Theoretical and Experimental Study on Gear-Coupling Contact and Loads Considering Misalignment, Torque, and Friction Influences

Yi Guo^{a,*}, Scott Lambert^a, Robb Wallen^a, Robert Errichello^b, Jonathan Keller^a

^a*National Renewable Energy Laboratory
15013 Denver West Parkway, Golden, CO, 80401, United States*

^b*GEARTECH
Townsend, Montana, United States*

Abstract

A new analytic model addresses the tooth contact and induced loads of gear couplings that are affected by misalignment, torque, and friction. The contact model accounts for Hertzian, bending, and shear deformations of coupling teeth considering crown modifications. For a specified torque and shaft misalignment, the model calculates the number of teeth in contact, tooth load, stiffness, stress, deformation, and safety factors. The tooth load distribution around the circumference compares well with high fidelity finite-element/contact-mechanics analyses. Simulation time with the analytic model is orders of magnitude less. Using the local contact characteristics, the model computes coupling loads that are primarily caused by the disrupted tooth contact and sliding friction caused by axial motions. This analytic model was validated by experiments. The load amplitude depends on the misalignment, torque, and friction. At low torque, coupling motion was induced by the eccentricity between the hub and sleeve even with nearly perfect alignment. This eccentricity was caused by its self-weight. When torque was larger than a threshold, the motion amplitude was greatly reduced. This torque threshold was analytically derived and validated by experiments.

Keywords: Gear coupling, contact, load, misalignment, friction

*Corresponding author. Tel.: +1-303-384-7187; Fax: +1-303-384-6901.
Email address: yi.guo@nrel.gov (Yi Guo)

Nomenclature	Meaning
A	Arc of gear tooth contact
b	Backlash
C	Single pair tooth stiffness
E	Young's modulus
F	Facewidth
f_c	Half tooth facewidth in contact
f_d	Distance from the tooth center to the contact point
F_c	Contact length of a single tooth
F_d	Distance between tooth center and contact point for a tooth
F_i	Maximum distance between tooth center and contact point
i_d	Design misalignment angle
L_s	Coupling shaft length
$M^{y'}, M^{z'}$	Total bending moments of a coupling around y' and z' axis
$M_d^{y''}, M_d^{z''}$	Single tooth bending moments around y'' and z'' axis
$M_s^{y'}, M_s^{z'}$	Total bending moments of a coupling around y' and z' axis at sensors
$M_t^{y''}, M_t^{z''}$	Total bending moments of a coupling around y'' and z'' axis
N	Number of teeth
$N_{c,d}$	Number of teeth in contact at the sleeve bottom
$N_{c,s}$	Number of teeth in contact at the sleeve sides
N_d	Normal tooth load to the coupling surface
N_s	Number of slices along the facewidth
P	Diametral pitch
P_d	Individual normal tooth load
P_d^s	Load on a single tooth slice along the facewidth
P_i	Maximum normal tooth load
P'_i	Load per unit length
P_t	Total mean normal tooth load

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Nomenclature	Meaning
P'_t	Mean normal tooth load
q	Load sharing ratio (number of teeth in contact)
R	Pitch radius
R_c	Hub root crown radius at pitch diameter
R_d	Distance from the hub center to the contact area in the z' direction
R_F	Hub face crown radius (normal plane)
S_{c2}	Design sleeve circular space width
S_{c2}^*	Sleeve circular space width at the contact point
T	Transmitted torque
T_m	Restoring torque
t_{c1}, t_{c2}	Hub and sleeve tooth circular thickness
t_{c1}^*	Hub tooth circular thickness at the contact point
W_c	Coupling weight
W_d	Axial load that balances a coupling tooth
W_{sh}	Coupling shaft weight
w_{sh}	Coupling shaft weight per unit length
Y_1, Y_2	Hub and sleeve tooth AGMA form factor
Z_1, Z_2	Hub and sleeve elasticity factor
z_o	Maximum tooth separation
z_e	Elastic tooth deformation
α	Tooth position angle offset
β	Misalignment direction
ζ	Misalignment angle
ξ	Jam angle
δ_b	Tooth backlash
δ_d	Normal tooth deformation

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Nomenclature	Meaning
Δ_c	Accessible tooth root clearance
Δ_r	Design tooth root clearance
γ	Tilt angle
Γ	$\cos(\xi)$
ϕ	Normal pressure angle
θ	Turning angle
μ	Coefficient of friction
μ_{kl}, μ_l	Restoring function

1. Introduction

Gear couplings are a common machine component used extensively in a wide range of applications including aerospace, manufacturing, automotive, and wind power. Aside from transmitting the driving torque, crowned-tooth gear couplings accommodate some axial motion and angular misalignment between the two connecting shafts. An important aspect of gear couplings is the existence of forces and moments that are transmitted to the driveline when misaligned. Amplitudes of these loads have been reported to be between 12% and 16% of the drive torque for crowned- and straight-tooth couplings [1]. These forces can increase other component loads in the driveline and could lead to high vibration.

The National Wind Technology Center (NWTC) at the National Renewable Energy Laboratory (NREL) operates two multimewatt wind-energy-specific dynamometers. These dynamometers offer a unique opportunity to perform measurements of coupling performance. Gear couplings used in the NWTC’s 5-MW dynamometer testing facility are key components that accommodate angular misalignment and transmit torque from the dynamometer gearbox to the device under test during operation. Coupling-induced loads

are ultimately reacted by both the device under test and the dynamometer gearbox, contributing to the uncertainty in the loads applied to the wind turbine drivetrain. To better understand the coupling-induced loads and ensure safe operation of the machinery, this work investigated the performance and loads of gear couplings in the NWTC’s 5-MW dynamometer through a combined modeling and testing approach.

Two primary mechanisms generate coupling loads, including 1) offsets of tooth contact on the opposite sides of the coupling and 2) sliding friction caused by tooth axial motions. Depending on the instantaneous torque and misalignment, the number of teeth in contact and contact pattern of gear couplings vary significantly. Understanding these mechanisms requires the knowledge of the local contact characteristics at individual teeth. However, studies on coupling-induced loads did not include a tooth-contact model. Different from aforementioned approaches in calculating the coupling loads, the new model described in this paper is anticipated to yield quick and accurate results by efficiently addressing the local contact characteristics. The developed model has been validated by experiment conducted at the 5-MW dynamometer.

Early coupling load models made assumptions about the number of teeth in contact and contact pattern of gear couplings. Crease [1] analytically derived the induced loads of straight-tooth couplings and assumed only two teeth in contact. Gibbons [2] presented moment and force equations for straight-tooth, diaphragm, and annular-disk couplings using an alignment load factor that is a function of misalignment angle, assuming two teeth in contact. Elkholy and Alfares [3] provided a simplified mathematical model to estimate misalignment loads of gear couplings also based on the same assumptions. Later, Cura and Mura [4] conducted an experimental study and found that Elkholy and Alfares’s model greatly overestimated the coupling loads because of these simplifications. They also presented a numerical model that calculated the number of teeth in contact based on Hertzian tooth deformation affected by misalignment and tooth stiffness. The details of tooth-contact pattern across the circumference were not given. Mancuso [5] developed analytic models to calculate coupling-induced loads. The contact points were empirically estimated as one-

third of the tooth facewidth from the flank center. Experimental data collected from a small-scale test bench were included to compare against theoretical results. Later on, Clark [6] extended Mancuso’s model [5] to consider all the teeth in contact, which was suitable for high-speed couplings. Palazzolo et al. [7] compared the modeling results using Clark’s method against experimental data presented by Mancuso and found reasonable agreement at high misalignment angles. Although these coupling models have proven to be reasonably accurate in predicting loads for the studied couplings, the exclusion of a tooth-contact model has limited the applications of these models across industries that have a wide range of coupling size, torque, speed, and misalignment. Furthermore, the induced moments of misaligned couplings have components around two perpendicular axes because of the shifted tooth-contact pattern around the circumference caused by the pressure angle. These early studies have not considered the effects of pressure angle on the induced loads and, therefore, the load direction was not studied.

Existing contact models were not utilized for coupling load calculations. Contact modeling of gear couplings was first reported by Moked [8] and Renzo et al. [9]. These models investigated the contact characteristics of meshing teeth, clearance, load distribution, tooth deformation, and load capacity. Renzo et al. found the number of teeth in contact increased with the decreasing misalignment and has presented an approximated computation of the number of teeth in contact at different misalignment angles. Ma and Hou [10] derived the relative displacement and tooth load distribution of a crowned gear coupling at different misalignments. Alfares et al. [11] developed a computational contact algorithm to calculate the clearance distribution of misaligned gear couplings, and results were correlated with those in Nakashima [12] and Heinz [13]. Tjernberg [14] found that tooth-spacing errors affected the number of teeth in contact during operation. Wink and Nakandakari [15] investigated the effects of tooth-spacing errors on the number of teeth in contact and axial load factor, based on the iterative procedure recommended by the American Gear Manufacturers Association (AGMA) 927 [16] and International Standards Organization (ISO) 6336-1 [17]. Guo et al. [18] developed an analytic model to evaluate the contact characteristics of

gear couplings that considered the effects of torque, misalignment, tooth modification, and various heat-treatment methods.

Several commercial computational packages address the contact characteristics of gear couplings. However, the various tools used for design yielded different reaction forces within the mechanical system [18]. Finite-element tools are generally regarded as accurate but at a high computational cost that limits their ability to be applied over a large design space or to problems requiring real-time execution. Hong et al. [19] developed a combined finite-element and surface-integral contact model to calculate tooth load distributions of clearance-fit gear couplings. This model considered manufacturing tooth indexing errors that commonly exist for gear couplings; however, this computational model required considerable analysis time, which made the usage less practical. Later, Hong et al. [20] established a hybrid analytical-computational model to address this problem for side-fit gear couplings. The tooth compliance caused by bending, shear, contact, and torsion was treated analytically whereas the deformation between elastic bodies was solved computationally. This approach improved computational efficiency by one order of magnitude. Although computational models and contact analysis tools exist to calculate coupling-tooth contact, the design of these components is still largely based on past experience and very little operational data are available.

The objective of this study was to address both the local contact characteristics and global-induced loads of gear couplings that can execute in a fraction of the time and thus allow comparison with real-time experimental data. Experiments conducted at the NWTC's 5-MW dynamometer test facility validated the developed coupling model. The tooth-contact model was verified by correlating results with finite-element/contact-mechanics analysis. For a given torque and shaft misalignment, this analytical model quickly yielded insights into relationships between the spline design parameters and resulting tooth loads; bending, contact, and shear stresses; and safety factors considering various heat treatment methods. Using the local contact characteristics, the model then computed the coupling loads that were primarily caused by the disrupted tooth contact by misalignment and slid-

ing friction as a result of axial motions. The analytic model also calculated the maximum misalignment that a gear coupling can accommodate and the minimum torque that was required to eliminate the initial eccentricity between the coupling sleeve and hub caused by its self-weight.

2. NREL Dynamometer Coupling Shaft and Load Application Device

The configuration of the dynamometer coupling-shaft assembly is shown in Figure 1. The crowned-tooth couplings at each end of the 2.8-meter-long shaft share many key design dimensions and yet differ in the allowable misalignment and backlash. Table 1 details the design parameters for each coupling. The sleeve on the driving end of the shaft has a flange that connects it to the gearbox output shaft. During operation, this sleeve rotates with the coupling shaft and any movement is strictly a result of very small deflections in the gearbox and its mounting structure. The coupling shaft is comprised of two crowned-tooth hubs bolted to the shafting. The shafting is split in the middle to accommodate a torque transducer (Figure 2). The driven sleeve is attached to the shafting inside a servo-hydraulic load-application device (LAD). In operation, the LAD can be used to apply forces and moments to the device under test to reproduce wind-loading conditions. Deflections of the device under these loads results in misalignments of the shafting and crowned-tooth couplings. The amount of coupling misalignment is therefore a function of the device stiffness and the forces applied to it.

The LAD uses hydrostatic bearings attached to hydraulic actuators to achieve nearly frictionless shaft rotation while applying forces and moments in the other five degrees of freedom on the device under test. Although the load path for the LAD is through the device under test, moments and forces generated by the crowned-tooth coupling are not in the load measurement path. The range of motion of the LAD is listed in Table 2.

(a)

(b)

(c)

Figure 1: (a) NWTC's 5-MW dynamometer test facility, *photo by Mark McDade, NREL 28218*; (b) dynamometer test bench layout; and (c) coupling gear teeth, *photo by Mark McDade, NREL*.

3. Experiment and Instrumentation

The instrumentation on the shaft assembly is shown in Figure 2. Five measurements were used to compute the magnitude and direction of each of the coupling misalignments.

Parameters	Upwind Coupling	Downwind Coupling
Number of teeth	100	100
Module (1/mm)	10.16	10.16
Normal pressure angle ($^{\circ}$)	20.00	20.00
Pitch diameter (mm)	1,016.00	1,016.00
Base diameter (mm)	954.73	954.73
Outer diameter (mm)	1,036.32	1,036.32
Root diameter (mm)	988.77	987.95
Center distance (mm)	0.00	0.00
Circular backlash (mm)	0.64	1.97
Circular tooth thickness (mm)	15.55	16.03
Tool tip radius (mm)	4.27	3.18
Facewidth (mm)	127	127.00
Lead modification (mm)	0.78 at tooth edges	1.20 at tooth edges
Profile tip relief (mm)	0.46 starting at 23.6° roll angle	0.46 starting at 23.6° roll angle

Table 1: Specified Geometry of Gear Couplings in NWTC’s 5-MW Dynamometer.

Direction	Motion Range
x (axial)	± 38 mm
y (lateral)	± 30 mm
z (vertical)	± 30 mm
θ_y (rotation about lateral)	$\pm 1.75^{\circ}$
θ_z (rotation about vertical)	$\pm 1.75^{\circ}$

Table 2: Range of Motion of the LAD.

A Heidenhain ERM 220 angular encoder provides 2,048 line counts per revolution of the shafting, resulting in an accuracy of ± 4 minutes of shaft rotation. Four Omega linear

variable differential transformer (LVDT) displacement transducers were used to derive the axial and angular displacement between the sleeve and hub of the couplings. The LVDT was arranged such that there are two transducers at each coupling spaced 90 degrees of rotation apart from one another. This arrangement allows for computation of shaft axial displacement from changes in the mean value of the signals. Angular misalignment was calculated using the minimum and maximum values, whereas direction can be obtained from the phase shift of the signals.

Figure 2: Instrumentation on the coupling shaft. *Solidworks model by Scott Lambert, NREL*

At three locations along the shaft, six sets of type-II full-bridge strain gauges were used to measure bending in three axial and two azimuthal locations along the rotating shaft assembly. The bridge configuration for the bending channels measures only bending and rejects both axial and torsional strains. Using these signals and the encoder, the bending moment in the nonrotating frame was converted from the strains in the rotating frame. As a result, a bending-moment distribution along the shaft was calculated. Prior to testing, a series of calibration forces were applied to the shaft assembly to determine the appropriate slopes and offsets for each of the bending channels. Using the bending signals and known geometry from the shaft, an estimation of friction was calculated.

Friction coefficient during operation was estimated using a combined testing and mod-

eling approach. In [5], the moment out of plane of the misalignment can be derived as:

$$M_t^{z''} = \frac{T [\sin(\varsigma) - \mu \cos(\varsigma)] \sqrt{R^2 - \left[\frac{R}{3} \sin\left(\frac{A}{2}\right)\right]^2}}{R [\cos(\varsigma) - \mu \sin(\varsigma)]} \quad (1)$$

The $\sqrt{R^2 - \left[\frac{R}{3} \sin\left(\frac{A}{2}\right)\right]^2}$ term is dropped because it is approximately 1.0. Similarly, $\mu \sin(\varsigma)$ is approximately 0. The final form of Eq. 1 is:

$$M_t^{z''} = \frac{T [\sin(\varsigma) - \mu \cos(\varsigma)]}{R \cos(\varsigma)} \quad (2)$$

where $M_t^{z''}$ is calculated using experimental data of shaft bending moments by:

$$M_t^{z''} = \sin(\phi + \beta) \left[M_s^{y'} + W_c \cos(\gamma) L_s + \frac{w_{sh} \cos(\gamma) L_s^2}{2} \right] + \cos(\phi + \beta) M_s^{z'} \quad (3)$$

Therefore, with measured torque, misalignment, and shaft-bending signals, the friction coefficient is estimated as:

$$\mu = T \frac{\sin(\varsigma)}{\cos(\varsigma)} - R \left\{ \sin(\phi + \beta) \left[M_s^{y'} + W_c \cos(\gamma) L_s + \frac{w_{sh} \cos(\gamma) L_s^2}{2} \right] + \cos(\phi + \beta) M_s^{z'} \right\} \quad (4)$$

The estimated friction coefficients under various torque and misalignment conditions are listed in the Appendix. Testing at various speeds, torque, and misalignments provided data to validate the analytic model.

4. Analytic Formulation Description

The analytic formulation described herein includes two important aspects: contact characteristics of individual teeth and induced loads transmitted to the driveline. The tooth-contact model, based on Guo et al.'s study in [18], calculates the number of teeth in contact; individual tooth load and contact area, stress, and safety factors at a given torque; and misalignment. The local contact characteristics provide crucial input for calculating the amplitude and direction of the induced loads of gear couplings. The induced loads by the coupling are primarily caused by a relocation of tooth contact by misalignment and sliding friction. Through the analytic nature of the model, solutions can be calculated two orders

of magnitude faster than higher fidelity models, making it very useful for parametric studies during early design stages and real-time health monitoring during operations. Assumptions that the analytic formulation makes include:

- The hub and sleeve shafts are considered rigid. This assumption is reasonable because the shafts have large thicknesses to resist shaft twist. This analytic model could be conservative compared to a model that accounts for compliance of the connecting shafts.
- Only angular misalignment is considered. This assumption is valid because there was no strong evidence that the coupling teeth were moving radially and contacting the roots for the studied test articles.
- The crowning of the coupling teeth leads to constant base pitch spacing in all planes.
- The bending and contact stiffnesses are derived at a specified, steady torque, and tooth contact occurs only on the drive-side.
- The coupling teeth are equally spaced circumferentially.
- Line contact at the pitch diameter is assumed to simplify the problem.

4.1. Local Tooth Contact

Coupling-tooth bending stiffness is estimated using an analytic approach [21, 18], considering combined tooth deformations caused by bending and compression. The actual bending stiffness may be different because the contact is conformal between two identical involute curves. The empirical values are based on measured deformations of gear teeth, rather than gear-coupling teeth, under known test loads in which the tooth contact is localized near the pitch line. The coupling-tooth bending stiffness is:

$$C = FE \frac{Z_1 Z_2}{Z_1 + Z_2} \quad (5)$$

where the elasticity form factors are:

$$Z_1 = \frac{Y_1}{0.76 + 7.25 Y_1} \quad Z_2 = \frac{Y_2}{0.76 + 7.25 Y_2}. \quad (6)$$

The total normal tooth load is:

$$P_t = \frac{T}{R \cos(\phi)} \quad (7)$$

where the pitch radius equals:

$$R = \frac{N}{2P}. \quad (8)$$

Without misalignment and eccentricity, all the teeth carry the same load caused by the transmitted torque and tooth-contact areas are concentrated at the tooth flank centers, as shown in Figure 3(a). When the coupling is misaligned, only a portion of the teeth are in contact with the sleeve and the tooth-load distribution changes. Some of the teeth move out of contact, as shown in Figure 3(b).

The individual normal tooth load [9] is assumed to be:

$$P_d = P_i \left[1 - \frac{z_o \left[\sin\left(\frac{2\pi}{N}j + \alpha\right) \right]^2}{z_e} \right] \quad (9)$$

where z is the circumferential distance proportional to the turning angle θ . The elastic tooth deformation equals:

$$z_e = \frac{P_i}{C}. \quad (10)$$

The maximum tooth separation occurs at a turning angle of 90° [9]:

$$z_o = \frac{R_c [\tan(\zeta)]^2}{2 \tan(\phi)} \quad (11)$$

where the hub root crown radius is:

$$R_c = R_F \cos(i_d) \sin(\phi). \quad (12)$$

The variable j denotes the tooth number and α is the offset that consists of the residue of qN if it is not an integer and the half tooth angle:

$$\alpha = \frac{qN - \text{int}(qN)}{4} + \frac{\pi}{N}. \quad (13)$$

Load on each tooth is distributed into N_S slices along the facewidth direction. Load on each slice is considered as:

$$P_d^s|_l = P_d \cos\left(\frac{\pi}{N_S}l\right) \Big|_{l=-\frac{N_S}{2}}^{l=\frac{N_S}{2}} = P_i \left[1 - \frac{z_o \left[\sin\left(\frac{2\pi}{N}j + \alpha\right) \right]^2}{z_e} \right] \cos\left(\frac{\pi}{N_S}l\right) \Big|_{l=-\frac{N_S}{2}}^{l=\frac{N_S}{2}}. \quad (14)$$

The maximum distance between the tooth center and the farthest tooth-contact point for all teeth is estimated by its misalignment angle:

$$F_i = R_F \sin(\zeta). \quad (15)$$

Sine function distribution is assumed. The distance between the tooth center and the contact point for any individual tooth in contact is derived with the amplitude of F_i :

$$F_d = F_i \cos\left(\frac{2\pi}{N}j + \alpha\right). \quad (16)$$

The tooth deformation in the circumferential direction depends on tooth load as:

$$\delta_d = \frac{P_d}{C}. \quad (17)$$

The width of each contact area on an individual tooth varies with tooth load:

$$F_c = 2\sqrt{R_F^2 - (R_F - \delta_d)^2}. \quad (18)$$

Therefore, the upper and lower boundaries of the contact area are defined as:

$$\left[F_d - 0.5F_c, \quad F_d + 0.5F_c \right]. \quad (19)$$

Because of the large pressure angle of gear-coupling teeth, the tooth that carries the maximum load does not align with the misalignment axis. Instead, it deviates from the misalignment axis by the amount of pressure angle. This tooth with the maximum load aligns with the z'' axis, as shown in Figure 3(b).

The contact model is solved using numerical iterations and assuming initial guesses for q . Let:

$$\begin{aligned} P'_t &= \frac{P_t}{N} \\ P_i &= \frac{P'_t}{q} \end{aligned} \quad (20)$$

therefore,

$$\frac{z_e}{z_o} = \frac{P_i}{Cz_o}. \quad (21)$$

Figure 3: (a) Coupling-tooth load profile without misalignment; (b) coupling-tooth load profile with misalignment; and (c) tooth contact area with misalignment.

The new value of q is calculated using the $\frac{z_e}{z_o} - q$ relation curves [9]:

$$\left\{ \begin{array}{ll} q = 0.36 \left(\frac{z_e}{z_o} \right) + 0.14 & 0.4 \leq \frac{z_e}{z_o} < 1.0 \\ q = \sqrt{1.195 + 1.75 \left(\frac{z_e}{z_o} \right) - \left(\frac{z_e}{z_o} \right)^2} - 1.033 & \frac{z_e}{z_o} < 0.4 \\ q = 1 - \frac{1}{2 \left(\frac{z_e}{z_o} \right)} & \frac{z_e}{z_o} \geq 1.0 \end{array} \right. \quad (22)$$

The above process iterates until the assumed q equals the calculated q from Eq. 22 within the specified tolerance (10^{-3} used in this study). The calculation process is detailed in [18].

4.2. Jam Angle

The jam angle defines the upper boundary of the coupling-shaft misalignment. Couplings must be designed such that their jam angle is larger than the maximum misalignment expected during operations as stated in AGMA 6123-B06 [22]; however, no simple guidance is given in the standard to calculate the jam angle. Figure 4 shows the coupling tooth that contacts on both the drive- and back-side of the sleeve at the jam angle. The circumferential

space between two adjacent teeth is limited by the amount of S_{c2}^* , that is:

$$2f_c \sin(\xi) + \frac{t_{c1}^*}{\cos(\xi)} = S_{c2}^* \quad (23)$$

where

$$f_c = 2R_F \sin\left(\frac{\xi}{2}\right) \quad (24)$$

$$t_{c1}^* = t_{c1} - 2R_F [1 - \cos(\xi)] \quad (25)$$

$$S_{c2}^* = S_{c2} - F \sin(\xi) [\sin(\phi)]^2 \quad (26)$$

where

$$S_{c2} = \frac{\pi}{P} - t_{c2}. \quad (27)$$

Inserting Eqs. (24) - (26) into Eq. (23) and letting $\Gamma = \cos \xi$, Eq. (23) becomes:

$$4\Gamma(1-\Gamma) \sqrt{\frac{1+\Gamma}{2}} R_F + \left\{ 2R_F - S_{c2} + F[\sin(\phi)]^2 \sqrt{1-\Gamma^2} \right\} \Gamma + (t_{c1} - 2R_F) = 0. \quad (28)$$

Therefore, the jam angle equals:

$$\xi = \cos^{-1}(\Gamma_0) \quad (29)$$

where Γ_0 is the solution of Eq. (23).

However, Eq. (28) is only valid when $f_c \geq \frac{F}{2}$. When $f_c \geq \frac{F}{2}$:

$$\xi = \sin^{-1}\left(\frac{F}{2R_F}\right). \quad (30)$$

The coupling could also jam in the radial direction when the relative displacement between the hub and sleeve exceeds the backlash. That is:

$$b = 2\Delta_c \sin(\phi) \quad (31)$$

where

$$\Delta_c = \frac{1}{2} F \sin(\xi) \quad (32)$$

Figure 4: Misalignment spline contact on both sides of the tooth flank.

$$b = S_{c2} - t_{c2}. \quad (33)$$

Therefore,

$$\xi = \sin^{-1} \left[\frac{S_{c2} - t_{c2}}{F \sin(\phi)} \right]. \quad (34)$$

The final form of jam angle is the lowest value of all the calculated angles under all three situations:

$$\xi = \min \left\{ \cos^{-1}(\Gamma_0), \sin^{-1} \left(\frac{F}{2R_F} \right), \sin^{-1} \left[\frac{S_{c2} - t_{c2}}{F \sin(\phi)} \right] \right\}. \quad (35)$$

The jam angle, in this formulation, is entirely determined by coupling geometric parameters. It can be enlarged primarily by adding the sleeve facewidth, increasing the coupling facewidth, decreasing hub-tooth thickness, or reducing the hub-face crown radius.

4.3. In-Plane and Out-of-Plane Bending Moments

The coupling stiffnesses in the radial and rotational directions are sufficiently high, such that they can be treated as rigid in this formulation; however, the coupling stiffnesses in the tilting directions are much lower to allow for the desired self-adjusting motion of the coupling shaft. When the coupling shaft tilts because of misalignment, the contact areas

on each tooth shift away from the center of the tooth flank, as shown in Figure 3(c). The distances from the contact areas to the tooth center vary across the circumference. These offsets between tooth-contact points create bending moments on the drive shaft.

Figure 5: Force diagram of a misaligned coupling tooth.

Figure 5 shows the force diagram of the coupling at equilibrium. A force balance of the coupling in the x' and z' direction leads to:

$$\begin{cases} W_d - \mu N_d \cos(\zeta) - N_d \sin(\zeta) = 0 \\ P_d + \mu N_d \sin(\zeta) - N_d \cos(\zeta) = 0. \end{cases} \quad (36)$$

Solving Eq. (36) results in:

$$W_d = \frac{P_d [\mu \cos(\zeta) + \sin(\zeta)]}{\cos(\zeta) - \mu \sin(\zeta)}. \quad (37)$$

Coupling-induced moments of a single tooth in contact around the z'' - z'' axis equals:

$$M_d^{z''} = W_d R_d \quad (38)$$

where

$$R_d = \sqrt{R^2 - f_d^2} \quad (39)$$

$$f_d = R_F \sin(\zeta) \cos \left[\left(\frac{2\pi}{qN} \right) j + \alpha \right], \quad j = 1, \dots, qN. \quad (40)$$

Coupling-induced moments of a single tooth in contact around the y'' - y'' axis equals

$$M_d^{y''} = P_d f_d \quad (41)$$

Summarizing the moment vector for all teeth in contact $(1, \dots, qN)$ and assuming the load distribution is symmetric about the misalignment axis results in:

$$M_t^{y''} = 4 \sum_{j=0}^{\frac{qN}{4}} P_i R_F \sin(\zeta) \cos\left(\frac{2\pi}{N}j + \alpha\right) \cos\left(\frac{2\pi}{qN}j + \alpha\right) \left\{ 1 - \frac{z_o [\sin(\frac{2\pi}{N}j + \alpha)]^2}{z_e} \right\} \quad (42)$$

and

$$M_t^{z''} = 4 \sum_{j=0}^{\frac{qN}{4}} P_i \cos\left(\frac{2\pi}{N}j + \alpha\right) \left[\frac{\mu \cos(\zeta) + \sin(\zeta)}{\cos(\zeta) - \mu \sin(\zeta)} \right] \left\{ 1 - \frac{z_o [\sin(\frac{2\pi}{N}j + \alpha)]^2}{z_e} \right\} \sqrt{R^2 - \left[R_F \sin(\zeta) \cos\left(\frac{2\pi}{qN}j + \alpha\right) \right]^2}. \quad (43)$$

Considering the effects of pressure angle and misalignment direction, the coupling-induced moments in the global coordinates are derived as:

$$M^{y'} = \cos(\phi + \beta) M_t^{y''} + \sin(\phi + \beta) M_t^{z''} \quad (44)$$

and

$$M^{z'} = -\sin(\phi + \beta) M_t^{y''} + \cos(\phi + \beta) M_t^{z''}. \quad (45)$$

The final forms of Eqs. (44) and (45) become:

$$M^{y'} = 4 \cos(\phi + \beta) \sum_{j=0}^{\frac{qN}{4}} P_i R_F \sin(\zeta) \cos\left(\frac{2\pi}{N}j + \alpha\right) \cos\left(\frac{2\pi}{qN}j + \alpha\right) \left\{ 1 - \frac{z_o [\sin(\frac{2\pi}{N}j + \alpha)]^2}{z_e} \right\} + 4 \sin(\phi + \beta) \sum_{j=0}^{\frac{qN}{4}} P_i \cos\left(\frac{2\pi}{N}j + \alpha\right) \left[\frac{\mu \cos(\zeta) + \sin(\zeta)}{\cos(\zeta) - \mu \sin(\zeta)} \right] \left\{ 1 - \frac{z_o [\sin(\frac{2\pi}{N}j + \alpha)]^2}{z_e} \right\} \sqrt{R^2 - \left[R_F \sin(\zeta) \cos\left(\frac{2\pi}{qN}j + \alpha\right) \right]^2} \quad (46)$$

$$\begin{aligned}
M^{z'} = & -4 \sin(\phi + \beta) \sum_{j=0}^{\frac{qN}{4}} P_i R_F \sin(\zeta) \cos\left(\frac{2\pi}{N}j + \alpha\right) \cos\left(\frac{2\pi}{qN}j + \alpha\right) \left\{ 1 - \frac{z_o \left[\sin\left(\frac{2\pi}{N}j + \alpha\right) \right]^2}{z_e} \right\} + \\
& 4 \cos(\phi + \beta) \sum_{j=0}^{\frac{qN}{4}} P_i \cos\left(\frac{2\pi}{N}j + \alpha\right) \left[\frac{\mu \cos(\zeta) + \sin(\zeta)}{\cos(\zeta) - \mu \sin(\zeta)} \right] \left\{ 1 - \frac{z_o \left[\sin\left(\frac{2\pi}{N}j + \alpha\right) \right]^2}{z_e} \right\} \\
& \sqrt{R^2 - \left[R_F \sin(\zeta) \cos\left(\frac{2\pi}{qN}j + \alpha\right) \right]^2}.
\end{aligned} \tag{47}$$

The bending moments at the load-sensor locations are derived as:

$$\begin{cases} M_s^{y'} = M^{y'} - W_c \cos(\gamma) L_s - \frac{w_{sh} \cos(\gamma)}{2} L_s^2 \\ M_s^{z'} = M^{z'}. \end{cases} \tag{48}$$

4.4. Minimum Torque to Center the Coupling

At very low torque, the coupling rests on the sleeve teeth because of its self-weight, which results in the eccentricity between the hub and sleeve. With this eccentricity, the coupling moves during rotation and the motion amplitude is one order of magnitude higher than that without eccentricity. With increasing torque, the lift force generated by the torque cancels out self-weight. The coupling motion diminishes after the torque exceeds a threshold. This torque threshold is the minimum restoring torque to lift the hub coupling and center it with the sleeve shaft. The restoring torque is mainly determined by the coupling geometry and can be analytically derived.

For the dynamometer low-speed shaft, the weight of the hubs and shaft must be accounted for. The sleeves are supported by the attached equipment, whereas the hub and shaft are supported by the sleeves. The potential contacting points between the hub and sleeve are illustrated in Figure 6. The actual contacting points are determined through a detailed tooth-deflection analysis.

The hub can rest at the bottom of the sleeve if there is inadequate root clearance. The allowable radial travel of the meshing teeth is linearly related to the backlash, as follows:

$$\Delta_c = \frac{\delta_b}{2 \sin(\phi)}. \tag{49}$$

Figure 6: Coupling resting on the meshing sleeve with potential contact zones.

Under this circumstance, the restoring torque can be calculated using:

$$T_m = \frac{(W_c + 0.5W_{sh})R}{N_{c,b} \cos(\phi)} \quad (50)$$

where $N_{c,b} = 2 + \sum_{k=\frac{N}{2}}^{\frac{3N}{2}} \mu_k$ denotes the number of teeth in contact at the sleeve bottom.

The quantity μ_k is the restoring function, derived as:

$$\begin{cases} \mu_k = 1, & \left[\delta_b - \frac{(W_c + 0.5W_{sh})R^2}{2 \cos \phi C} \right] > 2\Delta_c \cos\left(\frac{2\pi k}{N}\right) \sin(\phi) \\ \mu_k = 0, & \left[\delta_b - \frac{(W_c + 0.5W_{sh})R^2}{2 \cos \phi C} \right] \leq 2\Delta_c \cos\left(\frac{2\pi k}{N}\right) \sin(\phi), \quad k = \frac{N}{4}, \dots, \frac{3N}{4}. \end{cases} \quad (51)$$

Typically, the coupling teeth are designed with adequate root clearance $\Delta_c < \Delta_r$ such that contacting points are on the sides of the sleeve. Under this situation:

$$T_m = \frac{(W_c + 0.5W_{sh})R}{N_{c,s}} \quad (52)$$

where $N_{c,s} = 2 + 2 \sum_{l=0}^{\frac{N}{2}} \mu_l$ denotes the number of teeth in contact at the sleeve sides. The quantity μ_l is the restoring function, derived as:

$$\begin{cases} \mu_l = 1, & \delta_b \left[1 - \cos\left(\frac{2\pi l}{N}\right) \right] < \frac{(W_c + 0.5W_{sh})R^2}{2C} \\ \mu_l = 0, & \delta_b \left[1 - \cos\left(\frac{2\pi l}{N}\right) \right] > \frac{(W_c + 0.5W_{sh})R^2}{2C}, \quad l = 1, \dots, \frac{N}{2}. \end{cases} \quad (53)$$

5. Finite-Element/Contact-Mechanics Model Description

The finite-element/contact-mechanics (FE/CM) model, Transmission3D 2.2700/0.1195, assumes conformal tooth contact throughout the entire tooth flank. The surface contact is solved by a surface integral approach that analyzes the near-field contact mechanics by integrating, in the style of a Green's function, the solution for a point load on a half space over the contact area [23, 24]. Finite-element analysis calculates far-field elastic deformations starting a small distance away from the contact area. Matching of the contact deflections and FE solutions yields a combined contact solution for near-field surface deformations. The FE/CM models of gears and bearings were validated by experiments in [25, 26]. The FE/CM models include the flexibility of the coupling-tooth flank, rim, shaft, and tooth profile and lead modification. The FE/CM model of the upwind gear coupling is shown in Figure 7(a). Cubic elements are used to model the spline teeth and rim. The tooth-contact pattern of the coupling with a 0.5° misalignment angle is illustrated. As shown in Figure 7a, the tooth-contact patterns are located at the opposite sides of the coupling and they move toward the flank edges when the coupling is misaligned. Mesh convergence has been performed to improve accuracy and results are shown in Figure 7(b). Finite elements with an average size of 25 mm were selected for the following analysis.

6. Results and Discussion

The following sections evaluate the contact characteristics and induced loads of the couplings using the developed analytic model. Because of the lack of experimental data, results of the analytic formulation about the local tooth contact were compared to these outputs from the high-fidelity FE/CM model. The coupling-induced loads were correlated with measured experimental data.

6.1. Local Tooth Contact

The local contact of gear couplings is primarily characterized by the tooth rotational stiffness, tooth-load distribution, and jam angle across the circumference. The rotational

Figure 7: (a) The FE/CM model of the upwind coupling. Tooth-contact patterns with 0.5° misalignment are illustrated; (b) the mesh convergence study of the finite-element model. Current mesh size is denoted by the filled circle.

stiffness of a single tooth pair determines the tooth deformation and, therefore, the number of teeth in contact at a given torque and misalignment. The rotational stiffness calculated using the analytic model was compared against the FE/CM model for a different gear coupling [18].

Figure 8 and Figure 9 show the tooth-load distributions of the upwind gear coupling at selected misalignment angles, calculated using the analytic and FE/CM models. These models are in overall good agreement, each capturing the influence of misalignment on the tooth-load distribution. The number of teeth in contact and the maximum tooth load are generally higher for the FE/CM model than for the analytic model. This is especially evident at higher misalignment angles, which is a result of the inclusion of coupling-tooth and shaft elasticity and the subsequent additional compliance in the system.

When the coupling is perfectly aligned with the sleeve shaft, all of the teeth are in contact and the torque is transmitted evenly across the teeth, as shown in Figure 8(a) and

Figure 9(a). That is, the loads are centered on the flank and are equally distributed across all of the teeth. With misalignment, the number of teeth in contact decreases and the loads move toward the flank edges. The maximum load on a single tooth increases significantly, as shown in Figures 8(b)(c) and 9(b)(c). The contact pattern is symmetrically located at the opposite sides of the coupling. With a large misalignment that is close to the jam angle, less than 20% of the teeth are in contact. The contact areas are located at the flank edges and the load carried by a single tooth reaches the maximum, which is nearly four times the load without misalignment, as shown in Figures 8(d) and 9(d).

Figure 8: Upwind coupling-load distributions at (a) 0; (b) 0.1°; (c) 0.5°; and (d) 0.8° misalignment from the analytic model with applied torque of 1.5 MNm.

Figure 9: Upwind coupling-load distributions at (a) 0; (b) 0.1°; (c) 0.5°; and (d) 0.8° misalignment from the FE/CM model with applied torque of 1.5 MNm.

6.2. Jam Angle

The calculated jam angles of the couplings using the developed analytic and FE/CM models are compared in Table 3. Reasonably good agreement is present between these two models. The analytic model estimates the jam angle is 18% lower than the FE/CM model results, which can be caused by the different tooth-profile and structural flexibility considerations. The analytic model does not take into account the tooth profile modifications and the additional compliance in the system caused by the rim and shaft.

Jam angle ($^{\circ}$)	Upwind coupling	Downwind coupling
Analytic	0.9	1.8
FE/CM	1.0	2.2

Table 3: Jam Angles of the Upwind and Downwind Couplings

6.3. Coupling-Induced Loads

Combined torque and bending moments were applied to the coupling shaft through the dynamometer gearbox and LAD. The applied torque ranges from 0.1 MNm to 2.2 MNm. The reference torque used in the paper equals 1.5 MNm, which was selected arbitrarily. The bending moments applied by the LAD to the device under test ranged from -6.0 MNm to 6.0 MNm in two directions, M_y and M_z , respectively, which resulted in coupling misalignments up to 60% of its capacity. Experiments at these combined and static loading conditions were conducted at constant operational speeds ranging from 6.0 rpm to 15.0 rpm. The measured bending moments near the coupling were found to be almost independent of operating speed based on the data collected. Measured friction coefficients (in the Appendix) during the test were used during the load calculations. The friction coefficient is primarily affected by torque and misalignment. It increases with the torque decrease or the misalignment increase. The friction-induced loads are, therefore, significant at relatively large misalignment angles. Future experiments will investigate the effects of temperature on friction coefficients. Additionally, experimental data of the downwind coupling appear scattered at large misalignment angles, which was caused by system hysteresis of the test bench.

6.3.1. Effects of Misalignment

Bending moments around the y and z axes were applied by the LAD to misalign the coupling shaft in the θ_y and θ_z directions. Figures 10(a) and 10(b) show the misalignment amplitude and direction of the upwind and downwind couplings measured through the shaft-proximity sensors. Given the applied moments around the y axis ranges from -6.0 MNm

to 6.0 MNm and torque equals 1.0 MNm and 1.2 MNm, respectively, the applied moment caused the coupling misalignment of 17%—60% of its capacity. The misalignment of 17% at zero applied moment suggested an initial misalignment caused by assembly tolerances and structural deflections. As shown in Figure 10(a), the misalignment amplitude linearly changes with the applied moments as anticipated. The misalignment direction, however, deviates gradually from the misalignment direction, as shown in Figure 10(b). This unexpected behavior may be caused by the compliance of the large LAD system that resulted in certain cross-coupling effects among the six degrees of freedom. Although the applied moments were only around the y axis, the system responses of the coupling displacement were in multiple directions, mainly θ_y and θ_z .

Figure 10(c), Figure 10(d), Figure 10(e), and Figure 10(f) compare the modeling and experimental results of $M_s^{y'}$ and $M_s^{z'}$ of the upwind and downwind couplings under the same loading condition as Figures 10(a) and 10(b). The measured torque, misalignment amplitude, and direction are the input of the analytic model. The time-varying friction coefficient was selected based on the instantaneous torque and misalignment at each load case (in the Appendix). Good agreement is evident between the measured and calculated $M_s^{y'}$ and $M_s^{z'}$ for both couplings. The analytic model overpredicts the $M_s^{y'}$ of the downwind coupling at a large misalignment angle. This might be caused by the dynamic interaction of the downwind coupling with the LAD system. Further investigation is under way to understand this behavior.

Similar study was performed on the coupling loads with moments around the z axis applied by the LAD system. Figures 11(a) and 11(b) show the misalignment amplitude and direction of the upwind and downwind couplings measured through the shaft-proximity sensors. The applied moment caused the coupling misalignment of 0%—60% of its capacity. Figure 11(c)-Figure 11(f) compare the modeling and experimental results of $M_s^{y'}$ and $M_s^{z'}$ of upwind and downwind couplings. Excellent agreement between the measured and calculated $M_s^{y'}$ and $M_s^{z'}$ is present for the upwind coupling.

As shown in Figures 10 and 11, the coupling-induced moments can reach 14% of the

Figure 10: (a) Amplitude and (b) direction of measured coupling misalignment; upwind bending moments (c) M_y and (d) M_z with various measured misalignment; downwind bending moments (e) M_y and (f) M_z with various measured misalignment. Moments around the y axis were applied to the LAD ranging from -6.0 MNm to 6.0 MNm and torque equals 1.0 MNm and 1.2 MNm.

reference torque of the device under test. The downwind coupling-induced loads are greater than the upwind because of its higher absolute misalignment angle. Moreover, the global moment induced by the couplings $\sqrt{(M_s^{y'})^2 + (M_s^{z'})^2}$ for both couplings has a nearly linear relation with the misalignment amplitude, which agrees with the findings in [5, 7].

Figure 11: (a) Amplitude and (b) direction of measured coupling misalignment; Upwind bending moments (c) M_y and (d) M_z with various measured misalignment; downwind bending moments (e) M_y and (f) M_z with various measured misalignment. Moments around the z were applied to LAD ranging from -6.0 MNm to 6.0 MNm and torque equals 1.2 MNm.

6.3.2. Effects of Torque

Effects of torque on the induced loads of the downwind coupling, $M_s^{y'}$ and $M_s^{z'}$, are shown in Figures 12 and 13. The upwind coupling demonstrated similar behavior. Transmitted torque ranged from 10% to 95% of the reference torque of the device under test. Selected bending moments were applied by the LAD to achieve several specified misalignment angles

that equal $\pm 30\%$, $\pm 60\%$ of their capacities in the θ_y and θ_z directions. The modeling results match the experiments well. The coupling loads increase linearly with torque. The rate of increase, however, depends on the misalignment. The load dependency on torque increases with misalignment. It is anticipated that the maximum coupling loads can exceed 14% of the reference torque at its full misalignment capacity. Additionally, for wind applications, the drivetrain torque can be much greater than the reference torque in transient events, such as start-up, emergency stop, and braking [27]. The instantaneous loads induced by the couplings, therefore, can be greater than the presented values during these dynamic operations.

Figure 12: Downwind coupling bending moment M_y at various torque levels.

6.4. Effects of Restoring Torque on Coupling Motion

A torque ramp test was conducted at the dynamometer as shown in Figure 14. Torque was increased from 0.1 MNm to 2.2 MNm through 17 steps. The coupling motion amplitude changed dramatically during the torque ramp, as shown in Figure 15. Significant motion of the upwind and downwind couplings between the hubs and sleeves (nearly 0.6 mm in

Figure 13: Downwind coupling bending moment M_z at various torque levels.

the radial direction) occurred when torque was less than 1.2 MNm (178 s) and 1.3 MNm (202 s), respectively, compared to higher torque levels. The coupling motion diminished when torque exceeded these two thresholds (less than 0.05 mm). The coupling motion at low torque was caused by the eccentricity between the coupling hub and sleeve (Figure 6), resulting in an initial misalignment of 0.04° – 0.08° . When torque increased, the coupling that was initially resting on the meshing sleeve was lifted by the torque that canceled out its self-weight. Operating the coupling under low torque for a long duration could impair its health and, consequently, reduce the service life. The torque thresholds at 1.2 MNm and 1.3 MNm are the restoring torques that can lift the couplings and center themselves with the sleeve. These measured restoring torques are compared against the analytic solutions derived from Eqs. (50) and (52), as shown in Table 4. The agreement between the model and experiment is reasonable with a maximum difference of 15.2% for the upwind coupling. It has been observed that the sleeve of the upwind coupling is not perfectly round because

of manufacturing errors, which can also cause tooth spacing errors across the circumference. The analytic model assumes circular sleeve and equally spaced teeth, which can overpredict the restoring torque. Furthermore, the initial misalignment angles of both couplings are not zero. The analytical model assumes zero initial misalignment and thus cannot capture its influence on restoring torque.

Figure 14: Torque ramp test sequence conducted at the dynamometer.

Restoring torque (MNm)	Upwind coupling	Downwind coupling
Model	1.36	1.36
Experiment	1.18	1.32
Difference, %	+15.25	+3.03

Table 4: Upwind and Downwind Coupling Restoring Torque.

Figure 15: Upwind and downwind coupling motions and misalignment angles during the torque ramp.

7. Conclusions

A new analytic model for real-time design evaluation of gear couplings was developed. This model efficiently addresses both the local contact characteristics and global-induced loads of gear couplings that can be used in real-time health monitoring. Compared against previous models that calculated coupling loads, this model is more advanced by considering the local tooth contact and effects of pressure angle, which can significantly improve coupling design accuracy, reduce manufacturing cost, and decreases risks during experiments.

This model calculates the tooth-contact characteristics including number of teeth in contact, tooth-load distribution, stresses, and loads. Using the local contact characteristics, the model also predicts the induced loads that are affected by misalignment, torque, and friction. This model was validated by experiments conducted at the NWTC's 5-MW dynamometer for the coupling loads. Modeling results of tooth contact and jam angle have been correlated with FE/CM analyses.

Misalignment greatly affects the tooth-load distribution of gear couplings. When the

coupling is aligned, torque is shared equally across all of the teeth and the load is concentrated at the tooth-flank center. When the coupling is misaligned, the number of teeth in contact decreases and the maximum load on a single tooth increases. Moreover, the contact area deviates from the tooth-flank center and moves toward the tooth edges, symmetrically located on the opposite sides of the coupling. In the extreme, the teeth are edge-loaded and at risk of failure.

Coupling-induced loads are a combined effect of misalignment, torque, and friction. The load amplitude increases nearly linearly with misalignment and torque. The load direction, however, does not align with the misalignment axis. Instead, it shifts from the misalignment axis by the pressure angle. The maximum amplitude of the coupling loads is nearly 14% of the applied torque. Sliding friction caused by the axial motions contributes to the total loads induced by reference couplings. The friction coefficient increases with the torque decrease or the misalignment increase.

Coupling can have significant motions between the hub and sleeve at low torque. This coupling motion is induced by an initial eccentricity caused by its self-weight. By increasing the transmitted torque, this motion can be greatly reduced.

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Appendix: Friction Coefficient Matrix

Torque, MNm	Misalignment, $^{\circ}$								
	0	0.1	0.2	0.3	0.4	0.5	0.7	0.8	0.9
0.203	0.028	0.030	0.039	0.035	0.052	0.037	0.034	0.037	0.036
0.400	0.016	0.018	0.023	0.016	0.025	0.031	0.021	0.020	0.019
0.597	0.012	0.013	0.017	0.012	0.019	0.027	0.014	0.011	0.011
0.803	0.009	0.010	0.013	0.010	0.014	0.025	0.014	0.009	0.009
0.999	0.008	0.008	0.010	0.008	0.010	0.025	0.013	0.009	0.007
1.196	0.007	0.007	0.007	0.006	0.006	0.025	0.011	0.007	0.006
1.398	0.006	0.005	0.005	0.004	0.003	0.026	0.012	0.007	0.006

Table 5: Friction coefficient of the upwind gear coupling in NWTC's 5-MW dynamometer.

Torque, MNm	Misalignment, $^{\circ}$								
	0	0.1	0.2	0.3	0.4	0.5	0.7	0.8	0.9
0.205	0.013	0.015	0.017	0.020	0.016	0.026	0.022	0.018	0.016
0.397	0.008	0.009	0.010	0.013	0.009	0.015	0.017	0.012	0.009
0.598	0.005	0.006	0.007	0.009	0.007	0.010	0.015	0.007	0.005
0.803	0.004	0.005	0.006	0.007	0.005	0.007	0.014	0.008	0.004
0.998	0.004	0.004	0.005	0.005	0.004	0.005	0.014	0.007	0.004
1.196	0.003	0.004	0.004	0.004	0.003	0.003	0.014	0.006	0.003
1.397	0.003	0.003	0.003	0.003	0.002	0.002	0.014	0.006	0.003

Table 6: Friction coefficient of the downwind gear couplings in NWTC's 5-MW dynamometer.