Guided wave phased array beamforming and imaging in composite plates

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Abstract: This paper describes phased array beamforming using guided waves in anisotropic composite plates. A generic phased array algorithm is presented, in which direction dependent guided wave parameters and the energy skew effect are considered. This beamforming at an angular direction is achieved based on the classic delay-and-sum principle by applying phase delays to signals received at array elements and adding up the delayed signals. The phase delays are determined with the goal to maximize the array output at the desired direction and minimize it otherwise. For array characterization, the beam pattern of rectangular grid arrays in composite plates is derived. In addition to the beam pattern, the beamforming factor in terms of wavenumber distribution is defined to provide intrinsic explanations for phased array beamforming. The beamforming and damage detection in a composite plate are demonstrated using rectangular grid arrays made by a non-contact scanning laser Doppler vibrometer. Detection images of the composite plate with multiple surface defects at various directions are obtained. The results show that the guided wave phased array method is a potential effective method for rapid inspection of large composite structures.

Keywords: guided waves; phased arrays; anisotropic composite; array imaging; multiple defects; rapid inspection

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1. INTRODUCTION

Rapid inspection of large areas with nondestructive evaluation (NDE) methods is critical for ensuring operation ability and safety in aerospace industry, especially where safety related structural components are used [1, 2]. Advanced composite materials have been increasingly used in aerospace industry. The Boeing 787 Dreamliner has an airframe comprising nearly 50% carbon fiber reinforced plastic and other composites [3]. Though various NDE methods have been developed for metallic structures and proven effective, reliable and efficient evaluations for large composite structures are not yet well established [1, 4]. Complexity of the advanced composite material manufacturing and in-service maintenance present challenges in evaluation tools and methods [4].

Ultrasonic NDE is the technique to provide an invasive means to inspect the condition of a component. Among various NDE methodologies, ultrasonic method is directly sensitive to mechanical changes and can be used to directly assess the mechanical condition and integrity of the composite structure [4]. However it is commonly considered that NDE using bulk waves is time-consuming since it requires point-by-point measurement over the inspected area and therefore is not efficient for large area inspection. To address this deficiency, the guided waves (GW) based ultrasonic techniques have been studied, developed, and demonstrated great potentials on metallic structures [5-8]. Compared to bulk waves, GW can travel long distances within the waveguides with low energy loss [9, 10]. However, GW NDE is facing major challenges when being applied on composite structures [4]. The GW complexity caused by anisotropic and inhomogeneous properties in composite materials makes the traditional metal-based NDE methods inappropriate and sometimes even misleading [11-13].

When generating inspection results, an image of the structure being inspected often gives an efficient solution that quickly identifies and locates defects. Various imaging methods based on GW NDE have been explored including tomography [6, 14-16], sparse array [7, 17-24], synthetic aperture focusing technique [25-27], reconstruction algorithm for probabilistic inspection of damage (RAPID) [5, 28-30], and phased array [8, 31-64]. Among them, the phased array imaging is attractive since it uses sensors that are placed closed to each other in a compact format, steers the outputs of all sensors in a desired direction, and inspects the entire structure like a radar [31]. It hence allows for rapid inspection of large area with limited access. The additional advantages include reinforced wave energy in the steered direction, efficient and flexible control of the direction, improved signal-to-noise-ratio, and promising damage detection results [31].

Intensive study has been conducted on the GW phased array beamforming and damage detection on metallic plate like structures [8, 31-58]. A general beamforming algorithm for isotropic materials has been developed [31, 32] with investigation of beamforming optimization [33-36]. 1-D linear [37, 38] arrays and 2-D planar arrays [8, 31, 39] in various configurations have been designed and used for damage detection of hole or cracks with the arrays made of piezoelectric wafer sensors [31], piezoelectric paint sensors [39], or electromagnetic acoustic transducer (EMAT) [8].

Some researchers have started investigating the phased arrays for anisotropic composite materials [59-64]. Yan and Rose studied beamsteering of linear arrays in composite plates [59]. They found the traditional beamsteering technique for isotropic materials might fail in composite materials due to the anisotropic behaviors of the composite plates. Hence, they chose a quasi-

isotropic wave mode for beamsteering, which can suppress the influence of the anisotropic behavior. Rajagopalan et al. adopted an array of a single transmitter and multiple receivers (STMR) to locate a defect (hole) in a composite plate [60]. In their array imaging method, they used a weakly anisotropic wave mode and assumed that the phase and group velocity directions coincide locally. Later, Vishnuvardhan et al. used the STMR array to detect impact induced delamination damage in a quasi-isotropic composite plate [61]. Leleux et al. used ultrasonic phased array probes for long range detection of defects in composite plates [62]. Their method was limited to wave modes having phase and group velocities oriented in the same direction, where the skew angle was zero. Purekar and Pines investigated the capability of 1-D linear phased arrays in detecting delamination damage in a cross-ply composite plate [63]. They showed the array can detect damage at 0° direction, where the phase and group velocities had the same orientation. Osterc et al. investigated the beamsteering of 1-D linear arrays in composite laminates [64]. In their study, the exact phase velocity curve was used to develop a beamforming algorithm that accounted for non-omnidirectional guided wave propagation in anisotropic materials. It has found that compared to the array beamforming in isotropic plates, the array beamforming in anisotropic composite plates are more challenging. In anisotropic composite plates, guided wave parameters such as wavenumbers, phase velocities and group velocities are direction dependent due to the direction dependent physical properties of composite materials [11-13]. Moreover, the GW have energy skewness that the direction of group velocity is not always aligned to that of the phase velocity. Last but not least, the wave fronts of GW are no longer circular in composites. The traditional beamforming technique for isotropic materials may fail in composite materials due to the complexity involved with GW propagation.

In this paper, we investigate GW beamforming in anisotropic laminated composite plates. Based on the classic delay-and-sum principle, a generic formula of phased array beamforming in anisotropic composite plates is developed, in which the direction dependent guided wave properties are adopted. The beamforming is demonstrated by implementation of 2-D rectangular grid arrays. Beamforming with various array configurations of rectangular grid arrays are investigated. For the proof of concept, laboratory tests are performed using rectangular grid arrays made of scanning points of a non-contact scanning laser Doppler vibrometer (SLDV) for detecting surface defects. The results show that multiple defects at various directions can be successfully detected and the phased array method can be useful for rapid inspection of large composite structures. The remainder of this paper is organized as follows: Section 2 presents the formulation of phased array beamforming in anisotropic composite plates; Section 3 presents beamforming characterization of 2-D rectangular grid arrays; Section 4 presents the implementation and detection of multiple defects in a composite plate using arrays made of scanning points of a non-contact SLDV. Section 5 concludes the paper with novelties, discussions and planned future work.

2. GW BEAMFORMING IN COMPOSITE LAMINATES

In this section, we formulate general GW phased array beamforming in anisotropic composite laminates based on the classic delay-and-sum principle.

2.1. GW in composite laminates

When a guided wave with frequency ω and wavenumber $\mathbf{k}$ is generated from a source at the coordinate origin O in a composite plate, the wave arriving at the location $\mathbf{x}$ that is far away from the source (Figure 1a) can be expressed as [9, 10, 13],

$$u(t, \mathbf{x}) = Ae^{j(\omega t - \mathbf{k} \cdot \mathbf{x})} \quad (1)$$

where A is the amplitude, assuming independent of wave frequency. With the geometric relation illustrated in Figure 1a, we have,

$$\mathbf{k} \cdot \mathbf{x} = |\mathbf{k}| |\mathbf{x}| \cos \beta = k(\gamma) |\mathbf{x}| \cos \beta \quad (2)$$

with β being the angle between the wave propagation and wavenumber $\mathbf{k}$. Hence,

$$u(t, \mathbf{x}) = Ae^{j[\omega t - k(\gamma) |\mathbf{x}| \cos \beta]} \quad (3)$$

Using Eq.(1), for a source located at location $\mathbf{p}_m$, the wave resulted at the location $\mathbf{x}$ is,

$$u_m(t, \mathbf{x}) = Ae^{j[\omega t - \mathbf{k} \cdot (\mathbf{x} - \mathbf{p}_m)]} \quad (4)$$

In anisotropic composite laminates, GW parameters such as wavenumbers, phase velocities and group velocities are direction dependent, due to the direction dependent physical properties of composite materials [11-13]. Figure 1b plots the wavenumber curve $k(\gamma)$ and slowness curve $k(\gamma)/\omega$. As illustrated in Figure 1b, the wavenumber vector $\mathbf{k}$ is perpendicular to the wave front and the group velocity vector $\mathbf{c}_g$ is orthogonal to the wavenumber curve $k(\gamma)$ [11-13]. The angle γ of the wavenumber vector $\mathbf{k}$ is referred to as wavenumber angle. The angle θ of the group velocity vector $\mathbf{c}_g$ is referred to as group velocity angle (or energy propagation angle). The angle β between wavenumber angle γ and energy propagation angle θ is referred to as skew angle, with the relation $\beta = \gamma - \theta$. It can be seen when $\mathbf{c}_g$ is not parallel to $\mathbf{k}$, the skew angle β is not zero and hence the wave energy propagation direction is not perpendicular to the wave front.

2.2. Delay-and-sum beamforming

Consider an array with M identical elements located at $\{\mathbf{p}_m\}$ ($m=0, 1, 2, \dots, M-1$) which are geometrically close to each other. The phase center is defined as the origin O of the Cartesian

coordinate system, i.e., $\frac{1}{M} \sum_{m=0}^{M-1} \mathbf{p}_m = \mathbf{0}$. Each element serves as a wave source. When all elements generate waves with frequency ω and wavenumber vector $\mathbf{k}$ simultaneously, using Eq. (4) the total output (synthesized wave) of the array at location $\mathbf{x}$ can be derived as,

$$z(t, \mathbf{x}) = \sum_{m=0}^{M-1} A e^{j[\omega t - \mathbf{k} \cdot (\mathbf{x} - \mathbf{p}_m)]} = u(t, \mathbf{x}) \sum_{m=0}^{M-1} e^{j\mathbf{k} \cdot \mathbf{p}_m} \quad (5)$$

It is seen from Eq. (5) that the synthesized wave $z(t, \mathbf{x})$ is an amplification of the wave $u(t, \mathbf{x})$ emitted from the Origin. The amplification is controlled by the exponential component $\sum_{m=0}^{M-1} e^{j\mathbf{k} \cdot \mathbf{p}_m}$ in Eq. (5). Therefore, by adjusting the component $\sum_{m=0}^{M-1} e^{j\mathbf{k} \cdot \mathbf{p}_m}$, we can control the amplification. One way to adjust the component $\sum_{m=0}^{M-1} e^{j\mathbf{k} \cdot \mathbf{p}_m}$ is applying phase delays to all exponents. For example, to maximize the amplification $\sum_{m=0}^{M-1} e^{j\mathbf{k} \cdot \mathbf{p}_m}$ in a specific direction θ_s , we can apply phase delays $\Delta_m(\theta_s)$ to exponents and let all exponents be zero,

$$\sum_{m=0}^{M-1} e^{j[\mathbf{k} \cdot \mathbf{p}_m - \Delta_m(\theta_s)]} = \sum_{m=0}^{M-1} e^{j0} = M, \text{ where } \Delta_m(\theta_s) = \mathbf{k} \cdot \mathbf{p}_m \quad (6)$$

Using phase delays $\Delta_m(\theta_s)$, the amplification is maximized at the direction θ_s , i.e., generating a directional “beam”. θ_s is also known as steering angle. Eq. (6) shows phase delays $\Delta_m(\theta_s)$ depend on the m^{th} element’s position vector $\mathbf{p}_m$ and wavenumber vector $\mathbf{k}$.

In anisotropic composites, the wavenumber vector $\mathbf{k}$ depends on wave frequency ω and wavenumber angle γ_s . In addition, the geometry relation between wavenumber angle γ_s and steering angle θ_s is $\gamma_s = \theta_s + \beta_s$, where β_s is referred to as skew angle. Hence, phase delays $\Delta_m(\theta_s)$ in Eq.(6) can be further expressed as,

$$\Delta_m(\theta_s) = \mathbf{k}(\omega, \theta_s + \beta_s) \cdot \mathbf{p}_m \quad (7)$$

It can be seen from Eq.(7) the phase delay includes a frequency dependent term, $\mathbf{k}(\omega, \theta_s + \beta_s)$. Hence, it is frequency dependent and has the advantage of compensating dispersion effect (a.k.a. frequency dependent wave properties) by using frequency-related components during the delaying [8, 66]. This is otherwise not readily achievable in the commonly used time delay methods [31, 37, 59].

In addition to phase delays, weighting factors w_m can also be applied to delayed waves to further control the quality of beamforming [9, 65]. In a summary, with phase delays and weighting factors, the beamforming is represented as,

$$z(t, \mathbf{x}) = u(t, \mathbf{x}) \sum_{m=0}^{M-1} w_m e^{j[\mathbf{k} \cdot \mathbf{p}_m - \Delta_m(\theta_s)]} = u(t, \mathbf{x}) \sum_{m=0}^{M-1} w_m e^{j[\mathbf{k} - \mathbf{k}(\omega, \theta_s + \beta_s)] \cdot \mathbf{p}_m} \quad (8)$$

3. CHARACTERIZATION OF RECTANGULAR GRID ARRAYS

This section investigates 2-D rectangular grid arrays in composite plates. For array characterization, beamforming factor in terms of wavenumber and beam pattern in terms of wave propagation direction are used. With the beamforming factor and beam pattern, beamforming of rectangular grid arrays with different configurations are characterized and investigated in details.

3.1. Rectangular grid arrays

Elements in phased arrays can be arranged with various configurations, such as linear [37], rectangular [31], or spiral [39] arrangements. In this study, rectangular grid arrays with elements being uniformly placed on rectangular grids are investigated. Figure 2 illustrates a $P \times Q$ rectangular grid array (total number of elements: $M = P \times Q$) with its phase center as the coordinate origin O . The coordinates of the $(p, q)^{\text{th}}$ ($p = 0, 1, 2, \dots, P-1$ and $q = 0, 1, 2, \dots, Q-1$) element in the array are,

$$\mathbf{p}_{p,q} = \left(\left(p - \frac{P-1}{2} \right) d_x, \left(q - \frac{Q-1}{2} \right) d_y \right) \quad (9)$$

where d_x and d_y are array spacings in x and y directions, respectively. The array spans in x and y directions then are given as,

$$D_x = (P-1)d_x \quad \text{and} \quad D_y = (Q-1)d_y \quad (10)$$

The beamforming is implemented on an 8-ply $[0/45/90/-45]_s$ layup carbon fiber reinforced polymer (CFRP) composite plate (material properties in Table 1). The A_0 Lamb mode at 120 kHz is used for the beamforming. Figure 3a, 3b and 3c plot the wavenumber, slowness and phase velocity curves derived by using the semi-analytical finite element (SAFE) method [67]. The maximum wavenumber components in k_x and k_y directions are $k_{x,\max} = 0.55$ rad/mm and $k_{y,\max} = 0.68$ rad/mm, resulting in the minimum wavelengths $\lambda_{x,\min} = 2\pi/k_{x,\max} = 11.4$ mm and $\lambda_{y,\min} = 2\pi/k_{y,\max} = 9.2$ mm.

To study the beamforming, three array configurations (A1, A2, A3) listed in Table 2 are considered. Among them, (a) A1 and A2 have the same spacing, while the spans in A2 are twice of those in A1; (b) A2 and A3 have the same span, while the spacings in A3 are twice of those in A2.

3.2. Characterization of rectangular grid arrays

For beamforming characterization, the beamforming factor (BF) [9, 31] is adopted, which is given as,

$$BF = \frac{1}{M} \sum_{m=0}^{M-1} w_m e^{j[\mathbf{k} \cdot \mathbf{p}_m - \Delta_m(\theta_s)]} \quad (11)$$

Substituting Eq. (7) into Eq. (11), the beamforming factor becomes,

$$BF(\mathbf{k} | w_m, \theta_s) = \frac{1}{M} \sum_{m=0}^{M-1} w_m e^{j[\mathbf{k}-\mathbf{k}(\omega, \theta_s + \beta_s)] \cdot \mathbf{p}_m} \quad (12)$$

The beamforming factor in Eq. (12) can be interpreted as a function of wavenumber vector $\mathbf{k}$, denoted as $BF(\mathbf{k} | w_m, \theta_s)$ in the wavenumber domain. θ_s and w_m represent two parameters that can control the beamforming direction and beam shape. For 2-D GW, $BF(\mathbf{k} | w_m, \theta_s)$ evaluates the beamforming result at any wavenumber vector $\mathbf{k}$ in the k_x - k_y wavenumber plane.

Besides $BF(\mathbf{k} | w_m, \theta_s)$, a directional beam pattern is also introduced, which is a function of the wave propagation direction θ as,

$$BF(\theta | w_m, \theta_s) = \frac{1}{M} \sum_{m=0}^{M-1} w_m e^{j[\mathbf{k}(\omega, \theta + \beta) - \mathbf{k}(\omega, \theta_s + \beta_s)] \cdot \mathbf{p}_m} \quad (13)$$

where $\mathbf{k}(\omega, \theta + \beta)$ is the wavenumber dispersion relation of GW. The directional beam pattern $BF(\theta | w_m, \theta_s)$ evaluates the beamforming output *w.r.t.* the wave propagation direction θ .

3.2.1. Characterization of rectangular grid arrays using $BF(\mathbf{k} | w_m, \theta_s)$

Beamforming of three rectangular grid arrays in Table 2 is investigated by the beamforming factor $BF(\mathbf{k} | w_m, \theta_s)$ given in Eq.(12). By substituting element coordinates in Eq.(9) into Eq.(12), we can derive $BF(\mathbf{k} | w_{p,q}, \theta_s)$ for rectangular grid arrays,

$$BF(\mathbf{k} | w_{p,q}, \theta_s) = \frac{1}{P \cdot Q} \sum_{p=0}^{P-1} \sum_{q=0}^{Q-1} w_{p,q} e^{j[\mathbf{k}-\mathbf{k}(\omega, \theta_s + \beta_s)] \cdot \left((p - \frac{P-1}{2})d_x, (q - \frac{Q-1}{2})d_y \right)} \quad (14)$$

By substituting array spans in Eq.(10) into Eq.(14), $BF(\mathbf{k} | w_{p,q}, \theta_s)$ becomes,

$$BF(\mathbf{k} | w_{p,q}, \theta_s) = \frac{1}{\left(\frac{D_x}{d_x} + 1 \right) \left(\frac{D_y}{d_y} + 1 \right)} \sum_{p=0}^{\frac{D_x}{d_x}} \sum_{q=0}^{\frac{D_y}{d_y}} w_{p,q} e^{j[\mathbf{k}-\mathbf{k}(\omega, \theta_s + \beta_s)] \cdot \left(pd_x - \frac{D_x}{2}, qd_y - \frac{D_y}{2} \right)} \quad (15)$$

This equation shows the beamforming factor $BF(\mathbf{k} | w_{p,q}, \theta_s)$ also depends on the weighting factor $w_{p,q}$, steering angle θ_s , and array configuration parameters d_x , d_y , D_x and D_y .

In this study, uniform weighting ($w_{p,q} = 1$) is used. Figure 4a plots $BF(\mathbf{k} | w_{p,q} = 1, \text{no})$ of array A1 without applying delays as an intensity image in the k_x - k_y wavenumber plane. Four highlighted spots at $(0, 0)$, $(0, 4k_{y,\max})$, $(4k_{y,\max}, 0)$ and $(4k_{y,\max}, 4k_{y,\max})$ are present in the given region. These spots represent the local maxima of $BF(\mathbf{k} | w_{p,q} = 1, \text{no})$. When the array generates GW with wavenumbers at these local maxima, the array's output will be optimized. The sizes of spots are evaluated by the full width at one-half peak value (denoted as $FWHM_x$ and $FWHM_y$ for k_x and k_y directions, respectively) [65], given in Table 3. Smaller $FWHM$ signifies higher resolution as well as better directionality.

Note that Figure 4a only plots $BF(\mathbf{k} | w_{p,q} = 1, \text{no})$ in the wavenumber range $-1.0 \text{ rad/mm} \leq k_x \leq 3.7 \text{ rad/mm}$ and $-1.0 \text{ rad/mm} \leq k_y \leq 3.7 \text{ rad/mm}$. Indeed highlighted spots are present in a periodical pattern within the entire wavenumber domain. The periodical pattern can be perceived from the expression of beamforming factor in Eq.(15). The periods K_x and K_y of the periodical pattern in k_x and k_y directions are $2\pi/d_x$ and $2\pi/d_y$, respectively.

A wavenumber curve $k(\gamma)$ of the 120 kHz A_0 mode in the subject composite plate is also plotted in Figure 4a. It can be seen that no maxima of $BF(\mathbf{k} | w_{p,q} = 1, \text{no})$ falls on the wavenumber curve of the adopted wave mode. This indicates that if the array generates the 120 kHz A_0 mode without delaying, the array will not have maximized output.

In order to achieve maximized array output, phase delays are applied to relocate certain maxima on the wavenumber curve. Figure 4b plots $BF(\mathbf{k} | w_{p,q} = 1, \theta_s = 90^\circ)$ of array A1 with

phase delays of $(-0.02, 0.68) \cdot \mathbf{p}_{p,q}$. The delays are selected such that the local maxima located at $(0, 0)$ before the delays moves to the point $(-0.02, 0.68)$ rad/mm on the wavenumber curve.

Figure 5a and 5b plot $BF(\mathbf{k} | w_{p,q} = 1, \theta_s = 90^\circ)$ images of arrays A2 and A3, when phase delays $(-0.02, 0.68) \cdot \mathbf{p}_{p,q}$ are applied. *FWHM* values (in Table 3) of arrays A2 and A3 (with the same span) are the same and both smaller than those of array A1 (with a smaller span). This indicates the larger array span gives the smaller *FWHM* value.

Wavenumber periods of arrays A1, A2 and A3 are also derived, as listed in Table 3. The periods of arrays A1 and A2 (with the same array spacing) are the same, and both larger than those of array A3 (with a larger array spacing). Therefore, the smaller array spacing gives the larger wavenumber period.

The wavenumber period can affect the beamforming performance. For example, for the array A3, the $BF(\mathbf{k} | w_{p,q} = 1, \theta_s = 90^\circ)$ image in Figure 5b shows there are two intensified spots at $(-0.02, 0.68)$ rad/mm and $(-0.02, -0.68)$ rad/mm on the wavenumber curve $k(\gamma)$ of the 120 kHz A_0 mode. This means if array A3 generates a 120 kHz A_0 mode with phase delays $(-0.02, 0.68) \cdot \mathbf{p}_{p,q}$, the synthesized waves generated from the array will have two intensified components: (1) waves with the wavenumber $(-0.02, 0.68)$ rad/mm, and (2) waves with the wavenumber $(-0.02, -0.68)$ rad/mm, contradicting the single beam intention. Hence situations with more than one intensified components should be avoided since they give misleading beamforming results. Therefore, wavenumber periods should satisfy $K_x > 2k_{x,\max}$ and $K_y > 2k_{y,\max}$, i.e., the array spacings should satisfy $d_x < \lambda_{x,\min}/2$ and $d_y < \lambda_{y,\min}/2$.

3.2.2. Characterization of rectangular grid arrays using $BF(\theta | w_m, \theta_s)$

The beamforming can also be characterized in terms of the directional beam pattern $BF(\theta | w_m, \theta_s)$ given in Eq. (13). By substituting element coordinates in Eq.(9) into Eq.(13), we can derive,

$$BF(\theta | w_{p,q}, \theta_s) = \frac{1}{PQ} \sum_{p=0}^{P-1} \sum_{q=0}^{Q-1} w_{p,q} e^{j[\mathbf{k}(\omega, \theta + \beta) - \mathbf{k}(\omega, \theta_s + \beta_s)] \cdot \left((p - \frac{P-1}{2})d_x, (q - \frac{Q-1}{2})d_y \right)} \quad (16)$$

Or using array span and spacing as,

$$BF(\theta | w_{p,q}, \theta_s) = \frac{1}{\left(\frac{D_x}{d_x} + 1 \right) \left(\frac{D_y}{d_y} + 1 \right)} \sum_{p=0}^{\frac{D_x}{d_x}} \sum_{q=0}^{\frac{D_y}{d_y}} w_{p,q} e^{j[\mathbf{k}(\omega, \theta + \beta) - \mathbf{k}(\omega, \theta_s + \beta_s)] \cdot \left(pd_x - \frac{D_x}{2}, qd_y - \frac{D_y}{2} \right)} \quad (17)$$

Eq.(17) shows the beam pattern $BF(\theta | w_{p,q}, \theta_s)$ also depends on the weighting factor $w_{p,q}$, steering angle θ_s , and array configuration parameters d_x , d_y , D_x and D_y .

Figure 6a plots the directional beam pattern $BF(\theta | w_{p,q} = 1, \text{no})$ of array A1 using the 120 kHz A_0 mode without applying phase delays. The amplitude is seen low at all directions. This is consistent with $BF(\mathbf{k} | w_{p,q} = 1, \text{no})$ image in Figure 4a, where on maxima is on the dispersion curve of A_0 mode. To maximize the amplitude to a desired direction θ_s , phase delays are applied. Figure 6b plots beamsteering results toward 0° , 45° , 90° and 135° directions, by applying delays of $(0.55, -0.02) \cdot \mathbf{p}_{p,q}$, $(0.37, 0.51) \cdot \mathbf{p}_{p,q}$, $(-0.02, 0.68) \cdot \mathbf{p}_{p,q}$ and $(-0.37, 0.54) \cdot \mathbf{p}_{p,q}$, respectively.

To evaluate beamforming qualities at different directions, $FWHM$ values of array A1 are determined as given in Table 4, where the 0° beamforming has the smallest $FWHM$ value, thus, the best resolution among the four directions.

Directional beam patterns of arrays A2 and A3 at $\theta_s = 0^\circ, 45^\circ, 90^\circ$ and 135° have also been studied as given in Figure 7a and 7b. For $0^\circ, 45^\circ$ and 135° beamsteering angles, beam patterns of array A2 are nearly the same as those of array A3. However, for the 90° beamsteering direction, the beam pattern of array A3 in Figure 7b (dotted line) has an additional lobe at 265° with nearly the same amplitude and shape as the mainlobe at 90° . This spurious mainlobe (also known as grating lobe [9, 65]) is induced by large element spacing. The appearance of grating lobe in Figure 7b can also be confirmed in the $BF(\mathbf{k} | w_{p,q} = 1, \theta_s = 90^\circ)$ image (Figure 5b) where an additional intensified spot at $(-0.02, -0.68)$ rad/mm shows on the wavenumber curve.

The *FWHM* values of all arrays are also derived and compared in Table 4. *FWHM* values of arrays A2 and A3 are the same while smaller than those of array A1. Hence, arrays A2 and A3 offer better resolutions than array A1.

4. SLDV GW PHASED ARRAY

In this section, guided wave beamforming and damage detection in a laminated composite plate using rectangular grid arrays are implemented. A hybrid measurement system consisting of a surface bonded PZT wafer and a non-contact scanning laser Doppler vibrometer (SLDV) is used for GW actuation and sensing [68]. Phased arrays are constructed by selected points of a small SLDV scanning area. Using selected scanning points of the SLDV scan gives the flexibility of constructing arrays of various configurations with only one scan. The element spacing in SLDV arrays can be smaller than 0.1 mm [68], while the spacing in traditional arrays such as piezoelectric wafer arrays is usually limited by the sensor size. Four quartz rods are surface bonded on a composite plate at various angular positions as defects to be detected by the phased array method.

4.1. Experimental setup

Figure 8 shows the experimental setup of the GW sensing in a composite plate. The test specimen is an 8-ply $[0/45/90/-45]_s$ layup CFRP composite plate with dimensions of $610 \text{ mm} \times 610 \text{ mm} \times 2.54 \text{ mm}$. The material properties are listed in Table 1. A PZT wafer (APC 851: 7 mm diameter, 0.2 mm thickness) is installed to generate GW. The PZT center is set as the coordinate origin. GW are excited by a 3-cycle toneburst at 120 kHz generated from a function generator (model: Agilent 33522B) and amplified to 30V by a voltage amplifier (model: Krohn-Hite 1506). Four identical defects are simulated by bonding quartz rods of 10 mm high and 8 mm diameter (Q_1 , Q_2 , Q_3 and Q_4) on the plate surface at different angles 0° , 45° , 90° and 135° , as shown in Figure 8b. All rods are placed 100 mm away from the coordinate origin.

An SLDV (model: Polytec PSV-400-M2) is used to acquire the velocity wavefield of GW over a small scanning area ($45 \text{ mm} \times 45 \text{ mm}$ square) centered at the coordinate origin from the back side of the plate. From the small scanning area, selected scanning points are used to construct phased arrays for damage detection. The horizontal and vertical spatial resolutions of the scanning are both 0.1 mm. Based on the Doppler effect, the SLDV measures the GW velocity $v(t, \mathbf{x})$ along the laser beam over the scanning area, as a function of both time t and space $\mathbf{x}$. In the test, the laser beam is set normal to the plate such that the out-of-plane velocity is acquired.

4.2. SLDV measurements

Figure 9a and 9b plot wavefields in the $45 \text{ mm} \times 45 \text{ mm}$ scanning area measured by the SLDV, at $30 \mu\text{s}$ showing incident waves generated from the actuator and at $145 \mu\text{s}$ showing reflection waves from the four defects, respectively. For identifying the wave mode, wavenumber spectra of these wavefields are obtained by frequency-wavenumber analysis as detailed in [68-70]. The

theoretical wavenumber curve of A_0 mode derived by using the SAFE method is provided to confirm the existence of A_0 wave mode. Both spectra show their wavenumber components are on the curve of A_0 mode. The comparisons in Figure 9c and 9d verify the theoretical wavenumber curve derived by SAFE method. Moreover, the comparison confirms that both incident and reflection waves are A_0 mode.

4.3. Array beamsteering and imaging

The phased array is constructed using SLDV scanning points at selected locations $\{\mathbf{p}_m\}$ ($m=0, 1, 2, \dots, M-1$), whose phase center satisfies $\frac{1}{M} \sum_{m=0}^{M-1} \mathbf{p}_m = \mathbf{0}$. From the time-space wavefield $v(t, \mathbf{x})$ acquired by the SLDV, the signal at the m^{th} array point ($\mathbf{p}_m$) can be denoted as $v_m(t) = v(t, \mathbf{p}_m)$. Its frequency spectrum can be derived using the Fourier transform, as:

$$V_m(\omega) = \mathcal{F}[v_m(t)] = \int_{-\infty}^{\infty} v_m(t) e^{-j\omega t} dt \quad (18)$$

Using the frequency spectrum $V_m(\omega)$, we can derive the beamforming of the array in frequency-space representation $Z(\omega, \mathbf{x})$:

$$Z(\omega, \mathbf{x}) = \sum_{m=0}^{M-1} w_m V_m(\omega) e^{j[-\varphi(\omega, \mathbf{x}) - \Delta_m(\omega, \mathbf{x})]} \quad (19)$$

where,

$$\Delta_m(\omega, \mathbf{x}) = \mathbf{k}(\omega, \gamma) \cdot \mathbf{p}_m, \text{ and } \varphi(\omega, \mathbf{x}) = -2\mathbf{k}(\omega, \gamma) \cdot \mathbf{x} \quad (20)$$

$\Delta_m(\omega, \mathbf{x})$ is the phase delay applied to the m^{th} array point for beamsteering. $\varphi(\omega, \mathbf{x})$ represents the spatial phase shift. As guided waves travel from the PZT to the damage and then back to the array, they undergo a spatial phase shift $\varphi(\omega, \mathbf{x})$. Thus, $-\varphi(\omega, \mathbf{x})$ is applied in Eq. (19) in order to compensate such a spatial phase shift. The spatial phase shift $\varphi(\omega, \mathbf{x})$ and the phased delay $\Delta_m(\omega, \mathbf{x})$ both include the frequency dependent term $\mathbf{k}(\omega, \gamma)$. Hence, by considering the frequency

dependence, the dispersion effect (frequency dependent wave properties) can then be taken into consideration.

Using inverse Fourier transform, the frequency-space representation $Z(\omega, \mathbf{x})$ is transformed back to the time-space domain, as:

$$z(t, \mathbf{x}) = \mathcal{F}^{-1}[Z(\omega, \mathbf{x})] = \frac{1}{2\pi} \int_{-\infty}^{\infty} Z(\omega, \mathbf{x}) e^{j\omega t} d\omega \quad (21)$$

where $z(t, \mathbf{x})$ represents the array beamforming in time-space representation. An inspection image of the plate is then acquired at $z(t=0, \mathbf{x})$ or building an intensity image by defining the pixel value at location $\mathbf{x}$ as:

$$I(\mathbf{x}) = |z(t=0, \mathbf{x})| \quad (22)$$

4.4. Array imaging results

From the wavefields measured in the scanning area by the SLDV, multiple scanning points can be selected to construct phased arrays. The selected scanning points are serving as sensing elements of phased arrays. By this means, we can construct phased arrays of different configurations by using scanning points at different locations.

The array performance is highly related to the array configuration, such as array spacing and number of elements, as presented previous work [31, 37]. For example, the array spacing is usually set at a half wavelength of the selected guided wave [31, 37]. With the same spacing, a larger number of elements gives better array performance though longer data acquisition time is needed [31]. With these considerations, three phased arrays (A1, A2, and A3 listed in Table 2) with different configurations (spacings and numbers of elements) are investigated, to show how guided wave phased array beamforming is affected by array configurations. Among them, (a) A1

and A2 have the same spacing, while the A2 has more elements than A1; (b) A2 and A3 have the same number of elements, while the spacing in A3 is twice of that in A2. The beamforming results of these three arrays have been theoretically studied in subsection 3.2. In this section, the three arrays are experimentally investigated.

From the 45 mm×45 mm scanning area, 21×21 points are chosen to construct the array A2 (its configuration is given in Table 2). Using the phased array method presented in subsection 3.2, a synthesized time-space wavefield $z(t, \mathbf{x})$ is constructed. The synthesized wavefield $z(t=0, \mathbf{x})$ at 0 μ s (in Figure 10a) shows four reconstructed wave packets in 0°, 45°, 90° and 135° directions.

Using the pixel definition given in Eq. (22), an intensity image of the plate is reconstructed using the synthesized wavefield $z(t=0, \mathbf{x})$, as shown in Figure 10b. It clearly indicates the presence of four defects at various directions as by presenting four highlighted areas in the image, with localization errors less than 5 mm. Please note the damage at 0° has the highest magnitude which is due to the fact that the guided waves have strongest propagation along this direction in the subject composite plate.

Figure 11a and 11b also give intensity images generated by arrays A1 and A3. By comparing the imaging results of arrays A1, A2 and A3 in Figures 11a, 10b and 11b, it can be found that arrays A2 and A3 give four intensified areas (indicating the four defects) with smaller sizes than array A1. It means the imaging results of arrays A2 and A3 have better resolution than array A1. This is consistent with the $BF(\theta|w_m, \theta_s)$ comparison in subsection 3.2, which shows that $BF(\theta|w_m, \theta_s)$ plots of arrays A2 and A3 have smaller *FWHM* and better angular resolution than array A1. Although arrays A2 and A3 nearly give the same imaging result at the four defects, the imaging result of array A3 shows an additional intensified area around the

location (-10, -100) mm. This additional area is induced by the spatial aliasing, and consistent with the $BF(\theta|w_m, \theta_s)$ plot of the array A3 (in Figure 7b) which shows a grating lobe at 265° direction.

5. CONCLUSIONS

In this paper, we have presented a generic guided wave phased array beamforming method for anisotropic composite laminates. The method is implemented with a non-contact SLDV guided wave sensing system and successfully detects multiple defects at various locations. Investigation on effects of array configurations on beamforming and damage detection has also been conducted.

The novelty of this work is multifold. First, the generic beamforming formula uniquely considers the direction-dependent guided wave properties and the energy skew effect; thus it allows for phased array beamforming in anisotropic composites without taking any quasi-isotropic assumption. Second, 2D phased array beamforming characterization in frequency domain has been conducted. The phase delay in frequency domain offers the advantage of compensating the guided wave dispersion effect through the use of frequency-dependent wavenumber. Third, the phased array method is validated through a non-contact SLDV system which allows for the flexibility of using selected points from the SLDV scan to construct phase arrays of various configurations. The high spatial resolution feature of the SLDV system also significantly reduces array element spacing that gives the potential of using guided waves with smaller wavelengths to detect smaller defects in composite structures. Last, we have shown that the present method can detect multiple defects at various locations in the composite plate, which is less studied in the literature to the best knowledge of the authors.

Despite the success of phased array beamforming and multiple defects detection in a composite plate presented here, further studies are still needed. Besides the selected 120 kHz A0 mode used in the current study, additional work needs to be done with guided waves at other frequencies or different wave modes in highly anisotropic composite plates. The effectiveness of the present method should be also evaluated with realistic defects such as impact induced delamination in composite plates. In addition, direction dependent guided wave attenuation in the composite plates should be investigated and considered in the beamforming. The potential of weighting factors to improve the beamforming quality could also be investigated.

6. REFERENCES

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Figure 1 Schematics of geometric relations of GW in composite laminates: (a) the geometric relation of GW (in the far field) generated by sources at different locations; (b) wavenumber and slowness curves. The geometric relation between the wavenumber vector $\mathbf{k}$ and the group velocity vector $\mathbf{c}_g$ is given in figure b.

Figure 2 Schematic of a $P \times Q$ rectangular grid array

Figure 3 (a) wavenumber, (b) slowness and (c) phase velocity plots of the 120 kHz A_0 mode in the $[0/45/90/-45]_s$ CFRP composite plate.

Figure 4 $BF(\mathbf{k} | w_{p,q}, \theta_s)$ images for the array A1: (a) $BF(\mathbf{k} | w_{p,q} = 1, \text{no})$ without phase delays, and (b) $BF(\mathbf{k} | w_{p,q} = 1, \theta_s = 90^\circ)$ with phase delays of $(-0.02, 0.68) \cdot \mathbf{p}_{p,q}$. The solid white curve is the wavenumber curve $k(\gamma)$ of the 120 kHz A_0 mode in the $[0/45/90/-45]_s$ CFRP composite plate.

Figure 5 $BF(\mathbf{k} | w_{p,q} = 1, \theta_s = 90^\circ)$ images when phase delays $(-0.02, 0.68) \cdot \mathbf{p}_{p,q}$ are applied: (a) for array A2, and (b) for array A3. The solid white curve is the wavenumber curve $k(\gamma)$ of the 120 kHz A_0 mode in the $[0/45/90/-45]_s$ CFRP composite plate.

Figure 6 $BF(\theta | w_{p,q}, \theta_s)$ plots for array A1: (a) $BF(\theta | w_{p,q} = 1, \text{no})$ plot without beamsteering, and (b) $BF(\theta | w_{p,q} = 1, \theta_s)$ plots when beamsteering angles are $\theta_s = 0^\circ, 45^\circ, 90^\circ$ and 135° .

Figure 8. Experimental setup for the detection of multiple defects in a CFRP composite laminate: (a) photo of test setup, (b) front side of the test specimen, and (c) schematic of the sensing layout.

Figure 9. SLDV measurements: (a) and (b) are wavefields in the $45 \text{ mm} \times 45 \text{ mm}$ scanning area at $30 \mu\text{s}$ showing incident waves and at $145 \mu\text{s}$ showing reflection waves, respectively; (c) and (d) are wavenumber spectra of incident and reflection waves at 120 kHz . The solid white line is the wavenumber curve of the A_0 mode at 120 kHz .

Figure 10. Phased array imaging results for array A2: (a) synthesized wavefield $z(t=0, \mathbf{x})$, and (b) intensity image $|z(t=0, \mathbf{x})|$.

Figure 11

Figure 11. Phased array imaging results: (a) for array A1, and (b) for array A3.

Table 1 Material properties of a single ply.

ρ (kg/m ³)	E ₁ (GPa)	E ₂ (GPa)	E ₃ (GPa)	G ₁₂ (GPa)	G ₁₃ (GPa)	G ₂₃ (GPa)	ν_{12}	ν_{13}	ν_{23}	Thickness (mm)
1577.8	129.6	8.7	8.7	4.8	4.8	2.9	0.30	0.30	0.34	0.3175

Table 2 Configurations of three rectangular arrays for beamforming study

	Number of elements $P \times Q$	Spacing d (mm)	Span D (mm)
Array A1	11×11	$d_{x,1} = d_{y,1} = \lambda_{y,\min}/4 = 2.3$	$D_{x,1} = D_{y,1} = 23$
Array A2	21×21	$d_{x,2} = d_{y,2} = \lambda_{y,\min}/4 = 2.3$	$D_{x,2} = D_{y,2} = 46$
Array A3	11×11	$d_{x,3} = d_{y,3} = \lambda_{y,\min}/2 = 4.6$	$D_{x,3} = D_{y,3} = 46$

Table 3 $BF(\mathbf{k} | w_m, \theta_s)$ characteristics of the three rectangular grid arrays

	$FWHM$ (rad/mm)	Wavenumber periods, K (rad/mm)
Array A1	$FWHM_{x,1} = FWHM_{y,1} = 0.30$	$K_{x,1} = K_{y,1} = 2\pi/d_{x,1} = 2\pi/d_{y,1} = 4k_{y,\min} = 2.72$
Array A2	$FWHM_{x,2} = FWHM_{y,2} = 0.15$	$K_{x,2} = K_{y,2} = 2\pi/d_{x,2} = 2\pi/d_{y,2} = 4k_{y,\min} = 2.72$
Array A3	$FWHM_{x,3} = FWHM_{y,3} = 0.15$	$K_{x,3} = K_{y,3} = 2\pi/d_{x,3} = 2\pi/d_{y,3} = 2k_{y,\min} = 1.36$

Table 4 $FWHM$ values of directional beam patterns $BF(\theta|w_{p,q},\theta_s)$ at different directions of θ_s .

	$\theta_s=0^\circ$	$\theta_s=45^\circ$	$\theta_s=90^\circ$	$\theta_s=135^\circ$
Array A1	14°	31°	35°	35°
Array A2	7°	16°	18°	18°
Array A3	7°	16°	18°	18°