Observer-based Periodically Intermittent Control for Linear Systems via Piecewise Lyapunov Function Method

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Abstract

This paper focuses on the observer-based periodically intermittent control issue for linear systems by utilizing piecewise Lyapunov function method. Based on the concept of intermittent control, the description of an observer-based periodically intermittent controller is initially proposed. Then, stability of the corresponding periodically intermittent control system is analyzed by resorting to piecewise Lyapunov function method. Here, the so-called piecewise Lyapunov function means that Lyapunov function on control time intervals differs from the one on free time intervals. Besides, the existence of an observer-based periodically intermittent controller is converted into the feasibility of linear matrix inequalities. It is worth pointing out that, in comparison with an observer-based continuous controller, the designed observer-based periodically intermittent controller can still retain satisfactory performance with shorter control task execution time. Finally, an illustrative simulation example is given to show the validity and superiority of the result.

Keywords: periodically intermittent control; observer; stabilization; piecewise Lyapunov function.

1. Introduction

Intermittent control, as a transition between continuous control and impulsive control, emerges in control field. Its characteristic lies in that the control signal is imposed on a plant during certain nonzero time intervals, but is off during other nonzero time intervals. Due to its convenient implementation and high efficiency, intermittent control has been applied to various fields such as secure communication, medical treatment, air-quality control, transportation, and so on.

Recently, much attention has been paid to intermittent control technique to investigate two types of problems: stabilization (see references [1]-[8]) and synchronization (see references [9]-[17]). For the former, the description of a periodically intermittent state feedback controller is introduced in [1], based on which an exponential stability criterion of a periodically intermittent control system is obtained by using Lyapunov function theory. Paper [3] studies the exponential stabilization for a class of delayed chaotic neural networks by periodically intermittent control, and the exponential stabilization criterion is established by Lyapunov function and Halanay inequality. In [5], a periodically intermittent state feedback controller is designed to stabilize a class of uncertain nonlinear time-delay systems, and Lyapunov-Krasovskii functional method is adopted to seek the exponential stabilization conditions. For the latter, paper [9] focuses on the synchronization of chaotic systems via intermittent feedback method, and a Lyapunov function is constructed to deduce a synchronization criterion. In [13], the complete synchronization of chaotic neural networks with time delays is explored by intermittent control with two switches in a control period, and Lyapunov stability theory is used to derive some sufficient conditions for complete synchronization. Paper [15] deals with the exponential synchronization for chaotic systems with time delay by means of periodically intermittent state feedback control, and Lyapunov-Razumikhin methodology is utilized to deduce a new exponential synchronization criterion.

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Whether for the former or the latter, Lyapunov function or functional plays a critical role during the procedure of derivation. However, it should be observed that, in above literatures, the constructed Lyapunov function or functional on control time intervals is the same with the one on free time intervals, which leads to more conservativeness of the deduced results. This inspires the authors to adopt piecewise Lyapunov function or functional to analyze the stability of a periodically intermittent control system. The main idea of piecewise Lyapunov function or functional discussed in literatures [18, 19, 20, 21] is to construct different Lyapunov functions or functionals on time interval sequence. In comparison with Lyapunov function or functional, piecewise Lyapunov function or functional overcomes the limitation of Lyapunov function or functional and presents more generality. Therefore, in this paper, Lyapunov function is replaced by piecewise Lyapunov function to establish less conservative stability and stabilization criteria. Here, the so-called piecewise Lyapunov function means that Lyapunov function on control time intervals differs from the one on free time intervals.

On the other hand, for the stabilization issue, periodically intermittent state feedback controllers are usually considered to stabilize the concerned systems. However, in most practical situations, system states are unmeasurable. Literatures [22, 23, 24, 25, 26] do much work on this issue. In [22], an observer-based state feedback controller is addressed to stabilize a class of nonlinear time-delay systems subjected to input and output time-varying delays. Delay-dependent observer-based H_∞ finite-time control problem is addressed in [24] for switched systems with time-varying delay. It should be pointed out that these papers focus on the design problem of an observer-based state feedback controller and do not involve in an observer-based periodically intermittent control issue.

Motivated by above two aspects, we are concerned with the observer-based periodically intermittent control in this paper for linear systems by utilizing piecewise Lyapunov function method. The main contributions of this paper lie in the following three aspects. Firstly, based on the concept of intermittent control, the description of an observer-based periodically intermittent controller is proposed. Secondly, piecewise Lyapunov function is constructed to analyze the stability of the corresponding periodically intermittent control system. The so-called piecewise Lyapunov function means that Lyapunov function on control time intervals differs from the one on free time intervals. Thirdly, the existence of an observer-based periodically intermittent controller is converted into the feasibility of linear matrix inequalities. Compared with an observer-based continuous controller, the designed observer-based periodically intermittent controller is still capable of retaining satisfactory effects with shorter control task execution time.

Notation: In this paper, notations are fairly standard. R^n denotes the n dimensional Euclidean space, $\|\cdot\|$ refers to the Euclidean vector norm, and $R^{n \times m}$ represents the field of $n \times m$ dimensional real matrices. The superscript T and -1 stand for the transposition and the inverse of a matrix, respectively. Matrix $A > 0$ (< 0 , ≥ 0 , ≤ 0) means that A is positive definite (negative definite, positive semi-definite, negative semi-definite). $\star$ stands for the symmetric term in a symmetric matrix. In addition, $\lambda_m(P)$ ($\lambda_M(P)$) denotes the minimum (maximum) eigenvalue of matrix P .

2. Problem Formulation and Preliminaries

Consider the following linear system

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t), & t > 0, \\ y(t) = Cx(t), \\ x(0) = x_0, \end{cases} \quad (1)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input, $y(t) \in R^q$ is the measured output. $A \in R^{n \times n}$, $B \in R^{n \times m}$, $C \in R^{q \times n}$ are the known real constant matrices. x_0 is the initial state vector.

Assumption 1. When $u(t) = 0$, system (1) is unstable.

Assumption 2. The state vector $x(t)$ is unmeasurable.

Assumption 3. (A, B) is completely controllable and (A, C) is completely observable.

Assumption 4. Matrix C is of full row rank.

Here, the description of an observer-based periodically intermittent controller is proposed as

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + L(t)(y(t) - \hat{y}(t)) + Bu(t), \\ \hat{y}(t) = C\hat{x}(t), \\ \hat{x}(0) = \hat{x}_0, \\ u(t) = K(t)\hat{x}(t), \end{cases} \quad (2)$$

where

$$L(t) = \begin{cases} L_1, & t \in [kT, kT + \tau), \\ L_2, & t \in [kT + \tau, (k+1)T), \end{cases} \quad (3)$$

$$K(t) = \begin{cases} K, & t \in [kT, kT + \tau), \\ 0, & t \in [kT + \tau, (k+1)T), \end{cases} \quad (4)$$

$k = 0, 1, 2, 3, \dots$, $\hat{x}(t) \in R^n$ is the estimation of $x(t)$, $\hat{y}(t) \in R^q$ is the observer output, $K \in R^{m \times n}$ is the controller gain matrix, $L_1 \in R^{n \times q}$, $L_2 \in R^{n \times q}$ are the observer gain matrices, $T > 0$ denotes the control period, and $\tau > 0$ is the control width satisfying $\tau < T$.

Remark 1. In [1], the description of a periodically intermittent state feedback controller is introduced. Based on it, stabilization issues via periodically intermittent control cause widespread concern (see references [1]-[8]). After careful observation, it can be found that, in above literatures, system states are required to be measurable, which is often unavailable in practice. In contrast, an observer-based periodically intermittent controller can overcome this drawback, the description of which is first proposed in form of (2).

Remark 2. In general, corresponding to the description (4) of intermittent controller gain matrix $K(t)$, one easily thinks that the observer gain matrix $L(t)$ should be defined as

$$L(t) = \begin{cases} L_1, & t \in [kT, kT + \tau), \\ 0, & t \in [kT + \tau, (k+1)T). \end{cases} \quad (5)$$

However, different from an intermittent controller which is designed to retain satisfactory effects with shorter control task execution time, an observer is utilized to estimate system states. So it is no need to define the observer gain matrix $L(t)$ like the description (4) of intermittent controller gain matrix $K(t)$. Besides, description (3) presents more generality than description (5) since the latter is just a special case of the former.

Remark 3. As $\tau \rightarrow T$, the observer-based periodically intermittent controller (2) will reduce to the observer-based continuous controller

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + L_1(y(t) - \hat{y}(t)) + Bu(t), \\ \hat{y}(t) = C\hat{x}(t), \\ \hat{x}(0) = \hat{x}_0, \\ u(t) = K\hat{x}(t). \end{cases} \quad (6)$$

This shows that the observer-based continuous controller (6) is an extreme case of the observer-based periodically intermittent controller (2).

Combing (2) into (1) yields the following periodically intermittent control system (7)

$$\begin{cases} \dot{\tilde{x}}(t) = \tilde{A}_1 \tilde{x}(t), & t \in [kT, kT + \tau), \\ \dot{\tilde{x}}(t) = \tilde{A}_2 \tilde{x}(t), & t \in [kT + \tau, (k+1)T), \end{cases} \quad (7a)$$

$$(7b)$$

where

$$\begin{aligned} \tilde{x}(t) &= \begin{bmatrix} \hat{x}^T(t) & e^T(t) \end{bmatrix}^T, \\ e(t) &= x(t) - \hat{x}(t), \\ \tilde{A}_1 &= \begin{bmatrix} A + BK & L_1 C \\ 0 & A - L_1 C \end{bmatrix}, \\ \tilde{A}_2 &= \begin{bmatrix} A & L_2 C \\ 0 & A - L_2 C \end{bmatrix}. \end{aligned}$$

Obviously, the initial state vector of system (7) can be expressed as $\tilde{x}(0) = \tilde{x}_0 = \begin{bmatrix} \hat{x}_0^T & x_0^T - \hat{x}_0^T \end{bmatrix}^T$.

Remark 4. Periodically intermittent control system (7) can be viewed as a switched system with time-dependent switching signals. Correspondingly, the analysis and synthesis of periodically intermittent control system (7) can resort to the existing theories of switched systems (see references [27, 28, 29, 30, 31]).

To make this brief more readable, the following definition is necessary.

Definition 1. System (1) is said to be stabilized via an observer-based periodically intermittent controller (2) if the resulted periodically intermittent control system (7) is asymptotically stable.

In this paper, our purpose is to seek some sufficient conditions which ensure the existence of an observer-based periodically intermittent controller (2) to stabilize system (1).

3. Stability Analysis

In this section, we aim at establishing the stability criterion of periodically intermittent control system (7) by resorting to piecewise Lyapunov function.

Theorem 1. For given constants $\alpha > 0, \beta > 0, \mu \geq 1$, if there exist matrices $P_1 > 0, P_2 > 0$, scalars $T > \tau > 0$ such that the following inequalities hold

$$P_1 \tilde{A}_1 + \tilde{A}_1^T P_1 + \alpha P_1 \leq 0, \quad (8)$$

$$P_2 \tilde{A}_2 + \tilde{A}_2^T P_2 - \beta P_2 \leq 0, \quad (9)$$

$$P_1 - \mu P_2 \leq 0, \quad (10)$$

$$P_2 - \mu P_1 \leq 0, \quad (11)$$

$$\alpha\tau - \beta(T - \tau) - 2\ln\mu > 0, \quad (12)$$

then periodically intermittent control system (7) is asymptotically stable.

Proof. Choose the piecewise Lyapunov function (13)

$$V(\tilde{x}(t)) = \begin{cases} \tilde{x}^T(t) P_1 \tilde{x}(t), & t \in [kT, kT + \tau), \\ \tilde{x}^T(t) P_2 \tilde{x}(t), & t \in [kT + \tau, (k+1)T), \end{cases} \quad (13a)$$

$$(13b)$$

which implies

$$a\|\tilde{x}(t)\|^2 \leq V(\tilde{x}(t)) \leq b\|\tilde{x}(t)\|^2, \quad (14)$$

where $a = \min_{i=1,2}\{\lambda_m(P_i)\}$, $b = \max_{i=1,2}\{\lambda_M(P_i)\}$.

When $t \in [kT, kT + \tau)$, subsystem (7a) is activated. From (8), taking the derivative of (13a) with respect to t along the trajectory of subsystem (7a) and adding $\alpha V(\tilde{x}(t))$ yield

$$\dot{V}(\tilde{x}(t)) + \alpha V(\tilde{x}(t)) = \tilde{x}^T(t)(P_1 \tilde{A}_1 + \tilde{A}_1^T P_1 + \alpha P_1) \tilde{x}(t) \leq 0,$$

that is,

$$\dot{V}(\tilde{x}(t)) \leq -\alpha V(\tilde{x}(t)), \quad t \in [kT, kT + \tau). \quad (15)$$

Similarly, when $t \in [kT + \tau, (k+1)T)$, subsystem (7b) is activated. From (9), taking the derivative of (13b) with respect to t along the trajectory of subsystem (7b) and subtracting $\beta V(\tilde{x}(t))$ yield

$$\dot{V}(\tilde{x}(t)) - \beta V(\tilde{x}(t)) = \tilde{x}^T(t)(P_2 \tilde{A}_2 + \tilde{A}_2^T P_2 - \beta P_2) \tilde{x}(t) \leq 0,$$

that is,

$$\dot{V}(\tilde{x}(t)) \leq \beta V(\tilde{x}(t)), \quad t \in [kT + \tau, (k+1)T). \quad (16)$$

113 Near the instant $t = kT$, relationship of $V(\tilde{x}((kT)^+))$ and $V(\tilde{x}((kT)^-))$ can be calculated from (10) as

$$\begin{aligned}
& V(\tilde{x}((kT)^+)) \\
&= \tilde{x}^T((kT)^+)P_1\tilde{x}((kT)^+) \\
&= \tilde{x}^T(kT)P_1\tilde{x}(kT) \\
&\leq \mu\tilde{x}^T(kT)P_2\tilde{x}(kT) \\
&= \mu V(\tilde{x}((kT)^-)).
\end{aligned} \tag{17}$$

114 Near the instant $t = kT + \tau$, relationship of $V(\tilde{x}((kT + \tau)^+))$ and $V(\tilde{x}((kT + \tau)^-))$ can also be deduced from (11) as

$$\begin{aligned}
& V(\tilde{x}((kT + \tau)^+)) \\
&= \tilde{x}^T((kT + \tau)^+)P_2\tilde{x}((kT + \tau)^+) \\
&= \tilde{x}^T(kT + \tau)P_2\tilde{x}(kT + \tau) \\
&\leq \mu\tilde{x}^T(kT + \tau)P_1\tilde{x}(kT + \tau) \\
&= \mu V(\tilde{x}((kT + \tau)^-)).
\end{aligned} \tag{18}$$

115 In the sequence, the estimate value of $V(\tilde{x}(t))$ can be computed from (15)-(18). The detailed process is presented
116 as follows.

117 1) When $t \in [0, \tau)$, it follows that

$$\begin{aligned}
V(\tilde{x}(t)) &\leq e^{-\alpha t} V(\tilde{x}_0), \\
V(\tilde{x}(\tau^-)) &\leq e^{-\alpha \tau} V(\tilde{x}_0).
\end{aligned}$$

118 2) When $t \in [\tau, T)$, one obtains that

$$\begin{aligned}
V(\tilde{x}(t)) &\leq e^{\beta(t-\tau)} V(\tilde{x}(\tau^+)) \\
&\leq \mu e^{\beta(t-\tau)} V(\tilde{x}(\tau^-)) \\
&\leq \mu e^{-\alpha\tau+\beta(t-\tau)} V(\tilde{x}_0), \\
V(\tilde{x}(T^-)) &\leq \mu e^{-\alpha\tau+\beta(T-\tau)} V(\tilde{x}_0).
\end{aligned}$$

119 3) When $t \in [T, T + \tau)$, we derive that

$$\begin{aligned}
V(\tilde{x}(t)) &\leq e^{-\alpha(t-T)} V(\tilde{x}(T^+)) \\
&\leq \mu e^{-\alpha(t-T)} V(\tilde{x}(T^-)) \\
&\leq \mu^2 e^{-\alpha(t-T)-\alpha\tau+\beta(T-\tau)} V(\tilde{x}_0), \\
V(\tilde{x}((T + \tau)^-)) &\leq \mu^2 e^{-2\alpha\tau+\beta(T-\tau)} V(\tilde{x}_0).
\end{aligned}$$

120 4) When $t \in [T + \tau, 2T)$, it is easy to address that

$$\begin{aligned}
V(\tilde{x}(t)) &\leq e^{\beta(t-T-\tau)} V(\tilde{x}((T + \tau)^+)) \\
&\leq \mu e^{\beta(t-T-\tau)} V(\tilde{x}((T + \tau)^-)) \\
&\leq \mu^3 e^{-2\alpha\tau+\beta(T-\tau)+\beta(t-T-\tau)} V(\tilde{x}_0), \\
V(\tilde{x}(T^-)) &\leq \mu^3 e^{-2\alpha\tau+2\beta(T-\tau)} V(\tilde{x}_0).
\end{aligned}$$

121 By induction,
 122 5) when $t \in [kT, kT + \tau)$, one deduces from (12) that

$$\begin{aligned}
 V(\tilde{x}(t)) &\leq e^{-\alpha(t-kT)} V(\tilde{x}((kT)^+)) \\
 &\leq \mu e^{-\alpha(t-kT)} V(\tilde{x}((kT)^-)) \\
 &\leq \mu^{2k} e^{-\alpha(t-kT)-k\alpha\tau+k\beta(T-\tau)} V(\tilde{x}_0) \\
 &\leq \mu^{2k} e^{-k\alpha\tau+k\beta(T-\tau)} V(\tilde{x}_0) \\
 &= e^{-k\alpha\tau+k\beta(T-\tau)+2k \ln \mu} V(\tilde{x}_0) \\
 &= e^{-(\alpha\tau-\beta(T-\tau)-2 \ln \mu) \cdot k} V(\tilde{x}_0) \\
 &= e^{-\frac{\alpha\tau-\beta(T-\tau)-2 \ln \mu}{T} \cdot kT} V(\tilde{x}_0) \\
 &\leq e^{-\frac{\alpha\tau-\beta(T-\tau)-2 \ln \mu}{T} (t-\tau)} V(\tilde{x}_0), \\
 V(\tilde{x}((kT + \tau)^-)) &\leq \mu^{2k} e^{-(k+1)\alpha\tau+k\beta(T-\tau)} V(\tilde{x}_0),
 \end{aligned} \tag{19}$$

123 6) and when $t \in [kT + \tau, (k+1)T)$, the following inequality holds from (12)

$$\begin{aligned}
 V(\tilde{x}(t)) &\leq e^{\beta(t-kT-\tau)} V(\tilde{x}((kT + \tau)^+)) \\
 &\leq \mu e^{\beta(t-kT-\tau)} V(\tilde{x}((kT + \tau)^-)) \\
 &\leq \mu^{2k+1} e^{\beta(t-kT-\tau)-(k+1)\alpha\tau+k\beta(T-\tau)} V(\tilde{x}_0) \\
 &\leq \mu^{2(k+1)} e^{-(k+1)\alpha\tau+(k+1)\beta(T-\tau)} V(\tilde{x}_0) \\
 &= e^{-(k+1)\alpha\tau+(k+1)\beta(T-\tau)+2(k+1) \ln \mu} V(\tilde{x}_0) \\
 &= e^{-(\alpha\tau-\beta(T-\tau)-2 \ln \mu) \cdot (k+1)} V(\tilde{x}_0) \\
 &= e^{-\frac{\alpha\tau-\beta(T-\tau)-2 \ln \mu}{T} \cdot (k+1)T} V(\tilde{x}_0) \\
 &\leq e^{-\frac{\alpha\tau-\beta(T-\tau)-2 \ln \mu}{T} t} V(\tilde{x}_0) \\
 &\leq e^{-\frac{\alpha\tau-\beta(T-\tau)-2 \ln \mu}{T} (t-\tau)} V(\tilde{x}_0).
 \end{aligned} \tag{20}$$

124 From (19) and (20), one derives

$$V(\tilde{x}(t)) \leq e^{-\frac{\alpha\tau-\beta(T-\tau)-2 \ln \mu}{T} (t-\tau)} V(\tilde{x}_0), \quad t \geq 0,$$

125 which, together with (12), implies that system (7) is asymptotically stable. The proof is completed. $\square$

126 **Remark 5.** The purpose of Assumption 3 is to ensure that inequality (8) has a feasible solution. Specifically, complete
 127 controllability of (A, B) guarantees that eigenvalues of $A + BK$ can be assigned arbitrarily, which implies that there
 128 exists matrix K such that $\lambda_i(A + BK) < 0, i = 1, 2, \dots, n$. Similarly, complete observability of (A, C) ensures that
 129 eigenvalues of $A - L_1C$ can be assigned arbitrarily, which means that there exists matrix L_1 such that $\lambda_i(A - L_1C) <$
 130 $0, i = 1, 2, \dots, n$. Thereby, there exist matrices K and L_1 such that $\tilde{A}_1 = \begin{bmatrix} A + BK & L_1C \\ 0 & A - L_1C \end{bmatrix}$ is Hurwitz, which
 131 further implies that there exists $P_1 > 0$ such that $P_1\tilde{A}_1 + \tilde{A}_1^T P_1 < 0$. If parameter $\alpha > 0$ is enough small, $P_1\tilde{A}_1 +$
 132 $\tilde{A}_1^T P_1 + \alpha P_1 \leq 0$ holds, that is, the feasibility of inequality (8) is ensured. In fact, from the above analysis, Assumption
 133 3 can be relaxed to the assumption that both the uncontrollable part of (A, B) and the unobservable part of (A, C) are
 134 asymptotically stable.

135 **Remark 6.** In Theorem 1, values of parameters α, β and μ need to be set in advance, which will be facilitated by
 136 analyzing each parameter's meaning. Specifically, inequalities (14) and (15) yield $\|\tilde{x}(t)\| \leq \sqrt{\frac{b}{a}} e^{-\frac{1}{2}\alpha(t-kT)} \|\tilde{x}(kT)\|$,
 137 $t \in [kT, kT + \tau)$ which implies that parameter α denotes the double of the state norm decay rate estimate of subsystem
 138 (7a) on time interval $[kT, kT + \tau)$. Similarly, $\|\tilde{x}(t)\| \leq \sqrt{\frac{b}{a}} e^{\frac{1}{2}\beta(t-kT-\tau)} \|\tilde{x}(kT + \tau)\|$, $t \in [kT + \tau, (k+1)T)$ can be inferred
 139 from inequalities (14) and (16). Thus, parameter β stands for the double of the state norm increase rate estimate of
 140 subsystem (7b) on time interval $[kT + \tau, (k+1)T)$. In addition, from (17) and (18), parameter μ represents the change
 141 rate of $V(\tilde{x}(t))$ near $t = kT$ and $t = kT + \tau$.

Remark 7. In literatures [1]-[17], the constructed Lyapunov function or functional on control time intervals is the same with the one on free time intervals, which, to some extent, leads to that these results obtained in the above papers share a restriction, that is, $P_1 = P_2 = P$. To relax it, piecewise Lyapunov function (13) is constructed in the proof of Theorem 1, which results in less conservative conditions, allowing $P_1 \neq P_2$.

When $\mu \equiv 1$, the following corollary can be addressed by Theorem 1.

Corollary 1. For given constants $\alpha > 0, \beta > 0$, if there exist matrix $P > 0$, scalars $T > \tau > 0$ such that the following inequalities hold

$$\begin{aligned} P\tilde{A}_1 + \tilde{A}_1^T P + \alpha P &\leq 0, \\ P\tilde{A}_2 + \tilde{A}_2^T P - \beta P &\leq 0, \\ \alpha\tau - \beta(T - \tau) &> 0, \end{aligned}$$

then periodically intermittent control system (7) is asymptotically stable.

Proof. Denoting $P = P_1 = P_2$, this corollary can be deduced by Theorem 1. This proof is completed. $\square$

4. An Observer-based Periodically Intermittent Controller Design

Due to the existence of unknown matrices K, L_1 in $\tilde{A}_1$ and L_2 in $\tilde{A}_2$, inequalities (8) and (9) in Theorem 1 are nonlinear which can not be solved by Linear Matrix Inequalities Control ToolBox in MATLAB. Thus, in this section, much efforts are devoted to converting nonlinear matrix inequalities into linear matrix inequalities. Simultaneously, controller gain matrix K and observer gain matrices L_1, L_2 can be calculated by solving the deduced linear matrix inequalities.

Assumption 4 shows that the singular value decomposition of matrix C is of the form

$$C = U \begin{bmatrix} S & 0 \end{bmatrix} V^T, \quad (21)$$

where $S \in R^{q \times q}$ is a diagonal matrix with positive diagonal elements, $U \in R^{q \times q}$ and $V \in R^{n \times n}$ are unitary matrices.

Theorem 2. For given constants $\alpha > 0, \beta > 0, \mu \geq 1$, if there exist matrices $X_{11} > 0, X_{12} > 0, X_{21} > 0, X_{22} > 0, W, W_1, W_2$, scalars $T > \tau > 0$ such that the following inequalities hold

$$\begin{bmatrix} \Phi_{11} & W_1 C \\ \star & \Phi_{12} \end{bmatrix} \leq 0, \quad (22)$$

$$\begin{bmatrix} \Psi_{21} & W_2 C \\ \star & \Psi_{22} \end{bmatrix} \leq 0, \quad (23)$$

$$X_{21} - \mu X_{11} \leq 0, \quad X_{22} - \mu X_{12} \leq 0, \quad (24)$$

$$X_{11} - \mu X_{21} \leq 0, \quad X_{12} - \mu X_{22} \leq 0, \quad (25)$$

$$\alpha\tau - \beta(T - \tau) - 2 \ln \mu > 0, \quad (26)$$

where

$$\Phi_{11} = AX_{11} + X_{11}A^T + BW + W^T B^T + \alpha X_{11}, \quad (27)$$

$$\Phi_{12} = AX_{12} + X_{12}A^T - W_1 C - C^T W_1^T + \alpha X_{12},$$

$$\Psi_{21} = AX_{21} + X_{21}A^T - \beta X_{21},$$

$$\Psi_{22} = AX_{22} + X_{22}A^T - W_2 C - C^T W_2^T - \beta X_{22},$$

then periodically intermittent control system (7) is asymptotically stable, that is, system (1) can be stabilized by an observer-based periodically intermittent controller (2) with controller gain matrix $K = WX_{11}^{-1}$ and observer gain

matrices $L_1 = W_1 \hat{X}_{12}^{-1}$, $L_2 = W_2 \hat{X}_{22}^{-1}$, where

$$\hat{X}_{12} = US \hat{X}_{121} S^{-1} U^T, \quad (28)$$

$$X_{12} = V \begin{bmatrix} \hat{X}_{121} & 0 \\ \star & \hat{X}_{122} \end{bmatrix} V^T, \quad (29)$$

$$\hat{X}_{22} = US \hat{X}_{221} S^{-1} U^T, \quad (30)$$

$$X_{22} = V \begin{bmatrix} \hat{X}_{221} & 0 \\ \star & \hat{X}_{222} \end{bmatrix} V^T. \quad (31)$$

Proof. From (21), (28) and (29), $CX_{12} = \hat{X}_{12}C$ holds, which, together with $L_1 = W_1 \hat{X}_{12}^{-1}$, implies $W_1 C = L_1 \hat{X}_{12} C = L_1 C X_{12}$. Replacing $W_1 C$ with $L_1 C X_{12}$, one can rewrite inequality (22) as

$$\begin{bmatrix} \Phi_{11} & L_1 C X_{12} \\ \star & \Phi_{13} \end{bmatrix} \leq 0, \quad (32)$$

where Φ_{11} is defined in (27), $\Phi_{13} = AX_{12} + X_{12}A^T - L_1 C X_{12} - (L_1 C X_{12})^T + \alpha X_{12}$.

Pre- and post-multiplying both sides of (32) by $\begin{bmatrix} X_{11}^{-1} & 0 \\ \star & X_{12}^{-1} \end{bmatrix}$ and denoting $P_{11} = X_{11}^{-1}$, $P_{12} = X_{12}^{-1}$, $K = WX_{11}^{-1}$, it follows that

$$\begin{pmatrix} \Phi_{14} & P_{11} L_1 C \\ \star & \Phi_{15} \end{pmatrix} \leq 0, \quad (33)$$

where

$$\Phi_{14} = P_{11}(A + BK) + (A + BK)^T P_{11} + \alpha P_{11},$$

$$\Phi_{15} = P_{12}(A - L_1 C) + (A - L_1 C)^T P_{12} + \alpha P_{12}.$$

Defining $P_1 = \begin{bmatrix} P_{11} & 0 \\ \star & P_{12} \end{bmatrix}$, inequality (33) can be written as (8).

Similarly, (23) can be written as (9) by defining $P_2 = \begin{bmatrix} P_{21} & 0 \\ \star & P_{22} \end{bmatrix}$. Further, (24) and (25) can also be expressed as (10) and (11), respectively. According to Theorem 1, system (7) is asymptotically stable, that is, system (1) can be stabilized by the designed observer-based periodically intermittent controller. The proof is completed. $\square$

Remark 8. Values of parameters α , β , and μ in Theorem 2 determine whether the solution of inequalities (22)-(26) exists or not. Therefore, it is of significance to choose proper values of parameters α , β , and μ . Since some direct and indirect links exist among parameters α , β , and μ , how to set them is still a challenge even though each parameter's meaning is known in Remark 6. Here, the cut-and-try method is adopted.

Step 1. The lower bound β^* of parameter β can be obtained by fixing a small value of β and then increasing it gradually until inequality (23) has a feasible solution.

Step 2. Take the values of α , β and μ from $(0, +\infty)$, $[\beta^*, +\infty)$ and $[1, +\infty)$, respectively.

Step 3. Solve inequalities (22)-(25). If there exists a feasible solution, go to Step 4. Otherwise, back to Step 2.

Step 4. Take the values of control width τ and control period T satisfying $\tau < T$ and inequality (26). It ends.

Remark 9. Obviously, the feasible region of an observer-based periodically intermittent controller (2) is related to the feasible solution domain of inequalities (22)-(26). Control period T , independent of inequalities (22)-(25), exists in (26). After fixing values of parameters α , β and μ such that inequalities (22)-(25) have a feasible solution and taking value of τ satisfying $\tau > \frac{2\ln\mu}{\alpha}$, the set of values of T satisfying $\tau < T < \frac{\alpha+\beta}{\beta}\tau - \frac{2\ln\mu}{\beta}$ inferred from (26) determines the feasible region of the designed observer-based periodically intermittent controller (2).

Remark 10. As $\tau \rightarrow T$, the designed observer-based periodically intermittent controller (2) in Theorem 2 will degenerate to the observer-based continuous controller (6). In the case, condition (22) is the stabilization condition of system (1) with the observer-based continuous controller (6) (the other conditions (23)-(26) are redundant).

Remark 11. The output signal of the observer-based continuous controller (6) exists all the time, which makes an actuator work in entire process and causes resource waste. In contrast, an observer-based periodically intermittent controller (2) designed in Theorem 2 is still capable of retaining satisfactory performance with shorter control task execution time, which can be fully reflected in Example 1.

Remark 12. When system states are unmeasurable, periodically intermittent controllers explored in [1]-[8] fail to work while the observer-based periodically intermittent controller (2) designed in Theorem 2 can still work well. This point can also be shown in Example 1.

Similarly, when $\mu \equiv 1$, the following corollary can be derived by Theorem 2.

Corollary 2. For given constants $\alpha > 0, \beta > 0$, if there exist matrices $X_1 > 0, X_2 > 0, W, W_1, W_2$, scalars $T > \tau > 0$, such that the following inequalities hold

$$\begin{aligned} \begin{bmatrix} \Phi_1 & W_1 C \\ \star & \Phi_2 \end{bmatrix} &\leq 0, \\ \begin{bmatrix} \Psi_1 & W_2 C \\ \star & \Psi_2 \end{bmatrix} &\leq 0, \\ \alpha\tau - \beta(T - \tau) &> 0, \end{aligned}$$

where

$$\begin{aligned} \Phi_1 &= AX_1 + X_1 A^T + BW + W^T B^T + \alpha X_1, \\ \Phi_2 &= AX_2 + X_2 A^T - W_1 C - C^T W_1^T + \alpha X_2, \\ \Psi_1 &= AX_1 + X_1 A^T - \beta X_1, \\ \Psi_2 &= AX_2 + X_2 A^T - W_2 C - C^T W_2^T - \beta X_2, \end{aligned}$$

then periodically intermittent control system (7) is asymptotically stable, that is, system (1) can be stabilized by an observer-based periodically intermittent controller (2) with controller gain matrix $K = WX_1^{-1}$ and observer gain matrices $L_1 = W_1 \hat{X}_2^{-1}, L_2 = W_2 \hat{X}_2^{-1}$, where

$$\begin{aligned} \hat{X}_2 &= US \hat{X}_{21} S^{-1} U^T, \\ X_2 &= V \begin{bmatrix} \hat{X}_{21} & 0 \\ \star & \hat{X}_{22} \end{bmatrix} V^T. \end{aligned}$$

Proof. Denoting $X_1 = X_{11} = X_{21}, X_2 = X_{12} = X_{22}$, this corollary can be deduced by Theorem 2. This proof is completed. $\square$

5. An Example

In this section, an example is provided to verify the validity and superiority of the developed result. Consider system (1) with parameters

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \end{bmatrix}.$$

When $u(t) = 0$, the above system is unstable. (A, B) is completely controllable and (A, C) is completely observable. Matrix C is of full row rank. Assume that system state vector $x(t)$ is unmeasurable.

In this assumption, the methods proposed in [1]-[8] can not be utilized to stabilize the given system. In contrast, Theorem 2 can still work well. The detailed process is shown as follows.

According to the steps in Remark 8, one takes $\alpha = 1$, $\beta = 2.1$, $\mu = 1.2$, and solves inequalities (22)-(25) in Theorem 2. Values of matrices X_{11} , X_{12} , X_{21} , X_{22} , W , W_1 , W_2 can be calculated. Choose $T = 1$, $\tau = 0.8$ satisfying $\tau < T$ and inequality (26). These computed results, together with $K = WX_{11}^{-1}$, $L_1 = W_1\hat{X}_{12}^{-1}$, $L_2 = W_2\hat{X}_{22}^{-1}$, inequalities (28)-(31), yield values of controller gain matrix K , observer gain matrices L_1 and L_2 as follows

$$K = \begin{bmatrix} -2.4814 & 1.1113 \end{bmatrix}, \quad L_1 = \begin{bmatrix} 1.7912 & 0.1164 \end{bmatrix}^T, \quad L_2 = \begin{bmatrix} 0.2156 & -0.6655 \end{bmatrix}^T.$$

For $x_0 = \begin{bmatrix} 0.6 & -0.8 \end{bmatrix}^T$, $\hat{x}_0 = \begin{bmatrix} 0.5 & -0.3 \end{bmatrix}^T$, curves of system state vector $x(t)$, state estimated error vector $e(t)$, and observer-based periodically intermittent controller output $u(t)$ are depicted in Figure 1, Figure 2 and Figure 3, respectively.

From Figure 1, state vector of the given system tends to the origin under the designed observer-based periodically intermittent controller (2). The state estimated errors converge to zero from Figure 2, which shows that the designed observer works well. The output signal of the designed observer-based periodically intermittent controller exists on time intervals $[k, k+0.8)$ and disappears on time intervals $[k+0.8, k+1)$, $k = 0, 1, 2, 3, \dots$ in Figure 3, which coincides with the definition of intermittent control.

In addition, with controller gain matrix K , observer gain matrix L_1 and initial state vectors x_0 , $\hat{x}_0$, curves of system state vector $x(t)$ and observer-based continuous controller output $u(t)$ are depicted in Figure 4 and Figure 5, respectively. In comparison with the observer-based continuous controller, the designed observer-based periodically intermittent controller can still retain satisfactory performance from Figure 1 and Figure 4 with shorter control task execution time from Figure 3 and Figure 5.

6. Conclusion

This paper has addressed the observer-based periodically intermittent control issue for linear systems by piecewise Lyapunov function method. Based on the concept of intermittent control, the description of an observer-based periodically intermittent controller has been initially proposed. Then, stability of the corresponding periodically intermittent control system has been analyzed via piecewise Lyapunov function method. Besides, the existence of an observer-based periodically intermittent controller has been converted into the feasibility of linear matrix inequalities. Finally, a simulation example has been provided to show the validity and superiority of the result.

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Figure 1: The states of the given system with the designed observer-based periodically intermittent controller (2)

Figure 2: The state estimated errors of the given system with the designed observer-based periodically intermittent controller (2)

Figure 3: The output signal of the designed observer-based periodically intermittent controller (2)

Figure 4: The states of the given system under the designed observer-based continuous controller with controller gain matrix K , observer gain matrix L_1

Figure 5: The output signal of the designed observer-based continuous controller with controller gain matrix K , observer gain matrix L_1