Quality Assessment of Volume Compression Approaches Using Isovalue Clustering

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Abstract

Visualization of volumetric data has been widely used for exploration of data from scientific simulations and biomedical imaging. Despite advances of GPU-assisted rendering, which has become the state-of-art in direct volume rendering, still many volumetric data sets are too large to be visualized interactively. Therefore, compression-domain rendering approaches are used in visualization processes in order to reduce the amount of data sent to the GPU and thus speed up interactive visualization. Hence, reliable tools to assess the quality of the reconstructed 3D data are of great importance, influencing the effectiveness of the visualization. However, numerical error analysis approaches such as mean-squared-based metrics are often inconsistent with perceived visual quality. We propose a structural volume quality assessment approach for 3D scalar volume based on the human visual system (HVS). Our approach consists of two stages: First, we provide an interactive tool for extracting significant volume features via isosurfaces from a 3D scalar field using an isovalue classification process. Second, we propose a structural volume quality assessment (VQA) metric that employs representative isosurfaces as benchmark structures. For this purpose, we use a recently developed perceptual-based mesh quality metric [1] to assess the visual quality of compressed 3D scalar fields. Our experiments on a number of benchmark data sets suggest that, compared to existing methods, the proposed isovalue classification approach provides a more distinct set of isosurfaces that are more representative of the complexity of the data sets. We examine a number of widely used compression approaches, namely, discrete wavelet transform, discrete cosine transform, and tensor approximation, to establish the utility of our volume quality assessment approach.

Keywords: Volume quality assessment, volume rendering, isosurfaces, volume compression approaches, isovalue clustering.

1. Introduction

Volume visualization has become an indispensable tool in volumetric data analysis and continues to facilitate discovery in a broad range of applications from scientific computing (e.g., simulations) to biomedical imaging (e.g., CT and MR). Direct visualization algorithms make use of a user-defined transfer function to assign color and opacity to the scalar field data while indirect visualization requires the user to provide specific data values for extracting and rendering an isosurface. In both paradigms, the user's input directly impacts the features selected for visualization.

Direct Volume Rendering (DVR) [2] approaches allow scientists to visually inspect and explore a volume by simulating emission, absorption, and scattering of light as it travels through the volume rather than explicitly extract geometric surfaces from the data. In visualization of large-scale data sets, DVR approaches are often limited by the memory and computing resources needed for rendering; hence, compression approaches are commonly integrated to the visualization process [3].

Compression approaches use compact data representations, which require fewer bits for encoding than what is needed for the original volume. To generate a compact representation, a routine decomposes a data set through a mathematical framework or learned model into a set of *bases* and *coefficients*, which

are usually less error-prone to data reduction. Lossless algorithms are those that ensure maximum accuracy in the reconstruction and guarantee zero error. Lossy algorithms produce an *approximation* of the reconstructed volume up to a tolerated approximation error. Compact representations are often initially lossless and *dense*. However, higher compression ratios can be achieved by thresholding or encoding the coefficients. A recent example for an advanced encoding strategy has been proposed with fixed-rate compression of floating point coefficients [4]. Since data reduction becomes an essential step in large-scale data visualization, as recently shown with lossy compression of seismic tomography [5], reliable tools to assess the quality of the reconstructed 3D data are of great importance. The quality assessment of the compressed data directly affects the chosen compression rates and influences the usability and effectiveness of the visualization.

The volumetric approximation quality is either measured in the reconstructed data set, i.e., in the *data domain*, or in the aggregated final 2D visualization, i.e., in the *image domain*. In case of volume visualization, the approximation quality is usually measured numerically in the data domain in terms of *rate-distortion*, which is a ratio between the achieved approximation quality and the data reduction level. Frequently, the approximation quality is measured using the mean-squared error (MSE) between the original and the approximation. However,

the MSE measures a distortion rather than the perceived visual quality. The limitations of the MSE have been noted in image processing [6], mesh processing [7], as well as for comparing 3D compression approaches [8].

One important characteristic of volume rendering is that only subsets of the volumetric data, i.e., features of interest, are selected and visualized in the final image [9]. A volumetric data set contains a large number of isosurfaces at different isovalues of the scalar field. In contrast, the volumetric structure is only characterized by a finite number of representative isosurfaces that segment the data into important structures. The reconstruction quality of these representative isosurfaces directly portrays the visualization of compressed data. For a similar reason, Ljung et al. [10] proposed to evaluate the distortion of compressed data in the rendered images since the distortion of the reconstructed (whole) volume has a reduced correspondence to the final image. That is, large subsets of the volume will give little or no contribution to the rendering.

In our approach, we propose to directly assess the quality of the reconstructed volume in the *data-domain*. Instead of conducting quality assessment on the whole volume, we leverage the characteristics of volume rendering and use representative isosurfaces as benchmark structures to evaluate the visual quality of compressed 3D scalar fields. Specifically, we propose a two-stage approach to conduct volume quality assessment (VQA). First, we cluster isovalues of a given data set and identify its representative isosurfaces (see Fig. 1(a)). Second, we propose a structural VQA metric that examines the quality of compressed volumetric data by evaluating its representative isosurfaces using the tensor-based perceptual distance measure (TPDM) [1] that simulates human visual system (HVS) (see Fig. 1(b)).

The paper is organized as follows. In Section 3.1, we discuss the choices of candidate isosurface similarity metrics for our proposed isovalue classification algorithm, presented in Section 3.2. In Section 3.3, a set of experiments analyzing our proposed approach on simulation and real data sets is conducted. Finally, in Section 4, we demonstrate a perceptual VQA metric using representative isosurfaces to assess the visual quality of compressed 3D scalar fields.

2. Related Work

2.1. Salient isovalues

Bruckner and Möller [11] classified volumes by defining a similarity metric for the analysis of salient isovalues of a given 3D scalar field data set. The similarity between two specific isosurfaces is measured by the mutual information between their respective distance transforms (viewed as probability density functions). Their isosurface similarity map (ISM) provides the similarity information for any pair of isosurfaces present in a given data set. Based on the ISM, they apply a recursive priority queue algorithm to identify the most representative isosurface, i.e., the isosurface that is most similar to all other isosurfaces. The efficacy of the distance transform for accurately representing the geometry of an isosurface highly depends on

its sampled resolution and accurate reconstruction of the geometry from the distance transform is a challenging task [12]. Moreover, the values of the distance transform that are relevant to detailed surface geometry are only in the proximity of the isosurface. Therefore, geometry-relevant information are not well represented in a global measure (e.g., histogram) of the distance transform.

2.2. Compression-domain DVR

Numerous approaches have been proposed in compression-domain DVR, as can be looked up in [3]. Examples of compression approaches using *pre-defined bases* are the *discrete Fourier transform* (DFT) [13], the *discrete cosine transform* (DCT) [14], and the *discrete wavelet transform* (DWT) [15]. The DFT decomposes the data into the frequency domain by using sine and cosine waves as basis. DFT is known to be receptive to periodic structures. DCT represents a simplified version of the DFT as it uses only cosine waves and outputs only real values. The DWT is decomposing the data with more complex functions not only into the frequency domain, but also gives access to the spatial domain. Traditionally, the DWT is applied in a multi-resolution fashion, what makes the DWT well-fitted and a popular approach for multi-resolution visualization applications.

Compression approaches using *learned bases* are *dictionary* elements such as vector quantization (VQ) [16] or sparse coding (SC) [17], and *tensor approximation* (TA) [18]. VQ learns dictionary elements, i.e., small-sized volumes, that can be used to represent a scalar field through a lookup table. In SC, the general idea of VQ is further developed which uses linear combinations of dictionary elements to represent a data set. TA is basically a higher-order principal component analysis that statistically evaluates the major data components and represents them in vector bases that are mapped to a volume through coefficients. The learned bases approaches learn a compact structure of the data set by minimize the energy within a data set such that as few coefficients as possible are needed. In our work, we propose a structural VQA metric to compare those compression approaches.

2.3. Quality assessment of compressed data

MSE is a commonly used technique for numerical error analysis which is often limited by its inconsistency with perceived visual quality. In image processing, the limitations of MSE-like error analysis are addressed by designing metrics that simulate the HVS more closely. The HVS is sensitive to visual signals such as luminance, contrast, frequency content, or interactions between different signal components, what is modeled with the *contrast sensitivity function* (CFS). On this basis, the Structural Similarity (SSIM) index [19], triggered numerous image quality assessment (IQA) approaches and a review of the approaches based on SSIM is gathered in [20].

The most recent IQA tools focus on contour-based approaches [21, 22, 23, 24], which are motivated by the fact that the HVS can detect the significant underlying features, even in lower-quality images. These approaches can be of particular

Figure 1

value when working with lossy data, where only approximations are available, but features that are relevant to the target visualization need to be preserved. The advances in IQA stress the usefulness of tools that simulate the HVS in order to avoid cumbersome evaluation by a user/expert for every single image.

To the best of our knowledge, structural VQA, analogous to IQA, have not yet been proposed for 3D scalar field data. To fill this gap, we propose a VQA metric that uses significant benchmark structures representing relevant data features to assess the visual quality of compressed 3D scalar fields. To achieve this goal, we rely on algorithms that classify a 3D volumetric data set into relevant distinct structures, represented with isosurfaces.

The closest work to VQA is the one on saliency maps [25], a model to help users find salient and most relevant areas in 2D or 3D visualizations. In contrast to our proposed work, their output is a visual 2D map rather than an aggregated numerical value as tool output. Other methods treat data reductions as sources of uncertainty, and quantifying the impact of uncertainty has been studied in the context of isosurface visualization [26, 27] as well as DVR [28].

3. Isovvalue Classification

3.1. Analysis of isosurface similarities

An isosurface (level-set), S_α , of a trivariate function f is the locus of all points $\mathbf{x} = (x, y, z)$ in space where f attains the value α :

$$S_\alpha = \{\mathbf{x} \in \mathbb{R}^3 | f(\mathbf{x}) = \alpha\} \quad (1)$$

Given samples of f on a Cartesian grid, isosurface extraction algorithms such as the Marching Cubes [29] produce a triangulation that approximates S_α . The isovalues of interest for a given data set depend on the range of f . They are usually a finite set of isovalues selected depending on the quantization level of the data. For instance, for byte valued data sets, the range of isovalues is typically $\mathcal{I} = [0, 255]$; however, non-integer isovalues may be of interest for a particular data set. Often many isosurfaces of a given data set are similar and exhibit the same features; therefore, we are often only interested in a small subset of all isovalues that represents the diversity of the features present in the data set.

In order to obtain a representative set of isovalues, we first identify whether two given isovalues correspond to surfaces

(extracted using Marching Cubes algorithm) that represent similar features from the underlying volumetric data. Such two isovalues form a group of similar surfaces. On the other hand, when the surfaces, corresponding to two isovalues, represent different features of the data set they are to form different groups.

We employ the notion of Hausdorff distance to discern isosurfaces of a data set and define similarities within a range of isovalues. The Hausdorff distance has been used for shape analysis (e.g., [30]). The key advantage of Hausdorff distance is that it allows us to express the difference between any two arbitrary surfaces, connected or not. Spectral or histogram-based approaches are difficult to employ for isosurfaces since they are often composed of disconnected (open) surfaces [31]. Reference-to-distortion mesh quality metrics [7] require a minimum degree of shape correspondence between two meshes. Some of those metrics assume the same mesh connectivity, while others perform approximations to map two similar meshes by sampling points between them. Therefore, the shape deformation or reference-to-distortion metrics can not be used to measure similarities between vastly different isosurfaces encountered in 3D scalar field data.

Formally, we denote the distance between two isovalues $\alpha, \beta \in \mathcal{I}$ as the Hausdorff distance between their respective isosurfaces:

$$isodist(\alpha, \beta) = d_H(S_\alpha, S_\beta). \quad (2)$$

Here $d_H(S, S') = \max(d(S, S'), d(S', S))$ refers to the symmetric mean Hausdorff distance where $d(S, S') = \sup_{\mathbf{x} \in S} \inf_{\mathbf{y} \in S'} \|\mathbf{x} - \mathbf{y}\|$. The classical Hausdorff distance is the maximum of the smallest distances between two isosurfaces S_α and S_β , which is an asymmetric distance. Depending on the geometry of the two surfaces this can lead to situations where $isodist(\alpha, \beta) \gg isodist(\beta, \alpha)$ or vice versa. A small one-directional Hausdorff distance does not necessarily describe surfaces that represent the same features in the volumetric data. Therefore, we use the symmetric Hausdorff distance as it is commonly used in shape analysis [30]. Since the maximum Hausdorff distance is sensitive to outliers in one surface, we also use the mean Hausdorff distance, which is the arithmetic mean of all shortest distances between two surfaces.

The similarity map for a range of isovalues $\mathcal{I}$ can be pre-computed for any pair of isovalues in $\mathcal{I}$. Accordingly, this map depicts the relationship between any two surfaces present in the volumetric data. In the remainder of this paper, we refer to the map based on the Hausdorff distance as HDM and to the map

based on mutual information of the distance transform [11] as ISM.

3.2. Classification of isovalues

Considering a set of isovalues $\mathcal{I}$, the goal is to select a small subset, $\mathcal{K} \subset \mathcal{I}$, that represents the diversity of shapes in the surfaces corresponding to $\mathcal{I}$. The set of selected isovalues $\mathcal{K}$ can be used for the exploration of features in the data set or designing transfer functions for volume rendering. We can view this as a quantization problem where the range of isovalues $\mathcal{I}$ is quantized to a small set $\mathcal{K}$. Unlike in traditional quantization, the data points (isovalues) do not have a trivial vector space embedding (e.g., sum of two isovalues represents a surface that is possibly unrelated to the two isosurfaces). However, we can consider the quantization problem as the discretization of the space of isovalues equipped with the distance as defined in Eq. (2). This analogy allows us to employ Centroidal Voronoi Tessellation (CVT) to partition the isovalue space, an approach that is widely used in quantization and clustering. In particular, we can employ Lloyd’s algorithm for K-means clustering in forming partitions of the isovalue space.

The search for the optimal set of isovalues $\mathcal{K}$ is an iterative process similar to the Lloyd’s algorithm; however, the cluster membership and centroids are determined based on the HDM. Moreover, we may use other notions of distance such as the ISM [11] for this process. We initialize $\mathcal{K}$ to a set of isovalues $\mathcal{K} := \{\mu_1, \dots, \mu_K\}$, with $K = |\mathcal{K}|$ representing the number of desired representative isovalues. The choice of initial isovalues can be uniform throughout $\mathcal{I}$ or guided by isosurface area histograms. The iterations for finding the optimal $\mathcal{K}$ are composed of these two update steps:

- **Isovalue Membership Update:** For each isovalue $\alpha \in \mathcal{I}$ find the closest centroid isovalue:

$$\mu = \arg \min_{\mu \in \mathcal{K}} \text{isodist}(\alpha, \mu). \quad (3)$$

Assign α to C_μ – the cluster represented by μ .

- **Centroid Isovalue Update:** For each centroid, $\mu \in \mathcal{K}$, collect all isovalues $\alpha \in \mathcal{I}$ that have been assigned to μ to form the cluster C_μ . Update the centroid μ by the isovalue whose isosurface is as close as possible to the rest of the cluster members.

$$\mu = \arg \min_{\alpha \in C_\mu} \sum_{\beta \in C_\mu} \text{isodist}(\alpha, \beta) \quad (4)$$

As every update step reduces the Hausdorff distances between the surfaces corresponding to isovalues in a cluster, the iterations are guaranteed to converge to a minimum cluster assignment configuration. Similar to Lloyd’s algorithm for K-means clustering (and in CVT), the convergence to a global minimum is more difficult to guarantee and requires a number of random initializations [32, 33].

3.3. Evaluation

In [34], we have shown that the Hausdorff metric-based K-means clustering outperformed the previous approach [11] in terms of identifying representative isovalues. In this section, we briefly review the evaluation experiment.

We conducted a detailed analysis on an exemplar head CTA dataset by varying (1) the classification approach, and (2) the isosurface similarity metric. In order to evaluate the results, we manually analyzed all the isosurfaces S_α extracted at the isovalues $\mathcal{I} = [1, 255]$ and defined feature groups that contain distinctive features of the CT head data set.

The manual visual feature inspection was performed by two observers and resulted in seven feature groups as illustrated in Fig. 2: isovalues 1–10 depict the patient head including clothes, side props and a bandage, isovalues 11–29 show the patient head with the side props only; isovalues 30–55 represent the skin and only partial side props; isovalues 56–68 correspond to soft tissues mostly below the skin; isovalues 69–101 represent the skeleton and vessels; isovalues 102–149 correspond to partial bones and vessels; and isovalues 150–255 reveal details such as two teeth, the eyeballs, and the prominent vessels with contrast liquid. The group 102–149 covers a wide range of isosurfaces that represent varying degrees of partial bone/vessel tissue. For visualization purposes, this group does not reveal any semantic meaning. The group 150–255 includes almost empty isosurfaces, which depict scarce representations of the detected features in that group.

We performed classification experiments using (a) the (maximum) HDM, (b) the mean HDM (mHDM), and (c) the inverse ISM (iISM) as underlying distance metric. The classification was run with $K = 13$ clusters and the initial centroids $\mathcal{K} := \{10, 30, 50, \dots, 230, 250\}$. For the studied data set, this number was picked such that we have enough clusters for representing each feature group. Concordant classification experiments were conducted to find the first K most representative isosurfaces, as described in [11].

All classification experiments resulted in some repeated representatives found for certain feature groups. In general, the classification from [11] outputs more repeated representatives than the ones from K-means clustering. In conclusion, the representatives from the proposed isovalue classification using the HDM are closest to the manual inspection with respect to the maximum number of distinct features. We refer readers to [34] for details of the experiments.

3.4. Discussion of clustering approaches

We performed an extensive exploration of varying clustering approaches before we decided to use the simple K-means approach. In fact, in unsupervised learning applications, K-means is still often the tool of choice as it is simple to implement, deterministic, scales well for large amounts of data, and converges quickly. However, for K-means, the ground truth number of K clusters needs to be known a priori what is often difficult or an unknown. We focused on clustering approaches that estimate the number of K clusters.

The first concept that picked up that idea of estimating the number of K clusters is attributed to Pelleg & Moore [35] who

group 1-10	group 11-29	group 30-55	group 56-68	group 69-101	group 102-149	group 150-254
head clothes props bandage	head props	skin	soft tissues	skeleton vessels	partial bones vessels	details: teeth eyeballs vessels

Figure 2: Feature groups of the CT head data set obtained by manual inspection. The first row depicts the isovalue ranges for each group. The second row contains the description of each feature group. The third row displays the exemplar isosurfaces from each feature group.

proposed the X-means algorithm. X-means searches for the most likely number of X clusters in an iterative splitting process around the standard K-means algorithm. After every K-means run, X-means measures the score of each (parent) cluster and the score of the two child clusters running K-means with $K=2$ on the subset corresponding to the parent cluster. Dependent on the score value, X-means continues either with the parent or the two child clusters.

When using X-means for the proposed isovalue classification, we most of the times did not arrive at a splitting of the clusters. Often, one of the children had a particularly bad likelihood score that prevented the parent cluster from splitting. We explored X-means using likelihood score estimators such as the Bayesian Information Criterion (BIC) [35] and the Minimum Noiseless Description Length (MNDL) scores [36], which both rely on Gaussian distributed data. We assume that X-means achieved poor results on our data, because we cannot assume a Gaussian distribution of the isovalue similarity.

The issue of finding the number of clusters in an automated process for non-Gaussian distributed data was, for example, addressed with the DP-means clustering approach proposed by Kulis & Jordan [37]. DP-means is similar to K-means in its formulation, but it is derived from a Dirichlet process (DP) mixture model.

We explored using DP-means, e.g., for the CT head data set where we received a set of centroid isosurfaces $\mathcal{K} := \{4, 24, 52, 73, 87, 108, 119, 141, 144, 208\}$ on the HDM with $\lambda = 0.1942$. A number of centroid isovalues $\mathcal{K} := \{4, 24, 52, 87, 108, 208\}$ are overlapping with the results of K-means. However, the interesting feature group from isovalues 56–68 is missing. Similar results were obtained with other data sets. Since DP-means is dependent on the processing order of the points, we further explored varying processing orders such as increasing/decreasing isovalue order, isosurface area-based processing order, or alternating processing orders.

Since the results did not significantly improve the clustering results, we decided to use the most simple clustering approach, i.e., K-means, and let the user explore the number of K clusters in an interactive process with a graphical tool. In fact, using DP-means means that the clustering depends on the distance parameter λ , which is responsible for when to open new clusters, which again requires an explorative process with user interac-

Dataset	K-means clustering	
	# Iterations	Time
CT head		
(128 × 128 × 115)	3	0.16s
Fuel		
(64 × 64 × 64)	3	0.14s
Hydrogen		
(128 × 128 × 128)	5	0.11s

Table 1: The performance of K-means on the HDMs (255×255) of the exemplar data sets. System configuration: Intel Core i7 3.6 GHz CPU, 16 GB RAM.

Besides, for a naive user, choosing λ is not necessarily more intuitive than choosing K.

Finally, we performed a robustness check of K-means centroids when varying K. Fig. 3 (a) shows the resulting centroids from K-means runs for $\mathcal{K} = \{7, 8, \dots, 12\}$ on the three isosurface similarity metrics. The darker a bar, the more hits were found for a centroid isovalue. Analyzing Fig. 3 (a), we concluded that K-means gives most robust results on the HDM meaning that many centroids were chosen independent from K. Similar to the results from the previous section, the feature group 150–255 is least robust with respect to a single centroid. We explain this with the same argument as previously that this feature group has no clear semantic distinction of shapes. Furthermore, as the number of clusters ranges from 7–12, some new clusters are only formed with higher Ks. As the feature group 150–255 comprises many isovalues, this range is more likely to give birth to new centroids.

Fig. 3 (b) shows the centroid distribution for DP-means with varying $\lambda = \{0.202, 0.206, 0.212, \dots, 0.298\}$ resulting in 7–12 clusters. Even though the feature group 150–255 has more robust results with DP-means compared to K-means, it is less robust in the other visually more important feature groups. Therefore, the centroid robustness check for DP-means leads to the same conclusion as before, namely, that DP-means does not give enough advantages in order to choose it instead of K-means. Therefore, we use in our continuing experiments K-means on the HDM as isovalue classification approach.

3.5. Performance

In order to identify representative isovalues, we developed a tool where users can interactively explore the data by starting with a relatively large K to find the expected number of distinct features and then eliminate similar shapes found by K-means. Thus, the performance of K-means for isovalue classification determines the efficiency of such interactive exploration process. Table 1 lists the performance of K-means clustering on the HDMs of the exemplar data sets. The clustering works efficiently and K-means converges quickly, typically within 3–5 iterations as shown in the experiments. Note that the resolution of the data does not directly influence the performance of K-

Figure 3: Robustness check for K-means and DP-means. The centroid isovalues are represented by the vertical bars. The darker a bar, the more hits were found for a centroid isovalue.

means clustering; however, it will increase the pre-computation time of the similarity maps.

4. Volume Quality Assessment

Our motivation to design a new VQA originates from challenging problems that arise in comparing compression approaches applied to 3D scalar fields, as known from compression-domain DVR. In this section, we demonstrate how we connect structural quality assessment to the introduced isovalue classification.

4.1. Motivation of the proposed VQA approach

Despite the known significant shortcomings in identifying visually relevant information [6], numerical error analysis tools such as MSE or SNR are still widely used for visual quality assessment in 3D scalar fields. In the image processing community, the concept of structural similarity index (e.g., SSIM [19]) has led to the development of numerous approaches that assess the data quality using structural information that are relevant to the HVS, which in return leads to quality measures that correlate more closely with the human visual perception. A good overview of HVS inspired perceptual quality assessment approaches can be found in [20]. Analogous structural information approaches have been applied to mesh quality assessment (MQA), summarized in [7]. To the best of our knowledge, however, similar structural VQA approaches have not yet been proposed.

Inspired by structural IQA and MQA, we propose to assess the visual quality of 3D scalar fields based on a limited set of significant benchmark structures. Our work is motivated by Pinto et al. [22] who measure the contour degradation as basis for an IQA measure. This idea is based on the sensitivity of the HVS to edges, which makes it effective on identifying contours (or high contrasts) to recognize structures. In fact, the HVS is able to find contours and structures even on low-quality images. We propose to selectively measure the volume

quality of 3D scalar fields at significant 3D benchmark isosurfaces. In particular, we suggest to select the benchmark contours such that they represent a wide range of distinct shapes contained in a 3D scalar field. Connecting this idea to the introduced isovalue classification in Sec. 3.2, we propose to select the benchmark contours specifically at the representative isosurfaces (RIs), which each represent a set of distinct features in the data.

In order to measure the degradation of an isosurface when compression is applied, we use structural MQA. The basic idea behind MQA is to define visual quality metrics that reflect the subjective judgement of the HVS. One approach, the Mesh Structural Distortion Measure (MSDM) [38] can be seen as an extension of the SSIM to 3D meshes. Basically, the MSDM assesses differences in curvature statistics (amplitudes) computed through a local moving window over the two meshes. The MSDM has one limitation, namely that the two meshes under comparison need to consist of the same mesh topology, i.e., the same number of vertices and connectivity. However, meshes often undergo transformations such as compression that produce meshes of different topology. Two recently developed MQAs address this bottleneck. The Multiscale Mesh Structural Distortion Measure (MSDM2) [39] performs a fast asymmetric matching between the vertices of the two meshes. Moreover, Gaussian-weighted curvature statistics are computed at multiple scales to produce a global distortion map. The Tensor-based Perceptual Distance Measure (TPDM) [1] has taken the idea of MSDM2 one step further by incorporating further structural information, namely, the surface principal directions, defined by the orthogonal directions of the minimum and maximum curvatures. Additionally, the TPDM simulates the visual masking effect through a roughness-based weighting of local distances. Finally, the TPDM score is obtained through Minkowski pooling of the weighted local tensor distances throughout the mesh surface.

For the VQA, we compute the degradation of the benchmark contours using the most recently developed TPDM, which offers a good correspondence to the HVS verified with subject-

486 tive databases. As we are extracting meshes on compressed
 487 3D scalar field data, we rarely have meshes with the same con-
 488 nectivity and hence rely on a MQA metric that uses a vertex
 489 projection step for the analysis of the visual quality.

490 4.2. Definition of the proposed VQA

491 The basic workflow of the definition of the proposed VQA
 492 for 3D scalar fields is illustrated in Fig. 1(b). That is, we com-
 493 press a 3D scalar field $\mathcal{A} \in \mathbb{R}^{x,y,z}$ and reconstruct it to its ap-
 494 proximation $\tilde{\mathcal{A}} \in \mathbb{R}^{x,y,z}$. The approximation quality of $\tilde{\mathcal{A}}$ de-
 495 pends on the chosen compression approach as well as on the
 496 chosen compression ratio. In order to define the visual qual-
 497 ity of $\tilde{\mathcal{A}}$, we extract from the reference $\mathcal{A}$ and the distortion
 498 $\tilde{\mathcal{A}}$ a set of K benchmark isosurfaces S at the learned represen-
 499 tative isovalues $\mathcal{K} := \{\mu_1, \dots, \mu_K\}$, with $K = |\mathcal{K}|$, as obtained
 500 from Sec. 3.2. For every corresponding isosurface pair S_{μ_k} and
 501 $\tilde{S}_{\mu_k}$, we individually assess the mesh quality $M_{QTPDM}(S, \tilde{S})$ as
 502 computed with the TPDM. For the final VQA, we aggregate the
 503 $M_{QTPDM}(S, \tilde{S})$ for every isosurface pair using the arithmetic
 504 mean to receive one single volume quality score:

$$VQ(\mathcal{A}, \tilde{\mathcal{A}}) = \frac{1}{K} \sum_{\mu_k \in \mathcal{K}} 1 - M_{QTPDM}(S_{\mu_k}, \tilde{S}_{\mu_k}) \quad (5)$$

505 The scores of the TPDM are normalized between $[0, 1]$,
 506 where the lower the score, the better the quality. Since the SSIM
 507 and the SNR use a quality assessment of the higher the score
 508 the better the visual quality, we inverted the TPDM scores in
 509 our aggregation to the $VQ(\mathcal{A}, \tilde{\mathcal{A}})$ to make the comparison be-
 510 tween different VQA approaches more intuitive. Hence, the
 511 $VQ(\mathcal{A}, \tilde{\mathcal{A}})$ scores show a higher quality the closer to 1.

512 To further connect the proposed VQ score to the perceptual
 513 visual quality of compressed data, we employ a *psychometric*
 514 *function*, which plots subjective visual sensations against VQ
 515 scores. The subjective responses to various degrees of com-
 516 pression are obtained from four observers. Each observer is
 517 exposed to a set of RIs at various compression levels. At each
 518 time, the subject is asked to compare the rendered approxima-
 519 tion with the ground truth and assign a five-scale quality score
 520 to the underlying approximation. The subjective quality score
 521 for the compressed volume is obtained from the mean quality
 522 scores of its RIs from all observers. Then, we conduct a psycho-
 523 metric fitting between the VQ scores and the subjective scores
 524 using the cumulative Gaussian psychometric function [1]:

$$p = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt \quad (6)$$

525 where x is the raw VQ score. The parameters $\mu = 0.132$ and
 526 $\sigma = 0.034$ are obtained through a non-linear least squares fit-
 527 ting using the VQ scores and subjective scores of the CT head
 528 data set. The psychometric function maps each VQ score to a
 529 probability of visual quality between 0 (certainty of low visual
 530 quality) and 1 (certainty of high visual quality), which is easier
 531 to be understood by users. As shown in Fig. 4, the same psycho-
 532 metric function is used for other experimented data sets, where
 533 the plots show a good generalization of the fitted psychometric
 534 function.

Figure 4: Psychometric function for the proposed quality metric plotted with VQ score and p pairs of all the experimented data sets with the compressions at level 100%, 75%, 54%, 37%, 25%, 15%, 8%, 4%, 1.5%, and 0.5%.

535 4.3. Experiments with compression-domain VQA

536 For the purpose of analyzing the proposed VQA, we mea-
 537 sured the reconstruction quality of three compressed exemplar
 538 3D scalar fields. For approaches to compression we have chosen
 539 DCT as a representative for frequency-domain transform,
 540 DWT as a representative for state-of-the-art multi-resolution
 541 decomposition, and TA as an example of a recent approach us-
 542 ing a learned model. The initial decomposition of the 3D scalar
 543 field was performed as a lossless process. The data reduction is
 544 then applied on the decomposed/learned models. To be precise,
 545 besides the thresholding of coefficients, we did not apply any
 546 further data compression approaches such as run-length encod-
 547 ing or floating point quantization. The coefficient thresholding
 548 for DCT and DWT was performed such that the least signifi-
 549 cant coefficients were removed. For TA, the data reduction was
 550 performed with Tucker tensor rank truncation [40].

551 For the comparison, we matched the number of coefficients
 552 (NCs) for DCT, DWT and TA. We purely calculated the NCs,
 553 i.e., the DCT coefficients, the DWT coefficients and the core
 554 tensor coefficients for TA. Elements of bases such as the
 555 wavelet or the tensor bases do not contribute to our reported
 556 NCs. Our chosen data reduction levels are approximately at
 557 100%, 75%, 54%, 37%, 25%, 15%, 8%, 4%, 1.5%, and 0.5%
 558 of NCs compared to the original number of scalar field values,
 559 which correspond to the NCS of a systematic tensor rank trun-
 560 cation. For the VQA, we reconstructed the 3D scalar fields for
 561 DCT, DWT and TA at every data reduction level. The resulting
 562 VQAs were analyzed with respect to (a) the visual degradation
 563 of benchmark contours, and (b) the measured visual quality.
 564 The visual degradation was conducted on the RIs obtained from
 565 the isosurface clustering. The VQA was measured with the
 566 MSE-based Signal-to-Noise Ratio (SNR), the 3D SSIM [41],
 567 and the proposed VQA.

Figure 5: Visual differences of the three compression approaches on the isosurface 52 of the CT head data set. Red circles indicate significant visual differences.

Figure 6: The distance maps of TPDM between the original isosurface 52 and the isosurfaces in Fig. 5, warmer colors represent higher local distance values, i.e., lower reconstruction quality.

4.3.1. Medical data

As a first exemplar scalar field, we analyzed the previously used CT head data set. For this purpose, we visually inspected the contour degradation of the RIs from K-means on the HDM. Fig. 5 illustrates the visual degradation (rows) of the isosurface/3D contour of the skin at RI=52 when increasing the data reduction (columns) using DCT, DWT, and TA. Some notable visual differences are circled in red. The first few data reduction levels show only minor differences. For example, for TA, the nose starts to have a small artifact from data reduction level of 25% on. The skin pattern differs quite strongly, especially for compression levels of 8% and below. There, the behavior of the three compression approaches is well visible. Notably, the DWT contours reveal an axis aligned pattern resulting from

the applied wavelets along the three spatial axis, while TA contours show a more compact shape and a smoother skin as a result of extracting principal components of the shape. The DCT contours contain a noise pattern what is also typical for a high DCT compression. Fig. 6 shows the distance maps produced by TPDM between the original contour at RI=52 and the contours in Fig. 5 where warmer colors represent higher local distortions. The TPDM maps are quite consistence with the visual inspection (e.g., at compression level 8%, the nose artifact of TA is clearly shown as red).

In Fig. 7, we put the proposed VQA scores in relation to SNR and SSIM to analyze the degradation during increased data reduction levels for DCT, DWT and TA. In general, the curves follow a similar pattern. Interestingly, the SSIM mea-

Figure 7: Comparison of the SNR, the SSIM, and the proposed VQA for assessing visual quality of volumes compressed with DCT, DWT and TA. The proposed VQA gives more insight into differences until approximately 50% of data reduction, in particular as opposed to SSIM.

Figure 8: Psychometric function for the proposed quality metric plotted with VQ score and p pairs of the compressions (from left to right) at level 100%, 25%, 8%, 1.5%, and 0.5%. The RIs at isovalue 52 serve as references for the visual degradation of different compressions. The p values and the perceived degradations are linearly correlated.

Figure 9: Evaluation of the proposed VQA. Isosurface 52 extracted from the original volume (a), from DCT compression at data reduction level of 8% (b), and from DWT compression at data reduction level of 1.5% (c).

sures a different ranking of the quality where it is DWT, TA, DCT opposed to the quality ranking by SNR and our proposed VQA where it is DWT, DCT, TA. Referring back to Fig. 5 and Fig. 8, it is challenging to give a clear answer whether DCT or TA should be ranked as higher visual quality. However, until a data reduction level of 8%, DWT clearly performs better. For the data reduction levels of 1.5% and 0.5% we notice different degradation in the isosurface, which cannot be depicted with SSIM and the proposed VQA.

Fig. 8 plots the psychometric function where the square markers (from left to right) indicate compressions at level 100%, 25%, 8%, 1.5%, and 0.5%. We note that the probability of high visual quality p effectively reflects the perceived degradation, where $p > 0.8$ ($VQ > 0.9$) corresponds to compressions with good visual quality and the strong visual degradations at the two rightmost compressions are indicated by low p values. The worst VQ score is around 0.8 which corresponds to bad visual quality with $p \approx 0$.

To evaluate the benefit of the proposed VQA, we show an example where the proposed VQA gives more insights than

SSIM. In Fig. 9 (b) and (c), we have illustrated two compressed volumes, having the same SSIM scores, whose reconstructed isosurfaces are of significantly different visual quality. Only the proposed VQ scores depict the visual differences of the reconstructed isosurfaces. The psychometric function (6) maps $p = 0.533$ to (b) and $p = 0.286$ to (c). To further demonstrate the effectiveness of assessing volume quality using benchmark isovalues, we computed the VQA score, defined in Eq. 5, but using isovalues from equidistant sampling over the isosurface

Figure 10: Visual differences of the three compression approaches on the isosurface 15 of the fuel data set. Red circles indicate significant visual differences.

Figure 11: Comparison of the SNR, the SSIM, and the proposed VQA for assessing visual quality of volumes compressed with DCT, DWT and TA.

Figure 12: Comparison of the SNR, the SSIM, and the proposed VQA for assessing visual quality of volumes compressed with DCT, DWT and TA. The proposed VQA shows more differences for the medium data reduction levels.

range. As a result, a higher score is assigned to the volume in Fig. 9 (c) although it has a lower visual quality. Since the equidistant sampling may select unstable noisy isosurfaces in the computation of VQA, it is less suitable to accurately assess the volume quality.

4.3.2. Simulation data

The next scalar field we analyzed is a simulation of a fuel injection into a combustion chamber ($64 \times 64 \times 64$, 8-bit). It basically shows the absence/presence of air with density values in a 3D scalar field. The degradation of isosurface 15 of a selected RI is depicted in Figs. 10. In general, all the compression approaches perform quite well; nevertheless, there are some subtleties. While TA offers a nearly perfect reconstruction for all data reduction levels, especially DWT has some irregular artifacts. It is also nicely visible that DCT is receptive to periodic patterns what can be seen in the symmetric data reduction artifacts.

Looking at the VQA curves of the three compression ap-

proaches, shown in Fig. 11, we observed again similar curve patterns of the three VQAs, where the SNR scores lay above 20 decibel and the TPDM scores are quite below the 0.2 psychometric fitting score. All VQA curves show a perfect reconstruction of DWT until 8%; TA and DCT show a perfect quality from 37% on for SNR and the proposed VQA. For lower percentages, TA is consistently above DWT and DCT. In the proposed VQA, the differences between the VQAs on 5% are more similar than in SSIM, while all VQAs show DWT and TA to be closer to each other for 4% and 1.5%.

The last 3D scalar field we analyzed with respect to VQA is a simulation of the spatial probability electron distribution in an hydrogen atom (magnetic field) of size $128 \times 128 \times 128$, 8-bit. The hydrogen data sets achieves very high quality visual reconstructions, which is reflected with all VQA scores, as shown in Fig. 12. Moreover, the differences on the extracted surfaces such as in Fig. 13 for RI=26 are hardly visible. Still, the proposed VQA shows again more differences for the medium data reduction levels.

4.4. VQA conclusions and future work

In conclusion, we proposed a VQA based on relevant benchmark 3D contours to measure the visual quality degradation of 3D scalar fields. Even though we measure only a scarce set of isosurfaces, we achieve comparable VQA scores that often even give more details for smaller data reduction amounts. The visual degradation in contours is a powerful information change with respect to significant structures in scalar fields. While this work only demonstrates a possible future direction of VQA for scalar fields, its details still need to be explored. For example, varying MQA measures could be compared in the future. It also

Figure 13: Degradation of isosurface 26 extracted on scalar fields from DCT, DWT, and TA. Red circles indicate significant visual differences.

needs to be investigated further how well the vertex projection steps works for isolated components in isosurfaces. Our experiments indicate positive results; however, there is no guarantee that this approximation works for all data sets, see, e.g., [1] page 17. The proposed VQA scores match well with the psychometric fitting performed in [1] and the SNR values. However, for low visual qualities, none of the VQAs is so far able to describe the pattern of the visual differences such as in Fig. 5 (bottom row). Despite those limitations, we regard this demonstrated VQA application on RIs as an important step leading structural VQA for 3D scalar fields into directions already taken in related disciplines such as IQA and MQA.

Finally, the proposed comparisons by varying three compression approaches and three VQA measures show that the choice of compression approach is highly application and data dependent. Knowing the structure of the data and understanding how much data reduction is desired, gives clues onto the choice of data compression. For example, DWT looks favorable for lower compression ratios as well as real data such as CT. For simulation data, TA is especially interesting for high data reduction levels.

5. Summary

We have presented a novel isovalue classification algorithm that extracts representative surfaces from a 3D scalar field data set. Compared to previous work, we were able to detect more distinct shapes with a set of isosurfaces illustrating the most representative features in a 3D scalar field data set. We have proposed a structural VQA based on benchmark isosurfaces to assess the visual quality of the compressed 3D scalar fields. The VQA builds upon the proposed isovalue classification algorithm. We have also demonstrated the utility of our proposed VQA using three widely-used compression approaches (i.e., DWT, DCT and TA).

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(a) Workflow of the isovalue classification

(a) Workflow of the proposed VQA