Two dimensional modeling of helical structures, an application to simple strands

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Abstract

Herein, we elaborate a generic, planar finite element model for the simulation of the mechanical response of helical structures to axial and torsional strain. We verify the modeling approach over a wide range of helix angles using closed-form expressions. Thereafter, we present a case study application addressing the structural response of single layer engineering strands. Finally, we analyze the model's computational merits.

Keywords: Helix, Finite element method, Axial strain, Torsional strain

1. Introduction

Helical structures are encountered in nature (e.g. collagen) as well as in numerous engineering applications with ropes, lifting or power line cables and tissue engineering scaffolds being indicative examples.

The mechanical response of helical constructions has been primarily characterized by means of closed-form analytical expressions, while it is only in the last few decades that the notable development of numerical methods has provided new engineering tools for their analysis and design. Nevertheless, the modeling techniques appearing in the literature have been mainly based on beam or volume elements. The latter quickly incur a high number of *degrees of freedom (dofs)*, which respectively entail a significant computational power cost.

Most of the analytical models appearing in the literature based their formulation on thin beam theory considerations. In particular, early works characterized the helix stiffness by taking into account the response contribution arising from the helix cross section axial stiffness EA , as the work of Hruska indicates [1, 2]. Subsequent works, emphasized on the role of the helix cross section torsional

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stiffness GJ , incorporating the corresponding contribution in the provided helix closed-form stiffness expressions [3, 4]. Later on, Costello presented a linearized theory that was based on three generalized strain parameters accounting for helix bending as an additive response mechanism to the above broached ones [5]. To that extent, the variations of the helix curvature, twist and lay angle were taken into account. Velinsky studied the necessity of resorting to non-linear formulations for the helix structural response to be sufficiently characterized [6], concluding that for *service* strains, the omission of higher order contributions is utterly justified. Furthermore, Raoof and Kraincanic [7] conducted theoretic parametric studies and used experimental data to assess the range of validity of thin rod and orthotropic sheet theory based models, as a function of the helical strand size and geometric configuration. Moreover, Utting et Jones [8, 9] presented a beam theory based model along with a large set of experimental data, while Sathikh et al. [10] were the first to provide a model that complied with the *Betti-Maxwell* symmetry principle for the two-times-two linearized structural stiffness matrix describing the mechanical response of helices to axial and torsional strain. Their derivations were grounded on a four generalized strain model provided by Ramsey [11].

On the numerical modeling side, most of the presented numerical models were based on volume or beam elements. In particular, Jiang et al. [12, 13, 14] used a strand slice to simulate the response of helical bodies to axial strain, making use of standard volume elements. Nawrocki and Labrosse [15] emphasized on the effect that the helix kinematic considerations have on the stiffness terms of single layer helical strands. Cartraud and Messenger [16] applied homogenization theory to beams with periodic micro-structures to simulate the response of helical bodies, applying their modeling approach to single layer engineering strands, while Messenger and Cartraud [17] presented a modeling technique based on volumetric finite elements. In more recent works, Usabiaga and Pagalday [18] introduced a modeling approach for the simulation of axial and torsional helix loading, while Ghoreishi et al. [19] pointed out limitations of several analytical schemes. Finally, the impact of kinematic constraints arising from the braiding pattern of multilayer helical assemblies was elaborated, providing bounds for the construction's stiffness [20].

While a large number of volume or beam element based modeling schemes of helical structures has been presented, planar modeling techniques have only recently attracted researcher's attention. In particular, Treyss presented a SAFE method extension to helical waveguides where the propagation of elastic waves was addressed makingg use of a helix plane [21]. Thereupon, Frikha et al. simulated the structural response of a single layer helical strand to axial straining, grounding their formulation on a helix special twisted basis [22].

1.1. Structure of the present work

The present work is based upon Karathanasopoulos et al. [23], with the current work extending the modeling approach to account for torsional strains. What is more, an application to single layer engineering strands is provided, along with a quantification of the computational cost of the numerical model.

More specifically, in the current work we introduce a generic scheme for the planar finite element modeling of helical structures in Section 2, using a kinematically based method for the loading description. Thereupon, we compare the numerically retrieved structural response for a single helical body to closed-form analytical expressions over a wide range of helix angles for validation purposes. In Section 3, we present an application to simple engineering strands. Thereafter, we discuss on the model’s computational merits (Section 4) and conclude in Section 5.

2. Model development

2.1. Helix geometry

The three dimensional helix geometry is described making use of the $\mathbf{n}$, $\mathbf{b}$, $\mathbf{t}$ *Serret-Frenet* basis:

$$\mathbf{X}(x_n, x_b, s) = \mathbf{R}(s) + x_n \mathbf{n} + x_b \mathbf{b}, \quad -r \leq x_n, x_b \leq r \quad (1)$$

where r denotes the radius of the helical body and $\mathbf{R}(s)$ stands for the position vector of the helix centerline, defined as follows:

$$\mathbf{R} = \begin{Bmatrix} a \cos \varphi \\ a \sin \varphi \\ b \varphi \end{Bmatrix} \quad \varphi = \frac{s}{\gamma} \quad \gamma = \sqrt{a^2 + b^2} \quad \theta = \arctan \left(\frac{b}{a} \right) \quad (2)$$

In Eq. 2, a and b are intrinsic helix parameters, namely the centerline position of the helix and the helix rise along its axis per unit helix angular evolution φ . The latter define the curvature κ and tortuosity τ of the helical structure:

$$\kappa = \frac{a}{\gamma^2} \quad \tau = \frac{b}{\gamma^2} \quad (3)$$

The general basis of Eq. 1 is elaborated in [Appendix A](#). On the left side of Fig. 1, a *3D* helix reconstruction is provided, while on its right part, a two dimensional helix cross section mesh is depicted.

Figure 1: a) Helix and coordinate conventions b) Helix cross section mesh

2.2. Kinematic and constitutive equations

All mechanical formulations are described in a general curvilinear basis, for which the linearized kinematic equations are defined in index or in matrix notation as follows [24]:

$$\epsilon_{ij} = \frac{1}{2} (u_i|_j + u_j|_i) = \frac{1}{2} (u_{i,j} + u_{j,i}) - u_k \Gamma_{ij}^k, \quad \boldsymbol{\epsilon} = (\mathbf{L}_{12} + \mathbf{L}_3) \mathbf{u}_{123} \quad (4)$$

where the notation $(1, 2, 3)$ has been used for the derivation with respect to $\mathbf{x}_n$, $\mathbf{x}_b$ and $\mathbf{s}$ respectively, while the expansion of the linear operators $\mathbf{L}_{12}$ and $\mathbf{L}_3$ is provided in [Appendix B](#). The linear elastic constitutive law is defined as [25]:

$$\sigma^{ij} = C^{ijkl} \epsilon_{kl} \quad (5)$$

with superscripts and subscripts denoting respectively contravariant and covariant base components, [Appendix A](#). The stress tensor is defined in vector notation as $[\sigma^{11} \sigma^{22} \sigma^{33} \sigma^{23} \sigma^{13} \sigma^{12}]^T$ while the strain tensor is given as $[\epsilon^{11} \epsilon^{22} \epsilon^{33} 2\epsilon^{23} 2\epsilon^{13} 2\epsilon^{12}]^T$. Finally, the elasticity tensor $\mathbf{C}$ for an elastic and isotropic material is expressed as follows [26]:

$$C^{ijkl} = \frac{\nu E}{(1+\nu)(1-2\nu)} g^{ij} g^{kl} + \frac{E}{2(1+\nu)} (g^{ik} g^{jl} + g^{il} g^{jk}) \quad (6)$$

where E and ν stand respectively for the *Young* modulus and for the *Poisson* ratio. The definition of the contravariant metric tensor $[g^{ij}]$ entering the elasticity tensor is elaborated in [Appendix A.1](#).

2.3. Displacement field and strain definition

The planar model simulates the response of the helical body by defining its loading upon the concept of macro strains. Macro strains are described in the usual Cartesian reference basis. Related global deformations are expressed in terms of the applied macro strains $\bar{\boldsymbol{\epsilon}}_{xyz}$. The macro strains are compatible with the displacement field $\bar{\mathbf{u}}_{xyz}$. The total solution of a specified problem consists of two parts:

$$\mathbf{u}_{xyz} = \mathbf{u}_{xyz}^{FEM} + \bar{\mathbf{u}}_{xyz} \quad (7)$$

namely the finite element solution part $\mathbf{u}_{xyz}^{FEM}$ and the prescribed displacement field part $\bar{\mathbf{u}}_{xyz}$, while the definition of Eq. 7 can be accordingly viewed in its Curvilinear mapping as follows:

$$\mathbf{u}_{123} = \mathbf{u}_{123}^{FEM} + \bar{\mathbf{u}}_{123} \quad (8)$$

Implementing the kinematic relations of Eq. 4 in Eq. 8, the following expression for the mechanical strains is retrieved:

$$\boldsymbol{\epsilon}_{123}^M = \boldsymbol{\epsilon}_{123}^{FEM} + \bar{\boldsymbol{\epsilon}}_{123} \quad (9)$$

2.4. Variational formulation

We elaborate a variational formulation form, starting from the total potential energy Π of a linear elastic system, composed of the deformation energy U and of the externally applied work W , as follows:

$$\begin{aligned}\Pi &= U - W \\ &= \frac{1}{2} \int_{\Omega} \boldsymbol{\epsilon}^{M^T} \mathbf{C} \boldsymbol{\epsilon}^M d\Omega - \int_{\Gamma_{\sigma}} \mathbf{u}^T \hat{\boldsymbol{\sigma}} d\Omega - \int_{\Gamma_u} \hat{\mathbf{u}}^T \boldsymbol{\sigma} d\Omega ,\end{aligned}\quad (10)$$

In the above introduced total potential, the external work arises from the prescribed stresses $\hat{\boldsymbol{\sigma}}$ and displacements $\hat{\mathbf{u}}$ on the respective domain boundaries as well as from the prescribed *macro* strains contributing to the volume integral. Discretizing the domain upon element-wise defined shape functions Φ , the following expression for the mechanical strains is obtained:

$$\begin{aligned}\boldsymbol{\epsilon}_{123}^M &= (\mathbf{L}_{12} + \mathbf{L}_3) \mathbf{u}_{123} \\ &= \mathbf{L}_{12} \Phi^T \tilde{\mathbf{u}}_{123} + \bar{\boldsymbol{\epsilon}}_{123} = \mathbf{B} \tilde{\mathbf{u}}_{123} + \bar{\boldsymbol{\epsilon}}_{123}\end{aligned}\quad (11)$$

Where the analytical part of Eq. 7, describing the externally applied strains is elaborated in Section 2.5. By substituting the above expression in the energy potential Π of Eq. 10 and by letting its variation with respect to the solution parameters $\tilde{\mathbf{u}}_{123}$ vanish, the following system of equations arises:

$$\mathbf{K} \tilde{\mathbf{u}}_{123} = \mathbf{r}, \quad \mathbf{K} = \sum_{k=1}^N \int_{\Omega_k} \mathbf{B}^T \mathbf{C} \mathbf{B} d\Omega_k \quad (12)$$

where the right-hand side, or force vector, $\mathbf{r}$ is given as follows:

$$\mathbf{r} = - \sum_{k=1}^N \int_{\Omega_k} \mathbf{B}^T \mathbf{C} \bar{\boldsymbol{\epsilon}}_{123} d\Omega_k + \sum_{k=1}^N \int_{\Gamma_{\sigma_k}} \Phi^T \hat{\boldsymbol{\sigma}} d\Gamma_k \quad (13)$$

The formulation of Eq. 12 allows for the specification of different mechanical strains. We hereby concentrate on the proposal and verification of axial and torsional strain, elaborated in Section 3.2.

2.5. Planar mesh load application scheme

2.5.1. Axial strain

We prescribe a homogeneous axial strain field $\bar{\epsilon}_z$ along the global Cartesian Z axis, defined through the centerline of the helical body. The latter is compatible with the following displacement field:

$$\bar{\mathbf{u}}_{xyz} = \begin{Bmatrix} \bar{u}_x \\ \bar{u}_y \\ \bar{u}_z \end{Bmatrix} = \bar{\epsilon}_z \begin{Bmatrix} 0 \\ 0 \\ z \end{Bmatrix} = \frac{bs\bar{\epsilon}_z}{\gamma} \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} \quad (14)$$

Computing the local curvilinear displacement components along with the respective displacement gradients - for a transformation angle $\varphi = 0$ in Eq. A.10 - we retrieve the following:

$$\bar{\mathbf{u}}_{123} = \tau s \begin{Bmatrix} 0 \\ a \\ b \end{Bmatrix} \bar{\epsilon}_z, \quad \nabla \bar{\mathbf{u}}_{123} = \tau \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & a \\ 0 & 0 & b \end{bmatrix} \bar{\epsilon}_z \quad (15)$$

Evaluation of the kinematic Eq. 4 provides the following applied strain components:

$$\bar{\epsilon}_{123} = [0 \quad 0 \quad b\tau \quad a\tau \quad 0 \quad 0]^T \bar{\epsilon}_z \quad (16)$$

2.5.2. Torsional strain

We consider a homogeneous helix twist ω' , defining the helix rotation per unit helix development length $L_z = bs/\gamma$, so that the absolute rotation ω increases linearly with the local curvilinear coordinate s , as follows:

$$\omega(s) = \omega' \frac{b}{\gamma} s \quad (17)$$

The applied torsional strain is compatible with the following applied displacement field:

$$\bar{\mathbf{u}}_{xyz} = \begin{Bmatrix} \bar{u}_x \\ \bar{u}_y \\ \bar{u}_z \end{Bmatrix} = \frac{b}{\gamma} s \begin{Bmatrix} -Y \\ X \\ 0 \end{Bmatrix} \omega' \quad (18)$$

The curvilinear displacement components - for a transformation angle $\varphi = 0$ in Eq. A.10- are thereupon computed as follows:

$$\bar{\mathbf{u}}_{123} = \tau s \begin{Bmatrix} -bx_b \\ -b(a - x_n) \\ b\tau x_b^2 + (a - x_n)^2 \end{Bmatrix} \omega' \quad (19)$$

with the displacement gradients given as:

$$\nabla \bar{\mathbf{u}}_{123} = \tau \begin{bmatrix} 0 & -bs & -bx_b \\ bs & 0 & -bx_n^a \\ -2x_n^a s & 2bs\tau x_b & b\tau x_b^2 + (x_n^a)^2 \end{bmatrix} \omega' \quad (20)$$

where in Eq. 20, the abbreviation $x_n^a = a - x_n$ has been used. Evaluating the kinematic Eq. 4 yields the following imposed total strain field expression:

$$\bar{\epsilon}_{123}^T = \begin{Bmatrix} 0 \\ 0 \\ \tau (b\tau x_b^2 + (a - x_n)^2) \\ -\tau b(a - x_n) \\ -\tau bx_b \\ 0 \end{Bmatrix} \omega' \quad (21)$$

3. Verification

3.1. Analytical closed-form solutions

We model the helix axial and torsional strain structural response considering the global force and moment resultants F_z and M_z , related through the following system of equations:

$$\begin{bmatrix} F_z \\ M_z \end{bmatrix} = \begin{bmatrix} \kappa_{\epsilon_z \epsilon_z} & \kappa_{\epsilon_z \omega'} \\ \kappa_{\omega' \epsilon_z} & \kappa_{\omega' \omega'} \end{bmatrix} \begin{bmatrix} \bar{\epsilon}_z \\ \omega' \end{bmatrix} \quad (22)$$

We subsequently cite the closed-form expressions provided by Sathikh et al. [10], characterizing the stiffness term expressions of Eq. 22 as functions of the extensional, torsional and bending helix cross section stiffness, EA , GJ and EI respectively:

$$\begin{aligned} \kappa_{\epsilon_z \epsilon_z} &= EA s^3 + (GJ c^2 + EI s^2) c^4 s / a^2 \\ \kappa_{\omega' \epsilon_z} &= EA a s^2 c + [GJ s^2 - EI (1 + s^2)] s^2 c^3 / a \\ \kappa_{\omega' \omega'} &= EA a^2 c^2 s + EI c^2 s (1 + s^2)^2 + GJ s^7 \end{aligned} \quad (23)$$

where the abbreviations $s = \sin \theta$ and $c = \cos \theta$ have been employed, θ being the helix angle.

3.2. Planar modeling verification

A normalized form of the stiffness expressions is employed for the terms comparison so that a non-dimensional form arises, the normalization carried out as follows:

$$\kappa_{\epsilon_z \epsilon_z}^* = \frac{F_z}{\bar{\epsilon}_z EA}, \quad \kappa_{\omega' \epsilon_z}^* = \frac{M_z}{\bar{\epsilon}_z EA a}, \quad \kappa_{\omega' \omega'}^* = \frac{M_z}{\omega' EA a^2} \quad (24)$$

In Fig. 2 the numerically retrieved axial, coupling and torsional stiffness terms are compared with the closed form solutions of Eq. 23 over the entire range of helical angles. For the verification, a helix index of $a/r=10$ ($r=0.1$) has been employed and a *Poisson* ratio of $\nu=0.3$.

Figure 2: Stiffness terms comparison

The results verify the accuracy of the modeling approach for all stiffness terms, any relevant discrepancies lying below 0.5%.

3.3. Application to simple helical strands

The model is subsequently used to simulate the structural response of a single layer engineering strand, its layer comprised of 6 helical bodies. Table 1 summarizes the material and geometric specifications of the construction:

Table 1: Simple strand material and geometric properties

	E (KN/mm ²)	r (mm)	ν (-)	θ (°)
<i>Core</i> (c)	197.9	1.970	0.30	90.0
<i>Helix</i> (h)	197.9	1.865	0.30	80.8 77.8 73 70

The modeling results are compared to the ones provided by analytical [10] and numerical modeling schemes [19, 22]. Fig. 3 depicts the normalized stiffness term predictions for the axial $\kappa'_{\epsilon_z \epsilon_z}$, coupling $\kappa'_{\omega' \epsilon_z}$ and torsional stiffness $\kappa'_{\omega' \omega'}$ of the helical strand (Table 1) - the normalization carried out with respect to $A_c + 6A_h$, $A = 6A_h a$ and $6EA_h a^2 + GJ_c$ accordingly -.

Figure 3: Simple helical strand stiffness comparison

Fig. 3 indicates that all modeling predictions are in very good agreement for steep helix angles ($\theta = 80.8|77.8^\circ$), with their maximum relative difference being below 2%. Nevertheless, the lower the helix angle that the helical layer follows is ($\theta = 73.8|70^\circ$), the more compliant the numerically retrieved response is, with analytical closed-form solutions overestimating the constructions' stiffness. In particular, the purely axial stiffness $\kappa'_{\epsilon_z \epsilon_z}$ is overestimated by a maximum of 9% for a strand with a helix layer angle of 70° , as the comparison with the volume finite element prediction suggests.

Fig. 4 below depicts the normal stress distribution of the strand when subject to axial strain, for a helical layer angle of 80.8° :

Figure 4: Single layer helical strand

4. Computational aspects

The computational *complexity* of finite element models is a critical parameter for the analysis of large structural systems that commonly require a significant amount of *degrees of freedom*. Complexity assessment determines whether a task can be solved using available computational power while it provides an estimate of the time demanded. The asymptotic complexity of a linear finite element algorithm - noted as $\mathbf{O}$ - when lower order contributions are neglected, converges to [27]:

$$\mathbf{O}_{FEM} = \mathbf{O}(NW^2) \quad (25)$$

where N stands for the number of nodes and W for the bandwidth of the discrete model following a direct solution method. Elaborating on the simplified expression of Eq. 25, we can deduce a rough estimate of the relevant computational power requirements for a volumetric (v) and a planar (pl) modeling approach. To that extent, we consider a helical body that is discretized along its length with n prismatic elements, connected with identical cross sections. For a total of N_h helical bodies modeled, we retrieve:

$$\mathbf{O}_v = 4n N_h \mathbf{O}_{pl} \quad (26)$$

where the factor 4 stands for the duplication of the bandwidth that the connection of the lowest node of the one section to its adjacent one induces. Setting as unity the planar modeling approach complexity ($\mathbf{O}_{pl} = 1$), a graphical representation of the scaling factor between the two modeling approaches can be reconstructed, depicted in Fig. 5:

Figure 5: Computational complexity comparison

Fig. 5 suggests that the more detailed the geometric approximation (n) or the larger the helical assembly (N_h), the higher the computational resources that volumetric modeling requires, so that complexity correlation factors mounting up to orders of magnitude are retrieved, already for rather low values of the parameters involved. The latter testifies to the merit of the approach, in particular when iterative numerical patterns (*Optimization, Bayesian*) are to be combined, in which substantially higher values than the ones provided in Fig. 5 apply.

5. Conclusion

A two dimensional finite element model simulating the mechanical response of helical bodies to axial and torsional strain has been elaborated. A model verification was provided, by comparing the planar modeling numerical results to analytical closed-form stiffness expressions. Moreover, the structural response of a single layer helical strand was analyzed and confronted to established analytical and numerical models. Finally, the computational merits of the numerical approach were explicated.

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