values for Y_i in each layer are given by (Minsley, 2011):

$$Y_i = \frac{u_i}{j \omega \mu_i} \quad (6)$$

The frequency-domain EM data, $\mathbf{D}$, are usually reported as parts-per-million of the primary field for a number of discrete frequencies and can be expressed as (Minsley, 2011):

$$\mathbf{D}(\omega) = \frac{\mathbf{H}(\omega) - \mathbf{H}^0(\omega)}{\mathbf{H}^0(\omega)} \times 10^6 \quad (7)$$

Where $\mathbf{H}$ is the total magnetic field in eqs. 2 and 3, and H^0 is the free-space magnetic field.

In this right-handed coordinate system, z is positive downwards and x is in the survey direction. Equation (3) is specifically for an x -oriented magnetic dipole (Ward and Hohmann, 1987). However, for a horizontal coplanar system such as the GEM-2 with magnetic dipoles oriented in the y -direction, equation (3) can also be used but with $x=0$ and y (used to calculate r) equal to the transmitter-receiver offset.

2.2 Inverse Method: 1D Laterally Constrained Inversion

The inversion of multi-frequency EM data is an ill-posed problem in the sense that small uncertainties in the data may cause large uncertainties in the final solution which is non-unique. In order to overcome this issue, we have implemented a version of the Gauss-Newton nonlinear least-squares method with damping (Marquardt, 1963). FEMIC seeks solutions that minimize the following objective function:

$$\min\{\varphi(m) = U_1 + U_2 + U_3\} \quad (8),$$

where,

$$U_1 = \|\mathbf{W}_d \mathbf{d}_{meas} - \mathbf{W}_d \mathbf{d}_{est}\|_p^p \quad (9),$$

$$U_2 = \frac{\beta}{2} \|\mathbf{W}_m (\mathbf{m}_i - \mathbf{m}_{ref})\|^2 \quad (10)$$

$$(10),$$

$$U_3 = \alpha \mathbf{Z} \quad (11),$$

where U_1 is the L_p norm of data misfit (p is an integer denoting the norm order),

$\mathbf{d}_{meas}$ is a vector of measured field data, $\mathbf{d}_{est}$ is the model vector of the simulated data, $\mathbf{W}_d$ is the data weighting matrix where $\mathbf{W}_d = \text{diag}(1/s)$, $\mathbf{s}$ is a vector of standard deviations for each measurement; U_2 represents the model constraint term, where $\mathbf{m}_{ref}$ is the starting model, β is the regularization parameter, arbitrary chosen, that

balances the influence of the data misfit and model roughness terms on the final solution, $\mathbf{m}_i$ is the current conductivity model and $\mathbf{W}_m$ is the model regularization matrix with the following form (Pidlisecky et al., 2007):

$$\mathbf{W}_m = \mathbf{W}_u^T ||\beta_x \cdot \mathbf{G}_x + \beta_y \cdot \mathbf{G}_y + \beta_z \cdot \mathbf{G}_z||^2 \mathbf{W}_u \quad (12)$$

where $\mathbf{W}_u$ is a vector of model weights to allow a user to include a priori information in the inversion, β is the smoothness weight and $\mathbf{G}$ is the second derivative finite difference matrix and the subscripts denote spatial direction. The final term in the objective function U_3 , represents the positivity constraint imposed via a logarithmic barrier (∞) (Farquharson et al., 2003), in order to force all solutions to fall between user-defined minimum (m_{min}) and maximum conductivity values (m_{max}), where the diagonal barrier matrix $\mathbf{Z} = [\ln(\mathbf{m} - m_{min}) + \ln(m_{max} - \mathbf{m})]$.

The updated model ($\mathbf{m}_{est}$) at the new iteration is given by the following Levenberg-Marquadt scheme:

$$\mathbf{m}_{est} = [(\mathbf{W}_d \mathbf{J})^T \mathbf{W}_d \mathbf{J} + \beta \mathbf{W}_m^T \mathbf{W}_m + \alpha \mathbf{Z}^T \mathbf{Z}]^{-1} (\mathbf{W}_d)^T \mathbf{W}_d \mathbf{d} \quad (13)$$

Where $\mathbf{J}$ is the Jacobian or sensitivity matrix of partial derivatives of the simulated measurements with respect to model parameters. The sum of objective functions, $U_1 + U_2 + U_3$, is minimized iteratively until the optimization reaches one of the following convergence criteria: [1] the change in model conductivity is less than a predefined tolerance level, [2] the root-mean-squared (RMS) error is less than a predefined tolerance level, or [3] the user defined maximum number of iterations is reached.

To assess the quality of the inverted models, we include functionality to estimate the model cell sensitivity and model resolution values across the model space. The model cell sensitivity values, $\mathbf{S}$, are a relative measure of how different model cells across

the model space are influenced by the data where lower values indicate less sensitivity. It is defined as (Kemna et al., 2002; Nguyen et al., 2009):

$$S = (J^T W_d' W_d J) \quad (14)$$

The model resolution matrix, R_m , depicts to what degree the recovered conductivity values are determined by the starting model (we refer the reader to e.g., Gunther et al., 2003 for further details about the resolution matrix estimation). It is defined as (Gunther et al., 2003):

$$R_m = ((W_d J)'(W_d J) + \beta W_m' W_m)^{-1} (W_d J)'(W_d J) \quad (15)$$

In addition to the model cell sensitivity and model resolution matrix options, FEMIC contains an option for estimating the maximum depth of investigation (discussed in section 3.5).

[3] FEMIC Approach

The FEMIC source code is written in Matlab, thus enabling execution across multiple platforms for which different versions of Matlab are available. In addition, the code is easy to read and modify, allowing for future translation for execution under other programming environments such as Python and Octave. By setting software options, the user can configure parallelized inversion (with the parallel toolbox in Matlab), select the dimension of the problem (i.e., 1D, 2D or 3D), load data and an initial model, apply optional filters, calibrate the raw data, apply constraints to the inversion process, and assess inversion results through inspection of sensitivity and maximum depth-of-investigation plots; these various options are coded in separate functions or subroutines, which are easily accessed through the GUI. Moreover, the GUI allows the user to pre-process the data, speed up the inversion time, and explore different solutions using different regularization weights. Figure 2 shows the processing flow chart for the code. The user input includes [1] a raw dataset of in-phase and quadrature values and an estimated error for each data point; [2] an optional

conductivity model (e.g. from a 2D DC resistivity survey) when using the calibration and filtering option (Minsley et al., 2012a), [3] a starting model, and [4] model convergence parameters and inversion settings. The code also requires the number of layers, layer thicknesses, constraint weights in the case of defining prior information in the initial model, and regularization parameters. In addition, the user has the option to display the depth of investigation, model cell sensitivity and model resolution along with the final solution. After loading all the required inputs, the user starts the inversion and the progress at each iteration is displayed.

3.1 Parallelization

FEMIC takes advantage of recent advances in computing power and can be run in parallel on multi-processor machines to speed up the inversion process. This option is fully functional in the case of multiple individual 1D problems, whereas in 2D and 3D problems the parallelization is only applied on the forward modeling portion of the inversion because certain computational tasks, such as application of lateral constraints, cannot be distributed independently among different processors. The user is allowed to specify any number of processors to use during the inversion up to the maximum number of processors available on the machine. We evaluated the performance benefit of the parallel-processing option in FEMIC, by estimating the speedup curve for a fixed data input of size (n) and for different number of processors (p) as follows (Kim, 2009):

$$Speedup(n, p) = \frac{time(n)_{serial}}{time(n, p)_{parallel}} \quad (16)$$

The speedup curve shows a logarithmic dependence on the number of cores, and the estimated average percentage increase per processor is 8% (Figure 3).

3.2 Data conditioning

a. Noise assessment and filtering

Random errors affect all types of geophysical data, and must be accounted for to avoid inversion artifacts caused by over-fitting noise in the data. For EM data, random errors are often estimated by looking at the scatter in data measured when the instrument is left undisturbed at one location. The standard deviation of errors can be incorporated into the data weighting matrix, W_d , in Eq. (9) before inversion. Alternatively, random noise can be attenuated before inversion using a moving average or other spatial filtering process. However, the outcome of these filters depends largely on the size of the user-defined spatial smoothing window. Another approach discussed by Minsley et al. (2012a) is the use of a principal component analysis (PCA) filter. The PCA filter does not have an inherent spatial smoothing window, but rather takes advantage of the fact that EM data are correlated across frequencies (e.g., Kellett and Bauman, 2004). Uncorrelated signal at any spatial wavelength identified through the PCA analysis is treated as random noise. The PCA filtering approach is included in FEMIC.

b. Calibration approach

It is increasingly recognized that inversion of multi-frequency EM data benefits from calibration (e.g., Mester et al., 2011; Minsley et al., 2012). In many cases, systematic errors result in data that cannot fit any realistic conductivity model. Inversion of uncalibrated data can therefore lead to biased models and inaccurate interpretations. Minsley et al. (2012a) recently proposed an approach that uses DC resistivity information to calibrate EM data before inversion. In this approach, raw multi-frequency EM measurements are adjusted with complex offset and gain factors so as

to achieve consistency with the synthetic response of a model derived from inversion of DC resistivity data $\mathbf{d}_{obs}^*(\mathbf{x}, \mathbf{f}) = \mathbf{T}^*(\mathbf{f})[\mathbf{d}_{cal}^*(\mathbf{x}, \mathbf{f}) + \mathbf{B}^*(\mathbf{f})]$ (17)

Calibration factors ($\mathbf{T}^*$ and $\mathbf{B}^*$) are derived for each frequency ($\mathbf{f}$) using a least squares algorithm that minimizes the difference between the left and right hand sides of Equation 16, where $*$ indicates that all variables are complex numbers with real and imaginary parts. EM data simulated from the DC model ($\mathbf{d}_{cal}$) and observed data ($\mathbf{d}_{obs}$) vary as a function of distance along a survey line ($\mathbf{x}$) and frequency. The calibration factors for each frequency represent the best-fit factors over the entire profile. Details of the inversion algorithm to calculate the calibration parameters can be found in Minsley et al. (2012a).

Application of the derived calibration parameters is used to generate a calibration-corrected dataset ($\mathbf{d}_{corr}$) according to

$$\mathbf{d}_{corr}^*(\mathbf{x}, \mathbf{f}) = \frac{\mathbf{d}_{obs}^*(\mathbf{x}, \mathbf{f})}{\mathbf{G}^*(\mathbf{f})} - \mathbf{B}^*(\mathbf{f}) \quad (18)$$

Using the calibration-corrected result ensures that the data are consistent with realistic earth models; however, this approach also assumes that the DC resistivity model used for calibration is accurate. Any errors in the DC resistivity model used for calibration of the EM data will propagate through the inversion procedure, producing errors in the EM inversions.

3.3 Inversion options

In this first version of FEMIC, we have focused on widely used Tikhonov regularization. The objective function in this method includes both data and model misfit terms. To implement Tikhonov regularization in FEMIC, it is necessary for the model misfit term in the objective function (eqs. 9-11) to penalize deviation from a

reference model. Under this option, the regularization parameter, β , is fixed during the entire inversion in order to minimize computational cost.

3.4 Inclusion of prior information

In the case where prior information is available, FEMIC allows definition of the conductivity and depth of known subsurface units in the initial model. In addition, the user can penalize the inversion against deviations from this prior information by adjusting the values of the model weighting term in equation (10). The model weighting takes values between 0 (no constraint) and 1 (hard constraint).

3.5 Image appraisal

The depth of investigation (DOI) associated with a FDEM survey depends on numerous factors including measurement quality, the Earth's conductivity structure, system elevation and orientation, intercoil spacing, and frequencies for which data are available. Meaningful interpretation of inversion requires use of image appraisal strategies, such as the DOI index (Oldenburg and Li, 1999; Christiansen and Auken, 2012), which is the depth to which the model parameters are controlled by the data rather than by the model constraints. We have implemented in FEMIC the DOI estimation approach described by Oldenburg and Li (1999) using the Tikhonov regularization technique described in section 3.6. Following the approach of Oldenburg and Li (1999), the inversion is performed for the same data using two different homogenous reference models. Based on the inversion results, m_1 and m_2 , for the two reference models, m_{1r} and m_{2r} , respectively, the DOI index ($RI(x, z)$) is calculated as:

$$RI(x, z) = \frac{\log_{10}(m_1(x, z)) - \log_{10}(m_2(x, z))}{\log_{10}(m_{1r}) - \log_{10}(m_{2r})} \quad (20)$$

RI will approach zero at locations where the two inversions produce the same result regardless of the value of the reference model (i.e., where the result is strongly controlled by the data), whereas it will approach unity at locations where the inversions achieve the values of the reference models regardless of the data. The maximum DOI is the depth where R becomes less than some user-specified cutoff, for which Oldenburg and Li (1999) suggest 0.2, i.e., where the difference between two inversions is within 20% of the difference between the reference conductivity models. FEMIC allows the calculation of the maximum DOI at a user-defined location within the model domain e.g., where the total percentage change of the measured data for all frequencies is above a user defined percentage (e.g., 20%), and interpolates the results across the model space.

[4] FEMIC GUI

The FEMIC GUI parameter input screen is separated into four partitions: Inverse Solution, Field Input Data, Initial Model Parameters and Model/Inversion Parameters (Figure 4). The Inverse Solution partition contains selections for the problem dimension and inversion assessment method. The Field Input Data partition is where the user uploads format-specific data files and instrument-specific frequency information. Calibration and subsequent PCA based filtering with DC resistivity data can also be performed here in which case the filtered dataset will be used in the inversion, rather than the original input dataset (Details about data input formats can be found in the provided code manual).

Within the Initial Model Parameters partition, information can be input in two ways: 1) a format-specific file can be uploaded and 2) the ‘Create Layered Model’ will create an initial model with a user-specified number of layers and input conductivities equal to the number of soundings in the data input file. After this input, the user has

the option to review initial models at each sounding and modify the number of layers and/or the constraint weights associated with a particular layer. Initial models can be saved in this panel for future inversions.

Model / Inversion Parameters can be uploaded from a format-specific file or manually entered. Minimum and maximum conductivity values and barrier parameter values (equation 11), regularization parameter values, directional smoothing (i.e. used only for 2D or 3D problems), and convergence criteria are input in this partition of the GUI.

The user can enter the number of processors locally available before selecting to start an inversion. All input files and their detailed description can be found within the Matlab source code and the accompanying manual provided as supplementary material.

[5] Example Applications

In this section, we demonstrate the use of FEMIC for both synthetic and field-experimental datasets to show different aspects of the code's functionality. Both synthetic and field data sets were inverted using the Tikhonov regularization approach and the default inversion parameters as shown in Figure 4.

5.1. 2D Synthetic example

A synthetic example is provided for which the true distribution of conductivity is known. We calculated the synthetic quadrature component for a 2D model that includes sharp lateral conductivity changes using the 1D forward modeling component of FEMIC (Figure 5a). The true subsurface model is 25 m long and 14 m deep, and the upper most part of the model is divided into two side by side layers, whereas the rest of model space is homogeneous. The first layer extends from surface

361 to 2 m deep, having a width of 14.7 m and conductivity of 100 mS/m, the second
362 layer extends from surface to 4 m deep, having a width of 10.3 m and conductivity of
363 200 mS/m, and the third layer lies directly underneath the upper two layers and has a
364 conductivity value of 10 mS/m. The assigned conductivity values of different layers
365 are reasonable for unconsolidated soils. This model was used to simulate a dataset
366 acquired by a GEM-2 instrument, with two vertical dipole receivers spaced 1.66 m
367 apart and the transmitter operating at frequencies of 1530, 3930, 8250, 13590, 23070,
368 33030, 47970 Hz. The synthetic dataset was calculated every 1 m along the transect
369 using the vertical magnetic dipole mode. To simulate a realistic dataset collected at
370 typical sites, the data were contaminated with 3% Gaussian noise. Figures 5b & 5c
371 show the calculated noise-free dataset and the noisy data.

372 Prior to performing the FEMIC inversion, the noisy terrain conductivity data were
373 filtered using the data filtering option in FEMIC. As shown in Figure 5d, the results
374 of this pre-inversion process suppress the noise and improve the quality of the data.
375 The resulting filtered data were subsequently used in the inversion process.

376 The inversion for the synthetic conductivity data was carried out using the Tikhonov
377 regularization approach and using the default inversion parameters in the FEMIC GUI
378 (see Figure 4). To assess the influence of noise on the inversion performance, the
379 inversion was tested using noise free data, noisy data and filtered data. In all
380 inversions, the starting model was assigned a homogenous conductivity of 60 mS/m.
381 Figure (6c) shows the inverted model for the noise free data. The recovered
382 conductivities and depths of the three layers show the same structures and
383 conductivity values as the true model. Furthermore, the predicted data are highly
384 consistent with the measured data (data misfit 0.05%). However, the inversion was
385 not successful in recovering the boundary between the upper two horizontal layers. As

expected, the DOI estimation over the conductive layer (4 m) is much shallower than away from it (12 m) because of the strong attenuation of the EM field where conductivity is higher. The inversion results for the noisy data are shown in Figure 7. Although the inversion significantly minimizes the data misfit (2.8%), the inverted model contains artifacts particularly in the third layer. In addition, it shows unrealistic conductivity variation especially in the low resistivity layer between 0-15 m. However, the inversion was successful in recovering three distinctive layers. The DOI estimation shows unrealistic horizontal variability, but it is much shallower over the conductive layer than away from it. The inversion results of the filtered data (Figure 8) are significantly improved compared with the noisy data (data misfit 1.92%). The inverted model shows fewer artifacts and the recovered conductivity and layer geometry is more consistent with the true model. Furthermore, as expected, the predicted DOI values across the model space in all three cases are consistently shallower in the conductive areas than in the resistive ones.

5.2. 2D Field example

In order to demonstrate the efficiency of the FEMIC code on field datasets, we acquired GEM-2 along 28 m profile in Laramie (WY), and we precisely recorded the locations where ground surface resistivity likely changes along that line, in order to test if the inversion would capture such lateral changes (Figure 9a). The GEM-2 data consists of 51 soundings that were collected using the vertical magnetic dipole mode and at four different frequencies. The frequency values (20370, 33030, 63090, and 93090 Hz) were selected to achieve high vertical near surface resolution. Prior to performing the FEMIC inversion, the raw field data were filtered using the data filtering option in FEMIC. We did not calibrate the EM data using the acquired resistivity data, since the resistivity model covered shallower depth (~ 5 m deep) than

the EM model. The inversion was subsequently carried out on the filtered data, and assuming a homogenous starting model of 75 mS/m.

Figure (9c) shows the inverted model for the GEM-2 data. The recovered conductivities along the upper most model space are in good agreement with the recorded surface soil changes. The rest of the model space generally shows low conductivity values, indicative of soil with different degrees of moisture content and/or types. Most obvious, the right side of the model space at depth of 2 m shows a fairly high resistivity layer (~ 100 Ohm m) that disappears towards the left hand side of the model space. In addition, that layer contains a high conductivity lens at a depth of 8 m.

5.3 3D Field example

To demonstrate the 3D and parallel-processing capabilities of FEMIC, we present inversion results for field surveys collected at a research site in interior Alaska, USA. A multi-frequency GEM-2 unit with 1.66-m antenna separation was used to collect a total of ~ 21 (~ 9 km in 2011 and ~ 12 km in 2012) line-km of EM data around Twelvemile Lake, where USGS and other groups are investigating permafrost/aquifer interaction (e.g., Jepsen et al., 2012; Briggs et al., 2014). EM data were collected at 7 frequencies (1530 Hz, 8250 Hz, 23070 Hz, 33030 Hz, 47970 Hz, 63090 Hz, and 93090 Hz) about every 0.25 m along each transect. The objective of the geophysical surveys was to map the distribution of shallow permafrost around Twelvemile Lake. Climate warming appears to drive the drying of this lake, and new permafrost appears to form in the dried lake margin. Geophysical results have the potential to help elucidate the complex feedbacks between climate warming, ecological succession (i.e., vegetation growth in the lake margin), and lake/aquifer interaction.

The entire EM data were first calibrated using two DC resistivity models (350 m long for the 2011 data, 180 m long for the 2012 data) and then filtered using the PCA approach. The filtered data were inverted using Tikhonov regularization. Inversion results are shown in Figure 10. The datasets were inverted separately; the inversion of the 2012 data took about 20 hrs on an 8 core (48 GB of ram) Linux-based (Ubuntu) desktop utilizing the 3D option of the Femic code.

The depth of investigation is about 14 m, and thus we limit our interpretation to results from depths shallow than this. Recovered resistivity values are higher (~ 350 Ohm m) in regions covered by spruce, where old permafrost is expected, as well as under patches of shrub growth in the dried lake margin, where new permafrost is forming. The EM inversions fill in huge gaps in space between points where sparse direct information on permafrost presence is available from cores and frost probing. The EM results, in general, show lateral resistivity variations that are fairly consistent with the DC resistivity results (Figure 11). Additionally, they are consistent with a conceptual model for permafrost distribution at the research site (Briggs et al., 2014).

[6] Discussion

We have demonstrated our code on a 2D synthetic study as well as on 2D and quasi-3D field datasets collected at sites with different near subsurface geologic conditions. The synthetic study suggests that the quality of the inversion results depends on the noise level of the data. However, the DOI analysis appears to be useful in separating image content resulting from real subsurface structures from regions where image content is associated with inversion artifacts. The field studies demonstrate that the code produces models with spatially consistent conductivity structures. For example, the 2D field study shows surface conductivity structure that is fairly consistent with the expected surface conductivity distribution at the site (Fig. 9a vs. 9d). In addition, the

intersecting profiles in Figure (10) show similar resistivity changes at the intersecting locations.

We recognize processing-time limitations of FEMIC particularly when applied to large datasets. Firstly, the performance of the code when inverting large data sets will probably be slow, particularly if used on desktop computers with a limited number of processors. To address this, we have provided a completely open source, i.e., public-domain code that is easy to use and read to allow for recoding in a higher speed programming environment (e.g., Python) or for parallelizing for use on supercomputers. Secondly, choosing the optimum regularization parameter for each sounding from a set of user-defined parameters, using the L-Curve or discrepancy principle, significantly increases the overall inversion time and seems impractical for large datasets. This could be addressed by first running the L-Curve or discrepancy principle on a small subset of the data in order to identify a representative set of the optimum regularization parameters. After that, the entire dataset could be inverted using the estimated parameters and the fixed regularization approach. Finally, the forward modeling component in FEMIC is 1D, which obviously provides less accurate theoretical data when dealing with 2D and 3D than a multi-dimensional forward modeling code.

In addition, as a result of this limitation, the code would likely produce inaccurate models at the transition zones between sharp lateral conductivity variations in a highly heterogeneous environment, mainly because it is hard to model such conditions in 1D. Under such conditions, to ensure accurate final models, it is advisable to use prior information in the starting model (e.g., from other geophysical data or borehole logs) along with high constraining weights (e.g., > 0.5) in order to ensure that the final model does not deviate much from the starting model. Despite these limitations,

given that we provide the source code for FEMIC, it is straightforward to substitute the forward modeling routine.

FEMIC provides users with the flexibility to try different initial models, regularization parameters and constraining weights in an easy and quick way. That flexibility is particularly useful when wanting to efficiently determine the optimum inversion setting for a particular dataset, and it makes the code a learning tool to study how different inversion parameters and starting models affect the final solution. Furthermore, the code is designed to facilitate modification by users to include additional inversion and forward modeling options, or to allow different data formats for other tools.

A number of improvements are possible for future modifications of FEMIC. For example, one improvement would be implementation of a function to automate the estimation of the optimum regularization parameters for the entire data set prior to carrying out the inversion using e.g., the L-curve method. Secondly, a function could be included to estimate the data weighting vector that is used during inversion, perhaps based on correlating the consistency of the data collected at the same location using different frequencies. Finally, replacing the 1D forward modeling component in FEMIC with 2D and 3D codes would improve the performance of FEMIC for 2D and 3D problems.

[5] Conclusions

We have presented a Matlab-based code, FEMIC, for forward and inverse modeling of multi-frequency electromagnetic data. The code takes advantage of multi-processor capabilities on local machines to speed up different data processing steps, and it allows applying 2D and 3D lateral regularization constraints. In addition, the forward modeling component in FEMIC takes into account the elevation of the transmitter,

making the code suitable for processing both ground-based and air-borne data sets. FEMIC includes tools for pre-processing, filtering, calibration, inversion, image appraisal, and visualization. Source code is provided as material supplementary to this paper to facilitate modification and extension by others. The 2D and 3D synthetic and field examples were presented to highlight different code options. Finally, we have discussed the limitations of the current code and ways to overcome them. The data processing capabilities within FEMIC makes it suitable to process noisy data sets as well as data that are collected at sites with different geologic conditions.

[6] Acknowledgments

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List of Figure Captions

Figure 1: EM data acquisition and measured subsurface parameters (a & b) GEM2 data acquisition using two field setups, and (c) 2D subsurface discretized model used in FEMIC. The data are collected at individual locations, but are jointly processed using a 1D forward modeling approach and a 1D laterally and vertically constrained inversion (C_L and C_Z represent appropriate lateral and vertical constraints respectively). The number of layer conductivities (σ) and thicknesses (h) at each individual location are specified by the user (m , t , l , and n).

Figure 2: FEMIC flow chart

Figure 3: The speedup curve for the parallel computation option in FEMIC code

Figure 4: Different functions in the FEMIC GUI. The inverse solution section allows the user to specify the desired number of parallel calculations during inversion, the inverse method, the dimension of the problem and optional DOI and sensitivity calculations. The field input data partition allows the user to upload the raw data and an optional resistivity model to calibrate/filter the input data. The model parameters partition allows the user to specify initial models and any prior information. Finally the model/inversion section lets the user specify the desired inversion parameters and stopping criteria.

Figure 5: 2D forward modeling results (a) true subsurface model, (b) noise free synthetic data, (c) 3% noise data, and (d) filtered data based on ‘c’ using Minsley et al. (2012a) approach.

Figure 6: Inversion results for the synthetic noise free data (a) measured and estimated data, and (b) inverted model assuming homogenous starting model of 60 mS/m (the horizontal line depicts the maximum DOI).

Figure 7: Inversion results for the synthetic noisy data (a) measured and estimated data, (b) inverted model assuming homogenous starting model of 60 mS/m (the horizontal line depicts the maximum DOI).

Figure 8: Inversion results for the synthetic filtered data (a) measured and estimated data, and (b) inverted model assuming homogenous starting model of 60 mS/m (the horizontal line depicts the maximum DOI).

Figure 9: Inversion results of the Wyoming case study (a) surface soil changes along the surveyed line (b) measured data, (c) estimated, (d) inverted EM model assuming homogenous starting model of 75 mS/m (the horizontal black line depicts the maximum DOI, varies from ~ 6.3 m to 10 m), and (e) inverted seismic refraction model.

Figure 10: Inversion results of the Alaska research site (a) 3D laterally constrained inversion results, and (b) zoomed in resistivity model.

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Figure 11: Example inversion results of 2D EM and resistivity data sets (a) EM model, and (b) resistivity model. To enhance content clarity, a google earth KMZ file for this Figure is supplied as supplementary material to this paper.