Tensor Based Geology Preserving Reservoir Parameterization With Higher Order Singular Value Decomposition (HOSVD)

Sardar Afra^a, Eduardo Gildin^{b,*}

^a*Electrical and Computer Engineering Dept., Texas A&M University,
College Station, Texas, USA*

^b*Petroleum Engineering Dept., Texas A&M University,
College Station, Texas, USA*

Abstract

Parameter estimation through robust parameterization techniques has been addressed in many works associated with history matching and inverse problems. Reservoir models are in general complex, nonlinear, and large-scale with respect to the large number of states and unknown parameters. Thus, having a practical approach to replace the original set of highly correlated unknown parameters with non-correlated set of lower dimensionality, that captures the most significant features comparing to the original set, is of high importance. Furthermore, de-correlating system's parameters while keeping the geological description intact is critical to control the ill-posedness nature of such problems. We introduce the advantages of a new low dimensional parameterization approach for reservoir characterization applications utilizing multilinear algebra based techniques like higher order singular value decomposition (HOSVD). In tensor based approaches like HOSVD, 2D permeability images are treated as they are, i.e., the data structure is kept as it is, whereas in conventional dimensionality reduction algorithms like SVD data has to be vectorized. Hence, compared to classical methods, higher redundancy reduction with less information loss can be achieved through decreasing present redundancies in all dimensions. In other words, HOSVD approximation results in a better compact data representation with respect to least square sense and geological consistency in comparison with classical algorithms. We examined the performance of the proposed parameterization technique against SVD approach on the SPE10 benchmark reservoir model as well as synthetic channelized permeability maps to demonstrate the capability of the proposed method. Moreover, to acquire statistical consistency, we repeat all experiments for a set of 1000 unknown geological samples and

*Corresponding author
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Email addresses: afra@tamu.edu (Sardar Afra), eduardo.gildin@pe.tamu.edu
(Eduardo Gildin)

provide comparison using RMSE analysis. Results prove that, for a fixed compression ratio, the performance of the proposed approach outperforms that of conventional methods perceptually and in terms of least square measure.

Keywords:

Higher order SVD, Tensor based algorithms, Reservoir parameterization, History matching, Geology preserving parameterization.

1. Introduction

In reservoir management, the problems that one faces in controlling petroleum systems are very complex with respect to non-linearity and uncertainty in their models, which mathematically described by large-scale dynamical systems, at which fine scale discretization and massive computational machinery are very often required to perform simulation and optimization. Furthermore, porous media properties such as conductivity or permeability contain many small scales and uncertainties. For instance, in fractured media, the small scales can be much smaller compared to the field scales. It is computationally expensive to solve forward problems directly, and in particular, it is prohibitively expensive to solve many such forward problems as in the case of uncertainty quantification and controller design. Thus, it is of paramount interest to represent large scale and complex physical models of high dimensionality, e.g., reservoirs, with a reliable one of low dimensionality so that a better forecast of flow and transport within porous media can be obtained through forward reservoir simulation [1, 2, 3].

A very challenging problem in reservoir engineering is to estimate petrophysical properties of porous rock such as permeability and porosity, by means of reservoir production history. The dynamic system measurements cannot be forecasted employing static models and thus the process of adjusting geological properties to match the production data, i.e., history matching, became a routine practice [4, 2]. Having precise predictive reservoir model is of high importance to predict future production profile of reservoir. The main idea is to optimize reservoir production (e.g., hydrocarbons) by delaying water production and minimizing fluid injection during the reservoir life-cycle. This can be accomplished by setting up a model-based optimization and parameter estimators for proper account of uncertainties. Thus, providing a powerful reservoir characterization method significantly impact the amount of hydrocarbons extracted and, in turn, the volume of global reserves by increasing the recovery factor of conventional reservoirs by only

43 a small fraction. An intensive works have been done on the various aspects
44 of history matching and a solid library of problems and solutions has been
45 created [1].

46 Although, having a reliable and accurate reservoir model is highly tied
47 to providing a fine detailed representation of the reservoir, high resolution
48 model results in a large scale inverse problem both in the parameter space
49 (permeability, porosity) and state-space (pressures, saturations). Therefore,
50 it is of great importance to reparameterize and reduce the dimensionality of
51 the model to acquire efficiency in computational cost. Also, it is beneficial
52 to perform geologically consistent reservoir parameter adjustments as data
53 is being assimilated in the history matching process. This is a daunting
54 task to be performed as one is bounded by the number of data points, i.e.,
55 measurements, and very often the inverse problem has more unknowns that
56 can uniquely be estimated from the available data. To this end, having a
57 practical approach to reduce the number of reservoir parameters in order
58 to represent the reservoir model with one of lower dimensionality is of high
59 interest.

60 A number of studies has been conducted to reduce computational ef-
61 forts in reservoir simulation and history matching [5, 6, 7, 8], as well as
62 introducing new applications in multiple point geostatistics [9, 10, 11], and
63 reservoir characterization through integrated parameterization [2, 12]. The
64 methodologies basically worked in two fronts: reducing the number of states
65 using model reduction techniques, and reducing the number of unknown
66 parameters in a process known as parameterization [13, 14]. Among all
67 of these methods, a common approach that is applied to history match-
68 ing is proper orthogonal decomposition (POD)[15]. In POD, it is necessary
69 to carry out an eigen-decomposition of the random field covariance matrix
70 which is expensive for large models. Furthermore due to the vectorization
71 of the snapshots data in the POD computation, many features may be lost
72 in the reduced space [16, 17, 18].

73 Here, we introduce a tensor based approach by means of multi-linear
74 algebra, namely the higher order singular value decomposition (HOSVD)
75 [17, 18, 19, 20], to reduce high dimensionality of the reservoir property space.
76 HOSVD based techniques have been addressed in literature for many dif-
77 ferent applications including image processing [21, 22], face recognition [23],
78 data analysis [24, 25], among others. The application of HOSVD to flow
79 problems has been minimal due to the lack of fast and reliable tools for
80 the decomposition computations. In recent years, however, new tools and
81 algorithms have been developed that show promising results in large-scale
82 multilinear algebra computations [26, 27]. We treat property ensembles as

83 a high-dimensional tensor, and by means of tensor algebra (HOSVD) we
 84 show that the reduced models preserve better geometric features keeping
 85 them more intact during the reduced basis computations. This plays a piv-
 86 otal role in reservoir history matching as the properties in consideration
 87 have geological meaning. In other words, HOSVD has the ability to reduce
 88 the size of the model and reconstruct it accurately while keeping higher
 89 order statistical information and geological characteristics of the reservoir
 90 model. Conventional parameterization algorithms need to vectorize original
 91 replicates to obtain basis in process of dimensionality reduction which may
 92 results in losing geological consistency. In HOSVD, we take the snapshots
 93 in a 2D or 3D approach and stack them up into a tensor form and by ten-
 94 sor decomposition reduced bases are obtained. One of the advantages to
 95 the tensor based approaches is their capability to capture redundancy in
 96 all modes, i.e., dimensions, unlike classic SVD which only captures redun-
 97 dancy and noise in one dimension. Eager reader could refer to authors' other
 98 published work on integrating HOSVD based parameterization into history
 99 matching process [12].

100 **2. Methodology**

101 This section briefly states the parameterization problem and explains
 102 essential tools for performing permeability parameterization in the present
 103 work.

104 *2.1. Parameterization*

105 Reduction in the number of gridblocks in description of permeability in-
 106 formation is of central importance in history matching problems. Indeed,
 107 model reduction in this case can be achieved by reparameterizing the per-
 108 meability field or transforming it to a lower dimensional space by means
 109 of permeability parameterization. This can be analogous to treating the
 110 permeability field as an image compression problem. A map from space of
 111 correlated variables into an uncorrelated space of lower dimension is known
 112 as multilinear analysis. For image compression purposes, multilinear tech-
 113 niques are applied to a single known image and the compressed form of
 114 the image is obtained by truncating basis in the lower dimension space. A
 115 significant task in image processing procedures is to preserve important fea-
 116 tures of the image along with keeping best basis and achieving an efficient
 117 compression ratio.

118 Parameterization techniques have been studied in subsurface modeling
 119 since the 1960's in the form of zonation. In a broad sense, parameterization

can be divided into two main groups [14]: spatial and transform domain. In the spatial case, one aims at identifying spatial regions (zones) that can be thought as homogeneous pieces of the subsurface, and this can be assigned a single constant property during the inversion process. Although it seems logical to find such zones, problems with geologic discontinuities at the boundary of such regions make the implementation difficult. On the other hand, transform domain methods, can overcome these issues by means of introducing geological spatial correlations into the parameterization process. Several techniques have been developed in recent years [14], including the classical principal component analysis, discrete cosine transform (DCT), and the discrete wavelet transform. In all of these cases, the objective is to find an *optimal* basis selection that can explain the main variability in the parameter fields in a predefined norm. In this paper, we will address some of the shortcomings of the PCA approach.

In the parameterization by PCA case, one aims to reduce dimensionality of the parameter space, which defines the geological description of the reservoir, using the SVD operator. One of the main drawbacks of the SVD framework is the fact that both states and the geological parameters need to be vectorized for the computations of what is known as *the snapshot matrix*. This, in turn, leads to the loss of geological continuity when the parameters are reconstructed for the reservoir model simulation. We will address this issue by extending the projection framework using HOSVD techniques [20, 12], defined in the next section.

2.1.1. Higher Order SVD

One can decompose a $(I_1 \times I_2 \times \cdots \times I_N)$ -order tensor in the similar fashion as in the singular value decomposition for matrices. In fact, every $(I_1 \times I_2 \times \cdots \times I_N)$ -order tensor can be expressed as the following tensor product:

$$\mathcal{T} = \mathcal{C} \times_1 U^1 \times_2 U^2 \times_3 \cdots \times_N U^N \quad (1)$$

wherein, $\mathcal{C}$ is an *all-orthogonal* and ordered tensor referred to as *core tensor*, which represents coefficients matrix in regular SVD, and $U^{(n)}$ s are the bases (factor) matrices (which are usually orthogonal) and can be thought of as the principal components in each mode. Moreover, one could derive a matrix representation form of HOSVD as follow:

$$\mathcal{T} = U^n \cdot C_n \cdot (U^{n+1} \diamond U^{n+2} \diamond \cdots \diamond U^N \diamond U^1 \diamond U^2 \diamond \cdots \diamond U^{n-1}) \quad (2)$$

in which $\diamond$ denotes Kronecker product of two matrices. Due to the *orthogonality* property, HOSVD is essentially unique. In summary, computing

146 HOSVD of a tensor is basically given by Eq. 2. As a matter of fact, the
 147 n -mode singular matrix U^n can simply be found by calculating the left sin-
 148 gular matrix of unfolded form of $\mathcal{T}$. Thus, computing HOSVD is equivalent
 149 to reapplying SVD, N times. Eventually, using the Eq. 2, one could simply
 150 compute the core tensor $\mathcal{S}$.

151 2.1.2. Truncated HOSVD

Making the analogy of the approximation of a matrix by one of lower rank using SVD, one can come up with a truncated HOSVD in similar fashion. Indeed, in order to reduce the order of a given tensor, one could select only those basis that conserve more energy in each mode, which always are corresponded to the larger singular values in SVD. Mathematically, given the tensor $\mathcal{T} \in R^{I_1 \times I_2 \times \dots \times I_N}$, we seek to find

$$\min_{\hat{\mathcal{T}}} \|\mathcal{T} - \hat{\mathcal{T}}\|_F^2. \quad (3)$$

152 wherein, $\hat{\mathcal{T}} \in R^{I_1 \times I_2 \times \dots \times I_N}$ is the approximated tensor of the order of $(r_1, r_2, \dots, r_N)$
 153 which $r_i < I_i$. Therefore, the problem of finding the larger *mode- n* singular
 154 values of a tensor turns into the problem of finding the best lower rank ap-
 155 proximation of a tensor which is a optimization problem. Unlike the regular
 156 SVD, this problem does not have a closed form solution. A very common ap-
 157 proach to obtain a truncated HOSVD is to use the alternating least squares
 158 (ALS) method, proposed by Kroonenberg and de Leeuw [28, 26]. [Algorithm](#)
 159 [1](#) provides a detailed and step by step illustration of the procedure that has
 160 been developed in the present work. Figure 1 depicts HOSVD algorithm
 161 pipeline as well.

162 3. Experiment Description

163 Having a reliable compression method is paramount in history match-
 164 ing. However, it is not sufficient since, as it was mentioned previously, the
 165 permeability image in reservoir history matching problem is unknown. The
 166 other important characteristic of a useful compression method for reservoir
 167 history matching is the capability of the basis to reconstruct an unknown
 168 image with optimum coefficients. In the present experiments, the basis are
 169 obtained using prior knowledge of the reservoir permeability. In other words,
 170 we use a set of training samples to compute the required set of basis that
 171 is necessary for reconstruction and estimation. Using the basis from the
 172 reduced dimension space allow us to do both estimation and dimensional-
 173 ity reduction of the unknown permeability maps at the same time. Here,

Algorithm 1 Re-Parameterization Through HOSVD

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1: procedure 1. TRAINING AND TEST SAMPLE GENERATOR
2:   Initialization;
3:   Load Static data and geological continuity characteristics (e.g., variogram etc.)
4:   Two-point geostatistical permeability maps → call SGSIM subroutine;
5:   Multiple-point geostatistical permeability maps → call SNESIM subroutine;
6: procedure 2. PARAMETERIZATION
7:   MATLAB code:  $[coreTensor, Basis] = \text{hosvd\_als}(permMaps, reducedRank)$ 
8:   INPUT:  $m$ -order tensor  $\mathcal{T}$ 
9:   OUTPUT: Core tensor  $\mathcal{C}$  and matrices  $\mathbf{U}^1, \mathbf{U}^2, \dots, \mathbf{U}^M$ 
10:  Initiate  $\mathbf{U}_0^1, \mathbf{U}_0^2, \dots, \mathbf{U}_0^M$ ;
11:  while not converged do
12:     $\hat{\mathbf{U}}_i^m = \mathcal{T} \times_1 \mathbf{U}_{i-1}^1 \times \dots \times_{m-1} \mathbf{U}_{i-1}^{m-1} \times_{m+1} \mathbf{U}_{i-1}^{m+1} \times \dots \times_M \mathbf{U}_{i-1}^M$ ;
13:     $\max_{\mathbf{U}^m \in R^{J_m \times R_m}} f \equiv \|\mathcal{U}_i^m \times_m \mathbf{U}^{mH}\|$ ;
14:     $\gamma_i^m = f\left((\mathcal{U}_i^m)_{max}\right)$ ;
15:     $\mathcal{U}_i^m = (\mathcal{U}_i^m)_{max}$ ;
16:  end while
17: procedure 3. MAIN-FORWARD MODEL SIMULATION
18:  Compute space basis using training set → call procedure 2: HOSVD(.)
19:  for Test Set do
20:    Generate parameterized maps using basis computed in step 17
21:    Perform forward reservoir simulation using the parameterized maps
22:    True permeability maps
23:    Re-parameterized permeability maps
24:    Calculate dimensionless RMSE using production data (WPR, OPR and BHP)
25:  end for
26:  Generate RMSE's probability density function (PDF) curves

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Figure 1: HOSVD parameterization algorithm workflow.

we repeated the experiments for all samples to validate our results and the corresponding *root mean square error of the probability density functions*, namely RMSE of PDFs are discussed later. Furthermore, an appropriate permeability parameterization and compression method not only has to yield consistent permeability maps but also should result in the similar response of the highly nonlinear and complex reservoir system.

3.1. Statistical analysis

To analyze the reconstruction performance of the SVD and HOSVD we calculate the observation root mean square error (RMSE) of each experiment. RMSE of j^{th} observation (each well's BHP, oil or water rate) is defined:

$$RMSE_j = \sqrt{\frac{1}{N} \sum_{i=1}^N \left(\hat{d}_j^i - d_j^i \right)^2} \quad (4)$$

181 where i is the time step index. Also, the original and estimated observation
 182 are represented by d and $\hat{d}$, respectively. In our computations, we do not
 183 subtract the mean of the snapshots. It has been shown that this may not
 184 make a difference in the case of history matching [13]. Also, we work with
 185 the $\ln(K)$, natural logarithm of the permeabilities, and thus no negative
 186 values of permeability is possible in our decomposition.

To represent a single observation RMSE for each experiment simplifies comparison of the permeability reconstruction performance of SVD and HOSVD. Since observation RMSEs have different units, we need to make dimensionless RMSEs. To achieve this purpose we utilized the original observation's L^2 norm. Finally, we define the total dimensionless observation RMSE as follows:

$$RMSE_{total} = \sum_j \frac{RMSE_j}{\|d_j\|_2} \quad (5)$$

187 3.2. Note on the comparison between HOSVD and SVD

188 Given an ensembles of training images A consists of I_3 images of size
 189 $I_1 \times I_2$, one can reduce the size of original space to a lower rank utilizing
 190 the most significant principal components of the original space. Employing
 191 the first k singular values, representation of the data using PCA consists of
 192 k eigenvectors $\Phi \in \Re^{I_1 I_2 \times k}$ along with reduced representations $P = \Re^{I_3 \times k}$
 193 and consequently the number of scalars required to represent the data is
 194 $k(I_1 I_2 + I_3)$.

195 For HOSVD approximation, a tensor $\mathcal{T} \in \Re^{I_1 \times I_2 \times I_3}$ is estimated and
 196 by a core tensor $\mathcal{S} \in \Re^{R_1 \times R_2 \times R_3}$ and three basis matrices, $U^{(1)} \in \Re^{I_1 \times R_1}$,
 197 $U^{(2)} \in \Re^{I_2 \times R_2}$ and $U^{(3)} \in \Re^{I_3 \times R_3}$. Then $R_1 R_2 R_3 + I_1 R_1 + I_2 R_2 + R_3 I_3$
 198 scalars required to represent the reduced space employing HOSVD.

In order to conduct a fair performance comparison between SVD and HOSVD, we adopted compression ratio criterion defined as follows:

$$\beta = \frac{\alpha}{I_1 \times I_2 \times I_3} \quad (6)$$

199 where α is the number of scalars required for representing of the ensemble of
 200 images. Table 1 summarizes the discussion for different algorithms. For the
 201 sake of simplicity and without loss of generality, we assume $R_1 = R_2 = R$.

Table 1: Comparison between PCA and HOSVD in data representation factors

Method	Decomposition	# of scalars
HOSVD	$\hat{\mathcal{T}} = \hat{\mathcal{S}} \times_1 U^{(1)} \times_2 U^{(2)} \times_3 U^{(3)}$	$R_3(R^2 + I_3) + R(I_1 + I_2)$
PCA	$\hat{A}_t = \Phi.P$	$k(I_1 I_2 + I_3)$

After applying projection the best reduced ranked approximation, e.g., (R, R, γ) -rank, of the original tensor is computed. The value of γ is calculated so that the constant compression ratio assumption is fulfilled.

4. Results and discussion

To explore HOSVD-based parameterization properties and evaluate its capability to represent the feature space though exploiting redundancies in all modes, a comprehensive performance analysis is conducted for two different sets of experiments. All results have been compared to those of classic SVD with respect to RMSE measure and for a fixed compression ratio.

4.1. Experiment 1: Synthetic model

We apply tensor decomposition approach with ensemble of 2D permeability fields to illustrate how this method can provide low-dimensional geologically realistic permeability-field parameterizations suitable for history matching. A synthetic model is generated utilizing multiple-point geostatistical simulation algorithm, namely the single normal equation simulation (SNESIM)[29]. A training image is selected to draw fluvial channel replicates. The training image 2(a) along with two generated samples are illustrated in figure 2. This training image has $470 \times 470 \times 1$ pixels and contains two facies types, the first facies is low permeability as a background and the second one is the high permeability channels. The low permeability value is 500md while the high permeability value is 10,000 md. A training set, consists of permeability realizations, of size 1000 is generated to represent the geological structure. Each realization contains 45×45 grid blocks. Different shapes and geometry of the channel in these realizations are the major uncertainty in the unknown permeability field.

Each permeability realization is a 45×45 map of $30 \times 30 \times 30$ feet grid block size and the reservoir assumed to act as a 2D two-phase fluid flow problem. A water flooding scenario similar to [13, 30] is utilized here to simulate the fluid flow problem. Forward simulations are performed using

232 ECLIPSE [31]. Overall, we conducted three different sets of forward simu-
 233 lations. In the first setup the ensemble of original permeability samples are
 234 used as an input to the simulator. The other two setups are designed and
 235 implemented to feed simulator with ensembles of reduced order permeabil-
 236 ity samples, using SVD and HOSVD respectively, as input. As discussed
 237 before, since we utilized 2-*fold* CV with no repetition, thus, we performed
 238 three sets of 4×1000 forward reservoir simulations to complete error estima-
 239 tion analysis. A randomly drawn permeability realizations along with cor-
 240 responding reduced ordered representatives using classic SVD and HOSVD
 241 are shown in Fig. 2(b) and 2(c). All 45 producers are operating with a con-
 242 stant bottom-hole pressure (BHP) of 2900 psi and horizontal wells with 45
 243 ports is considered injecting water with a constant injection rate to secure
 244 a total of 1 pore volume of water is injected through the simulation time
 245 of 1 year. As stated previously, porosity is not parameterized in this work
 246 and therefore is fixed at 0.22. Table 2 summarizes all information regarding
 247 simulation description and reservoir specifications.

248 We applied HOSVD-ALS algorithm to permeability replicates as a train-
 249 ing set to find the basis function employed to reduce the dimension of the
 250 original images' space for compression purposes. First, HOSVD-ALS is
 251 utilized to provide truncated basis matrices and a coefficient core tensor.
 252 Based on the HOSVD-ALS approach, the reduction procedure requires the
 253 selection of basis which express the map with higher accuracy, e.g., those
 254 which capture more significant features. Here, the following scenario is per-
 255 formed to obtain re-parameterized permeability representations using both
 256 SVD and HOSVD. In parameterization with the classic SVD, all samples
 257 are vectorized and stacked into a matrix namely the training set, and SVD
 258 is performed to find the space basis required for the reconstruction step.
 259 Then, the singular vectors associated with the largest 55 singular values of
 260 the covariance matrix, which conserve the most of the energy, are selected
 261 as reduced order space's basis. In the SVD case we used those selected
 262 basis to compute reduced order representation of the original permeability
 263 maps to reduce the dimensionality of original space. This is not the case
 264 in the HOSVD, as we keep the snapshots *as it is*. For the HOSVD param-
 265 eterization, the best $(15, 15, \gamma)$ -*rank* representation of the original tensor
 266 is approximated and then Kronecker product is performed in all simula-
 267 tions to compute required coefficients for the reconstruction step. Utilizing
 268 truncated basis results in a compressed reconstructed version of known per-
 269 meability maps. As mentioned before, all experiments are run for a fixed
 270 compression ratio. To conduct a fair comparison between classic SVD and
 271 HOSVD, γ is calculated as 135 to satisfy fixed compression ratio of 0.0822.

272 Figures 2(b) and 2(c) also show that HOSVD not only outperforms clas-
 273 sic SVD in the measure of RMSE, but also overcomes its representation
 274 perceptually, i.e., if we consider permeability realizations as images. One
 275 can see that HOSVD-based parameterization is able to better capture the
 276 high and low permeability edges compared to the SVD-based method. It
 277 is evident from the results that HOSVD-ALS has a promising power to
 278 compress an image while preserving geological features along with spacial
 279 characteristics of permeability maps. In addition to preserving the most im-
 280 portant features, the low computational cost is the other significant property
 281 of HOSVD. A number of intensive studies in machine learning applications
 282 have addressed the computational cost of tensor based algorithms and many
 283 efficient techniques, i.e., in computational cost and memory sense, to com-
 284 pute *Rank-R* approximation of a tensor [18, 32, 25, 21]. In fact, in practical
 285 reservoir parameterization problems with system of order of millions, low
 286 computational cost and the ability to reduce to a lower rank become im-
 287 portant. Eager reader can refer to [19, 20] for more detailed analysis and
 288 discussion about the capabilities of HOSVD parameterization.

289 Comparing oil production rate curves in Figure 3(a) illustrates the ad-
 290 vantage of using HOSVD over the classic SVD even more particularly once
 291 the SVD response fails to follow the trend of original response. Figure 3(b)
 292 presents water production rates. Interestingly, the SVD based parameter-
 293 ization causes an earlier breakthrough comparing to the original response
 294 and that of HOSVD.

295 An important characteristic of a powerful compression method for reser-
 296 voir history matching is the capability of the basis to reconstruct an unknown
 297 image with optimum coefficients. In the present experiments, basis are ob-
 298 tained using prior knowledge of reservoir permeability. In other words, we
 299 use a set of training samples to compute required set of basis that is necessary
 300 for reconstruction and estimation part. As mentioned before, we repeated
 301 this procedure for all samples utilizing *2-fold* CV with no repetition, also
 302 known as holdout method, to validate our results and corresponding RMSE
 303 PDFs are discussed in the next section. Here, we presented a randomly
 304 drawn samples to compare the results. The reader can refer to [33] for more
 305 details on cross validation techniques.

306 In order to statistically demonstrate the advantage of HOSVD over SVD
 307 we applied both methods to a set of size 2000 including permeability real-
 308 izations. In each step of cross validation error estimation, a set of size 1000
 309 samples is left out to be used as the test set. In order to reduce the vari-
 310 ability associated with the cross validation method, we repeated this folding
 311 process 10 times with different folds. We also repeated the forward reservoir

312 simulation for all permeability samples within the training and test set to
 313 statistically compare the performance of HOSVD-based and classic SVD-
 314 based parameterization in reproducing the original response of the system.
 315 Figure 4(a) which includes permeability RMSE PDFs clearly shows that
 316 HOSVD results in considerably less reconstruction error than SVD in the
 317 parameterization process. This convey the fact that HOSVD-ALS has a
 318 promising power to compress a permeability image while preserving geo-
 319 logical features along with spatial characteristics of the permeability map.
 320 Figure 4(b) express the probability density function of total dimensionless
 321 observation RMSE using the results of all forward simulations. Correspond-
 322 ing statistics are concluded in table 3.

Figure 2: Reconstructed multiple point statistics samples utilizing reduced order basis (extracted features) through HOSVD and SVD: (a) Training image used to generate 1000 unconditional realizations; (b) The left column represents true samples. The middle shows reconstructed samples employing truncated HOSVD best (15,15)-rank approximation. The right shows reconstructed samples employing truncated SVD for $k = 55$; (c) The left represents true samples. The middle shows reconstructed samples employing truncated HOSVD best (5,5)-rank approximation. The right shows reconstructed samples employing truncated SVD for $k = 5$.

Figure 3: Production curves for synthetic 45×45 Example. System responses are plotted for original map as well as classic SVD and HOSVD based parameterized maps: (a) Oil production rate curves of all producing wells. ; (b) Water production rate curves of all producing wells for the second sample.

4.2. Experiment 2: SPE10 model

To prove the advantages of the proposed parameterization method, we employ the top 5 layers of SPE10 benchmark [34]. A two-point geostatistical simulation algorithm is utilized to generate 1000 permeability realizations. Here, to represent the geological structure, we incorporated patterns from top 5 layers of SPE10 to an exponential variogram model with correlation length properties equal to 100 and 80 in the x and y directions, respectively. Moreover, a proper permeability covariance matrix is constructed in order to assure each map includes the desired permeability range in the top 5 layers of SPE10 model. Each permeability realization is a $60 \times 220 \times 5$ map of $33 \times 33 \times 33$ feet grid block size. We run ECLIPSE reservoir simulator for a 3D two-phase (oil and water) black oil during 360 days or 6 time intervals of 60 days. All the simulation setup and parameterization procedure are done in a similar fashion to the synthetic example. Again, we conducted three distinct sets of forward simulation experiments for original, classic SVD, and HOSVD parameterized maps. A randomly drawn true permeability realization along with corresponding reduced ordered representative using classic SVD and HOSVD is shown in Fig. 5. Eight producers are operating along with 15 injectors constructing a network of eight inverted 5 spot configuration all in a waterflooding scenario. As stated previously, porosity is not reparameterized in this work and therefore assumed to be distributed uniformly and fixed at 0.22. Furthermore, 1 pore volume of water is injected through the simulation time of 1 year.

Figure 4: Probability density functions (PDF) of expected error, i.e., a training-set based error estimation like cross validation used for synthetic 45×45 Example: (a)PDFs of permeability reconstruction RMSE for SVD and HOSVD; (b)PDFs of total dimensionless observation RMSE for SVD and HOSVD.

Figures 5–8 summarize all results for a randomly picked replicate out of SPE10 ensembles. We plotted bottom hole pressure curves for 9 selected injectors as well as oil and water production rates for 6 producers for the sake of simplicity to compare results and values for the selected samples. Similar to the synthetic example, Figure 5 illustrates that HOSVD not only outperforms SVD in terms of permeability RMSE, but also overcomes it in perceptual sense. Moreover, the capability of reducing rank in all modes is assisting HOSVD to become more efficient than SVD in the manner of memory and computational costs as well even more than the previous example due to the higher dimension nature of the second example. Figure 6 shows the bottom hole pressure curves for the original samples and corresponding HOSVD-based and SVD-based reparameterize versions. In all injectors the corresponding results of HOSVD-based parameterization tends to follow the trend of original response both in shape and values. One may see that in injector number 3, 12 and 13 the HOSVD response perfectly fits to the true response. In fact, it is obvious from the results that utilizing HOSVD-based parameterization not only result in better reduced order versions perceptually and in terms of RMSE, but also produces similar responses to that of original maps which is more important in our purpose.

Oil production rates are depicted in figure 7(a). One can simply compare the HOSVD method results with that of SVD method. Again, it is apparent from the results that even for the simpler permeability map with a non-complex trend, HOSVD-based parameterization curves outperforms those of classic SVD parameterization. For instance, in the producers 1,

Table 2: Reservoir description and fluid properties for experiments 1 and 2.

Property	45x45	SPE10
Simulation	Two-phase	Two-phase
Simulation Time-day	360	360
Grid blocks	45×45	$60 \times 220 \times 5$
Grid block-size-ft	$30 \times 30 \times 30$	$33 \times 33 \times 33$
Porosity- <i>uniform</i>	0.22	0.22
Rock compressibility- psi^{-1}	$3.0E - 6$	$3.0E - 6$
Geometry	2D	3D
Depth-ft	2000	4000
Injectors	1	15
Producers	4	8
Initial reservoir pressure- psi	3100	6000
Producers constant BHP- psi	2900	5900
Water saturation	0.3	0.3

2 and 7, the SVD-based results are way off in values and trend comparing to those of original and HOSVD-based. Figure 7(b) displays water production rate curves as well. In contrast to the first example, we do not observe breakthrough in any of the producers during the simulation period. Although, no breakthrough happened in the experiments, the difference between HOSVD and classic SVD is noticeable specifically for more complex trends in permeability maps.

In order to statistically demonstrate the advantage of HOSVD over SVD we applied both methods to 2000 permeability samples employing 2-fold CV with no repetition. All experiments are run for a fixed compression ratio. To conduct a fair comparison between classic SVD and HOSVD, γ is calculated as 634 to satisfy fixed compression ratio of 0.0592. In fact, the best (15, 15, 634)—rank approximation of the tensor is computed through HOSVD-ALS and the 55 most significant singular values are picked for regular SVD. Figure 8(a) which includes permeability RMSE PDFs, clearly shows that HOSVD results in considerably less reconstruction error than SVD in parameterization process. This conveys the fact that HOSVD-ALS has a promising power to compress an image while preserving geological features along with spacial characteristics of permeability map. Figure 8(b) express the probability density function of total dimensionless observation RMSE using the results of all forward simulations. It should be noted that 90% of total dimensionless observation RMSE values lay within the shown

Table 3: Error distribution statistical parameters summary for the experiments.

	Method	Mean	Median	Mode
45x45	HOSVD	10.53	0.1245	0.116
	Classic SVD	57.1368	0.6482	0.5291
SPE10	HOSVD	0.3437	0.3274	0.3232
	Classic SVD	0.7382	0.7204	0.6873

interval in Fig. 8 and 4. The statistics of the probability density functions
corresponding to the SPE10 example summarized in table 3.

Figure 5: First permeability sample along with its reconstructed versions by SVD and HOSVD. SPE10 example.

4.3. Running Time Comparison

Comparison between classic SVD and HOSVD with respect to running time is of high importance in order to evaluate their performance as well as their efficiency. As mentioned previously, we generate two ensemble of 1000 permeability realizations employing two-point geostatistical simulation algorithm (SGSIM) and an exponential variogram with specific assumed covariance and correlation parameters. It should be noted that variogram

Figure 6: Bottom hole pressure curves of all injectors for the first sample. System responses are plotted for original map, classic SVD and HOSVD based parameterized maps.

401 properties are set independently for 45×45 and SPE10 cases. The below
 402 results are generated as follows. All 1000 samples in each ensembles are
 403 considered as training sets of known permeability maps. Then, we apply
 404 classic SVD and HOSVD based parameterization method to the ensembles
 405 and observe running time of each scheme first for computing basis and coef-
 406 ficients of the parameter space and second for compression process (reducing
 407 the order of each map). The results are summarized in the table 4. One
 408 must note that for SPE10 example, we repeated a similar fashion for each
 409 layer of every realizations separately and independently. Then the mean of
 410 all running times is reported to acquire statistical consistency. Results show
 411 that HOSVD method outperforms classic SVD with respect to running time
 412 when computing the basis and coefficients as well as reconstructing step.

Table 4: Running Time Comparison between HOSVD and classic SVD (in seconds).

Method	Computing Time	Reconstructing Time
HOSVD	2.0639	37.1983
Classic SVD	39.5381	456.4611

413 5. Conclusion

414 Reducing the dimensionality of parameter space, e.g., the space of per-
 415 meability or porosity realizations, is of central interest in reservoir history

416 matching and simulation specifically when the problem is of very high or-
 417 der (millions of grid blocks). A high reduction in size of parameter space,
 418 decreases the number of unknown geological properties that have to be es-
 419 timated through the reservoir history matching procedure. Furthermore, a
 420 daunting task in reservoir characterization is to choose the principal features
 421 of the parameter space, i.e., those that conserve most of the energy, in order
 422 to improve the estimation. One must note that a powerful parameterization
 423 method not only provides better basis, but also does so in a time efficient
 424 manner. To this end, having a powerful parameterization method is of high
 425 importance in reservoir simulation and inverse problems.

426 The results of the present work illuminate the promising power of HOSVD
 427 to reparameterize and capture the important geological features of the per-
 428 meability realizations. Moreover, as a powerful parameterization and com-
 429 pression method, HOSVD regenerates similar petrophysical property rep-
 430 resentations, e.g., permeability. It also reproduces same response of the
 431 highly nonlinear and complex reservoir system. In other words, the pro-
 432 posed approach employed in this work can strongly capture all important
 433 spatial features and all spatial geological characteristics in an efficient man-
 434 ner with respect to the size of low dimensionality representations space,
 435 running time and performance perspectives, comparing to regular SVD as
 436 failed to preserve spatial features due to vectorization. Indeed, HOSVD re-
 437 parameterization offers a more consistent, e.g., geologically, reduced order
 438 representation of the parameter space as one can observe from the RMSE
 439 plots. Also, to acquire statistical consistency, all experiments were repeated
 440 utilizing 2-fold CV with no repetition. Comparing running time, proves that
 441 HOSVD overcomes classic SVD in running time as well. One must note that
 442 the provided results can easily be reproduced for the case of porosity re-
 443 parameterization. To conclude, the present work indicates that HOSVD
 444 provides a promising tool for permeability parameterization in reservoir
 445 characterization required for history matching processes.

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(a)

(b)

Figure 7: Production curves for SPE10 Example. System responses are plotted for original map as well as classic SVD and HOSVD based parameterized maps: (a) Oil production rate curves of all producing wells. ; (b) Water production rate curves of all producing wells for the second sample.

Figure 8: Probability density functions (PDF) of expected error, i.e., a training-set based error estimation like cross validation used for SPE10 Example: (a)PDFs of permeability reconstruction RMSE for SVD and HOSVD; (b)PDFs of total dimensionless observation RMSE for SVD and HOSVD.

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