Mixed integer multi-objective optimization of composite structures with frequency-dependent interleaved viscoelastic damping layers

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ABSTRACT

The optimal design of composite structures with frequency-dependent interleaved viscoelastic damping layers is addressed in this paper. The design objective of simultaneously maximizing structural modal damping and frequency and minimizing weight is considered. The design problem is formulated as a mixed integer multi-objective optimization problem and solved by evolutionary algorithm. A layerwise finite element model is used. The Pareto-optimal solutions are obtained for two applications. The results show that the approach is quite useful in integrally designing such kind of composite structures. In particular, it is shown that the inserting position and the material type are two important design parameters.

KEYWORDS: viscoelastic damping; mixed integer multi-objective optimization; layerwise finite element; evolutionary optimization; composite structures

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1. INTRODUCTION

Fiber reinforced composites are extensively used in aerospace, civil and automotive industries due to their high specific stiffness, high specific strengths and tailorable anisotropic elastic properties. Although fiber-reinforced composites exhibit higher energy dissipation capacity than common metals, the amount of damping is still not sufficient for many applications especially under severe vibration circumstances [1]. In past decades, a number of works [2-6] to improve the damping of composite structures have been concentrated on inserting viscoelastic damping layers between the base composite laminate. By this hybrid technique, the damping capacity of composite laminate can be increased many times by the shear deformation induced between the soft damping layer and the stiff constraining layer. However, due to interleaving soft material layers within stiff laminate, the mechanical analysis and design optimization of these structures becomes difficult and challenging.

Many works have been done on dynamic analysis of viscoelastic laminated composite structures. For example, DiTaranto and Blasingame [7] proposed analytical solutions for composite viscoelastic beams. Rao et al. [8] presented closed-form solutions for resonance frequencies and modal loss factors of laminated composite beams with multiple viscoelastic layers by the energy formulation and Ritz method. Zhou et al. [9] analytically studied the effects of various geometry and physical parameters on the resonance frequencies and modal loss factors of a composite tube with embedded viscoelastic layers. Finite element method is also used for analyzing viscoelastic sandwich structures by many researchers. Lu et al. [10] compared finite element to experimental results for three layered damped viscoelastic composite plates. Rikards et al. [11] studied the damping features of laminated visco-elastic sandwich beams using finite elements. Rao et al. [12] proposed a finite element based

modal strain energy method to analyze laminated composite beams with embedded multiple damping layers. Three dimensional solid elements were used to model the viscoelastic material (VEM) and shell elements to discretize the elastic layers in their study. Akoussan et al. [13] elaborated a model for multilayered laminated viscoelastic sandwich plates and proposed a new solution procedure to perform parametric studies of damping characteristics of these structures. Zhang et al. [14] developed a detailed 3D finite element model for predicting the modal loss factor of laminated composite beams with integral viscoelastic layers. The frequency dependence of viscoelastic damping materials was considered. It is concluded that to accurately represent the strain energy induced by the shear of the viscoelastic layer, a full 3-D solid finite element model or a mixed solid/shell composite element model is generally necessary. Although those full 3D or mixed solid/shell finite element models can effectively describe the shear deformation inside the viscoelastic layer, a cumbersome and time consuming mesh generation process are commonly required. Moreover, the developed FE model is not well suited for the optimal design procedure due to the need to completely regenerate three dimensional FE mesh when changing the geometric parameters of damping treatment.

In recent years, Reddy's layerwise theory [15], which is based on a piece-wise description of displacement field, has been applied to develop finite element beam/plate/shell formulations for analyzing viscoelastic laminated structures. Zapfe et al. [16] presented a discrete layer finite element for the dynamics analysis of viscoelastic laminated beams. A discrete layer annular finite element was proposed by Wang et al. [17] for vibration and damping analysis of a three-layered composite annular plate with a viscoelastic mid-layer. Moreira et al. [18] developed a generalized layerwise

plate finite element for multi-layer viscoelastic damped structures. A full layerwise shell finite element considering frequency dependent material properties was proposed by Oh et al. [19] for cylindrical laminates with viscoelastic layer. Araujo et al. [20] presented a finite element model for composite laminates with frequency dependent viscoelastic core using mixed layerwise approach, by considering a high order shear deformation theory to represent the displacement field of viscoelastic layer and a first order shear deformation theory for the displacement files of adjacent laminated face layers. Ferreira et al. [21] presented a layerwise finite element model based on Carrera's Unified Formulation for the analysis of composite laminated plates with a viscoelastic core.

It can be found that a significant simplification in the spatial finite element modelling of multilayer structures can be achieved by using a layerwise finite element model since it maintains a 2-D type data structure. In addition, the element stiffness matrix can be computed much faster for layerwise element by performing in-plane and out-of-plane integration separately [22]. In this paper, to simplify the modelling and optimization process, a quadrilateral plate finite element based on the layerwise theory is developed for modelling hybrid composite laminates with interleaving viscoelastic layers.

Design optimization is an important procedure to ensure achieving desired structural mechanical and damping capacities for hybrid viscoelastic composite laminates. Some researchers formulate this design problem as a single-objective optimization problem. Araújo et al. [20, 23] conducted a single objective constrained optimization for the maximization of modal loss factor of viscoelastic laminated composite structures, using a gradient based algorithm and mixed layerwise finite element

model. Montemurro et al. [24] presented a genetic algorithm based optimization technique for the design of damping properties of hybrid elastomer/composite laminates. The only optimization objective was to maximize the first N modal loss factors. Le Maout et al. [25] maximized damping loss factors of a hybrid elastomer/composite sandwich plate taking the total number of layers, their respective thicknesses, their fiber orientations, and the position of the visco-elastic core, and the stacking sequence as design variables. A layerwise optimization method was proposed by Li et al. [26] to design the damping capacity of laminated rectangular plates under general edge conditions using the analytical optimization technique. Montemurro et al. [27] proposed a two-level procedure for the global optimization of the damping behavior of composite laminated plates with elastomer patches.

The presence of soft viscoelastic layers in composite laminate increases the structural damping capacity, while it changes the overall mechanical properties of the composite structure and increases the weight [28]. In real engineering practice, these conflicting design performances need to be considered simultaneously. Therefore, it is natural to formulate a multi-objective optimization problem instead because multi-objective optimization can balance all aspects of design performance and output a set of flexible solutions [29]. Recently, the multi-objective optimization of viscoelastic laminated structures gradually attracts some attention. Multi-objective optimization of viscoelastic laminated composite structures based on genetic algorithm and ABAQUS finite element model was presented by Araújo et al. [23]. Madeira et al. [30, 31] presented a multi-objective method for optimization of viscoelastic laminated sandwich structures for minimizing weight and material cost and maximizing the modal loss factors. The new developed derivative-free algorithm, called direct multi-search (DMS) [50] are used as the optimization tool in their work. Xu et al. [32] proposed a

multi-objective optimization model for constrained layer damping structures, and only frequency-independent material properties were considered. Hamdaoui et al. [33] presented a multi-objective optimization approach to choose the most appropriate frequency dependent viscoelastic material for a three-layered rectangular composite beam for low mass and high damping.

In this paper, we present an optimal design method for hybrid composite laminates with interleaved viscoelastic damping layers under general edge conditions using a mixed integer multi-objective optimization method. A quadrangular plate finite element based on the layer-wise theory is developed to model multi-layered hybrid composite laminates under general edge conditions. Frequency-dependent material properties are considered. An evolutionary multi-objective algorithm handling discrete design variables is implemented to find a global optimum. The aim is to obtain designs that give simultaneously low mass, high natural frequency and high modal loss factors. In Section 2, the layer-wise plate finite element model is presented and validated. The optimal design formulation and implementation of the evolutionary multi-objective algorithm are presented in Section 3. In Section 4, the optimization cases are presented. The results are given and discussed in Section 5.

2. LAYERWISE FINITE ELEMENT MODELLING

2.1 Layerwise plate finite element

A quadrilateral layerwise plate element is developed and applied to model viscoelastic hybrid multilayered composite laminates. The kinematics is described by the layerwise theory based on the first-order shear deformation assumption in each layer, which was also used in Ref. [34]. Fig.1 represents a section of an n -layer hybrid composite laminate. According to the layerwise theory, each layer is modelled as a thick plate fulfilled to the first-order shear deformation theory. The middle surface of the bottom layer is chosen as the reference plane and its displacement components are the reference displacement components. The displacement of the other points on the plate is defined with reference to the displacement of the reference plane.

FIG. 1 Kinematic model of the layer-wise theory

Since the first-order shear deformation is assumed in each layer, the in-plane displacement components show a zig-zag behavior through plate thickness. The displacement of each point in the upper layer can be obtained by summing up the displacement of the corresponding point on the

reference plane and the displacement caused by zig-zag effect, and applying displacement continuity conditions at layer interfaces, defined by

$$\{\mathbf{u}\}_k = \begin{Bmatrix} u_k \\ v_k \\ w_k \end{Bmatrix} = \begin{Bmatrix} u_0 + \frac{h_1}{2} \beta_1^x + \sum_{j=2}^{k-1} h_j \beta_j^x + \frac{h_k}{2} \beta_k^x + z_k \beta_k^x \\ v_0 + \frac{h_1}{2} \beta_1^y + \sum_{j=2}^{k-1} h_j \beta_j^y + \frac{h_k}{2} \beta_k^y + z_k \beta_k^y \\ w_0 \end{Bmatrix} \quad (1)$$

where u_0 , v_0 , and w_0 are displacement components of a point at the reference plane, β^x and β^y are the rotations of the normal to the layer mid-plane about the y - and x - axes, and h_k is the k -th layer thicknesses, $z_k \in \left[-\frac{h_k}{2}, \frac{h_k}{2}\right]$, respectively. Based on Eq.1, a generalized displacement field can be defined as

$$\{d\} = \{u_0, v_0, w_0, \beta_1^x, \beta_1^y, \dots, \beta_k^x, \beta_k^y, \dots, \beta_n^x, \beta_n^y\}^T \quad (2)$$

Eq.1 can be written into a compact form

$$\{\mathbf{u}\}_k = [T]_k \{d\} \quad (3)$$

with

$$[T]_k = \begin{bmatrix} 1 & 0 & 0 & \frac{h_1}{2} & 0 & \dots & h_j & 0 & \dots & \frac{h_k}{2} + z_k & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \frac{h_1}{2} & \dots & 0 & h_j & \dots & 0 & \frac{h_k}{2} + z_k & 0 & \dots \\ 0 & 0 & 1 & 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 & 0 & \dots \end{bmatrix} \quad (4)$$

The strain field of the k -th layer is given by

$$\{\boldsymbol{\varepsilon}\}_k = [L] \{\mathbf{u}\}_k \quad (5)$$

and the differential operator matrix $[L]$ is

$$[L] = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \end{bmatrix} \quad (6)$$

The linear elastic constitute relationship is considered for the k -th layer, defined by

$$\{\boldsymbol{\sigma}\}_k = [D]_k \{\boldsymbol{\varepsilon}\}_k \quad (7)$$

The stress-strain relation for each layer made of frequency-dependent orthotropic or isotropic material can be written in the form of Eq. (8):

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11}^*(i\omega) & Q_{12}^*(i\omega) & 0 & 0 & 0 \\ Q_{12}^*(i\omega) & Q_{22}^*(i\omega) & 0 & 0 & 0 \\ 0 & 0 & Q_{44}^*(i\omega) & 0 & 0 \\ 0 & 0 & 0 & Q_{55}^*(i\omega) & 0 \\ 0 & 0 & 0 & 0 & Q_{66}^*(i\omega) \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{23} \\ \varepsilon_{13} \\ \varepsilon_{12} \end{Bmatrix} \quad (8)$$

It should be noted that Eq. (8) is written with respect to the principal direction of the layer and the zero transverse normal stress situation is considered. In Eq. (8), $i = \sqrt{-1}$, σ_{ij} are stress components, ε_{ij} are strain components, and Q_{ij}^* are frequency-dependent complex elastic coefficients related to the engineering properties as the follows:

$$\begin{aligned} Q_{11}^*(i\omega) &= \frac{E_1^*(i\omega)}{1-\nu_{12}\nu_{21}}, \quad Q_{22}^*(j\omega) = \frac{E_2^*(i\omega)}{1-\nu_{12}\nu_{21}}, \\ Q_{12}^*(j\omega) &= \frac{\nu_{12} \times E_2^*(i\omega)}{1-\nu_{12}\nu_{21}}, \quad Q_{44}^*(i\omega) = G_{23}^*(i\omega), \\ Q_{55}^*(i\omega) &= G_{31}^*(i\omega), \quad Q_{66}^*(i\omega) = G_{12}^*(i\omega) \end{aligned} \quad (9)$$

The frequency dependent complex elastic moduli E_1^* , E_2^* and the complex shear moduli G_{23}^* , G_{13}^* and G_{12}^* can be obtained experimentally for a given range of frequencies.

The kinetic energy of the hybrid multi-layer plate structure can be obtained by the sum of the kinetic energy of individual single layer, i.e.,

$$\Pi^K = \sum_{k=1}^n \frac{1}{2} \rho_k \int_{\Omega_k} \{\dot{\mathbf{u}}\}_k^T \{\dot{\mathbf{u}}\}_k d\Omega_k \quad (10)$$

where $\{\dot{\mathbf{u}}\}_k$ denotes the velocity field of the k -th layer and ρ_k is the mass density. The potential energy of the structure can be calculated by the sum of the potential energy of each individual layer, i.e.,

$$\Pi^P = \sum_{k=1}^n \frac{1}{2} \int_{\Omega_k} \{\boldsymbol{\sigma}\}_k^T \{\boldsymbol{\varepsilon}\}_k d\Omega_k \quad (11)$$

As shown in Fig. 2, a four-node isoparametric quadrangular finite element is proposed to discretize the structure. The shape functions of this element in parametric space are

$$N_i = \frac{1}{4}(1 + \xi_i \xi)(1 + \eta_i \eta) \quad i = 1, 2, 3, 4 \quad (12)$$

where ξ and η are local parametric coordinates.

FIG.2 A four-node quadrangular plate element

By applying Hamilton's principle, the finite element equations of motion for the hybrid multilayer plate can be deduced as

$$[M]\{\ddot{U}\} + [K]\{U\} = \{F\} \quad (13)$$

where $[M]$ and $[K]$ are global mass and stiffness matrices, respectively; $\{U\}$ denotes the vector of global degrees of freedom, and $\{F\}$ is the externally applied mechanical load vector. It is noted that the viscoelastic behavior of the material layer translates into a complex frequency dependent stiffness matrix $[K(i\omega)]$. Assuming harmonic vibrations, and for the free vibration problem, disregarding the excitation terms, Eq. (13) can be reduced to the following complex eigenvalue problem:

$$([K(i\omega)] - \lambda^*[M])\{\phi^*\} = \{0\} \quad (14)$$

where $\{\phi^*\}$ is the complex eigenvector and λ^* is the complex eigenvalue.

Note that since a frequency-dependent viscoelastic model is considered in this study, the resulting eigenvalue problem (Eq.14) is nonlinear and hard to be treated numerically. Efficient numerical procedures are required to consider that the global stiffness matrix $[K]$ depends nonlinearly on frequency. In the present study, the complex eigenvalue problem is iteratively solved using the well-known Implicitly Restarted Arnoldi Method (IRAM) [35] implemented in *eigs* function of Matlab with shift-invert transformation. The calculated complex eigenvalue can be written as

$$\lambda^* = \lambda' + i\lambda'' \quad (15)$$

Thus, the circular frequency ω and the modal loss factor η of the hybrid laminate can be calculated by the following formulas:

$$\omega = \sqrt{\lambda'} \quad \text{and} \quad \eta = \frac{\lambda''}{\lambda'} \quad (16)$$

2.2 Model validation

In this section, to validate the proposed finite element and complex eigenvalue solution procedure, numerical tests with constant and frequency dependent viscoelastic models analyzed in Ref. [36] are calculated. The results by the present analysis are compared to the results given in [36].

The first example is an aluminum/polymer/aluminum beam-plate. The polymer core material is represented by a constant complex modulus. The material and structural data of the sandwich beam-plate are listed in Table 1. The core's loss factor is varied as $\eta_c \in \{0.1, 0.6, 1.0, 1.5\}$. The present layerwise finite element model is used to investigate its vibration characteristics. The first four vibration modes of the sandwich beam-plate under SFSF and CFFF edge boundary condition (S, simply supported; C, clamped; F, free) for different core material loss factors are given and

compared to the results obtained in Ref. [36] in Table 2. Note that for comparison, the modal loss factor η_l is normalized to the input viscoelastic material loss factor η_c .

TABLE 1 Material and structural data of the sandwich beam-plate [36]

Elastic faces	Young modulus	$E_f=69000\text{MPa}$
	Poisson ratio	$\nu_f=0.3$
	Mass density	$\rho_f=2766\text{kgm}^{-3}$
	Thickness	$H=1.524\text{mm}$
Viscoelastic polymer core	Young modulus	$E_c=1.794\text{MPa}$
	Poisson ratio	$\nu_c=0.3$
	Mass density	$\rho_c=968.1\text{kgm}^{-3}$
	Thickness	$h=0.127\text{mm}$
Sandwich beam-plate	Length	$L=177.8\text{mm}$
	Width	$l=12.7\text{mm}$

TABLE 2 Modal frequencies and associated normalized loss factors corresponding to the first four vibration modes of the sandwich beam-plate with constant complex core's modulus and various core's material loss factors

η_c	Present method		Ref. [36]		Present method		Ref. [36]	
	SFSF beam-plate		SFSF beam-plate		CFFF beam-plate		CFFF beam-plate	
	$f_l(\text{Hz})$	η_l/η_c	$f_l(\text{Hz})$	η_l/η_c	$f_l(\text{Hz})$	η_l/η_c	$f_l(\text{Hz})$	η_l/η_c
0.1	148.49	0.3501	148.56	0.3500	64.3	0.281	64.3	0.281
	488.37	0.1954	489.28	0.1954	297.5	0.242	298.1	0.242
	1034.39	0.1066	1038.74	0.1066	745.7	0.154	749.0	0.153
	1794.26	0.0647	1807.57	0.0647	1396.3	0.088	1406.9	0.088
0.6	150.69	0.3327	150.75	0.3327	65.7	0.246	65.7	0.245
	489.64	0.1940	490.56	0.1940	300.0	0.232	300.6	0.232
	1035.07	0.1064	1039.43	0.1064	747.5	0.152	750.7	0.152
	1794.67	0.0646	1807.99	0.0646	1397.3	0.088	1407.8	0.088
1.0	154.40	0.3051	154.47	0.3051	67.7	0.202	67.7	0.202
	491.95	0.1915	492.87	0.1914	303.9	0.217	304.5	0.217
	1036.32	0.1060	1040.69	0.1060	750.6	0.150	753.9	0.149
	1795.42	0.0645	1808.75	0.0646	1399.0	0.087	1409.5	0.087
1.5	160.71	0.2625	160.77	0.2625	70.1	0.153	70.2	0.153
	496.37	0.1867	497.3	0.1867	310.0	0.197	310.6	0.197
	1038.75	0.1054	1043.13	0.1054	756.4	0.145	759.7	0.145
	1796.89	0.0644	1810.23	0.0644	1402.0	0.087	1412.6	0.086

The second example is a sandwich plate with a core made of 3M ISD 112, whose shear modulus is

frequency dependent [36]. The material and structural data of the sandwich plate are listed in Table 3.

The frequency-dependent model and properties of 3M ISD 112 at 27°C can be found in appendix.

TABLE 3 Material and structural data of the sandwich plate [36]

Elastic faces	Young modulus	$E_f=68900\text{MPa}$
	Poisson ratio	$\nu_f=0.3$
	Mass density	$\rho_f=2740\text{kgm}^{-3}$
	Thickness	$H=0.762\text{mm}$
Viscoelastic polymer core 3M ISD 112 at 27°C	Poisson ratio	see in Appendix
	Mass density	see in Appendix
	Thickness	$h=0.254\text{mm}$
	Shear modulus	see in Appendix
Sandwich plate	Length	$L=348\text{mm}$
	Width	$l=304.8\text{mm}$

TABLE 4 Modal frequencies and associated loss factors corresponding to the first four vibration modes the sandwich plate with 3M ISD112 frequency dependent core at 27°C

Boundary conditions	Present method		Ref. [36]	
	$f_l(\text{Hz})$	η_l	$f_l(\text{Hz})$	η_l
SSSS	53.75	0.218	53.77	0.213
	110.47	0.283	110.31	0.272
	126.74	0.295	126.72	0.283
	176.99	0.304	176.97	0.289
CCCC	83.08	0.254	83.01	0.246
	146.83	0.267	146.61	0.258
	168.86	0.267	168.92	0.257
	224.98	0.281	225.27	0.270
CSCS	66.77	0.238	66.71	0.231
	133.68	0.294	133.66	0.282
	137.40	0.265	137.11	0.256
	197.01	0.292	197.13	0.280
CFCF	46.74	0.223	46.62	0.218
	58.28	0.214	58.28	0.209
	111.75	0.248	112.25	0.238
	117.77	0.256	117.43	0.249

The first four vibration modes of the sandwich plate are calculated under SSSS, CCCC, CSCS, CFCF edge boundary conditions. Table 4 lists the first four vibration mode frequencies and modal

loss factors. For comparison, the results given in Ref. [36] are also listed.

It can be observed from Table 2 and Table 4 that the normal frequencies and modal loss factors calculated by the present finite element and solution procedure show good agreement compared to those of published results, providing validation of the proposed layerwise finite element analysis method.

3. OPTIMAL DESIGN METHOD

3.1 Optimization model

The main objective of this study is to find the optimal design of a multilayered hybrid composite laminate to ensure achieving high structural damping and satisfactory mechanical performance simultaneously. This kind of design problems are often complex and difficult to solve without considering several conflicting design objectives. For viscoelastic damped composite structures, the maximization of structural modal loss factor is usually considered as a main design objective. Meanwhile, many engineering applications require the control of the added weight to the structures, thus the minimization of structural weight is also considered. In addition, for the hybrid composite laminate with interleaved VEM layers, the minimal decrease of structural stiffness due to inserting soft VEM layers is necessary. Therefore, the objective functions used in this study are stated as follows:

- (1) Minimizing the opposite of the sum of modal loss factors of the first N modes. Thus, the first objective function is

$$\phi_1 = -\sum_{i=1}^N \eta_i \quad (17)$$

- (2) Minimizing the total structural mass m of the viscoelastic damping structure. Thus, the second objective function is

$$\phi_2 = m \quad (18)$$

- (3) The minimal decrease of stiffness can be expressed as the minimizing the opposite of the sum of modal frequencies of the first N modes, defined as

$$\phi_3 = -\sum_{i=1}^N f_i \quad (19)$$

It is worth noting that minimizing ϕ_1 or ϕ_3 is equivalent to maximizing $-\phi_1$ or $-\phi_3$.

The objective functions ϕ_1 , ϕ_2 and ϕ_3 , can be obtained by layerwise finite element analysis and solving the eigenvalue problems of Eq. (14) iteratively, according to a defined frequency dependent complex stiffness matrix and a real mass matrix.

The proposed layerwise finite element analysis is called in the optimization loop. Design variables related to ply material type, stacking sequence, and ply geometry can be considered simultaneously in design optimization. In the practical design optimization of composite laminates, orthotropic ply thickness is often fixed and ply angles are limited to discrete set such as 0° , $\pm 45^\circ$ and 90° in order to obtain a cost-effective design. Therefore, the viscoelastic material type, the fiber orientation of the orthotropic elastic layer, the inserting position of the VEM layer, and the thickness of VEM layer are taken as design variables in this study. It is noted the variables representing material type, fiber orientation, and VEM layer inserting position are all discrete variables. Since the vibration energy is dissipated through the shear strains induced in the soft VEM layer, the structural damping capacity is sensitive to the variation of the VEM layer thickness. Thus, the variable of the viscoelastic layer thickness is considered as a continuous variable, which can change continuously in a design interval. Hence, the optimization problem is a mixed optimization problem involving naturally continuous and discrete design variables.

In this optimization problem, since the main structural performance indexes are considered as design objectives, constraints are imposed only on some chosen design variables. For the VEM layer thickness variable, constraints are imposed by limiting the upper and lower bound of the variable. For the variable of the inserting position of the viscoelastic layer, the variable must be limited in the

physical feasible range of the structural geometric boundary.

3.2 Optimization algorithm

The formulated multi-objective optimization problem defined above can be written in a more general form as follows,

$$\begin{aligned}
& \text{minimize } \boldsymbol{\phi}(\mathbf{X}) = [\phi_1(\mathbf{X}), \phi_2(\mathbf{X}), \dots, \phi_m(\mathbf{X})] \\
& \text{subject to } \mathbf{X} = [x_1, x_2, \dots, x_n] \\
& \quad x_i^L \leq x_i \leq x_i^U \quad (i = 0, 1, \dots, n)
\end{aligned} \tag{20}$$

where $\mathbf{X}$ is the vector of design variables, m is the number of objectives, x_i^L and x_i^U are the lower and upper limits of the design variables, respectively, n is the total number of design variables. It is worth noting that the design variables are either continuous or discrete. For a multi-objective optimization problem, a set of points rather than single optimum is constructed representing trade-offs between different objectives. The optimal solution of multi-objective optimization is commonly called Pareto front or Pareto surface.

In recent years, global optimization algorithms based on evolutionary computation, which employ principles of natural selection, have been developed to solve multi-objective optimization problems [37-41]. The non-storing genetic algorithm, NSGA-II, is more powerful than the other algorithms since elitism and a crowded comparison operator are used. In this study, a new modified multi-objective genetic algorithm, called reference point based NSGA-II, proposed by Deb et al. in 2006 [42] is used. The main algorithm flowchart of NSGA-II is given in Fig. 3. Suppose the population size is N . NSGA-II first generates randomly an initial parent population P_t of size N . Then, an

offspring population Q_t of size N is created from the parent population P_t through genetic operators (selection, crossover and mutation). A new population of size $2N$ is formed by combining P_t and Q_t . The NSGA-II discriminates the combined population by fast non-dominated sorting [39], resulting in a set of non-dominated fronts $F_1, F_2, \dots, F_R$. Members of the next parent generation P_{t+1} are selected from solutions in F_1 preferentially, and then from the second non-dominated front F_2 . The selection process goes on until the size of P_{t+1} is N . Therefore the new population is filled by points of different non-dominated front, one at a time. This method ensures that the optimal solutions, the first non-dominated front F_1 , can be partially kept to the next generation. In generation $t+1$, the offspring population is created from P_{t+1} and the above process iterates until the maximal number of generations is reached.

FIG.3 Flowchart of NSGA-II algorithm

A Matlab[®] code implementing this algorithm is developed. The code can use real and integer mixed coding. It has the ability of parallel computational evaluations of objective functions, which is very powerful and useful in improving calculation's efficiency. Thanks to parallel computation, the running time has been reduced by 75% according to the results of numerical examples.

3.3 Implementation of the optimization algorithm

In this paper, we consider the optimization of hybrid laminated composites with viscoelastic cores. The design variables considered include the fiber orientation of orthotropic elastic layer, the inserting position of VEM layer, the viscoelastic material type and the thickness of VEM layer. It is noted that both continuous and discrete design variables are included. The classical binary representation is not used here; instead, a mixed integer and real coding strategy is proposed to form the chromosome that represents a population individual.

The encoding of these variables is performed as follows. In this paper, the fiber orientation may take any value from a set of 10 angles from -45° and 90° in steps of 15° . Therefore, the integers from 0 to 9 are used to represent each of the above ply angles. For an N -ply laminate, to satisfy the mid-plane symmetry constraint, only $N/2$ ply orientations are required to describe the entire laminate. If the symmetrical laminate having odd number plies, the half of the center ply can be considered that it lies on either side of the plane of symmetry. Therefore, the stacking sequence of the laminate is represented by $N/2$ genes. Each gene is an integer between 0 and 9. For example, the laminate $[90_2/45/-45/90_2]_s$ is encode as 9 9 6 0 9 9. The rightmost 9 corresponds to the layer closest to the laminate plane of symmetry.

The viscoelastic material layer can only be inserted between adjacent orthotropic elastic layers. As shown in Fig.4, for an N -ply composite laminate, due to the mid-plane symmetry constraint, there are only $N/2$ physical possible positions for inserting viscoelastic material layers. The integers from 1 to $N/2$ are used to each of these possible positions. The viscoelastic material type is also a discrete variable and coded by integer from 1 to n , where n is the number of types of material. The thickness of viscoelastic material layer is a continuous variable and represented by real values between its lower and upper bound.

FIG.4 Physical possible positions for inserting viscoelastic material layers

An individual in the population is formed by concatenating the fiber orientation substring with the inserting position substring, the viscoelastic material type substring and the viscoelastic layer thickness substring. This is identified as complete design string. As show in Fig. 5, for an instance, the number of orthotropic elastic plies is 12. Due to symmetry constraint, a substring consisted of six genes ($L_1 \sim L_6$) are used to represent the stacking sequence. A gene is used to represent candidate inserting position of VEM layer. An integer between 1 and 6 is used to represent each physical possible position. A gene is used to represent candidate material type. Six types of viscoelastic

material are considered and an integer between 1 and 6 is used to code it. A real value is used to represent the thickness of embedded VEM layers. The upper bound is 0.500 and the lower bound is 0.100.

Example for $N=12, n=6$

FIG.5 An example of Chromosome representation

The NSGA-II starts with the random generation of a population of design individuals. Then it is processed by means of genetic operators as described in section 3.2. The process is repeated until a stopping criterion is satisfied. The input parameters of the NSGA-II algorithm used in this paper are listed in Table 5.

TABLE 5 Input parameters of NSGA-II

Population size	Specified by user (default: 100)
Maximal number of generation	Specified by user
Selection type	Binary tournament selection
Crossover type	Arithmetic
Crossover fraction	0.8
Mutation type	Gaussian
Mutation fraction	0.3

4. OPTIMIZATION CASES

4.1 Case 1

The first case considered here is a symmetrically hybrid composite laminated plate with two interleaved viscoelastic layers. The in-plane dimension of the simply supported sandwich plate is $300\text{mm} \times 200\text{mm}$. The nominal thickness of each orthotropic ply is 0.25mm and the total number of orthotropic plies is fixed to 12. Thus, the total thickness of the plate without embedded viscoelastic damping layers is 3.00mm . Two viscoelastic layers are inserted to improve structural damping. The proposed layer-wise finite element model is applied to analyze the laminated composite structure. The sandwich plate is discretized by 16×10 layer-wise elements. The orthotropic layers are assumed to be graphite/epoxy (GE), and material properties of the unidirectional layer are listed in Table 6[43]. The isotropic viscoelastic material layers are frequency-dependent with an elastic modulus and loss factor referenced from experimental results [44] and characterized by Eq. (21) and (22). In addition, Poisson ratio $\nu=0.49$ and material density $\rho=750 \text{ kg/m}^3$ are used for the viscoelastic material.

$$\log E_v = 0.106 \log f + 1.52, E_v(\text{MPa}) \quad (21)$$

$$\eta_v = 39.4 - 5.56 \log f, \eta_v(\%) \quad (22)$$

TABLE 6 Material properties of graphite/epoxy unidirectional layer

E_1	119GPa	E_2	8.67GPa
μ_{12}	0.31	G_{12}	5.18GPa
G_{13}	5.18GPa	G_{23}	3.9GPa
ρ	1570Kg/m ³	η_{11}	0.118%
η_{22}	0.620%	η_{12}	0.812%
η_{13}	0.812%	η_{23}	0.846%

4.2 Case 2

The second case is a three-layered rectangular sandwich beam-plate with a single viscoelastic layer inserted in the middle plane. The in-plane geometry size of the beam is shown in Fig. 6, and the nominal total thickness is 3.175mm, including the elastic layers and viscoelastic layer. The sandwich beam is modeled using 28×2 layerwise elements and the boundary condition is of the type clamped-free. The material properties of the isotropic elastic layers are: $E=69\text{GPa}$, $\nu=0.3$, $\rho=2766 \text{ kg/m}^3$. For the viscoelastic layer in this example, six types of frequency and temperature dependent materials with different damping levels are considered. The detailed material model and properties are listed in Appendix. It is noted that the curve fitting parameters for experimental results of viscoelastic materials are all obtained at room temperature, ranging from 20°C to 30°C . Since the temperature changes are fairly tiny, they are ignored in this study.

FIG. 6 Geometry of a multi-layer viscoelastic cantilever beam plate

5. RESULTS AND DISCUSSIONS

5.1 Case 1

In the first optimal use case, the design objectives are chosen to maximize the first modal frequency and modal loss factor and to minimize the added mass of the structure simultaneously. Since the ply thickness of the fiber reinforced composite is fixed to 0.25mm, only the fiber angles of the orthotropic layers, the thickness of the embedded viscoelastic layer and the inserting positions of viscoelastic layers are taken as design variables. The thickness of the embedded viscoelastic layer, t_{vem} , varying from 0.1mm to 0.5mm, is encoded by real value. The inserting positions and the fiber angles are both discrete variables and are encoded by decimal integers. Due to symmetry, the integers 1 to 6 are used to encode the six candidate positions for inserting, and only the first six (from bottom to top) layers' ply angles are taken as design variables. It is assumed that the ply angle can vary from -45° to 90° with increment of 15° , and the integers from 0 to 9 are used to encode this variable. The population size in this case is specified as 300, while maximal number of generations is specified as 100.

The final three-dimensional Pareto front solutions after 100 generations are displayed in Fig. 7(a). In addition, three different projections of the resulted Pareto non-dominated solutions which are made in ϕ_1 - ϕ_3 , ϕ_1 - ϕ_2 , and ϕ_3 - ϕ_2 planes, are depicted in Fig. 7(b), (c) and (d), respectively. In these figures, some specific solutions are labeled and the resulting design configurations are listed in Table 7. In Fig 7(a), the set of the non-dominated solutions form the Pareto-optimal front in three dimensions is depicted. Solution A, B and C are three extreme solutions. Solution A is the best solution for damping ($\eta = 10.5\%$), but the worst for weight (with $m=45.00g$). Solution B is the best

solution for dynamic stiffness (with $f_1=331.27\text{Hz}$), but the worst for weight too. Solution C is the best solution for weight (the lightest with $m=9.00\text{g}$), but is the worst solution for damping ($\eta = 1.39\%$).

FIG.7 Pareto front solutions for multi-objective optimization of a laminated sandwich plate: (a) presenting the three-dimensional view; (b), (c) and (d) presenting three plane views.

The Pareto-optimal curves in Fig. 7(b)-(d) enable us to perform trade-off studies. For example, in Fig 7(b), solution C and D have identical minimum mass, and the first modal frequency is 14.85% larger

than that of D. However, the first modal loss factor of solution C is 35.82% of that of solution D. Solution A and solution G are of same stacking sequence except for the thicknesses of VEM layers, but solution A gives both higher loss factor and higher frequency than solution G. Solution C, D and K all achieve minimum added mass, but the damping and dynamic stiffness capacities are very different due to the different stacking sequence and the inserting positions of the viscoelastic damping layers. In addition, solution E, F, H, I and J correspond to intermediate configurations.

It is noted that the inserting positions have great influence on the dynamic properties of the hybrid structure as expected. For example, if the VEM layers are embedded near the middle plane of the laminated plate, the damped structure obtains higher loss factor. This trend can be verified by simply comparing solution A and solution B, or comparing solution I and solution J.

TABLE 7 Case 1: Pareto-optimal design configurations labeled in Fig. 7

Optimal solutions	t_{vem}	Stacking sequence	$\eta_1(\%)$	$m(g)$	$f_1(Hz)$
A	0.500	[90 ₅ /45/VEM/VEM/45/90 ₅]	10.50	45.00	324.80
B	0.500	[90 ₂ /VEM/90/75/90/45/45/90/75/90/VEM/90 ₂]	5.18	45.00	331.27
C	0.100	[90 ₂ /VEM/90 ₂ /60 ₄ /90 ₂ /VEM/90 ₂]	1.39	9.00	316.49
D	0.100	[-30/-45/60/45 ₂ /VEM/15 ₂ /VEM/45 ₂ /60/-45/-30]	3.88	9.00	275.56
E	0.479	[90 ₃ /VEM/90 ₂ /45 ₂ /90 ₂ /VEM/90 ₃]	7.54	43.14	329.63
F	0.494	[90 ₃ /75/VEM/75/45 ₂ /75/VEM/75/90 ₃]	9.44	44.49	327.25
G	0.358	[90 ₅ /45/VEM/VEM/45/90 ₅]	8.70	32.26	320.65
H	0.194	[-45 ₂ /45/60/75/VEM/45 ₂ /VEM/75/60/45/-45 ₂]	5.77	17.48	290.40
I	0.316	[75/90 ₃ /75/45/VEM/VEM/45/75/90 ₃ /75]	7.93	28.44	318.52
J	0.372	[90 ₃ /VEM/90/75/30 ₂ /75/90/VEM/90 ₃]	6.34	33.46	325.96
K	0.100	[-45/60 ₂ /75/90/VEM/15 ₂ /VEM/90/75/60 ₂ /-45]	3.73	9.00	304.39

Therefore, from this application it is clear that using a multi-objective optimization method to design frequency dependent composite laminated structure with interleaved viscoelastic layers allows for a great flexibility in the design process. It can handle multiple design objectives simultaneously and

the resulted non-dominated solutions can be analyzed from several perspectives to give required trade-off designs. The approach presented also provides a novel design tool for the integral dynamic design of those composite structures with hybrid frequency-dependent material layers.

5.2 Case 2

The design objectives in the second case are to maximize the first modal frequency and modal loss factor, and to minimize the total mass of the sandwich structure simultaneously. For simplicity, the two isotropic face layers are of same thickness, and the total thickness of the sandwich beam, t_{nom} , is fixed to 3.175mm. The thickness of the embedded viscoelastic layer, t_{vem} , varying from $0.1t_{nom}$ to $0.8t_{nom}$, is encoded by a real value. The material type of the viscoelastic core is also taken as design variable. The integers from 1 to 6, corresponding to the six kinds of viscoelastic materials in Appendix, are used to encode this variable. In this case, the population size is specified as 100, and maximal number of generations is specified as 100. The final Pareto front solutions after 100 generations are shown in Fig. 8(a) in three dimensions. Three projected views of Pareto front which are made in ϕ_1 - ϕ_2 , ϕ_1 - ϕ_3 , and ϕ_2 - ϕ_3 planes, are also shown in Fig. 8(b)-(d). Several Pareto-optimal solutions are presented in Table 8.

In this application, the aim is to choose the most appropriate viscoelastic material type and best geometric parameters. The results validate that the proposed method is a good tool to perform this task. It is found that some viscoelastic materials, including 3M ISD112 and EAR C-1002, are rejected during the process of optimization. This is because they provide low stiffness, bring more weight or poor damping ability compared to the other materials in this application. By analyzing the

Pareto-optimal solutions, we can find that the damping levels associated to ZN-1 are better than those associated to PVB, DM 9740 and GE.SMRD materials. However, the solutions associated with ZN-1 material give much lower dynamic stiffness. PVB is actually a kind of weak damping material, it can provide higher structural dynamic stiffness, but the structural damping is dozens of times lower than that of ZN-1. The solutions associated with DM 9740 and GE.SMRD can be regarded as compromises compared to PVB and ZN-1, because both of them provide moderate damping capacity and stiffness. Thus, by the presented method, the most appropriate frequency dependent viscoelastic damping material can be determined.

TABLE 8 Case 2: Pareto-optimal design configurations labeled in Fig. 8

Optimal solutions	Viscoelastic material	t_{vem}	$\eta_1(\%)$	$m(g)$	$f_1(Hz)$
A	ZN-1	2.540	64.74	9.53	34.41
B	ZN-1	2.540	64.74	9.53	34.41
C	PVB	1.661	1.03	13.20	89.70
D	PVB	2.236	0.95	10.91	86.34
E	PVB	2.540	0.82	9.70	79.78
F	GE.SMRD	1.108	17.84	15.66	81.29
G	GE.SMRD	1.825	21.56	12.97	80.65
H	GE.SMRD	2.540	17.02	10.28	72.65
I	DM 9740	1.244	7.12	15.71	85.01
J	DM 9740	1.783	7.92	13.93	84.16
K	DM 9740	2.190	7.44	12.58	80.32
L	ZN-1	1.386	48.80	14.21	42.33
M	ZN-1	0.318	28.49	18.54	62.63

In addition, for the sandwich configuration interleaved with ZN-1, the larger is the viscoelastic material layer thickness, the larger the structural damping. The relationship between them is monotonic. While for the sandwich configuration inserted with DM 9740 or GE.SMRD, the damping and frequency shows a non-monotonic variation with the thickness variation of the viscoelastic layer. After the inserting material type is chosen, the Pareto-optimal material thickness can easily found

according to the compromise of the three design objectives. Therefore, simultaneously determining the optimal material type and geometric parameters is achieved.

FIG.8 Pareto-optimal solutions for multi-objective optimization of a viscoelastic sandwich beam: (a) presenting the three-dimensional view; (b), (c) and (d) presenting three plane views.

6. CONCLUSIONS

In this paper, a multi-objective approach to the optimal design of composite structures with frequency-dependent interleaved viscoelastic layers for simultaneous high damping, low added mass and high dynamic stiffness has been presented. Layerwise plate finite element analysis is proposed and applied in the optimization loop. The design problem is formulated as a mixed integer multi-objective optimization problem. An efficient evolutionary algorithm, NSGA-II, is introduced to solve this optimization problem. Parallel computation is applied to improve the computational efficiency.

Non-dominated solutions have been obtained for a general composite plate with two interleaved viscoelastic damping layers and a sandwich beam with one inserting viscoelastic damping layer. In the first application, the Pareto-optimal design solutions with the laminate stacking sequence, damping inserting position and thickness as design variables are determined. In the second application, the most appropriate frequency dependent viscoelastic damping material type and its geometric parameter are determined simultaneously. Numerical applications validated the effectiveness of the proposed method and the obtained results can be very useful in the design such kind of composite structures.

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APPENDIX

Models and frequency-dependent properties of viscoelastic materials at reference temperature used in this study.

	Material	Complex modulus $G^*(\omega) =$	Fitting parameters	Reference Temperature	Other properties
1	PVB[33,45]	$G_\infty + (G_0 - G_\infty)(1 + (i\omega\tau)^{1-a})^{-b}$	$G_0=0.479\text{MPa}$, $G_\infty=235\text{MPa}$, $\tau=0.3979\text{s}$, $a=0.46$, $b=0.1946$	20°C	$\nu=0.49$ $\rho=999\text{ Kg/m}^3$
2	ZN-1[46]	$G_0(1 + \sum_{k=1}^{k=3} \hat{\alpha}_k \frac{(i\omega)^2 + 2\hat{\zeta}_k \hat{\omega}_k i\omega}{(i\omega)^2 + 2\hat{\zeta}_k \hat{\omega}_k i\omega + \hat{\omega}_k^2})$	$G_0=0.5542\text{MPa}$, $\hat{\alpha}_1=3.960$, $\hat{\zeta}_1=148.0$, $\hat{\omega}_1=896200\text{ rad/s}$ $\hat{\alpha}_2=65.69$, $\hat{\zeta}_2=12.16$, $\hat{\omega}_2=927800\text{ rad/s}$ $\hat{\alpha}_3=1.447$, $\hat{\zeta}_3=810.4$, $\hat{\omega}_3=716300\text{ rad/s}$	30°C	$\nu=0.49$ $\rho=970\text{ Kg/m}^3$
3	3M ISD112[47]	$G_0 \left(1 + \sum_{k=1}^{k=3} \frac{\Delta_k \omega}{\omega - i\Omega_k}\right)$	$G_0=0.5\text{MPa}$ $\Delta_1=0.746$, $\Omega_1=468.7\text{ rad/s}$ $\Delta_2=3.265$, $\Omega_2=4742.4\text{ rad/s}$ $\Delta_3=43.284$, $\Omega_3=71532.5\text{ rad/s}$	27°C	$\nu=0.5$ $\rho=1600\text{ Kg/m}^3$
4	DM 9740[48]		$G_0=1.4\text{MPa}$ $\Delta_1=7.262$, $\Omega_1=406.611\text{ rad/s}$ $\Delta_2=288181.234$, $\Omega_2=22490029.8\text{ rad/s}$ $\Delta_3=2.039$, $\Omega_3=46.750\text{ rad/s}$	Room Temperature	$\nu=0.49$ $\rho=1101\text{ Kg/m}^3$
5 ^a	GE.SMRD[49]	$G_0 + G_0(d - 1) \frac{(i\omega\tau)^\alpha}{(1+i\omega\tau)^\beta}$	$G_0=5.0\text{MPa}$, $d=36$, $\alpha=0.605$, $\beta=0.554$, $\tau=2.09 \times 10^{-4}\text{s}$	29°C	$\nu=0.49$ $\rho=1300\text{ Kg/m}^3$
6	EAR C-1002 [30,49]		$G_0=0.8\text{MPa}$, $d=1570$, $\alpha=0.566$, $\beta=0.558$, $\tau=7.23 \times 10^{-10}\text{s}$	25°C	$\nu=0.49$ $\rho=1300\text{ Kg/m}^3$

^a The density of GE.SMRD is estimated.

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