Apex Height Control of a Two-Mass Robot Hopping on a Rigid Foundation

Frank B. Mathis and Ranjan Mukherjee

*Department of Mechanical Engineering,
Michigan State University,
East Lansing, MI, 48824, USA
email: mukherji@egr.msu.edu*

Abstract

The spring loaded inverted pendulum (SLIP) model is commonly used to describe the dynamics of hopping robots. A fundamental limitation of the SLIP model is that it fails to account for impact with the ground; this is due to the fact that the leg is modeled as a massless spring. Assuming that the ground is rigid, a two-mass model with inelastic impact is proposed for more accurate description of hopping dynamics. A control scheme is developed to converge the maximum jumping height of the center-of-mass of the robot to a desired value. The control scheme utilizes feedback linearization in continuous time and update of a control parameter in discrete time. Simulation results are first presented to show the efficacy of the control scheme. Experimental results based on a voice-coil actuator show good match with simulation results despite actuator saturation.

Keywords: two-mass hopping robot, apex height control, inelastic impact, hybrid dynamics, hybrid control, periodic orbit

1. Introduction

Some of the early work on hopping robots can be credited to Seifert [1], who explored the idea of using a pogo stick for transportation on the moon; and Matsuoka [2], who investigated two-dimensional hopping in humans with a one-legged model. Following these initial investigations, a hopping gait was experimentally demonstrated by Raibert [3], [4] using a mass on a hydraulic piston; the control design used a spring like motion in the piston and a stabilizing torque at the hip joint. Schwind and Koditchev [5] proposed a lossless

model of Raibert’s planar hopper for the purpose of controlling its forward velocity. A closed form of the return map of a hop was derived and a controller was proposed for improving upon the gait of Raibert’s hopper. Saranli et al. [6] introduced the SLIP (spring loaded inverted pendulum) model of the hopper and argued its equivalence to the higher degree-of-freedom AKH (ankle-knee-hip) leg model. A deadbeat controller, which varied the spring parameters of the SLIP model at each hop, was designed to produce stable periodic motion. Recently, Cherouvim and Papadopoulos [7] developed a method for controlling both the forward speed and apex height using a hip torque; the leg was modeled as a massless spring in the stance phase.

Following the work by Schwind and Koditchev [5] and Saranli et al. [6], the SLIP model became very popular in the research community. Hyon used a combination of hydraulic actuators and springs to apply Raibert’s control method to a kneed robot [8]. Kajita used a resolved momentum approach to control a legged robot to a desired trajectory [9]. Ghigliazza et al. [10] and Holmes et al. [11] investigated passive stability of the SLIP model for various parameters. Poulakakis and Grizzle [12], [13] investigated the SLIP model with an asymmetric mass and compared its behavior with that of a three-link hopper.

The SLIP model is incapable of accounting for impulsive forces that are generated at the time of impact with the ground. To account for the impulsive forces, it is necessary to model the leg as a mass rather than a massless spring. The two-mass model is particularly important for the apex-height control problem where the loss of energy due to impact has to be taken into account. Dankowicz and Petrinen [14] and Burden et al. [15] addressed the problem of modeling and control a two-mass hopper, and impulsive systems in general. A two-mass hopper model was also investigated by Saitou et al. [16]; optimal control methods were used to maximize the jumping height in the presence of control constraints. Ishikawa et al. [17] used a port-controlled Hamiltonian method to control the energy of a two mass system to a desired level to maintain a maximum jumping height. Both Saitou et al. [16] and Ishikawa et al. [17] modeled the ground as a spring and neglected the the impulsive effect of ground impact. Mikhailova [18] proposed a speed-gradient method to control the energy of a two-mass hopping system but a spring was placed between the lower mass and the ground; a similar model was used to study lift-off dynamics by Aguilar et al. [19]. Yu and Iida [20] investigated a method for inducing hopping of a two-mass robot through rotational motion of a third mass connected to the upper mass by a spring.

In this paper we consider a two-mass hopping robot undergoing inelastic collision with the ground at each hop. We use feedback linearization to stabilize the internal dynamics. The periodic nature and stability characteristics of the resulting hybrid dynamic system is analyzed using Poincare maps [21] and the OGY¹ method of chaos control [22] is used to discretely adjust a system parameter and converge the apex height to its desired value. The validity of the control method is verified through simulations and experiments²; the parameters used in simulations are identical to those used in experimental hardware. The paper is structured as follows. In Section 2 we present the system dynamics for the flight phase, inelastic impact, and contact phase. In Section 3 we present a hybrid control strategy comprised of continuous-time controllers that stabilize the internal dynamics in the flight and contact phases and a discrete controller that stabilizes the periodic orbit for the desired apex height. Section 4 presents simulation results and experimental results are presented in Section 5. Section 6 contains concluding remarks and directions for future research.

2. Dynamics of Two-Mass Hopper

2.1. System Description

Consider the two-mass robot in Fig.1 (a), which is hopping on a rigid foundation. It is comprised of an upper mass m_1 and a lower mass m_2 , both of which are constrained to move in the vertical direction. The upper and lower masses are connected by a prismatic joint. The force applied by the actuator in the prismatic joint on the two masses is denoted by F ; it is assumed that this force can be actively controlled. The positions of the center-of-mass of m_1 and m_2 from the datum are denoted by x_1 and x_2 . The position of the center-of-mass of m_1 relative to that of m_2 is denoted by y and the height of the center-of-mass of m_2 from the base of m_2 is denoted by ℓ . The force of interaction between the lower mass and the ground is denoted by F_{ext} . The hybrid dynamics of the robot is comprised of three distinct phases: the flight phase, the impact phase, and the contact phase. These phases are described next.

¹A method introduced by Ott, Grebogi and Yorke for achieving stabilization of a periodic orbit [22].

²Preliminary simulation results appeared in our earlier work [23].

Figure 1: (a) Two-mass robot hopping on a rigid surface, (b) free-body diagrams of the two masses at an instant when the lower mass is in contact with the ground.

2.2. Flight Phase

During the flight phase, the robot is not in contact with the ground and therefore the following conditions hold:

$$x_2 > \ell \quad F_{\text{ext}} = 0 \quad (1)$$

From the free-body diagrams in Fig.1 (b), the equations of motion of the robot can be obtained as follows:

$$\ddot{x}_1 = -g + \frac{F}{m_1} \quad \ddot{x}_2 = -g - \frac{F}{m_2} \quad (2)$$

Using the relative displacement $y \triangleq (x_1 - x_2)$ to replace x_1 , an alternate form of the equations of motion are

$$\ddot{y} = \left[\frac{m_t}{m_1 m_2} \right] F \quad \ddot{x}_2 = -g - \frac{F}{m_2} \quad (3)$$

where

$$m_t \triangleq (m_1 + m_2)$$

is the total mass of the system.

2.3. Impact Phase

The impact phase refers to infinitesimal time intervals $t \in [t^-, t^+]$ during which the lower mass m_2 comes in contact with the ground and the condition $x_2 > \ell$ changes to $x_2 = \ell$. During these time intervals, we assume the following to be true:

Assumption 1: The collision of m_2 with the ground surface is inelastic.

Assumption 2: The control force F is not impulsive in nature.

The implications of these assumptions are:

$$x_1(t^+) = x_1(t^-) \quad \dot{x}_1(t^+) = \dot{x}_1(t^-) \quad (4)$$

$$x_2(t^+) = x_2(t^-) \quad \dot{x}_2(t^+) = 0 \quad (5)$$

If we use variables y and x_2 , (4) and (5) can be rewritten as

$$y(t^+) = y(t^-) \quad \dot{y}(t^+) = \dot{y}(t^-) + \dot{x}_2(t^-) \quad (6)$$

$$x_2(t^+) = x_2(t^-) \quad \dot{x}_2(t^+) = 0 \quad (7)$$

Using matrix notation, (6) and (7) can be written as follows:

$$X(t^+) = S_{\text{rf}} X(t^-) \quad (8)$$

where

$$X \triangleq \begin{bmatrix} y \\ \dot{y} \\ x_2 \\ \dot{x}_2 \end{bmatrix} \quad S_{\text{rf}} \triangleq \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (9)$$

2.4. Contact Phase

The period of time after the impact phase for which mass m_2 remains in contact with the ground is the contact phase. For the contact phase, we make the following assumption:

Assumption 3: The force exerted by the ground on mass m_2 is non-negative, i.e., $F_{\text{ext}} \geq 0$.

The above assumption follows directly from the unilateral nature of the constraint imposed by the ground. During contact, we have

$$x_2 \equiv \ell \quad \Rightarrow \quad \dot{x}_2 = \ddot{x}_2 = 0 \quad (10)$$

and therefore the robot is left with a single degree-of-freedom. By summing forces on the two masses and substituting (10), we get the equation of motion

$$\ddot{x}_1 = -g + \frac{F}{m_1} \quad (11)$$

and the expression for F_{ext} in terms of F :

$$F_{\text{ext}} = F + m_2g \quad (12)$$

There is freedom in the choice of the control force F but for contact to be maintained, it has to satisfy

$$F_{\text{ext}} \geq 0 \quad \Rightarrow \quad F \geq -m_2g$$

At the instant when the system switches from the contact phase to the flight phase, F_{ext} crosses zero and we have

$$F = -m_2g$$

2.5. Unified Dynamics: Flight and Contact Phases

Using (3), (10) and (11), the dynamics of the hopping robot for the flight phase and the contact phase can be written in an unified manner as follows:

$$\ddot{y} = \left[\frac{m_t}{m_1 m_2} \right] F - \frac{\lambda F_{\text{ext}}}{m_2} \quad (13)$$

$$\ddot{x}_2 = -g + \frac{(\lambda F_{\text{ext}} - F)}{m_2} \quad (14)$$

where λ is defined as follows:

$$\lambda = \begin{cases} 0 & : \text{Flight Phase} \\ 1 & : \text{Contact Phase} \end{cases} \quad (15)$$

and F_{ext} is defined by (12).

2.6. Apex Height

If z denotes the height of the center-of-mass of the hopping robot, we have

$$z = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = x_2 + m_f y, \quad m_f \triangleq (m_1/m_t) \quad (16)$$

The hopping robot will have multiple flight phases. For each flight phase, the apex height is defined as the maximum value of z , and is denoted by h .

3. Hybrid Control Based on the OGY Method

We present a hybrid controller comprised of a continuous controller and a discrete controller. The continuous controller modulates the actuator force F during the flight and contact phases. Based on the OGY method [22], the discrete controller alters a parameter of the closed-loop system discretely at the instants when the system switches from the contact phase to the flight phase.

3.1. Continuous Control Design

We define r to be the height of the center-of-mass of the robot relative to that of the lower mass m_2 . Using (13) and (16), it can be shown

$$\begin{aligned} r &\triangleq (z - x_2) = m_f y \\ \Rightarrow \quad \ddot{r} &= m_f \ddot{y} = \frac{1}{m_2} [F - m_f \lambda F_{\text{ext}}] \end{aligned} \quad (17)$$

Using (16) and (17), it can be shown that

$$\begin{aligned} \ddot{z} &= \ddot{x}_2 + \ddot{r} \\ &= -g + \frac{1}{m_t} \lambda F_{\text{ext}} \end{aligned} \quad (18)$$

Assuming that F_{ext} will be measured and available, the choice of the control input

$$F = m_f \lambda F_{\text{ext}} + m_2 v \quad (19)$$

results in

$$\ddot{r} = v \quad (20)$$

where v is the new control variable. The choice of v for the flight and contact phases are discussed next.

3.1.1. Flight Phase

The equations of motion are obtained using $\lambda = 0$ and (18) and (20) as follows:

$$\ddot{z} = -g, \quad \ddot{r} = v \quad (21)$$

From (21) it is clear that the dynamics of z is uncontrollable. To control r , we choose v as follows

$$v = -D\dot{r} - K(r - r_d), \quad r_d, K, D > 0 \quad (22)$$

where r_d is the desired equilibrium configuration of r , and K and D are proportional and derivative gains. The choice of v in (22) results in the controllable subsystem behaving like a mass-spring-damper with an asymptotically stable equilibrium $(r, \dot{r}) = (r_d, 0)$.

3.1.2. Contact Phase

The equations of motion are obtained using $\lambda = 1$ and (10), (18) and (20) as follows:

$$\ddot{z} = \ddot{r}, \quad \ddot{r} = v \quad (23)$$

The equation $\ddot{z} = \ddot{r}$ can be simplified to the form $\ddot{z} = \ddot{r} + \ell$; this is in agreement with the earlier observation that the system has a single degree-of-freedom. We choose the control variable v as follows:

$$v = \begin{cases} -D\dot{r} - K(r - r_d) & \dot{r} \leq 0 \\ -\nu D\dot{r} - K(r - r_d) & \dot{r} > 0 \end{cases} \quad (24)$$

where r_d , K and D were defined earlier and ν is a negative constant. In (24), D is chosen to satisfy

$$0 < D < 2\sqrt{K} \quad (25)$$

such that the controlled system behaves like an underdamped mass-spring-damper during its downward motion. During the upward motion, $\nu < 0$ results in negative damping; this provides the scope for adding energy to the system.

3.2. Hybrid Dynamics of Closed-Loop System

3.2.1. Flight Phase

The flight dynamics can be obtained using (16), (17), (21) and (22), as follows:

$$\begin{aligned} \ddot{y} &= -D\dot{y} - K \left[y - \frac{1}{m_f} r_d \right] \\ \ddot{x}_2 &= -g + m_f D\dot{y} + K(m_f y - r_d) \end{aligned} \quad (26)$$

3.2.2. Impact Phase

The impact phase dynamics is described by the mapping in (8).

3.2.3. Contact Phase

Using (16), (17), (23), and (24), the contact phase dynamics can be expressed as follows:

$$\ddot{y} = -\mu D\dot{y} - K \left[y - \frac{1}{m_f} r_d \right] \quad (27)$$

$$\ddot{x}_2 = 0$$

where μ is defined as

$$\mu = \begin{cases} 1 & : \dot{r} \leq 0 \\ \nu & : \dot{r} > 0 \end{cases}$$

3.3. Construction of First Return Map

We define the first return as the mapping over a single hop starting from liftoff and ending at the next liftoff. Without loss of generality, we consider the time of first liftoff to be $t = 0$, and Δt_1 , Δt_2 , and Δt_3 to be the durations of the flight phase, contact phase with $\dot{r} \leq 0$, and contact phase with $\dot{r} > 0$, respectively. For convenience, we define

$$T_1 = \Delta t_1, \quad T_2 = (\Delta t_1 + \Delta t_2), \quad T_3 = (\Delta t_1 + \Delta t_2 + \Delta t_3)$$

The states, defined in (9), at the end of the flight phase can be obtained from (26) as follows

$$X(T_1^-) = e^{A_f \Delta t_1} X(0) + B_f \quad (28)$$

where A_f and B_f are derived in Appendix A. Using (8), the states after impact with the ground can be described by the relation

$$X(T_1^+) = S_{rf} X(T_1^-) \quad (29)$$

Using (27), the states at the end of the contact phase can be shown to be

$$X(T_2) = e^{A_c \Delta t_2} X(T_1^+) + B_c \quad (30)$$

$$X(T_3) = e^{A_p \Delta t_3} X(T_2) + B_p \quad (31)$$

where A_c , B_c , A_p , and B_p are derived in Appendix A.

Let the discrete state variable ξ denote the free states of the system at the time of lift-off. The following three conditions are satisfied at the time of lift-off:

$$x_2 = \ell, \quad \dot{x}_2 = 0, \quad F_{\text{ext}}(y, \dot{y}) = 0$$

This leaves one independent variable, which we choose to be $\dot{y}$. At the beginning and end of the first hop, the discrete states are therefore

$$\xi(0) = \dot{y}(0), \quad \xi(1) = \dot{y}(T_3)$$

and they are related by the expression

$$\xi(1) = \mathbb{P}[\xi(0), \Delta t_1, \Delta t_2, \Delta t_3] \quad (32)$$

where $\mathbb{P}[\cdot]$ is the Poincare map. The time intervals Δt_1 , Δt_2 and Δt_3 can be solved using the following equations that hold for both the rigid and elastic foundations:

$$\begin{aligned} x_2(\Delta t_1) &= \ell \\ \dot{y}(\Delta t_1 + \Delta t_2) &= 0 \\ F_{\text{ext}}(\Delta t_1 + \Delta t_2 + \Delta t_3) &= 0 \end{aligned}$$

Therefore, the dependence of $\mathbb{P}$ on Δt_1 , Δt_2 and Δt_3 can be dropped in (32). For the k -th hop, we can now write

$$\xi(k+1) = \mathbb{P}[\xi(k)] \quad (33)$$

Definition: A point $\xi = \xi^*$ is a fixed point if $\mathbb{P}[\xi^*] = \xi^*$.

By linearizing (33), it can be shown that a fixed point ξ^* is asymptotically stable if and only if the spectral radius ρ of the Jacobian matrix is less than unity [22], *i.e.*,

$$\rho \left[\frac{\partial \mathbb{P}(\xi)}{\partial \xi} \bigg|_{\xi=\xi^*} \right] < 1 \quad (34)$$

For a given set of system parameters such as K , D , and ν , there exists a set of fixed points ξ^* ; each of these fixed points correspond to a single apex height h . Our goal is to choose a fixed point ξ^* and system parameters which correspond to our desired apex height. There is no guarantee that the fixed point ξ^* for our desired height will be asymptotically stable, and therefore, an additional discrete controller (OGY method) will be invoked to ensure stability of the fixed point.

3.4. Discrete Control Design

Although many system parameters can be varied, we choose to vary ν (a single parameter) for the sake of simplicity. With ν varying, our Poincare map will now be a function of both ξ and ν , and (33) can be rewritten as

$$\xi(k+1) = \mathbb{P}[\xi(k), \nu(k)] \quad (35)$$

If (ξ^*, ν^*) denotes a fixed point, linearization of the discrete dynamics in (35) in the neighborhood of this fixed point gives

$$\xi(k+1) = \mathbb{P}[\xi^*, \nu^*] + J_\xi^* [\xi(k) - \xi^*] + J_\nu^* [\nu(k) - \nu^*] \quad (36)$$

where

$$J_\xi^* \triangleq \left[\frac{\partial \mathbb{P}(\xi)}{\partial \xi} \right]_{\xi=\xi^*, \nu=\nu^*}, \quad J_\nu^* \triangleq \left[\frac{\partial \mathbb{P}(\xi)}{\partial \nu} \right]_{\xi=\xi^*, \nu=\nu^*}$$

By defining the errors in the states and parameter ν as

$$E(k) = [\xi(k) - \xi^*], \quad U(k) = [\nu(k) - \nu^*]$$

the linearized dynamics in (36) can be rewritten as

$$E(k+1) = J_\xi^* E(k) + J_\nu^* U(k) \quad (37)$$

By choosing the parameter ν according to the relation

$$U(k) = CE(k), \quad \Rightarrow \quad \nu(k) = \nu^* + CE(k) \quad (38)$$

where C is a constant matrix that satisfies

$$\rho[J_\xi^* + J_\nu^* C] < 1 \quad (39)$$

the fixed point (ξ^*, ν^*) is asymptotically stabilized.

To solve for a fixed point (ξ^*, ν^*) that corresponds to our desired apex height, we first numerically find a set of fixed points within a given domain. The domain is discretized using a mesh and the number of fixed points obtained depend on the resolution of the mesh. For each fixed point, we solve for the apex height and identify the fixed point which has the apex height closest to our desired apex height. Then, small variations of ξ and ν from this fixed point are used as initial estimates for a new fixed point whose apex height is closer to the desired value. The procedure is continued till (ξ^*, ν^*) are found to have an apex height sufficiently close to our desired value.

4. Simulation Results

The mass and length parameters of the hopper were assumed to be the same as those in our experimental setup, namely

$$m_1 = 2.668 \text{ kg}, \quad m_2 = 0.808 \text{ kg}, \quad \ell = 0.059 \text{ m} \quad (40)$$

The parameters of the continuous controller were chosen as

$$K = 600 \text{ s}^{-2}, \quad D = 5 \text{ s}^{-1}, \quad r_d = 0.0979 \text{ m} \quad (41)$$

The choice of r_d is the same as that used in experiments and will be explained later in section 5.2.

The discrete controller parameter ν requires us to choose a nominal value ν^* , where ν^* corresponds to the fixed point (ξ^*, ν^*) . A numerical search was used to find as many fixed points (ξ^*, ν^*) as possible. Each fixed point corresponds to a unique value of $h = h(\xi^*, \nu^*)$ and Fig.2 plots these fixed points in the h - ν plane. This plot is useful for choosing ν^* for a desired value of $h = h_{\text{des}}$. Our desired apex height was chosen as

$$h_{\text{des}} = 0.213 \text{ m} \quad (42)$$

From Fig.2, this corresponds to

$$\nu^* = -4.55 \quad (43)$$

Figure 2: Apex height h corresponding to periodic orbits of the system obtained with different values of ν .

Figure 3: Simulation results: Error between the actual lift-off state $\xi = \dot{y}$ and the desired lift-off state $\xi^* = \dot{y}^*$ at the beginning of the k -th hop, $k = 1, 2, \dots, 9$.

For these values of h_{des} and ν^* , the value of ξ^* was

$$\xi^* = \dot{y} = 0.76 \text{ m/s} \quad (44)$$

The gain of the discrete controller was chosen to place the pole at 0.1; the gain was found to be

$$C = -1.08 \quad (45)$$

Figure 4: Simulation results: Plot of the height of the upper mass x_1 , the lower mass x_2 , and the center-of-mass z , as a function of time.

Figure 5: Simulation Results: Plot of the force F applied by the actuator.

The initial conditions³ were assumed to be

$$\begin{aligned} x_1(0) &= 0.149 \text{ m}, & \dot{x}_1(0) &= 0.0 \text{ m/s} \\ x_2(0) &= 0.059 \text{ m}, & \dot{x}_2(0) &= 0.0 \text{ m/s} \end{aligned} \quad (46)$$

and the results are shown here in Figs.3, 4 and 5. Figure 3 plots the errors between the actual lift-off state $\xi = \dot{y}$ and desired lift-off state $\xi^* = \dot{y}^*$ as a discrete function of time; the hops are sequentially numbered using the integer variable k and the errors at the beginning of each hop are plotted for $k = 1, 2, \dots, 9$. It can be seen that the errors become negligible after three hops, *i.e.*, for $k \geq 4$. The positions of the upper mass, lower mass, and center-of-mass of the robot are plotted as a continuous function of time in Fig.4. The time intervals during which x_2 remains constant at its lowest value of $\ell = 0.059 \text{ m}$ indicate the contact phases. The robot lift-offs the ground at the end of each contact phase; the discrete instants of time corresponding to $k = 1, 2, \dots, 9$ in Fig.3 are marked by “•” in Fig.4. It is clear from Fig.4 that the apex height of the robot h (maximum value of z during flight phase) converges to its desired value $h_{\text{des}} = 0.213$ within three hops. The

³These initial conditions are the same as those used in experiments. The value of $x_2(0)$ was chosen to be equal to $\ell = 0.059 \text{ m}$ and the value of $x_1(0)$ corresponds to $y(0) = 0.09 \text{ m}$.

force applied by the actuator F , defined by (19), is shown in Fig.5. It can be seen that the actuator force increases significantly during the contact phase. Each contact phase is marked by “c” and the maximum value of the actuator force reaches approximately 200 N, which is slightly larger than 7.5 times the weight of the upper mass m_1 . In each contact phase, the upper mass changes direction from downward motion to upward motion. This change in direction results in the non-smooth control action described by (24), which is discernible from Fig.5. In the flight phase, the relative motion of the masses is designed to emulate the motion of an underdamped mass-spring damper system; this explains the oscillatory nature of the actuator force. At the beginning of the flight phase, the actuator force is negative since the upper mass lifts the lower mass off the ground.

Remark: The locus of fixed-points in Fig.2 is serpentine with multiple bends and branches. To move from one hopping height to another (both of which are on the locus) in a stable manner, it will be necessary to march through the locus rather than jump between the points. The hopping height has to be changed gradually and marching through the locus may require the control parameter ν to be increased and decreased cyclically.

5. Experimental Results

5.1. Description of Experimental Hardware

In the experimental setup, the two-mass hopper is comprised of a voice-coil actuator and two linear guides - see Fig.6. One linear guide is connected to the coil housing of the voice-coil actuator and together they make up the lower mass of the hopper. The other linear guide is connected to the cylindrical permanent magnet of the voice-coil actuator and together they make up the upper mass of the hopper. Both linear guides are mounted to the vertical rail; this constrains the upper and lower masses to move in the vertical direction and prevents collision between them. The voice-coil actuator, a product of Moticont⁴, is a linear motor, and for a commanded input current it outputs a force between the upper and lower masses with a gain of 10.6 N/A. It has a stroke length of 0.1334 m and has the capability to apply a continuous force of 21.5 N and an intermittent force of 68.1 N (10% duty cycle). The position of the two masses are measured by two linear

⁴Linear Voice Coil Motor Actuator, Model LCVM-051-165-01

Figure 6: Schematic (left) and photograph (right) of the experimental hardware.

encoders whose scale is mounted on the other vertical member; the encoders are a product of US Digital⁵ and they have a resolution of 120 lines/inch. The current in the voice-coil is provided by a motor controller⁶ powered by a 80 volt power supply. A dSpace board⁷, residing in a host personal computer, is used for data acquisition and real-time control. The mass and length parameters of the hopper are the same as those used in simulations - see (40).

5.2. Control Implementation in Experiments

The continuous controller defined by (19), (15), (22) and (24) requires the knowledge of the external force F_{ext} during the contact phase. The external force can be measured directly using a force sensor but such a sensor is not present in our experimental hardware. To overcome this problem, we substitute (12) in (19) to rewrite the control force in the contact phase as

⁵Transmissive optical encoder module, Model EM1-0-120-N

⁶A-M-C Servo drives - brushed, analog, panel mount, Model 25A8

⁷Single-board real-time hardware, Model DS1104

follows

$$\begin{aligned} F &= m_f(F + m_2g) + m_2v \\ \Rightarrow F &= \frac{m_2(m_fg + v)}{1 - m_f} \end{aligned} \quad (47)$$

It can be verified from (16) that $m_f < 1$; the force in (47) is therefore non-singular. The parameters of the continuous controller were chosen to be the same as those in simulation which are given in (41). The choice of the value of $r_d = 0.0979$ m can be explained as follows: In the shortest length configuration, the upper and lower masses of the voice-coil are separated by a distance of $y_{\min} = 0.064$ m. The maximum stroke length of the voice-coil is 0.1334 m but the force decays rapidly at the end of its stroke. The active stroke length of the voice-coil is the length over which the force can be accurately controlled; this is equal to $s_{\max} = 0.127$ m as per the specifications of the manufacturer. By choosing half the stroke length as the desired neutral position of the hopper, the value of r_d is determined as

$$r_d \triangleq m_f y_d = \frac{m_1}{(m_1 + m_2)} (y_{\min} + \frac{1}{2}s_{\max}) = 0.0979$$

The desired apex height was chosen to be the same as that in simulation - see (42). The fixed point (ξ^*, ν^*) and gain C for the discrete controller were chosen to be the same as those in (43), (44) and (45).

5.3. Comparison of Experimental Results with Simulations

The experimental results are shown in Figs.7 and 8 for the same initial conditions as those used in simulations, namely (46); the contact phases are marked by “c” in Fig.8. Figure 7 shows the positions of the upper mass, lower mass, and center-of-mass of the hopper as a continuous function of time; it can be seen that the apex height converges to its desired value in two hops. The control objective is achieved but there are differences between simulation and experimental results. A comparison of Figs.4 and 7 indicate difference in transient behavior. The apex height gradually increases to its desired value in simulations whereas it overshoots and converges to its desired value in experiments. The time-period for each hop is significantly different. In a period of 3 sec, the hopper completes seven hops in experiments but is on its ninth hop in simulations. This is due to the difference in the duration

Figure 7: Experimental results: Plot of the height of the upper mass x_1 , the lower mass x_2 , and the center-of-mass z , as a function of time.

of the contact phases. For example, the contact phase prior to the sixth hop⁸ is approx. 0.174 sec in simulation but approx. 0.239 sec in experiment. This is due to the saturation of the actuator force in experiments. A comparison of Figs.5 and 8 indicate that the peak actuator force is 200 N in simulation but only 150 N in experiments. Since the maximum actuator force is lower in experiments, it is applied for a longer duration in the contact phase such that the hopper can reach the same apex height.

As supplementary material, a video of the two-mass hopping robot has been uploaded as E-component on the journal website. The video shows the robot operating under the proposed control method; a section of the video is shown in slow motion to better illustrate the flight phase, inelastic impact, and the contact phase.

6. Conclusion

This paper presents a method for controlling the apex height of a two-mass hopping robot. The two-mass model is proposed as an improvement over the Spring Loaded Inverted Pendulum (SLIP) model in that it accounts

⁸At the time of the sixth hop, the transient phase is over and the hopper exhibits steady-state hopping behavior in both simulations and experiments.

Figure 8: Experimental Results: Plot of the force F applied by the actuator.

for inelastic impact with the ground and models the flight dynamics more accurately. The motion of the two-mass hopping robot is investigated in three phases: flight phase, inelastic impact, and contact phase. The dynamics of the robot is modeled during each phase and a continuous-time controller is designed to stabilize the internal dynamics using feedback linearization. The closed-loop hybrid dynamics of the robot is used to derive the first return map and periodic points are found numerically for different values of a control parameter. A periodic point is chosen based on a desired apex height and the control parameter is discretely updated at the instants prior to lift-off to stabilize the periodic point. Both simulation and experimental results are presented. Experiments were conducted using a voice-coil actuator and the simulations were carried out with parameter values identical to those in experiments. The experimental results corroborate the results obtained from simulations and both show the convergence of the apex height of the robot to its desired value within three hops. Our future work will investigate the applicability of this method to hopping on an elastic foundation and extension of the method to hopping robots with rotary joints and greater number of degrees-of-freedom.

Appendix - Derivation of the First Return Map

From (26), we can write

$$\dot{X} = A_f X + R_f \quad (48)$$

where

$$A_f \triangleq \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K & -D & 0 & 0 \\ 0 & 0 & 0 & 1 \\ m_f K & m_f D & 0 & 0 \end{bmatrix}, \quad R_f \triangleq \begin{bmatrix} 0 \\ (K/m_f)r_d \\ 0 \\ -g - K r_d \end{bmatrix}$$

The solution to the differential equation in (48) is given by (28) where A_f is defined above and

$$B_f = \int_0^{\Delta t_1} e^{A_f(\Delta t_1 - \tau)} R_f d\tau$$

For the contact phase with $\dot{r} \leq 0$, the dynamics in (27) can be written in the form

$$\dot{X} = A_c X + R_c \quad (49)$$

where $A_c = \bar{A}(1)$, and

$$\bar{A}(\mu) \triangleq \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K & -\mu D & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad R_c \triangleq \begin{bmatrix} 0 \\ (K/m_f)r_d \\ 0 \\ 0 \end{bmatrix}$$

The solution to the differential equation in (49) is given by (30) where A_c is defined above and

$$B_c = \int_0^{\Delta t_2} e^{A_c(\Delta t_2 - \tau)} R_c d\tau$$

For the contact phase with $\dot{r} > 0$, the dynamics in (27) can be written in the form

$$\dot{X} = A_p X + R_p \quad (50)$$

where

$$A_p \triangleq \bar{A}(\nu), \quad R_p \triangleq R_c$$

The solution to the differential equation in (50) is given by (31) where A_p is defined above and

$$B_p = \int_0^{\Delta t_3} e^{A_p(\Delta t_3 - \tau)} R_p d\tau$$

ACKNOWLEDGMENT

The support provided by the National Science Foundation, NSF Grant CMMI 0925055, is gratefully acknowledged.

- [1] H. S. Seifert, The lunar pogo stick, *Journal of Spacecraft and Rockets* 4 (7) (1967) 941–943.
- [2] K. Matsuoka, A model of repetitive hopping movements in man, *Fifth World Congress on Theory of Machines and Mechanisms* .
- [3] M. H. Raibert, H. B. Brown Jr., M. Chepponis, Experiments with balance with a 3D one-legged hopping machine, *International Journal of Robotics Research* 3 (2) (1984) 75–92.
- [4] J. K. Hodgins, M. N. Raibert, Adjusting step length for rough terrain locomotion, *IEEE Transactions on Robotics and Automation* 7 (3) (1991) 289–298.
- [5] W. Schwind, D. Koditschek, Control of forward velocity for a simplified planar hopping robot, in: *Proceedings., 1995 IEEE International Conference on Robotics and Automation*, vol. 1, IEEE, 691–696, 1995.
- [6] U. Saranli, W. Schwind, D. Koditschek, Toward the control of a multi-jointed, monoped runner, in: *Proceedings. 1998 IEEE International Conference on Robotics and Automation*, vol. 3, IEEE, 2676–2682, 1998.
- [7] N. Cherouvim, E. Papadopoulos, Control of hopping speed and height over unknown rough terrain using a single actuator, in: *Proc. IEEE International Conference on Robotics and Automation (ICRA)*, 2743–2748, 2009.
- [8] S. Hyon, T. Emura, T. Mita, Dynamics-based control of a one-legged hopping robot, *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering* 217 (2) (2003) 83–98.
- [9] S. Kajita, T. Nagasaki, K. Kaneko, K. Yokoi, K. Tanie, A hop towards running humanoid biped, in: *Proceedings. ICRA’04. 2004 IEEE International Conference on Robotics and Automation*, vol. 1, IEEE, 629–635, 2004.

- [10] R. M. Ghigliazza, R. Altendorfer, P. Holmes, D. Koditschek, A simply stabilized running model, *SIAM review* 2 (2005) 519–549.
- [11] P. Holmes, R. Full, D. Koditschek, J. Guckenheimer, The dynamics of legged locomotion: Models, analyses, and challenges, *Siam Review* 48 (2006) 207–304.
- [12] I. Poulakakis, J. Grizzle, Formal embedding of the spring loaded inverted pendulum in an asymmetric hopper, in: *Proceedings of the European Control Conference*, 2007.
- [13] I. Poulakakis, J. Grizzle, The spring loaded inverted pendulum as the hybrid zero dynamics of an asymmetric hopper, *IEEE Transactions on Automatic Control* 54 (8) (2009) 1779–1793.
- [14] H. Dankowicz, P. T. Piiroinen, Exploiting discontinuities for stabilization of recurrent motions, *Dynamical Systems* 17 (4) (2002) 317–342.
- [15] S. A. Burden, S. Rezven, S. S. Sastry, Model reduction near periodic orbits of hybrid dynamical systems, *IEEE Transactions on Automatic Control* 60 (10) (2015) 2626–2639.
- [16] Y. Saitou, T. Nagano, T. Seki, M. Ishikawa, S. Hara, Optimal high-jump control of linear 1-dof trampoline robot, in: *SICE 2002. Proceedings of the 41st SICE Annual Conference*, vol. 4, IEEE, 2527–2530, 2002.
- [17] M. Ishikawa, A. Neki, J. Imura, S. Hara, Energy preserving control of a hopping robot based on hybrid port-controlled Hamiltonian modeling, in: *Proceedings of 2003 IEEE Conference on Control Applications*, vol. 2, IEEE, 1136–1141, 2003.
- [18] I. Mikhailova, Energy-based state-feedback control of systems with mechanical or virtual springs, in: *Proc. IEEE International Conference on Robotics and Automation (ICRA)*, 2509–2514, 2013.
- [19] J. Aguilar, A. Lesov, K. Wiesenfeld, D. I. Goldman, Lift-off dynamics in a simple jumping robot, *Physical Review Letters* 109 (174301).
- [20] X. Yu, F. Iida, Minimalistic Models of an Energy-Efficient Vertical-Hopping Robot, *IEEE Transactions on Industrial Electronics* 61 (2) (2014) 1053–1062.

- [21] N. Boccara, Modeling complex systems, Springer Verlag, 2004.
- [22] E. Scholl, H. Schuster, Handbook of chaos control, Wiley-VCH, Weinheim, 2008.
- [23] F. B. Mathis, R. Mukherjee, Apex height control of a two-mass hopping robot, in: IEEE International Conference on Robotics and Automation (ICRA), Karlsruhe, Germany, 4785–4790, 2013.