Generation mechanism of nonlinear ultrasonic Lamb waves in thin plates with randomly distributed micro-cracks

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Abstract

Since the identification of micro-cracks in engineering materials is very valuable in understanding the initial and slight changes in mechanical properties of materials under complex working environments, numerical simulations on the propagation of the low frequency S_0 Lamb wave in thin plates with randomly distributed micro-cracks were performed to study the behavior of nonlinear Lamb waves. The results showed that while the influence of the randomly distributed micro-cracks on the phase velocity of the low frequency S_0 fundamental waves could be neglected, significant ultrasonic nonlinear effects caused by the randomly distributed micro-cracks was discovered, which mainly presented as a second harmonic generation. By using a Monte Carlo simulation method, we found that the acoustic nonlinear parameter increased linearly with the micro-crack density and the size of micro-crack zone, and it was also related to the excitation frequency and friction coefficient of the micro-crack surfaces. In addition, it was found that the nonlinear effect of waves reflected by the micro-cracks was more noticeable than that of the transmitted waves. This study theoretically reveals that the low frequency S_0 mode of Lamb waves can be used as the fundamental waves to quantitatively identify micro-cracks in thin plates.

Keywords: Ultrasonic nonlinearity, Lamb waves, Micro-cracks, S_0 mode,

1. Introduction

Material property degradation and defects often occur in engineering structures due to fatigue load. Thus it is necessary to develop non-destructive testing methods to evaluate the safety of engineering structures. Especially in the aerospace industry, to ensure the safety and durability of engineering structures, there is an increasing demand for the detection of micro-cracks at an early stage.

The linear ultrasonic testing technology has been widely used to detect cracks, holes, corrosion and other defects in materials, but it is only sensitive to severe defects [1-3]. When the defect is relatively small, it is difficult for conventional linear ultrasonic inspection methods to obtain the linear variation characteristics. For example, if there are closed micro-cracks in materials, the reflection and scattering at the defect cannot be inspected when the ultrasonic waves pass through them. Therefore, it is highly possible that the linear ultrasonic testing technology would fail to detect the closed micro-cracks [4, 5]. Compared with the linear ultrasonic testing technology, the nonlinear characteristics of the ultrasonic waves are of high sensitivity to the microstructure of the materials with a size much smaller than the wavelength of the ultrasonic waves. The change of the microstructure due to early material degradation, such as micro-cracks, dislocations and precipitates, can be measured by nonlinear ultrasonic testing technologies [6-11]. Thus, the early detection of material property degradation and micro-cracks can be realized [12-16]. The key to the nonlinear ultrasonic testing technology for damage inspection is the excitation and detection of the nonlinear effects of the ultrasonic waves. Based on different nonlinear effects, many different nonlinear ultrasonic techniques have been developed, including higher harmonics technique [17], sub-harmonic and DC response technique [18], nonlinear resonant ultrasound spectroscopy technique, and mixing resonant wave technique [8, 19]. Many studies have demonstrated that the nonlinear ultrasonic testing technology is an effective method to evaluate early material degradation and micro-crack initiation by analyzing the nonlinear effects of ultrasonic waves [20-22].

Because of the long propagation distance of the Lamb waves in a thin plate structure, nonlinear Lamb-wave detection techniques have attracted more and more attention in recent years [17, 23-31]. Many efforts have been devoted to the micro-crack detection by higher harmonics of Lamb waves. The interactions between elastic waves and micro-cracks with clapping mechanisms have been used to explain the higher harmonic generation of Lamb waves [14, 32-35]. It is believed that when elastic waves reach a micro-crack, the crack can be closed and opened under compression and tension; the compression part of the Lamb waves can penetrate the crack, while the tension part cannot. The higher harmonics are generated due to the apparent local stiffness changes at the crack region under tension and compression. However, it is challenging to detect the higher harmonics of Lamb waves due to the dispersion of Lamb waves. Deng and Lima have reported that obvious second harmonic can be observed only when phase velocities between the fundamental and double frequency Lamb waves are matched [23, 24]. At present, in most studies, S_1 and A_1 modes are selected as the fundamental waves [23, 24, 27, 36] because the condition of the phase velocity matching for these two modes can be easily satisfied. However, the research on S_0 and A_0 modes, which have higher energy, is still rarely reported [28]. Therefore, we investigated the mechanism of the second harmonic generation and propagation when S_0 mode was used as the fundamental waves. The generation condition of the second harmonic for S_0 mode was investigated by numerical modelling of S_0 mode propagation in a thin plate structure with distributed micro-cracks, and the quantitative relationship between the acoustic nonlinear parameter and the characteristics of micro-cracks was also studied. This work will provide a theoretical basis for nonlinear Lamb-wave detection techniques based on S_0 mode.

2. Linear Lamb waves

Lamb waves are most commonly used in acoustic guide-wave modes for thin plate structures, whose wavelength has the same magnitude as the thickness of the plate. The dispersion is the significant characteristic of Lamb waves, that is, the

velocity of propagation depends on not only the elastic constants and density of the material, but also the frequency of waves. Figure 1 shows the dispersion curves of Lamb wave propagation in a 2mm aluminum plate.

Lamb waves are always composed of various wave modes, which include at least two modes (S_0 and A_0 modes) in the low-frequency domain (0-800 kHz). It is known that the zero-order modes (S_0 and A_0 modes) carry more energy than the higher-order modes, with a smaller energy attenuation during propagation compared to the higher-order modes. Moreover, Fig. 1(a) shows that the phase velocity of S_0 mode basically slowly changes in the low-frequency domain (0-800 kHz) and high-frequency domain (1800-4000 kHz). Many studies have pointed out that one necessary condition to obtain obvious second harmonic is the same of phase velocities between the fundamental and double frequency Lamb waves. Therefore, S_0 mode has a great advantage in evaluating early material degradation and micro-crack initiation in materials. Here we investigated the potential of S_0 mode within the range of 800 kHz to detect the micro-cracks in thin plate structures.

(a)

(b)

Fig. 1. Dispersion curves of Lamb waves of a 2mm aluminum plate: (a) phase velocity; (b) group velocity

3. Acoustic nonlinearity parameter

A large number of experiments have demonstrated that the nonlinear ultrasonic detection technique based on second harmonics is an effective tool to detect the change of the microstructure of the metal materials [37]. The second harmonic waves can be generated when the fundamental ultrasonic waves pass through a material with changed microstructure. The amplitude of the second harmonic waves can be quantitatively evaluated by acoustic nonlinearity parameter.

For one dimensional case, the nonlinear stress-strain relationship based on the classical nonlinear elastic theory can be expressed as:

$$\sigma_{xx} = E_0 \varepsilon_{xx} (1 + \beta \varepsilon_{xx}) \quad (1)$$

where σ_{xx} is the stress, E_0 is the linear elastic modulus of materials, ε_{xx} is the strain, and β is the second-order classical nonlinearity parameter.

Combining the wave motion equation with Eq. (1), one dimensional nonlinear wave motion equation of longitudinal waves can be derived. Thus, the relationship between the nonlinear response (the amplitude of harmonic waves) and the classical

nonlinearity parameter can be obtained. The second-order classical nonlinearity parameter can be expressed as:

$$\beta = \frac{8A_2}{A_1^2 x \kappa^2} \quad (2)$$

where A_1 is the amplitude of fundamental waves, and A_2 is the amplitude of second harmonic waves, $\kappa = \omega/c_L$ is the wave number of fundamental waves, $\omega = 2\pi f$ is the angular frequency of fundamental waves, c_L is the velocity of longitudinal waves in materials.

However, contact acoustic nonlinearity caused by cracks is different from the classical acoustic nonlinearity. A variety of theories and models have been proposed to explain the contact acoustic nonlinearity, including the hysteretic model and bi-linear stiffness model. Qu *et al.* [14, 33, 34] improved the bi-linear stiffness model by considering different effective tensile and compressive moduli for the local micro-crack structure, and derived the acoustic nonlinearity parameter when ultrasonic waves propagate in a structure with micro-cracks:

$$\beta' = \frac{A_2}{A_1} \quad (3)$$

where the acoustic nonlinearity parameter β' is related to crack density, frequency of fundamental waves, propagation distance and friction coefficient. In this work, Eq. (3) has been used to investigate the relationship between micro-cracks and the acoustic nonlinearity parameter.

4. Numerical simulation

Since it is very challenging to directly obtain the nonlinear response when S_0 mode passes through micro-cracks by an analytic method, numerical simulations have been employed instead. A two-dimensional finite element method (FEM) model of a thin plate structure with distributed micro-cracks is built using the commercial FEM software ABAQUS (Version 6.14, Dassault Systèmes Simulia Corp., Providence, RI, USA) and the Python (Version 2.7, Python Software Foundation, available at

<http://www.python.org>).

The problem of Lamb waves of S_0 mode propagating through a thin plate structure is described in Fig. 2. The middle part of the whole region is the micro-crack damage zone. S_0 mode waves were applied using a dynamic displacement excitation on the left end of the model. The waves propagated along the positive direction of x , and interacted with the micro-cracks when they reached the micro-crack damage zone. As a result, the second harmonic waves of S_0 mode were generated. Because the size of micro-cracks is much smaller than the wavelength of S_0 mode waves, most waves passed through the micro-crack damage zone and were collected at position 2 (named as transmitted waves). Meanwhile, a very small portion of the waves was reflected by the micro-cracks and then propagated along the negative direction of x , which were collected at position 1 and named as reflected waves.

Fig. 2. Schematic of Lamb waves propagating through a region with micro-cracks

To investigate the influence of micro-cracks on the acoustic nonlinearity parameter, N micro-cracks were randomly distributed spatially in the middle part with the area A of the thin plate structure. And all the cracks had the same length of $2a$ (the range of a is from $10 \mu m$ to $200 \mu m$). So the crack density can be defined as $c = Na^2/A$, where c is a dimensionless parameter. In this study, there were two variables that affected the random distribution of micro-cracks: the center position and the orientation of cracks, which were described by the probability density function (PDF) with a uniformly random variable in the simulations.

A linear elastic constitutive law of aluminum (AL-6061-T6) was considered in the FEM analysis, and the material properties used in the simulations were

$\rho = 2704 \text{ kg/m}^3$, $E = 68.9 \text{ GPa}$ and $\nu = 0.33$. The size of the FEM model was $600 \text{ mm} \times 2 \text{ mm}$, and the length of the micro-crack damage zone was L (Note that the range of L in this paper is from 4 mm to 16 mm).

For the crack model, the surfaces of the micro-cracks were modeled as contact surfaces, which could be separated from, but cannot interpenetrate into each other. Meanwhile, the Coulomb law of friction was used to model the relative sliding of the surfaces, and the friction coefficient was μ . Therefore, by accounting for the frictional forces, the actual traction applied to the crack faces was thus given by [14]

$$\mathbf{q} = \sigma_0 \mathbf{n} + [\tau_0 + \mu \sigma_0 H(-\sigma_0)] H[\tau_0 + \mu \sigma_0 H(-\sigma_0)] \mathbf{s} \quad (4)$$

where σ_0 and τ_0 are the applied normal and shear stresses on the surfaces of the cracks, respectively; $H(\square)$ is the Heaviside step function; $\mathbf{n}$ is a unit vector normal to the crack face, and $\mathbf{s}$ is in the plane of the crack.

The dynamic displacement excitation applied to the left edge of the model can be expressed as $u(x, t) = A_0 \sin(2\pi ft) \times \sin(\pi ft/10)^2$, where A_0 is the amplitude of excitation signal ($1 \times 10^{-4} \text{ mm}$ in this study [12, 19, 28]); f is the frequency of excitation signal (ranging from $100 \text{ kHz} \sim 400 \text{ kHz}$) which was used to excite S_0 mode in the low frequency domain. The transmitted waves were collected at the right boundary of the micro-crack damage zone as shown in Fig. 2, and the reflected waves were collected at three positions, which are 0 mm , 50 mm and 100 mm left to the left boundary of the micro-crack damage zone, respectively.

Since at least 20 elements are required in the wavelength of the highest frequency waves for calculation precision, the element size for non-crack damage zone was set to $L_{\max} = 0.25 \text{ mm}$ to ensure at least 6 elements for each crack surface. The FEM model was constructed using the four-node plane strain (CPE4R) elements as shown in Fig. 3.

The central different method was used to simulate the propagation of the Lamb wave in the time domain, and the stable time increment was carefully chosen according to the time of stress waves passing through the minimum element to ensure

the efficiency of the simulations. In this case, the smallest stable time increment required for ABAQUS/Explicit solver was $\Delta t = 1 \times 10^{-9}$ sec. Since the higher harmonic waves are generated by the interactions between the excitation signals and micro-cracks, the stable time increment was set to $\Delta t = 5 \times 10^{-10}$ sec.

Moreover, it should be noted that the cracks in the FEM model were completely random. To eliminate the random effect, the results of numerous FEM models are averaged in this study. Figure 4 shows the average acoustic nonlinearity parameter versus the number of the FEM models. The acoustic nonlinearity parameter was more stable when the number of FEM models was larger than 50. To better eliminate the effect of the random distribution of micro-cracks, the results shown in the following section were averaged over 100 FEM models.

Fig. 3. FEM model of micro-crack damage zone: (a) distribution of micro-cracks; (b) finite element mesh (the red line represents one crack in the micro-crack damage zone)

Fig. 4. Acoustic nonlinearity parameter versus number of FEM models ($c = 0.015$, $f = 200\text{kHz}$, $\mu = 0.1$)

5. Result and discussion

Because of micro-cracks, the effective material parameters (effective moduli, effective Poisson ratio) of elastic solids have subtle changes under a dynamic loading, which can lead to a slight change of the phase velocity of Lamb waves after they pass through the micro-crack damage zone [14]. Meanwhile, because the wavelength λ (13-53 mm) of the low-frequency S_0 mode is much larger than the crack length (10-200 μm), most of the waves can pass through the micro-crack damage zone (transmitted waves), and only a very small portion of the waves are reflected by the micro-cracks (reflected waves). Both of the transmitted and reflected waves may contain the second harmonic waves. To the best of our knowledge, there is no relevant work to analyze this problem. Therefore, we set two locations in the FEM model to collect and analyze the transmitted waves and reflected waves respectively.

5.1 Attenuation of phase velocity and fundamental waves

The existence of micro-cracks not only leads to slight changes of phase velocity after they pass through the micro-crack damage zone, but also causes a small amount of reflected waves. Thus, the energy of fundamental waves will slightly decrease. Table 1 lists the attenuation of the phase velocity and fundamental waves for the

non-cracked model and the cracked model at 200 kHz. We can see that the phase velocity of Lamb waves for the cracked model decreased about 1.18% compared with the non-cracked model, and the energy of fundamental waves only decreased about 0.075%. Therefore, we assume that the influence of micro-cracks on fundamental waves can be ignored.

Figure 5 shows the normalized phase velocity at 200 kHz versus the crack density. The phase velocity of the cracked models with different crack density was normalized to the phase velocity of the non-cracked model. It can be seen that the phase velocity decreased linearly with the increase of crack density. However the decrease was insignificant. And we can infer that the effective moduli of a cracked elastic solid decreases as the crack density increases, leading to the decrease of phase velocity.

Table. 1. Phase velocity of Lamb waves and energy attenuation ($f = 200$ kHz)

	Phase velocity (m/s)	Attenuation of fundamental waves
Without micro-crack	5339	
With micro-crack ($c=0.015$)	5276	0.075%

Fig. 5. Normalized phase velocity versus crack density ($f = 200$ kHz)

5.2 Transmitted waves

Figure 6 shows the displacement contour of S_0 mode waves while propagating in a thin plate structure with micro-cracks at 200 kHz (U1 and U2 are the displacements in the x and y direction respectively). We can see clearly that the wavelength of Lamb waves was much larger than the crack length. And although the waves interacted with the micro-crack surfaces in the micro-crack damage zone, no obvious change was observed in the waveforms. Only a very weak scattering phenomenon occurred around the micro-cracks. Meanwhile, the 10,000 times zoomed deformed shapes can clearly demonstrate the micro-crack opening and closing statuses.

(a)

(b)

Fig. 6. Propagation of S_0 mode waves in a thin plate structure with micro-cracks ($c = 0.015$, $f = 200\text{kHz}$, $\mu = 0.1$). (a) displacement contour in the x direction (10,000 times zoomed deformed shapes in the dashed box); (b) displacement contour in the y direction

Figures 7 and 8 show the transmitted wave signals collected at position 2 in the x and y direction respectively, wherein the wave signals for the non-cracked case are marked by the solid line, and the wave signals for the cracked case are marked by the

dashed line. We can see the energy of the transmitted wave signals in the x direction is significantly higher than that of y direction. Moreover, the time-domain signals for the non-cracked case and the cracked case are mostly overlapped in x direction as shown in Fig. 7(a), which indicates that micro-cracks have negligible effects on the transmitted wave signals in the x direction. The time-domain signals for the non-cracked case and the cracked case in the y direction are also very similar as shown in Fig. 8(a), more obvious differences can be observed. The solid line and dashed line are mostly overlapped for the first half of wave packet. However, the fluctuation and deviation between the solid line and dashed line emerge at the latter half of wave packet, which indicates that the transmitted wave signals contain some wave signals at different frequencies. From the frequency analysis of transmitted wave signals (Fig. 7(b) and Fig. 8(b)), we found that the wave signals for the non-cracked case marked by the solid line only contained the 200 kHz wave signals (the fundamental waves), which indicates that the second harmonic waves cannot be generated for the non-cracked case; while the wave signals for the cracked case marked by the dashed line contained not only the 200 kHz wave signals, but also the 400 kHz wave signals (second harmonic waves). It should be noted that the amplitude of the second harmonic waves in the x direction is obviously smaller than that of the y direction, which can be inferred that the existence of micro-cracks is the critical factor for generating the second harmonic waves. A recent work shows that, for S_0 mode, material nonlinearity can cause the generation of second harmonic [28]. Here we found from the numerical simulations that, when using S_0 mode, micro-cracks could also generate the second harmonic waves of Lamb waves. Therefore, this study provides a foundation of new techniques to identify the micro-crack damage in the thin structure using S_0 mode.

(a)

(b)

Fig. 7. Transmitted wave signals in the x direction (a) time domain; (b) frequency domain

$$(c = 0.015, f = 200\text{kHz}, \mu = 0.1)$$

(a)

(b)

Fig. 8. Transmitted wave signals in the y direction (a) time domain; (b) frequency domain

$$(c = 0.015, f = 200\text{kHz}, \mu = 0.1)$$

Next, we further investigated the relationship between the acoustic nonlinearity parameter and the crack density as well as the propagation distance in the micro-crack damage zone. Figure 9 shows the acoustic nonlinearity parameter (defined by Eq. (3)) versus the crack density c . The crack density $c = Na^2/A$ is a dimensionless number [14, 33, 34], which can be used as a quantitative indicator for the acoustic nonlinearity

caused by micro-cracks. Because crack density can be affected by the parameter N and a , we consider two cases: one is to increase N and to keep a the same, another is to increase a and to keep N the same. In consideration of the random FEM models, the results of the two cases are basically consistent in Fig. 9. Meanwhile, when the excitation frequency and friction coefficient of the micro-crack surfaces remain unchanged, the acoustic nonlinearity parameter increased linearly with the increase of the crack density, and the acoustic nonlinearity in the y direction was noticeably higher than that of the x direction.

Figure 10 shows the acoustic nonlinearity parameter versus the propagation distance in the micro-crack damage zone. The acoustic nonlinearity parameter also increased linearly with the increase of the propagation distance in the micro-crack damage zone. It has been proven that when the phase velocity of the second harmonic Lamb waves is the same as that of the fundamental wave, the second harmonic Lamb waves increase linearly with the propagation distance. On the contrary, when their phase velocities are different, the amplitude of the second harmonic Lamb waves change as a sinusoidal oscillation form. For S_0 mode, the steadiness of the phase velocity in the low-frequency domain (0-800 kHz) is especially important, since it leads to the linear cumulative tendency of the acoustic nonlinearity parameter on the propagation distance.

In contrast to the classic acoustic nonlinearity, the factors affecting the acoustic nonlinearity also include the excitation frequency and friction coefficient of the micro-crack surfaces. In this study, the Coulomb law of friction was used to calculate the frictional force between the micro-crack surfaces. Figure 11 shows the acoustic nonlinearity parameter versus the friction coefficient. We can see that the acoustic nonlinearity parameter slightly increased with the increase of the friction coefficient. S_0 mode is more like a longitudinal wave in essence, which does not induce relatively severe sliding between the micro-crack surfaces, and thus does not cause strong friction behaviors [34].

Different from the classic acoustic nonlinearity parameter, the acoustic nonlinearity parameter caused by micro-cracks is dispersive. Figure 12 shows the

acoustic nonlinearity parameter versus the excitation frequency. It can be seen that the acoustic nonlinearity parameter of the x direction increased linearly with the excitation frequency, while the parameter of the y direction increased nonlinearly with the excitation frequency. The main reason is that the wavelength of S_0 mode decreases when the excitation frequency increases, and the wave with a smaller wavelength is more sensitive to the micro-crack. Therefore, the nonlinear interactions of Lamb waves with a smaller wavelength with the micro-cracks became stronger, and the second harmonic effect was more noticeable.

Fig. 9. Acoustic nonlinearity parameter versus crack density ($f = 200\text{kHz}$, $\mu = 0.1$)

Fig. 10. Acoustic nonlinearity parameter versus the propagation distance in the micro-crack

damage zone ($c = 0.015$, $f = 200\text{kHz}$, $\mu = 0.1$)

Fig. 11. Acoustic nonlinearity parameter versus friction coefficient ($c = 0.015$, $f = 200\text{kHz}$)

Fig. 12. Acoustic nonlinearity parameter versus excitation frequency ($c = 0.015$, $\mu = 0.1$)

5.3 Reflected waves

At present, most studies are focused on the second harmonic effect of transmitted waves. Although the second harmonic waves can be used to identify the damage of materials, it cannot precisely locate the damage region. The reason is that the second harmonic waves are obtained from frequency analysis and thus have no time-domain characteristic. To locate the micro-crack damage zone, the reflected waves were collected at position 1 as shown in Fig. 2. It is challenging to directly obtain the

reflected wave signals because it overlaps with the fundamental waves in the time domain. To solve this problem, the wave signals $X_0(t)$ obtained in the non-cracked case were set as the baseline data, and the wave signals obtained in the cracked case were marked as $X(t)$. Therefore, the reflected waves were obtained by $X_e(t) = X(t) - X_0(t)$.

The reflected wave signals in this study were considered as the amplitudes of wave motion in the x direction. Figure 13 shows the reflected wave signals in the time domain and frequency domain. It can be seen that the reflected wave signals were consistent even at different positions. Hence, the approximate position of the micro-crack damage zone could be estimated from the arrival time of the reflected waves. Compared with the transmitted wave signals in the x direction shown in Fig. 7, the intensity of the reflected wave signals was much smaller than that of the transmitted wave signals by one order of magnitude. However, it should be noted that noticeable second harmonic effect emerged in the reflected signals as shown in Fig. 13(b), and the acoustic nonlinearity parameter of reflected waves was clearly larger than that of transmitted waves as presented in Table 2. The results indicate that the reflected waves may be a better choice for identifying the micro-crack damage of materials.

(a)

(b)

Fig. 13. Reflected waves in the x direction: (a) time domain; (b) frequency domain

$$(c = 0.025, f = 200\text{kHz}, \mu = 0.1)$$

Table. 2. Comparison of acoustic nonlinearity parameter of transmitted waves and reflected waves

$$(c = 0.025, f = 200\text{kHz}, \mu = 0.1)$$

	β'
Transmitted wave (x -axis)	0.00456
Transmitted wave (y -axis)	0.01229
Reflected wave (x -axis)	0.16233

6. Conclusion

A numerical model containing randomly distributed micro-cracks was constructed to investigate the propagation phenomenon of low frequency S_0 Lamb waves by a Monte Carlo simulation method. Using the S_0 mode as fundamental waves, the ultrasonic nonlinear effect in terms of second harmonics caused by the micro-cracks was explored in details. The following conclusions have been drawn:

First, when the low frequency S_0 mode is used as fundamental waves, there was no second harmonic wave in thin plates without micro-cracks. It indicates that the low frequency S_0 mode can be used as fundamental waves to effectively identify micro-cracks in thin plates. Moreover, since the S_0 mode is the only mode existing

within the low excitation frequency domain, *i.e.*, 100-400 kHz, this technique can effectively avoid the interference of high-order modes, *e.g.*, S_1 and S_2 , thus increasing the signal-to-noise ratio.

Second, the influence of micro-cracks on the propagation of the S_0 fundamental wave was insignificant although its normalized phase velocity decreased linearly with the micro-crack density. At the same time, the equality condition between the phase velocity of the fundamental wave and that of the second harmonic wave could be achieved within the low frequency domain, *e.g.*, 100-800 kHz, which makes the quantitative analysis of the relationship between the generated second harmonics and the extent of material nonlinearity possible.

Third, we systematically investigated the relationship between the acoustic nonlinear parameter and some key factors, *i.e.*, micro-crack density, excitation frequency, friction coefficient of the micro-crack surfaces, and propagation distance in the cracking region. The results reveal that, the acoustic nonlinear parameter in the y -axis was much stronger than that in the x -axis; and it was linearly proportional to the crack density and propagation distance of waves in the cracking region. The acoustic nonlinear parameter increased significantly with the excitation frequency. However, it was not correlated with the friction coefficient of the micro-crack surfaces. Therefore, the acoustic nonlinear parameter can be used to effectively characterize the degeneration of material properties caused by micro-cracks. Furthermore, a proper increase of the excitation frequency can generate more significant second harmonics and improve the efficiency of this technique.

Finally, weak reflected waves in the x -axis could be induced by micro-cracks. We found that the region containing micro-cracks could be located by detecting these reflected waves at different positions. Although the intensity of the reflected waves was lower than that of the transmitted waves by one order of magnitude, these reflected waves contained more significant second harmonics. This finding provides a theoretical foundation for effective detection of micro-crack locations. We will further verify the results obtained in the present study by using an experimental measurement technique similar to that [19].

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