The influence of the elastic vibration of the carrier to the aerodynamics of the external store in air-launch-to-orbit process

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Abstract: The separation between the carrier and store is one of the most important and difficult phases in Air-launch-to-orbit technology. Based on the previous researches, the interference aerodynamic forces of the store caused by the carrier are obvious in the earlier time during the separation. And the interference aerodynamics will be more complex when considering the elastic deformation of the carrier. Focusing on the conditions that in the earlier time during the separation, the steady and unsteady interference aerodynamic forces of the store are calculated at different angle of attacks and relative distances between the carrier and store. During the calculation, the elastic vibrations of the carrier are considered. According to the cause of formations of the interference aerodynamics, the interference aerodynamic forces of the store are divided into several components. The relative magnitude, change rule, sphere of influence and mechanism of interference aerodynamic forces components of the store are analyzed quantitatively. When the relative distance between the carrier and store is small, the interference aerodynamic forces caused by the elastic vibration of the carrier is about half of the total aerodynamic forces of the store. And as the relative distance increases, the value of interference aerodynamic forces decrease. When the relative distance is larger than twice the mean aerodynamic chord of the carrier, the values of interference aerodynamic forces of the store can be ignored. Besides, under the influence of the steady interference aerodynamic forces, the lift characteristics of the store are worse and the static stability margin is poorer.

Keywords: Air-launch-to-orbit; interference aerodynamics; elastic vibration; dynamic response

I. Introduction

Air-launch-to-orbit is the method of launching rockets at altitude from a conventional horizontal takeoff aircraft, to carry payload to the earth orbit ^[1]. With this method, the space activities will become less reliant on the launch site support and launch window, and the launch costs will decrease ^[2-3]. In future, air-launch-to-orbit will be a rapid, flexible and economical way for space launching.

The separation between the carrier and store is one of the most important and difficult phases in air-launch-to-orbit technology. In order to ensure that the separation process could run smoothly, many separation methods are designed by the researchers. The common methods are captive on bottom method ^[4-6], internally carried method ^[7-8], towed method ^[9], etc. This paper turns to study the captive on bottom method. In this method, the payloads are carried under the wing. This method is relatively simple and has been used widely, such as the Scaled Composites White

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Knight Two/Space Ship Two.

During the store separation, the distance between the carrier and store is small, and the carrier will certainly influence the aerodynamics of the store. Besides, store separation is a kind of unloading process for the carrier. As the mass of the store is large, there is a big shocking force load on the carrier when separating the store. The shocking force is exciting force and will cause the carrier's vibration, which will make the interference aerodynamics of the store more complex. In order to ensure the launching process successful, the interferences aerodynamic forces of the store should be predicted accurately and understood deeply.

A great deal of work has been taken in the unsteady aerodynamic forces and flight path prediction of the store. In the early years, L. Formaggia et al. simulated the two-dimensional store separation problem numerically by coupling the two-dimensional Euler equation and rigid body dynamic equation ^[10]. With the great development of computer and CFD technology, the numerical simulation of dynamic responses of the three-dimensional store separation has become possible. A great deal of three-dimensional coupled solver based on the unsteady Euler equation and six-degree-freedom rigid body equations have been developed ^[11-14]. The coupled solvers have been used to solve the problems of air-to-air missile launching ^[15], cluster bomb separation ^[16], store separation ^[17], store trajectory optimization ^[18], store aerodynamic performance optimization ^[19] and store stability analysis ^[20]. In order to balance the simulation accuracy and performance, a quasi-steady approach has been implemented to model the dynamic responses of the store separation ^[21-22]. However, most of the studies mentioned above are the development, verification and application of the coupled solvers, and they pay little attention to the interference aerodynamic forces. Furthermore, those studies have ignored the effect of the elastic deformation and vibration of the carrier.

In our previous researches, the coupled functions of unsteady Navier-Stokes equations, six degrees of freedom dynamic equations and structural dynamic equations were solved simultaneously, and the store separations with the elastic deformation of the aircraft considered were simulated numerically. The previous study showed that the interference aerodynamics was obvious in the earlier time during the separation, and the dominant frequency of the elastic wing determined the aerodynamic forces frequencies of the store ^[23].

In this paper, focusing on the conditions in the earlier time during the separation, the steady and unsteady simulation methods are used to calculate the interference aerodynamic forces of the store at different angle of attacks (AOA) and relative distances between the carrier and store. The elastic vibrations of the carrier are considered in the simulation. According to the cause of formations of the interference aerodynamics, the interference aerodynamic forces of the store are divided into several components. The main purpose of this paper is to analyze relative magnitude, change rule and sphere of influence of the steady and unsteady interference aerodynamic forces components of the store quantitatively. What is more, this paper tries to explain the mechanism of production of the interference aerodynamics.

II. Research method

Research design

Assume that the aerodynamic forces of the store are comprised by the following components: 1. the aerodynamic forces caused by its geometry and flight attitude (expressed as F_1); 2. the interaction aerodynamic forces caused by the rigid carrier (expressed as F_2); 3. the interaction aerodynamic forces caused by the elastic deformation of the carrier (expressed as F_3). Here, F_3 can be divided into two parts artificially: the interaction aerodynamics caused by the static elastic deformation of the carrier (expressed as $F_{3,1}$) and the interaction aerodynamics caused by the dynamic elastic deformation of the carrier (expressed as $F_{3,2}$). In addition, assume that the aerodynamic forces components mentioned above can be added linearly. Thus, the aerodynamic forces of the store can be expressed as:

$$F_{\text{store}} = F_1 + F_2 + F_{3,1} + F_{3,2} \quad (1)$$

In this study, in order to extract the interference aerodynamic forces expediently, the change of the flight attitude of the store is ignored. In other words, the attitude angles and the displacements in x-direction and y-direction of the store are zero in all of the simulation states bellow. Therefore, F_1 is the aerodynamic forces caused just by the geometry of the store. In this study, F_1 is a function of AOA.

The steady numerical simulation method is used to study the interference aerodynamics of the store caused by the rigid carrier (F_2). In the study, the store is been locked in the different positions in z-direction. And the steady aerodynamic forces (F_{steady}) of the store are calculated. The steady aerodynamic forces F_{steady} contain F_1 and F_2 . When the distance between the store and the carrier is infinite, the interference aerodynamics will be sufficiently small, and the steady aerodynamic forces F_{steady} will equal to F_1 . The interference aerodynamic forces F_2 can be solved by the following formula:

$$F_2(\alpha, c) = F_{\text{steady}}(\alpha, c) - F_1(\alpha) = F_{\text{steady}}(\alpha, c) - F_{\text{steady}}(\alpha, \infty) \quad (2)$$

Here, c is the distance between the carrier and store, α is AOA.

The pseudo-steady and unsteady numerical simulation method is used to calculate the interference aerodynamics of the store caused by the elastic carrier (F_3). In this study, assume that the carrier is elastic, and the store is been locked in the different positions in z-direction. A concentrated load is applied on the elastic carrier. The magnitude of the load equals to the gravity of the store. The static deformation of the carrier can be simulated and the aerodynamic forces of the store (F_{def}) are calculated. F_{def} contain F_1 , F_2 and $F_{3,1}$:

$$F_{\text{def}}(\alpha, c) = F_1(\alpha) + F_2(\alpha, c) + F_{3,1}(\alpha, c) \quad (3)$$

$F_{3,1}$ can be expressed as:

$$F_{3,1}(\alpha, c) = F_{\text{def}}(\alpha, c) - (F_1(\alpha) + F_2(\alpha, c)) = F_{\text{def}}(\alpha, c) - F_{\text{steady}}(\alpha, c) \quad (4)$$

$F_{3,2}$ is the interference aerodynamic forces of the store under the dynamic elastic deformation of the carrier. Generally, the aeroelastic responses of the wing are vibration. During the simulation, the store is locked in the different positions in z-direction. At the initialization phase, a concentrated load is applied on the elastic carrier. The magnitude of the load equals to the gravity

of the store. Firstly, the static deformation of the carrier is calculated. Secondly, the concentrated load will disappear instantaneously, which is a shocking force loaded on the carrier. Under the action of the shocking force, the carrier will vibrate, and the unsteady aerodynamic forces responses of the store ($F_{\text{unsteady}}(\alpha, c, t)$) can be calculated. The unsteady aerodynamic forces can be expressed as:

$$F_{\text{unsteady}}(\alpha, c, t) = F_1(\alpha) + F_2(\alpha, c) + F_{3.1}(\alpha, c) + F_{3.2}(\alpha, c, t) \quad (5)$$

$F_{3.2}(\alpha, c, t)$ can be solved by the following formula:

$$F_{3.2}(\alpha, c, t) = F_{\text{unsteady}}(\alpha, c, t) - (F_1(\alpha) + F_2(\alpha, c) + F_{3.1}(\alpha, c)) = F_{\text{unsteady}}(\alpha, c, t) - F_{\text{def}}(\alpha, c) \quad (6)$$

To sum up, by separating the interference aerodynamics forces of the store, the relative magnitude, sphere of influence and mechanism of the aerodynamics components can be studied deeply.

Simulation method

In this paper, the GFSI (Generalized Fluid-Structure Interaction) solver is used to calculate the interference aerodynamics of the store caused by the rigid carrier and elastic carrier. The GFSI is a fluid-structure interaction solver developed by our research group, which can be used to solve the CFD/RBD problems and CFD/CSD problems. The GFSI has been got sufficient verification, and with high computing efficiency and calculation accuracy. In order to solve different dynamic problems flexibly, the framework of the solver code is modular design. The GFSI is been divided into several parts, such as Navier-Stokes equations solver (CFD module), six degrees of freedom dynamic equations solver (RBD module) and structural dynamic equations solver (CSD module). By calling different solver modules, the corresponding coupled equations are initiated in time-domain. The theoretic knowledge of CFD module and CSD module which have been employed in this paper are introduced bellow.

A. CFD module

The CFD module is used to calculate the aerodynamic forces of the aircraft. The governing equations are unsteady, three dimensional Navier-Stokes equations, and are described by the Arbitrary Lagrangian–Eulerian (ALE) method [14,24]. The conservation form of Navier-Stokes equations written in earth-fixed rectangular coordinate system is as follows:

$$\frac{\partial}{\partial t} \iiint_{\Omega} \mathbf{Q} dV + \iint_{\partial\Omega} \mathbf{F}(\mathbf{Q}, \mathbf{V}_{\text{grid}}) \cdot \mathbf{n} dS = \iint_{\partial\Omega} \mathbf{F}^V(\mathbf{Q}) \cdot \mathbf{n} dS \quad (7)$$

Where $\mathbf{Q} = [\rho, \rho u_x, \rho u_y, \rho u_z, e]^T$ is the vector of conservative variable,

$\mathbf{V}_{\text{grid}} = [u_{x\text{grid}}, u_{y\text{grid}}, u_{z\text{grid}}]^T$ is the velocity vector of the control volume due to elastic

deformation, $\mathbf{F}(\mathbf{Q}, \mathbf{V}_{\text{grid}})$ is inviscid flux vector, $\mathbf{F}^V(\mathbf{Q})$ is the viscous flux vector, Ω means the control volume, $\partial\Omega$ means the boundary of the control volume, $\mathbf{n}$ is the normal vector of the boundary face of a control volume. $\mathbf{F}(\mathbf{Q}, \mathbf{V}_{\text{grid}})$ and $\mathbf{F}^V(\mathbf{Q}) \cdot \mathbf{n}$ can be expressed as:

$$\mathbf{F}(\mathbf{Q}, \mathbf{V}_{grid}) = \begin{bmatrix} \rho \mathbf{V} \\ \rho u_i (\mathbf{V} - \mathbf{V}_{grid}) + P \\ (e_0 + P) \mathbf{V} + P \mathbf{V}_{grid} \end{bmatrix}, \mathbf{F}^V(\mathbf{Q}) \cdot \mathbf{n} = \begin{bmatrix} 0 \\ \sigma_{ij} n_i \\ (u_j \sigma_{ij} - q_i) n_i \end{bmatrix}$$

Where the variable σ_{ij} means the stress of the fluid, the subscript i means the direction of the acting surface and the subscript j means stress direction. q_i mean the distribution of the volumetric heat per unit mass in three directions. More details are given in Refs. [14, 25].

The state equations of ideal gas are expressed as follows.

$$P = (\gamma - 1) \left[e_0 - \frac{1}{2} \rho (\|\mathbf{V}\|)^2 \right] \quad (8)$$

$$T = \frac{P}{\rho R} \quad (9)$$

Where $\mathbf{V}$, P , ρ , T and e_0 are velocity vector, pressure, density, temperature and total internal energy per unit volume of the incoming flow respectively. γ means the specific heat ratio of the gas. The S-A turbulence model is employed to enclose the N-S equations [25].

A cell-centered finite volume method is employed based on unstructured hybrid meshes. Discrete equation (1) at a control volume and it can be expressed as:

$$\frac{d(\mathbf{Q}_n V_n)}{dt} = - \left[\mathbf{R}(\mathbf{Q}_n, \mathbf{V}_{grid}) - \mathbf{R}^V(\mathbf{Q}_n) \right] \quad (10)$$

Where V_n is the volume of the control volume signed as n , $\mathbf{R}(\mathbf{Q}_n, \mathbf{V}_{grid})$ and $\mathbf{R}^V(\mathbf{Q}_n)$ are residual values of the inviscid flux and the viscous flux respectively. The time derivative term $\frac{d(\mathbf{Q}_n V_n)}{dt}$ represents the unsteady characteristic of the control volume n . In this paper, the dual-time stepping method is used. While the pseudo-time variable τ is introduced, equation (10) becomes:

$$\frac{d(\mathbf{Q}_n^{k+1} V_n^{k+1})}{d\tau} = - \frac{3\mathbf{Q}_n^{k+1} V_n^{k+1} - 4\mathbf{Q}_n^k V_n^k + \mathbf{Q}_n^{k-1} V_n^{k-1}}{2\Delta\tau} - \bar{\mathbf{R}}(\mathbf{Q}_n^{k+1}, \mathbf{V}_{grid}^{k+1}) \quad (11)$$

Where,

$$\bar{\mathbf{R}}(\mathbf{Q}_n^{k+1}, \mathbf{V}_{grid}^{k+1}) = \mathbf{R}(\mathbf{Q}_n^{k+1}, \mathbf{V}_{grid}^{k+1}) - \mathbf{R}^V(\mathbf{Q}_n^{k+1}) \quad (12)$$

Where, the superscript k means the number of the time step. The conservative variables of the current time step $\mathbf{Q}_n$ can be solved with pseudo time iterative method. And in this paper, the Gauss-Seidel method is used for pseudo time marching and AUSM+ scheme is used for spatial discretization [14, 23-25].

The noslip boundary condition of cell-centred scheme is used for the solid wall [16, 21], and the relative velocity between the solid wall surface and the fluid is zero. In this paper, the dynamic mesh is used. As mentioned above, the velocity vector of the control volume is

$\mathbf{V}_{grid} = [u_{xgrid}, u_{ygrid}, u_{zgrid}]^T$, then the boundary condition can be expressed as:

$$\begin{cases} \rho_f = \rho_{cell} \\ u_{xf} - u_{xgrid} = 0 \\ u_{yf} - u_{ygrid} = 0 \\ u_{zf} - u_{zgrid} = 0 \\ P_f = P_{cell} \end{cases} \quad (13)$$

Where $\rho_f, u_f, v_f, w_f, P_f$ are density, velocity component and pressure on the solid wall. And ρ_{cell} and P_{cell} are the density and pressure of the cell adjacent to the wall.

The Riemann non-reflection boundary is employed in the farfield [25]. For the supersonic inflow, the variables on the boundary are only determined by the freestream values. Thus, for physical quantity q :

$$q_b = q_{input} \quad (14)$$

For the supersonic outflow, the variables on the boundary are determined by the solution inside the domain. Thus,

$$q_b = q_e \quad (15)$$

For the subsonic inflow, the boundary conditions are determined by the following equations:

$$\begin{cases} P_b = \frac{1}{2} \left\{ P_{input} + P_e + \rho_e c_e \left[\mathbf{n} \cdot (\mathbf{V}_e - \mathbf{V}_{input}) \right] \right\} \\ \rho_b = \rho_{input} + (P_b - P_{input}) / c_e^2 \\ \mathbf{V}_b = \mathbf{V}_{input} + \mathbf{n} \cdot (P_b - P_{input}) / \rho_e / c_e \end{cases} \quad (16)$$

And for the subsonic outflow, the boundary conditions are determined by the following equations:

$$\begin{cases} P_b = P_{input} \\ \rho_b = \rho_e + (P_e - P_b) / c_e^2 \\ \mathbf{V}_b = \mathbf{V}_e + \mathbf{n} \cdot (P_e - P_b) / \rho_e / c_e \end{cases} \quad (17)$$

Where, the subscript b means the boundary values; the subscript $input$ means the freestream values; the subscript e means the solution values inside the domain. The parameter c means speed of sound.

B. CSD module

When the deformation of the elastic body is much smaller than the reference length, the elastic deformation can be regarded as linear and expressed by mode method. The elastic displacement vector can be written as follows, where the first N order modes are considered.

$$\mathbf{w}(x, y, z, t) = \sum_{i=1}^N \Phi_i(x, y, z) \xi_i(t) \quad (18)$$

$\mathbf{w}(x, y, z, t)$ is the elastic displacement vector of structural deformation, $\Phi_i(x, y, z)$ is the i th-order mode, $\xi_i(t)$ is the generalized coordinate of the i th-order mode.

Derived from the Lagrange equations, the final structure dynamic equations based on the mode method can be expressed as:

$$\mathbf{M}\ddot{\xi} + \mathbf{G}\dot{\xi} + \mathbf{K}\xi = \mathbf{Q} \quad (19)$$

Where, $\mathbf{M}$ is the generalized mass matrix, $\mathbf{G}$ is the generalized structural damping matrix, $\mathbf{K}$ is the generalized stiffness matrix, and $\mathbf{Q}$ is the generalized force.

The structure dynamic equations are calculated by the second order Semi-Implicit Linear Multi-step Scheme (SILMS) [14, 26-27]. SILMS is the improvement of the Hybrid Linear Multi-step Scheme (HLMS), and has advantages of higher calculation precision, efficiency and robustness.

C. Coupling simulation

The loosely coupled method is used to solve the coupled equations of CFD/ CSD. For the unsteady simulation method, the unsteady N-S equation is coupled with the structure dynamic equations. And for the pseudo-steady simulation method, the steady N-S equation is coupled with the structure dynamic equations. Concretely, at the n th time step, the CFD module is called firstly to calculate the unsteady aerodynamic forces. And the aerodynamic forces are in earth-fixed coordinate system. Secondly, in terms of the CFD results, the aerodynamics forces in body-fixed coordinate system are computed through coordinate system transformation. Thirdly, with the aerodynamic forces solved above, call the CSD module to calculate the elastic deformation of the aircraft. Finally, the elastic deformation of the elastic aircraft are transformed to the earth-axis coordinate system, and superimposed to the surface grids of the aircraft. Then, update the grids with dynamic mesh technology to get ready for the next time step.

D. Dynamic mesh

The hybrid unstructured grid is used for spatial discretization. The grids near the wall are prism cells and the others are tetrahedral cells. The mesh deformation method is used to deal with the elastic deformation. And the weighted distance method is employed to distribute the displacements of surfaces. More details are given in Reference [14] and Reference [24].

Geometric model

In this study, the elastic carrier has been simplified as a wing. The wing is the same with AGARD445.6 wing. AGARD445.6 is designed by the Langley Research Center (NASA-LRC) to test the transonic flutter characteristics of elastic wing. The airfoil of AGARD445.6 is NACA65A004, and the aspect ratio is 1.644. In this study, the first four order structure modes are considered during the CFD/ CSD simulation [28].

The store is a tailless rigid delta wing aircraft designed by this paper. As shown in figure 1, the fuselage of the aircraft is revolution body with a length of 600mm and a diameter of 60mm. The wingspan is 364mm and the sweep angle is 70 deg. The distance between the mass center and the nose is 600mm.

Fig 1 The three view drawing of the store

III. Verification cases and grid convergence study

Verification of the CFD code

The ONREA M6 wing is employed to verify the CFD code in GFSI. In this case, the Mach number is 0.8395, the angle of attack (AOA) is 3.06 deg, and the Reynolds number is 1.127×10^7 . The S-A turbulence model is used to enclose the N-S equation. The grid of ONREA M6 wing is shown in figure 2.

Figure 3 shows the surface pressure coefficient result (C_p) of the wing at different position. The Ansys-Fluent results and experiment data are drawn in the figures for comparison. The results of the GFSI agree better with the experiment data ^[29] than the results of Ansys-Fluent. Therefore, the CFD code in GFSI is valid for similar problems.

Fig 2 Grid of M6 wing

a. $\eta/l=20\%$

b. $\eta/l=44\%$

c. $\eta/l=65\%$

d. $\eta/l=80\%$

e. $\eta/l=90\%$

f. $\eta/l=95\%$

Fig 3 Computed Cp profiles

Verification of the coupled CFD/CSD method

The dynamic responses of the standard AGARD445.6 wing are simulated to evaluate the accuracy of the coupled CFD/CSD simulation method used in this paper. At present, AGARD445.6 wing is

a standard verification case for dynamic simulation. The first four order modes are considered ^[28]. The incoming flow Mach number is 0.901 and the attack angle is 0 degree.

Figure 4 shows the surface grids of initial state. The surface cell number is 28 thousands and the volume cell number is 330 thousands. The S-A turbulence model is used to enclose the N-S equation. The dual-time stepping method is employed. To ensure the simulation precision, the real time step is the twentieth of the natural period of the highest order mode. The sub-iteration number is 100.

The generalized displacement responses are shown in figure 5. The non-dimensional flutter critical speed is about 0.357, which agree well with the experiment result 0.37 ^[28]. Therefore, the code and method this paper used can give a reliable result for the fluid-structure coupling simulation, and can be used for similar problems.

Fig 4 Surface grid of AGARD445.6 wing

Fig 5 The generalized displacement responses at different speed($Ma=0.901$)

Grid convergence study

The hybrid unstructured grid is used for spatial discretization. The grids nearby the wall are prism

cells and the others are tetrahedral cells. For the grid convergence study, three grids with different resolutions are generated. The grid statistics are summarized in table 1. The distance between the first prism cell and the wall is 10^{-5} m, and the parameter y^+ is less than 1 when the Reynolds number is $1.58e6$.

Fig 6 three grids with different resolutions

Table 1 Grid statistics		
	Wall surface cell number(thousand)	Volume cell number(million)
course	30.39	0.73
medium	62.30	1.65
fine	106.48	2.75

In this simulation, the Mach number is 0.7, the AOA is 2 deg, and the Reynolds number is $1.58e6$. The S-A model is employed. And the real time step is the twentieth of the natural period of the highest order mode. The sub-iteration number is 150. The interference aerodynamic forces of the store caused by the elastic wing are calculated. For different mesh resolutions, simulation results of the dynamic responses are shown in figure 7. In figure 7b, the results of the generalized displacement of the carrier with different grids are almost the same. However, as shown in figure 7a, the differences between the results of lift force responses of the course grid and the other grids are large, and the results of the medium grid have good agreement with the fine grid. Thus, when the density of the grids is refined, the spatial resolution of the solver is enhanced, and qualitative consistency of the numerical results performs well. As the results of the medium grid and fine grid are almost the same, and the medium grid has higher computing efficiency, the medium grid is selected for the following study.

a. results of lift response of the store with different grids

b. results of the generalized displacement of the carrier

Fig 7 results of grids convergence study

The error estimates of the simulation

For unsteady problems, the accumulated error should be considered. For the given allowable value of total error $S_{\max}$, the allowable number of time steps can be estimated by the following equation [30-31].

$$n_{\max} = (S_{\max} / S_{err})^2 \quad (20)$$

The code developed by our research group has 2-order numerical scheme [14]. As the hybrid unstructured grid is used, the mean ratios of the cell size in any directions are almost the same.

Then the mean ratios of the cell size can be expressed as $\Delta L = \sqrt[3]{V / c_n}$. There, V means the volume of the domain and c_n means the cell number. Then the S_{err} can be expressed as:

$$S_{err} = \left(\frac{\Delta L}{L_1}\right)^3 + \left(\frac{\Delta L}{L_2}\right)^3 + \left(\frac{\Delta L}{L_3}\right)^3 = \frac{V}{c_n} \left[\left(\frac{1}{L_1}\right)^3 + \left(\frac{1}{L_2}\right)^3 + \left(\frac{1}{L_3}\right)^3 \right] \quad (21)$$

The accumulated error is:

$$S = \sqrt{n} \times S_{err} \quad (22)$$

The ratio of maximal allowable number of time steps is defined by the following equation [30-31]:

$$R_s = n_{\max} / n \quad (23)$$

Consider the medium grid above mentioned, the error estimates are shown in Table 2 when the time step number is 650. When the allowable value of total error is 1%, the allowable number of time steps is 10890000. Because the time steps number is less than 650 in the following simulations, the error is below the given value in this paper.

Table 2 Error estimates

cell number	time step number	allowable number of time steps (allowable value of total error is 1%)	accumulated error	R_s
1650000	650	10890000	0.00007726	16753.846

IV. Research and analysis of the interference aerodynamics of the store caused by the rigid carrier

In this study, the Mach number is 0.7 and the Reynolds number is $1.58e6$. The AOA of 0deg, 2deg and 5deg are considered in the simulation. And the distance between the carrier and store of 0c, 0.1c, 0.2c, 0.3c, 0.4c, 0.5c, 1.0c, 2.0c and infinite are considered. Here, c is the mean aerodynamic chord. The S-A turbulence model is employed to enclose the N-S equation.

Figure 8 shows the static lift coefficient of the store at different simulation states. The horizontal coordinate means the distance between the carrier and store. Figure 9 shows the lift coefficient curves of the store. The horizontal coordinate is AOA.

As shown in figure 8, with the increase of the relative distance between the carrier and store, the lift coefficient of the store drops off primarily and then tends to the equilibrium constant. The equilibrium constant equals to value of infinity. Therefore, a preliminary conclusion can be

derived: with the increase of the relative distance, the aerodynamic interference between the carrier and store is decreased. The preliminary conclusion agrees with the common sense.

In figure 8, when the AOA is different, the variation trends of the lift coefficient of the store with the relative distance are different. At 0 deg, the lift coefficient of the store is largest when the relative distance is 0c, which is about 226 times the value of infinite distance. And with the increase of relative distance, the lift coefficient drops off and tends to the value of infinite distance. When the AOA is 2 deg, store's lift coefficient at the relative distance of 0c is about 1.2 times the value of infinite distance. With the increase of relative distance, lift coefficient of the store drops to the valley value firstly (at the relative distance of 0.3c and about 0.76 times the infinity value), and then increase to the value of infinite distance. When the AOA is 5 deg, store's lift coefficient at the relative distance of 0c is about 0.6 times the value of infinite distance. And the lift coefficient decreases to a minimum value (at the relative distance of 0.1c and about 0.56 times the value of infinite distance) and then increases with the increase of the relative distance between the carrier and store. When the lift coefficient is larger than the value of infinite distance, the interference aerodynamic force of the store is positive, and vice versa. To sum up, there are positive and negative interference aerodynamic forces at different AOA and relative distances, and the value of the interference aerodynamic force changes sensitively with change of AOA and relative distance.

In figure 9, the lift coefficient curve slope will drop off when the relative distance decrease. The slope of the lift coefficient curve of infinite distance is about 3.8 times the value of 0c relative distance. To put it in another way, when the relative distance between the carrier and store is small, the lift characteristics of the store are worse.

Fig 8 the static lift coefficient of the store at different simulation states.

Fig 9 the lift coefficient curves of the store at different relative distance.

However, what is the mechanism of the steady interference aerodynamic forces and what is the reason why the interference aerodynamics changes at different AOA and relative distance? This problem will be discussed below.

The store is a tailless rigid delta wing aircraft with a revolution body fuselage. A great amount of the lift is provided by the delta wing. At different flight state, the change of the lift is caused by the different of the surface pressure distribution of the delta wing. By observing and analysing the change of the surface pressure distribution, the problem mentioned above can be answered. A cross-section is made on the store, as shown in figure 10. The cross-section is parallel to the x-o-z plane and the distance between the cross-section and x-o-z plane is 0.07m. The surface pressure coefficient distributions on the cross-section are shown in figure 11.

Figure 11b shows the surface pressure coefficient distributions on the cross-section when AOA is 2 deg. With the increase of the relative distance between the carrier and store, the pressure coefficient distributions on the lower surface changes a little. However, on the upper surface of the store, the pressure coefficient distributions changes drastically. When the relative distance is 0c, the pressure on the upper surface of the leading edge is smaller than that on the lower surface, but the pressure on the upper surface of the trailing edge is much lower. Therefore, the lift is higher. With the increase of the relative distance between the carrier and store, the pressure on the upper surface of the leading edge decreases and the pressure on the upper surface of the trailing edge increases. And the change of the pressure at trailing edge is rapid. Then the lift decreases. When

the relative distance is $0.3c$, the lift is minimum. When the relative distance change from $0.3c$ to $2.0c$, the pressure at trailing edge changes a little, but the pressure on the upper surface of the leading edge increases. Then the lift increases.

Figure 11a shows the pressure coefficient distributions on the cross-section when the AOA is 0° . When the relative distance is $0c$, the pressure on the upper surface of the leading edge is relative high and the pressure on the upper surface of the trailing edge is much lower. With the increase of the relative distance between the carrier and store, the pressure on the upper surface of the leading edge decrease and the pressure on the upper surface of the trailing edge increase. When the relative distance tend to infinity, the pressure coefficient distributions on the upper and lower surface tend to the same. And the lift tends to 0.

To sum up, the change of the pressure distribution on the upper surface of the store leads to the change of its lift.

Fig10 the location of the cross-section ($Y=0.07m$)

a.AOA=0 deg

b.AOA=2 deg

Fig11 the Cp profiles of the store on the cross-section (Y=0.07m)

Figure 12 and figure 13 show the pressure and x-velocity contours on the cross-section ($Y=0.07\text{m}$) of the store at AOA of 2 deg respectively. In figure 12a, the high pressure on the stagnation point of the carrier influences the pressure on leading edge of the store. At the leading edge, the pressure on the upper surface of the store is higher than that on the lower surface. On the other hand, in figure 13a, there are accelerative effect in the flow field between the carrier and store. From the leading edge to the trailing edge, the thickness of the wing increases, and the space between the carrier and store decreases. The speed of the flow will increase and the static pressure decrease.

Comparing figure 12b with 12a, the relative distance increase, and the influence of the high pressure on the stagnation point of the carrier to the store wear off. Contrast figure 13b with 13a, the speed between the carrier and store decreases. Thus, the accelerative effect of the flow between the carrier and store weakens when the relative distance increases, and the pressure on the upper surface of the trailing edge increases.

In the analysis of the space flow fields, we can find that the high pressure on the stagnation point of the carrier and the accelerating effect of the flow between the carrier and store influence the pressure on the upper surface of the store, which led to the change of the interference aerodynamic forces of the store.

Fig12 pressure contours on the cross-section ($Y=0.07\text{m}$) when AOA=2deg

Fig13 x-velocity contours on the cross-section ($Y=0.07\text{m}$) when AOA=2deg

At the state where the relative distance between the carrier and store is small, with the action of the interference aerodynamics, the pressure on the upper surface of the store's leading edge increases and of the trailing edge decreases. Then, a nose-down pitching moment is produced. The magnitude of the interference pitching moment is very considerable as the relative distance is small. For example, when AOA is 2deg and relative distance is 0c, the total pitching moment of the store is about 2 times the value of infinite distance. With the increase of the relative distance, the interference pitching moment of the store decreases, and the total pitching moment of the store decreases (shows in figure 14).

In figure 15, the slopes of the pitching moment curves of the store are negative. The store is statically stable. However, with the decrease of the relative distance, the slopes of the pitching moment curves decrease. When the relative distance is 0c, the slopes of the pitching moment curves is -0.00016, which is 1/12 times the value of infinite distance. Thus, when the carrier and store are much close to each other, the static stability margin will be poor.

Fig14 the pitching moment coefficient of the store at different simulation states.

Fig15 the pitching moment coefficient curves of the store at different relative distance.

V. Research and analysis of the interference aerodynamics of the store caused by the static deformation of the carrier

In this study, the Mach number is 0.7 and the Reynolds number is 1.58×10^6 . The AOA is 2deg, and the relative distance between the carrier and store of 0.1c, 0.3c, 0.5c, 1.0c, 2.0c and infinity are considered. The S-A turbulence model is used to enclose the N-S equation. The dual-time stepping method is employed. To ensure the simulation precision, the real time step is the twentieth of the natural period of the highest order mode. The sub-iteration number is 150.

The interference aerodynamic lift coefficients of the store caused by the elastic carrier with static deformation are calculated ($Cl_{3.1}$) and shown in table 3. $Cl_{3.1}$ is solved through formula (4). Here, F_{steady} has been solved above, and F_{def} will be calculated in this section primarily. Figure 16 to 18 show the simulation results at the state when the AOA is 2 deg and relative distance is 0.3c. Figure 16 shows the static deformation of the carrier. Figure 17 shows the the lift coefficient response of the store. Figure18 show the generalized displacement response of the carrier.

When the aerodynamic load is applied on the carrier, the carrier is deformed, as shown in figure 16. In this simulation, the first four order structural modes are considered. The first and third order structural modes of AGARD445.6 wing are flexural modes. The second and fourth order structural modes of AGARD445.6 wing are torsional modes. In figure 18, the magnitude of generalized displacement of the first flexural mode is about ten times that of the first torsional mode, and the generalized displacement of the second flexural mode and second torsional mode are much

smaller. The first flexural mode is the leading mode of the carrier. Thus, the bend deformation is the most obvious and plays a leading role. Under the action of the aerodynamic load, the wing bends upward. The relation distance between the carrier and store increases by $0.0236c$. In table 3, lift coefficient of the store increases by 0.001691 . This conclusion agrees with the above results. In figure 8, at the state when AOA is 2° and relative distance is $0.3c$, the lift coefficient of the store will increase with the increase of the relative distance. Besides, the torsional deformation of the carrier will change its local AOA, which will cause the interference aerodynamics as well, although the value is very small (In this cases, the local AOA of the carrier has increased by 0.0095°). When the carrier is deformed, the change of the relative distance and local AOA cause the change of the aerodynamic forces of the store. This is the primary cause of the generation of the interference aerodynamic forces $F_{3.1}$.

Table 3 the lift coefficient of the store at different relative distance

	Cl_{steady}	Cl_{def}	$Cl_{3.1}$
0.3c	0.013176	0.014867	0.001691
1.0c	0.016116	0.016751	0.000636
2.0c	0.016526	0.016779	0.000253

Fig16 the static deformation of the wing when the relative distance is $0.3c$

Fig17 the lift coefficient response of the store when the relative distance is $0.3c$

Fig18 the generalized displacement response of the carrier when the relative distance is 0.3c

VI. Research and analysis of the interference aerodynamics of the store caused by the dynamic deformation of the carrier

The main purpose of this section is to calculate the interference aerodynamic forces $F_{3,2}(\alpha, c, t)$ of the store, when the relative distance between the carrier and store is 0.1c, 0.3c, 0.5c, 1.0c, 2.0c and infinity. $F_{3,2}(\alpha, c, t)$ can be solved by formula (6). In formula (6), $F_{\text{def}}(\alpha, c)$ has been solved above, and the unsteady aerodynamic forces responses of the store ($F_{\text{unsteady}}(\alpha, c, t)$) will be calculated in this section primarily. During the simulation, a concentrated load is applied on the elastic carrier. The magnitude of the load equals to the gravity of the store. Firstly, the static deformation of the carrier will be calculated. Secondly, the concentrated load will disappear instantaneously, which simulates the uploading of the concentrated force when the store separates. Thus, the concentrated load acted as the vibration exciting force of the carrier. As shown in figure 19, during the time of 0~0.0175s, the static deformation of the carrier and the lift coefficient of the store are convergent under the action of the concentrated load. At about 0.0175s, the concentrated load disappears instantaneously. And after 0.0175s, the dynamic deformation response of the carrier and the unsteady aerodynamic forces of the store can be calculated.

The unsteady lift coefficients responses of the store at different relative distance are shown in

figure 20. The oscillation of the unsteady lift coefficients responses of the store is convergent. The mean value and peak value of the lift coefficients responses of the store ($Cl_{unsteady}(\alpha, c, t)$) are shown in table 4. The mean value of $Cl_{unsteady}(\alpha, c, t)$ equals $Cl_{def}(\alpha, c)$. Therefore, the zero-mean-value of the unsteady aerodynamic forces responses are $Cl_{3,2}(\alpha, c, t)$.

The other component forces of the aerodynamics of the store have the similar characteristics. For example, when the AOA is 2deg and the relative distance is 0.3c, the mean value of drag coefficient $Cd_{unsteady}(2deg, 0.3c, t)$, side force coefficient $Cy_{unsteady}(2deg, 0.3c, t)$ and pitching moment coefficient $Cmy_{unsteady}(2deg, 0.3c, t)$, equal $Cd_{def}(2deg, 0.3c)$, $Cy_{def}(2deg, 0.3c)$ and $Cmy_{def}(2deg, 0.3c)$ respectively (as shown in table 5).

Fig19 the lift coefficient response of the store and the generalized displacement response of the first order mode of the carrier (AOA=2deg, relative distance=0.3c)

Fig20 the lift coefficient response of the store at different relative distance

Table 4 value of $Cl_{unsteady}(t)$ and Cl_{def}

Relative distance	0.1c	0.3c	0.5c	1.0c	2.0c
peak value of $Cl_{unsteady}(t)$	0.0324243	0.0233063	0.0204708	0.0191125	0.0177437
mean value of $Cl_{unsteady}(t)$	0.0163526	0.0148575	0.0152407	0.0167376	0.0167716
Cl_{def}	0.0163462	0.0148670	0.0152529	0.0167512	0.0167785

Table 5 comparison of $Cf_{unsteady}(t)$ and Cf_{def} when AOA is 2 deg and relative distance is 0.3c

	Cd	Cy	Cmy
mean value of $Cf_{unsteady}(t)$	0.008467	0.000271	-0.003220
Cf_{def}	0.008469	0.000272	-0.003221

When the AOA is 2deg, and the relative distance is 0.3c, the zero-mean-value of the unsteady lift coefficients responses of the carrier and store are shown in figure 21. The frequency and the variation trend of the two responses curves are almost the same. We guess that there is a ratio relationship between the two responses curves, and the ratio is unrelated to the time. The ratio relationship is defined as $k(c)$. In figure 21, the peak value of the lift coefficients responses of the carrier and store are -0.101797 and 0.008439 respective. Then, $k(0.3)$ can be solved:

$$k(0.3c) = -0.101797 / 0.008439 = -12.0622$$

When the zero-mean-value of the unsteady lift coefficients responses of the store are multiplied by $k(0.3c)$, we can get a new responses curve of the store, as shown in figure 22. The new curve agrees with the zero-mean-value of the unsteady lift coefficients responses of the carrier well. At different relative distances, the $k(c)$ are calculated and shown in Figure 23. The $k(c)$ is a function of c . With the increases of c , the k increases. When the relative distance is $2.0c$, $k(2.0c)$ is -91.4294 . Therefore, we can draw a conclusion that there is a ratio relationship k between the zero-mean-value of the lift coefficients responses of carrier and store, $k(c)$ is a function of c and is unrelated to the time. It is a good property for the system. With this conclusion, it is easy to build the interference aerodynamic forces models of the store.

Fig21 zero-mean-value of the lift coefficients responses of the carrier and store
(AOA = 2deg, relative distance = 0.3c)

Fig22 comparison of zero-mean-value of the lift coefficients responses of the carrier and the new responses curve of store (AOA =2deg, relative distance= 0.3c)

Fig23 k(c) at different relative distances

VII. Component analysis of the interference aerodynamics of the store

The aerodynamic forces of the store have been divided into different components. The main purpose of this section is to analyze the percentage of different component. F_{unsteady} is a function of time. Therefore, the peak value of F_{unsteady} is considered in the analysis. In this section, the AOA is 2 deg.

As shown in table 6 and figure 24, the geometric characteristic is the most important factor of the lift of the store. When the relative distance is 0.1c, the Cl_1/Cl_{store} is above 50% and the Cmy_1/Cmy_{store} is above 40%. And values of Cl_1/Cl_{store} and Cmy_1/Cmy_{store} increase with the increases of the relative distance. When the relative distance is 2.0c, the Cl_1/Cl_{store} is 98.2% and the Cmy_1/Cmy_{store} is 97.6%, and the interference aerodynamics is very small. On the other hand, the values of Cd_1/Cd_{store} are above 86% in all of the simulation states. And the change of the interference drag is smaller with the increase of the relative distance, as shown in figure 24b. Therefore, the sphere of influence of the interference aerodynamic forces of the store is 0~2.0c.

In table 6 and figure 24a, the interference aerodynamics of the store caused by the elastic deformation is obvious. When the relative distance is 0.1c, the $Cl_{3.2}/Cl_{\text{store}}$ is 49.59% and $Cmy_{3.2}/Cmy_{\text{store}}$ is 48.97%. Put it in another way, when the relative is small, the interference aerodynamics of the store caused by the elastic deformation of the carrier is about half of the total aerodynamic forces of the store. However, with the increase of the relative distance, the $Cl_{3.2}/Cl_{\text{store}}$ and $Cm_{3.2}/Cmy_{\text{store}}$ will decrease. And when relative distance is 2.0c, the value of $Cl_{3.2}/Cl_{\text{store}}$ and $Cmy_{3.2}/Cmy_{\text{store}}$ are only 5.44% and 6.15% respectively.

By contrast, the F_2 and $F_{3.1}$ are relative smaller, but cannot be ignored. For example, with the increase of the relative distance, the values of $Cl_2, Cl_{3.1}, Cmy_2$ and $Cmy_{3.1}$ first increase and then decrease. When the relative distance is 0.3c, Cl_2/Cl_{store} get to the peak value of -18.23%. When the relative distance is 0.5c, Cmy_2/Cmy_{store} gets to the peak value of -14.41%.

Table 6 percentage of the interference lift coefficient of store at different simulation state

	0.1c	0.3c	0.5c	1.0c	2.0c
Cl_1/Cl_{store}	53.74%	74.77%	85.12%	91.17%	98.20%
Cl_2/Cl_{store}	-7.33%	-18.23%	-16.66%	-6.85%	-5.07%
$Cl_{3.1}/Cl_{\text{store}}$	4.01%	7.26%	6.04%	3.33%	1.42%
$Cl_{3.2}/Cl_{\text{store}}$	49.59%	36.21%	25.49%	12.35%	5.44%

a.Cl

b.Cd

c.Cmy

Fig24 percentage of the interference aerodynamic forces coefficients of store at different simulation state

VIII. Conclusion

This paper focuses on the conditions in the earlier time during the store separation, and calculates the steady and unsteady interference aerodynamic forces of the store at different AOA and relative distances. According to the simulation results, some conclusions are obtained as follows.

1. With the decrease of the relative distance of the carrier and store, the lift coefficient curve slope and pitching moment curve slope decrease. The slope of the lift coefficient curve of infinite distance is about 3.8 times the value of 0c relative distance. And the pitching moment curve slope of infinite distance is about 12 times the value of 0c relative distance. Therefore, under the influence of the interference aerodynamic forces, the lift characteristics of the store are worse and the static stability margin is poorer.
2. The interference aerodynamics of the store caused by the elastic deformation is obvious. When the relative distance is 0.1c, the $Cl_{3.2}/Cl_{store}$ and $Cmy_{3.2}/Cmy_{store}$ are 49.59% and 48.97% respectively. In other words, when the relative distance is small, the interference aerodynamics of the store caused by the elastic deformation of the carrier is about half of the total aerodynamic forces of the store, which should be carefully considered in the engineering applications.
3. The sphere of influence of the interference aerodynamic forces of the store is 0~2.0c. When

the relative distance is $2.0c$, the value of Cl_1/Cl_{store} , Cd_1/Cd_{store} and Cmy_1/Cmy_{store} are 98.2%, 94.39% and 97.6% respectively, and the value of the interference forces are very small.

4. There is a ratio relationship k between the zero-mean-value of the lift coefficients responses of carrier and store, $k(c)$ is a function of c and is unrelated to the time. It is a good property for the system to build the lift forces models of the store easily.
5. The mean value of the interference aerodynamics responses of the store caused by the carrier with dynamic deformation equals the value of interference aerodynamics of the store caused by the carrier with static deformation.

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