On the Mindlin microelasticity in one dimension

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Abstract

Mindlin's microelasticity theory is reduced to the one-dimensional setting to examine its prediction of the response of microstructured materials on a pulse loading. The one-dimensional setting requires a minimal number of additional material parameters. Since analytical solutions may be found only in highly simplified asymptotic cases, numerical simulations are performed by means of a finite-volume numerical scheme modifying the wave-propagation algorithm. The calculated response of a material described by the Mindlin microelasticity is compared then with the corresponding response of an "equivalent" periodic laminate.

Keywords: Microstructure, solids, wave propagation, numerical simulation

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1. Introduction

Generalized continuum theories extend the conventional continuum mechanics by incorporating intrinsic microstructural effects into the description of the mechanical behavior of materials (Mindlin, 1964; Eringen and Suhubi, 1964; Green and Rivlin, 1964). Introduced in 60s by famous mechanicians these theories are still under development, as recent reviews show (Voyiadjis and Faghihi, 2014; Maugin, 2015). The advantage of the micromorphic theories is their compatibility with molecular dynamics (Chen and Lee, 2003a,b). The main objection for their use in engineering practice is a huge amount of material parameters needed to be determined in advance (Neff et al., 2014). Nevertheless, generalized continuum theories are applied to the computational dynamic characterization of microstructured solids (Gonella et al., 2011; Greene et al., 2012). It is also expected that they are promising for nanomechanics (Wang and Lee, 2010).

It is worth therefore to analyze the simplest possible one-dimensional situation with the minimal number of additional parameters. The idea is in the comparison of the prediction of the microelasticity

theory with results of the direct numerical simulation of a similar problem in the medium with a given periodic microstructure. In the case of a good agreement, it would be possible to assess the values of additional material parameters.

We choose the Mindlin microelasticity as the example of the generalized continuum theory due to its detailed presentation in the original paper by Mindlin (1964). An one-dimensional pulse propagation problem is considered as a benchmark.

The paper is organized as follows. First, the Mindlin microstructure theory is reduced to its one-dimensional version. Then the algorithm of computation of the dynamics of microstructured solids is described briefly. The calculated response of a material described by the Mindlin microelasticity on the pulse loading is compared then with the corresponding response of an "equivalent" periodic laminate. Conclusions are included in the last section of the paper.

2. Mindlin's microstructure theory

Our starting point is a reminder of the Mindlin microelasticity (Mindlin, 1964). In the Mindlin microstructure theory, the quadratic free energy density W for an isotropic material has the form

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(Mindlin, 1964)

$$\begin{aligned}
W = & \frac{1}{2}\lambda\varepsilon_{ii}\varepsilon_{jj} + \mu\varepsilon_{ij}\varepsilon_{ji} + \frac{1}{2}b_1\gamma_{ii}\gamma_{jj} + \\
& + \frac{1}{2}b_2\gamma_{ij}\gamma_{ij} + \frac{1}{2}b_3\gamma_{ij}\gamma_{ji} + g_1\gamma_{ii}\varepsilon_{jj} + \\
& + \frac{1}{2}g_2(\gamma_{ij} + \gamma_{ji})\varepsilon_{ij} + a_1\kappa_{iik}\kappa_{kjj} + \\
& + a_2\kappa_{iik}\kappa_{jjk} + \frac{1}{2}a_3\kappa_{iik}\kappa_{jjk} + \\
& + \frac{1}{2}a_4\kappa_{ijj}\kappa_{ikk} + a_5\kappa_{ijj}\kappa_{kik} + \\
& + \frac{1}{2}a_8\kappa_{ijj}\kappa_{kjk} + \frac{1}{2}a_{10}\kappa_{ijk}\kappa_{ijk} + \\
& + a_{11}\kappa_{ijk}\kappa_{jki} + \frac{1}{2}a_{13}\kappa_{ijk}\kappa_{ikj} + \\
& + \frac{1}{2}a_{14}\kappa_{ijk}\kappa_{jik} + \frac{1}{2}a_{15}\kappa_{ijk}\kappa_{kij},
\end{aligned} \tag{1}$$

where ψ_{ij} is the microdeformation tensor, a_i, b_j, g_k are material parameters, and

$$\kappa_{ijk} \equiv \partial_i \psi_{jk}, \quad \gamma_{ij} = \partial_i u_j - \psi_{ij}, \tag{2}$$

are the microdeformation gradient and the relative strain, respectively, ∂_i denotes space derivative. Accordingly, the Cauchy stress tensor is defined as

$$\begin{aligned}
\tau_{ij} \equiv \frac{\partial W}{\partial \varepsilon_{ij}} = & \lambda \delta_{ij} \varepsilon_{kk} + 2\mu \varepsilon_{ij} + \\
& + g_1 \delta_{ij} \gamma_{kk} + g_2 (\gamma_{ij} + \gamma_{ji}).
\end{aligned} \tag{3}$$

Keeping the nomenclature by Mindlin (1964), for the interaction tensor σ we have

$$\begin{aligned}
\sigma_{ij} \equiv \frac{\partial W}{\partial \gamma_{ij}} = & g_1 \delta_{ij} \varepsilon_{kk} + 2g_2 \varepsilon_{ij} + \\
& + b_1 \delta_{ij} \gamma_{kk} + b_2 \gamma_{ij} + b_3 \gamma_{ji}.
\end{aligned} \tag{4}$$

Double stress μ_{ijk} is defined as follows:

$$\begin{aligned}
\mu_{ijk} \equiv \frac{\partial W}{\partial \kappa_{ijk}} = & a_1(\kappa_{qqi}\delta_{jk} + \kappa_{kqq}\delta_{ij}) + \\
& + a_2(\kappa_{qqj}\delta_{ik} + \kappa_{qkq}\delta_{ij}) + a_3\kappa_{qqk}\delta_{ij} + \\
& + a_4\kappa_{iqq}\delta_{jk} + a_5(\kappa_{jqj}\delta_{ik} + \kappa_{qiq}\delta_{jk}) + \\
& + a_8\kappa_{qqj}\delta_{ik} + a_{10}\kappa_{ijk} + \\
& + a_{11}(\kappa_{kij} + \kappa_{jki}) + a_{13}\kappa_{ikj} + \\
& + a_{14}\kappa_{jik} + a_{15}\kappa_{kji}.
\end{aligned} \tag{5}$$

Equations of motion include the balance of linear momentum (no body forces) (Mindlin, 1964)

$$\rho \ddot{u}_j = \partial_i (\tau_{ij} + \sigma_{ij}), \tag{6}$$

and the evolution equation for the microdeformation

$$\frac{1}{3}\rho' d^2 \ddot{\psi}_{jk} = \partial_i \mu_{ijk} + \sigma_{jk}, \tag{7}$$

where ρ' is the density of the material, d is the length scale at the microlevel, "dot" denotes time derivative. For practical applications, 16 material parameters in addition to conventional Lamé coefficients is far too many. We will study the one-dimensional case to minimize the number of additional material parameters.

3. Unidirectional case

Considering first the unidirectional case, we assume that the displacement vector and the microdeformation tensor are functions of only one space variable

$$u_i = u_i(x_1, t), \quad \psi_{jk} = \psi_{jk}(x_1, t). \tag{8}$$

Non-zero components of the strain tensor are

$$\varepsilon_{11} = \partial_1 u_1, \quad \varepsilon_{12} = \partial_1 u_2 = \varepsilon_{21}, \quad \varepsilon_{13} = \partial_1 u_3, \tag{9}$$

whereas all other components are zero

$$\varepsilon_{31} = 0, \quad \varepsilon_{22} = 0, \quad \varepsilon_{33} = 0, \quad \varepsilon_{23} = \varepsilon_{32} = 0. \tag{10}$$

We restrict the consideration by the so-called longitudinal system of equations (Mindlin, 1964).

3.1. Longitudinal motion

Following Mindlin (1964), we have for the longitudinal motion

$$k_{11} \partial_1 \partial_1 u_1 - k_{12} \partial_1 \psi_{11}^D - k_{13} \partial_1 \psi = \rho \ddot{u}_1, \tag{11}$$

$$\begin{aligned}
& k_{21} \partial_1 u_1 + k_{22} \partial_1 \partial_1 \psi_{11}^D - k'_{22} \psi_{11}^D + \\
& + k_{23} \partial_1 \partial_1 \psi = 0.5 \rho' d^2 \ddot{\psi}_{11}^D,
\end{aligned} \tag{12}$$

$$\begin{aligned}
& k_{31} \partial_1 u_1 + k_{32} \partial_1 \partial_1 \psi_{11}^D + k_{33} \partial_1 \partial_1 \psi - \\
& - k'_{33} \psi = \rho' d^2 \ddot{\psi},
\end{aligned} \tag{13}$$

where $\psi = 1/3 \psi_{ii}$, $\psi_{11}^D = \psi_{11} - \psi$, and coefficients k_{ij} are expressed in terms of material parameters in the definition of the free energy density

$$k_{11} = \lambda + 2\mu + 2g_1 + 4g_2 + b_1 + b_2 + b_3, \tag{14}$$

$$k_{12} = k_{21} = 2g_2 + b_2 + b_3, \tag{15}$$

$$k_{13} = k_{31} = 3g_1 + 2g_2 + 3b_1 + b_2 + b_3, \tag{16}$$

$$k'_{22} = \frac{3}{2}(b_2 + b_3), \tag{17}$$

$$k'_{33} = 3(b_1 + b_2 + b_3). \tag{18}$$

Coefficients $k_{22}, k_{23}, k_{32}, k_{33}$ depend only on a_i and their exact form is not relevant here.

3.2. One-dimensional approximation

The further reduction can be obtained under assumption that the transverse components of the microdeformation tensor are much smaller than the longitudinal one. The one-dimensional approximation can be represented by means of asymptotic expansions for a small parameter $\epsilon \ll 1$

$$\begin{aligned}\psi_{11} &= \psi_{11}^0 + \epsilon \psi_{11}^1 + \dots, \\ \psi_{22} &= \epsilon \psi_{22}^0 + \dots, \\ \psi_{33} &= \epsilon \psi_{33}^0 + \dots\end{aligned}\quad (19)$$

In the zero-order approximation with respect to ϵ the deviatoric part of the microdeformation $\psi_{11}^D = 2\psi_{11}^0/3$ and the mean microdeformation $\psi = \psi_{11}^0/3$ are represented in terms of ψ_{11}^0 , and we have for equations of motion

$$k_{11}\partial_1\partial_1 u_1 - \frac{2k_{12} + k_{13}}{3}\partial_1\psi_{11}^0 = \rho\ddot{u}_1, \quad (20)$$

$$\begin{aligned}3k_{21}\partial_1 u_1 + 2k_{22}\partial_1\partial_1\psi_{11}^0 - 2k'_{22}\psi_{11}^0 + \\ + k_{23}\partial_1\partial_1\psi_{11}^0 = \rho'd^2\ddot{\psi}_{11}^0,\end{aligned}\quad (21)$$

$$\begin{aligned}3k_{31}\partial_1 u_1 + 2k_{32}\partial_1\partial_1\psi_{11}^0 + k_{33}\partial_1\partial_1\psi_{11}^0 - \\ - k'_{33}\psi_{11}^0 = \rho'd^2\ddot{\psi}_{11}^0.\end{aligned}\quad (22)$$

It should be noted that in the absence of microdeformation ($\psi_{11}^0 = 0$) the first of above equations should be reduced to the conventional one-dimensional equation of the linear elasticity, which yields

$$2g_1 + 4g_2 + b_1 + b_2 + b_3 = 0. \quad (23)$$

Next two equations should be identical, which means

$$\begin{aligned}k_{21} = k_{31}, \quad 2k'_{22} = k'_{33}, \\ 2k_{22} + k_{23} = 2k_{32} + k_{33}.\end{aligned}\quad (24)$$

As the result, the number of independent coefficients can be reduced

$$b_1 = 0, \quad g_1 = 0, \quad g_2 = -\frac{1}{4}(b_2 + b_3), \quad (25)$$

and coefficients k_{12} and k'_{33} are simplified to

$$k_{12} = -2g_2, \quad k'_{33} = -12g_2. \quad (26)$$

One-dimensional equations of motion

$$\rho\dot{v}_1 = (\lambda + 2\mu)\partial_1\epsilon_1 + 2g_2\partial_1\psi_{11}, \quad (27)$$

$$\begin{aligned}\frac{\rho'd^2}{3}\ddot{\psi}_{11} &= \frac{2k_{32} + k_{33}}{3}\partial_1\partial_1\psi_{11} - \\ &- 2g_2\epsilon_1 + 4g_2\psi_{11},\end{aligned}\quad (28)$$

can be represented in a more compact form

$$\rho v_t = (\lambda + 2\mu)\epsilon_x + A\psi_x, \quad (29)$$

$$M\psi_{tt} = C\psi_{xx} - A\epsilon + 2A\psi, \quad (30)$$

with $M = \rho'd^2/3$, $C = (2k_{32} + k_{33})/3$, $A = 2g_2$. The coefficient M can be associated with microinertia (Berezovski and Ván, 2016), the coefficient C plays the role of the Young modulus for microstructure, and the coefficient A provides the coupling between micro- and macro- motions.

4. Dimensionless variables

To be as general as possible, we will express the governing equations in the dimensionless form. First, we rewrite equations of motion (29)-(30) in the form of the system of first-order equations

$$\rho\frac{\partial v}{\partial t} = \frac{\partial}{\partial x}[(\lambda + 2\mu)\epsilon + A\psi], \quad (31)$$

$$\frac{\partial \epsilon}{\partial t} = \frac{\partial v}{\partial x}, \quad (32)$$

$$M\frac{\partial w}{\partial t} = C\frac{\partial \psi}{\partial x} - \int (A\epsilon - 2A\psi)dx, \quad (33)$$

$$\frac{\partial \psi}{\partial t} = \frac{\partial w}{\partial x}, \quad (34)$$

where w can be associated with a microvelocity since it plays the same role as the macrovelocity v .

For the further analysis we introduce dimensionless variables

$$U = \frac{u}{U_0}, \quad X = \frac{x}{L}, \quad T = \frac{c_0 t}{L}, \quad (35)$$

where U_0 and L are certain constants (e.g., wavelength of the initial excitation and macro length scale), and c_0 is the characteristic velocity.

Dimensionless strain ϵ^* and velocity v^* are introduced as follows:

$$\epsilon = \frac{\partial u}{\partial x} = \frac{\partial(U_0 U)}{\partial(LX)} = \frac{U_0}{L} \frac{\partial U}{\partial X} \equiv \frac{U_0}{L} \epsilon^*, \quad (36)$$

$$v = \frac{\partial u}{\partial t} = c_0 \frac{\partial(U_0 U)}{\partial(LT)} = \frac{c_0 U_0}{L} \frac{\partial U}{\partial T} \equiv \frac{c_0 U_0}{L} v^*. \quad (37)$$

To characterize a microstructure we also need to introduce a microscale l and another characteristic

velocity c_1 . Then for the dimensionless microdeformation ψ^* and the microvelocity w^* we have

$$\psi \equiv \frac{l}{L} \psi^*, \quad (38)$$

$$w \equiv \frac{c_1 l}{L} w^*. \quad (39)$$

The dimensionless stress σ^* is introduced as follows:

$$\sigma^* = \frac{\sigma}{\rho_0 c_0^2} = \frac{U_0}{L} \varepsilon^*. \quad (40)$$

In terms of dimensionless variables, the system of equations (31) - (34) reads

$$\frac{U_0}{L} \frac{\partial v^*}{\partial T} = \frac{\partial}{\partial X} \left(\sigma^* + A^* \frac{l}{L} \psi^* \right), \quad (41)$$

$$\frac{\partial \varepsilon^*}{\partial T} = \frac{\partial v^*}{\partial X}, \quad (42)$$

$$\frac{\partial \psi^*}{\partial T} = \frac{c_1}{c_0} \frac{\partial w^*}{\partial X}, \quad (43)$$

$$\begin{aligned} \frac{\partial w^*}{\partial T} &= \frac{c_1}{c_0} \frac{\partial \psi^*}{\partial X} - \\ &- \frac{\rho_0}{I^*} \frac{c_0}{c_1} \frac{A^*}{\delta} \int \left(\frac{U_0}{l} \varepsilon^* - 2\psi^* \right) dX, \end{aligned} \quad (44)$$

where $A^* = A/\rho_0 c_0^2$, $M = l^2 I^*$, $C = l^2 C^*$, and $\delta = l^2/L^2$ are introduced for the convenience and $c_1^2 = C^*/I^*$.

Thus, the problem under consideration is characterized by two geometrical dimensionless parameters U_0/L and l/L , by the relation between longitudinal speeds at micro- and macro- levels c_0/c_1 , by the ratio of densities ρ_0/I^* , and by the dimensionless coupling coefficient A^* .

5. Numerical scheme

It is clear that analytical solutions may be found only in highly simplified asymptotic cases. Therefore, we turn to numerical methods. We use the finite-volume numerical scheme modifying the wave-propagation algorithm (LeVeque, 1997) as described in (Berezovski et al., 2008; Berezovski, 2011)

$$\begin{aligned} (v^*)_n^{k+1} - (v^*)_n^k &= \frac{L}{U_0} \frac{\Delta T}{\Delta X} ((\Sigma^{*+})_n^k - (\Sigma^{*-})_n^k) + \\ &+ \frac{L}{U_0} \frac{\Delta T}{\Delta X} A^* \frac{l}{L} ((\Psi^{*+})_n^k - (\Psi^{*-})_n^k), \end{aligned} \quad (45)$$

$$(\varepsilon^*)_n^{k+1} - (\varepsilon^*)_n^k = \frac{\Delta T}{\Delta X} ((V^{*+})_n^k - (V^{*-})_n^k), \quad (46)$$

$$(\psi^*)_n^{k+1} - (\psi^*)_n^k = \frac{c_1}{c_0} \frac{\Delta T}{\Delta X} ((\Omega^{*+})_n^k - (\Omega^{*-})_n^k), \quad (47)$$

$$\begin{aligned} (w^*)_n^{k+1} - (w^*)_n^k &= \frac{c_1}{c_0} \frac{\Delta T}{\Delta X} ((\Psi^{*+})_n^k - (\Psi^{*-})_n^k) - \\ &- \frac{\rho_0}{I^*} \frac{c_0}{c_1} \frac{A^*}{\delta} \left(\frac{U_0}{l} (\bar{\varepsilon}^*)_n^k - 2(\bar{\psi}^*)_n^k \right) \Delta X \Delta T, \end{aligned} \quad (48)$$

where n and k denote space and time steps, respectively, overbars denote averaged quantities, which are introduced together with excess quantities both for macro- and microfields as follows:

$$\begin{aligned} \sigma^* &= \bar{\sigma}^* + \Sigma^*, & v^* &= \bar{v}^* + V^*, \\ \psi^* &= \bar{\psi}^* + \Psi^*, & w^* &= \bar{w}^* + \Omega^*. \end{aligned} \quad (49)$$

Note that ΔT is the dimensionless time step and ΔX is the dimensionless space step used in computations.

The values of excess quantities, which play the role of numerical fluxes at the boundaries between computational cells, are calculated by means of jump relations at the interfaces between computational cells

$$\left[\bar{\sigma}^* + \Sigma^* + \frac{A^* l}{L} (\bar{\psi}^* + \Psi^*) \right] = 0, \quad (50)$$

$$[\bar{v}^* + V^*] = 0, \quad (51)$$

$$\begin{aligned} &\left[\frac{c_1}{c_0} (\bar{\psi}^* + \Psi^*) \right] - \\ &- \left[\frac{(\Delta X)^2 \rho_0 c_0 A^*}{I^* c_1 \delta} \left(\frac{U_0}{l} \bar{\varepsilon}^* - 2\bar{\psi}^* \right) \right] = 0, \end{aligned} \quad (52)$$

$$[\bar{w}^* + \Omega^*] = 0, \quad (53)$$

complemented by the Riemann invariants conservation conditions at these interfaces (Berezovski et al., 2008; Berezovski, 2011)

$$\frac{U_0}{L} V_n^{*-} + \Sigma_n^{*-} = 0, \quad (54)$$

$$\frac{U_0}{L} V_{n-1}^{*+} - \Sigma_{n-1}^{*+} = 0, \quad (55)$$

$$\Omega_n^{*-} + \Psi_n^{*-} = 0, \quad (56)$$

$$\Omega_{n-1}^{*+} - \Psi_{n-1}^{*+} = 0. \quad (57)$$

It should be noted that the values of excess quantities are calculated exactly (Berezovski et al., 2008).

6. Numerical simulation

6.1. Reference solution

To have a reference solution, the propagation of a pulse in a one-dimensional medium, which can be represented as an elastic bar, is considered first. This bar is assumed homogeneous except of a region of length d , where periodically alternating layers of size l are inserted (Fig. 1a). The density and

Figure 1: Geometry of the bar.

longitudinal speed in the bar are chosen as $\rho = 4510 \text{ kg/m}^3$ and $c = 5240 \text{ m/s}$, respectively, which corresponds to a titanium alloy. The corresponding parameters for the material of the inhomogeneity layers are $\rho_1 = 2703 \text{ kg/m}^3$ and $c_1 = 5020 \text{ m/s}$, respectively (like for Al 6061). Initially, the bar is at the rest. The shape of the pulse before the crossing of the inhomogeneity region is formed by an excitation of the strain at the left boundary for an limited time period ($0 < t < 100\Delta t$)

$$u_x(0, t) = (1 + \cos(\pi(t - 50)/50)). \quad (58)$$

The length of the pulse ($U_0 = 100\Delta x$) is comparable with the size of inhomogeneity l . The pulse holds its shape up to the entering into the inhomogeneity region. The problem is solved directly, i.e., without application of the Mindlin microstructure theory, since the location and properties of substructure are prescribed.

Numerical calculations are performed by the numerical scheme described in Sec. 5 with $A^* = 0$, $L = 5000\Delta x$, $d = 2200\Delta x$. To demonstrate the influence of the size of inhomogeneity, the scattering of the pulse was computed for its various values, namely, for $l = 90, 120, 150$ and 180 space steps. The numerical solution shows that after the interaction with the periodic multilayer, the single pulse is separated into many reflected and transmitted

parts as it can be seen in Fig. 2. To visualize the distinction in the shape of leading pulses, the stress distribution is shown at 4800 time steps for the periodicity of inhomogeneity equal to 180 space steps, at 4700 time steps for the periodicity equal to 150 space steps, at 4600 time steps for the periodicity equal 120 space steps, and at 4500 time steps for the periodicity equal 90 space steps, respectively. In each case, there are two leading positive pulses with a following tail of smaller positive and negative pulses. The dispersion of the initial pulse occurs be-

Figure 2: Scattering of a pulse by a periodic multilayers.

cause of the successive reflections at each interface between the alternating layers.

6.2. Prediction of the Mindlin microelasticity

Now we consider the region of inhomogeneity by means of the Mindlin microelasticity using the same material and geometric parameters as in the case of periodic laminate (Fig. 1b). This means that the value of I^* is chosen to be equal to the density of inhomogeneity layers, the microscale l is equal to the size of these layers, and the longitudinal speed at the microscale c_1 is determined by the material of inhomogeneity layers. The total length L and the length of excitation U_0 are the same as in the reference solution. Therefore, the solution can be parameterized by the value of the coupling parameter A^* . Snapshots of the stress distribution for various values of the parameter A^* are presented in Fig. 3. The visualization of the resulting pulses is made by displaying the pulse for the value $A^* = -2.5$ at 4800 time steps, for the value $A^* = -2.0$ at 4500 time steps, for the value $A^* = -1.5$ at 4200 time steps, and the value $A^* = -1.0$ at 3900 time steps, respectively. It is clear that the amplitude of the pulse increases with the grow of the absolute value of the coupling parameter A^* , and the resulting positive pulse is accompanied by a negative disturbance. The negative sign of the parameter A^* follows from the requirement of convexity of

the free energy. As one can see, the prediction of

Figure 3: Pulse transmission predicted by Mindlin's microelasticity.

the Mindlin microstructure theory is qualitatively different from the reference solution for a periodic laminate.

A similar situation was observed in the comparison of the internal variable model (Berezovski, 2011) for microstructure with the direct numerical simulation of a periodic laminate (Berezovski et al., 2010a). The improvement of the microstructure model was achieved by the change of the sign in source terms in the right hand side of Eq. (30) (Berezovski et al., 2010a,b), which results in

$$\rho v_t = (\lambda + 2\mu)\varepsilon_x + A\psi_x, \quad (59)$$

$$M\psi_{tt} = C\psi_{xx} + A\varepsilon - 2A\psi. \quad (60)$$

Calculations using modified microstructure model

Figure 4: Pulse transmission predicted by the modified microstructure model.

(59)-(60) were performed for the same values of material parameters as in the case of the Mindlin microelasticity, and the visualization was made in the same way as previously. The corresponding result is shown in Fig. 4. As one can see, the use of the modified microstructure model (Berezovski and Berezovski, 2012) provides two positive pulses (their mutual amplitude and location can be adjusted by the choice of material parameters).

It is shown, however, that the alteration of the sign in the source terms leads to a non-convex free energy function (Berezovski and Berezovski, 2012). This may be allowed for a microstructure model (like in (Berezovski et al., 2010a)), and the resulting modification can be stable (Berezovski and Berezovski, 2012), but this is not permitted in the Mindlin microelasticity.

7. Conclusions

As it is demonstrated on the example of one-dimensional elastic pulse propagation, predictions of the Mindlin microstructure theory may be inefficient if the size of inhomogeneity is comparable with the pulse length. This means that the Mindlin microelasticity is still an approximation of the description of the behavior of materials, which is satisfactory for long wavelengths or for small size of inhomogeneities. It is also shown that the change of the sign of the source terms in the evolution equation for the microdeformation gives more acceptable results, at least qualitatively. The free energy density in the corresponding microstructure model is non-convex (Berezovski and Berezovski, 2012). However, the requirement for the strong convexity of the free energy density is not always fulfilled for any material (Cherkaev, 2004; Schröder and Neff, 2010).

The main issue for the disagreement between the predictions by the Mindlin microelasticity and results of direct numerical computation of the response of a periodic laminate on a pulse loading is the demand of the strong convexity of the free energy density in the Mindlin theory.

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