A study of the noncollinear ultrasonic-wave-mixing technique under imperfect resonance conditions

A. Demčenko^{1*} and L. Mainini¹

¹*Chair of Production Technology, Faculty of Engineering Technology, University of Twente,
7500AE, Enschede, The Netherlands.*

^{*}*Corresponding author, e-mail andriejus.demcenko@gmail.com, phone +37065444101*

V. A. Korneev²

²*Lawrence Berkeley National Laboratory, Berkeley, California 94720, United States.*

ABSTRACT

Geometrical and material property changes cause deviations in the resonant conditions used for noncollinear wave mixing. These deviations are predicted and observed using the $SV(\omega_1)+L(\omega_2)\rightarrow L(\omega_1+\omega_2)$ interaction, where SV and L are the shear vertical and longitudinal waves, respectively, and ω_1 , ω_2 are their frequencies. Numerical predictions, performed for the scattered secondary field in the far field zone, show three field features of imperfect resonance conditions: (1) rotation of a scattered beam, (2) decrease in the beam amplitude, and (3) beam splitting.

The response of the nonlinear ultrasonic wave mixing technique is verified experimentally in two ways: (1) detection of a kissing bond between two polyvinyl chloride (PVC) plates, and (2) detection of subsurface micro-cracks in polymethyl methacrylate (PMMA). A predominant decrease in nonlinear wave energy is observed in both experiments. Beam rotation and splitting is observed in the kissing-bond experiment, while a minor increase in the nonlinear wave energy up to 100% is observed in the micro-cracked PMMA specimen.

Keywords: nonlinear ultrasonics; resonance conditions; scattered field; kissing bond; subsurface micro-cracks.

I. Introduction

Present scientific and industrial needs require the use of acoustic measurement systems with a high sensitivity to different material properties, such as cracks and micro-cracks [1, 2], fatigue [3, 4], ageing [5, 6], and bonding state [7, 8, 9, 10]. Nonlinear ultrasonics enable investigators to fulfill this requirement when linear ultrasonic techniques become inefficient or are insensitive to particular material properties [11]. This study is focused on bulk-wave nonlinear-ultrasonic-measurement techniques

Detection of micro-cracks induced by fatigue or during machining processes is a typical application of nonlinear ultrasonics [1, 2, 12], often based on harmonic generation [13] or various modulation [14, 15, 16, 17] techniques. The previous work on the subject shows that harmonic amplitudes (harmonic technique cases) and sideband amplitudes (nonlinear wave modulation techniques) grow with increasing damage to the material [13, 14, 15, 16, 17]. Recently, a collinear mixing technique has been reported with application to micro-crack detection [18] and damage evaluation in concrete [19]. It is useful to note that both techniques, harmonic generation and collinear wave mixing, are special cases of the wave-mixing technique [20]. In the first case (harmonic generation), harmonic generation occurs between two waves of the same frequency propagating in one direction. During this interaction, nonlinear waves with multiple harmonics are generated. In the second case (collinear wave mixing), the nonlinear interaction occurs between two initial waves of different frequencies. The initial waves can propagate either in parallel or in opposite directions. In a general case, depending on the frequency ratio, nonlinear waves of sum and difference are generated for initial wave frequencies [20]. Possibility for frequency separation is the advantage that the collinear wave-mixing technique has over the harmonic generation technique because the harmonic generation can possibly occur in the places other than the specimen.

A more sophisticated nonlinear-ultrasonic-measurement technique, known as the noncollinear wave-mixing technique, has not as yet been used for nondestructive testing of engineering structures, and the reported results deal with laboratory tests. The basic idea of this technique is that of a nonlinear wave interaction between two initial waves intersecting at a non-zero angle. During the interaction, a nonlinear wave of sum or of difference the initial wave frequencies is generated. Special resonance conditions must be satisfied for the wave interaction, which are necessary, but not sufficient ones [20]. These resonant conditions tie together the frequencies and wave vectors of interacting and generated waves. The noncollinear wave-mixing technique has various advantages over harmonic generation and collinear mixing techniques, which have been fully covered in previous studies [21, 22], so they are not repeated here.

Despite the fact that the fundamentals of the noncollinear wave mixing theory were developed in the 1960s [23], there is still too little information about the practical applications of this technique to various scientific and industrial fields. Pioneering experiments were dedicated to illustration of the theory without application to material inspection [24, 25]. However, recently it has been shown that nonlinear ultrasonics, based on noncollinear wave mixing, shows a high potential for evaluation of fatigue in metals [26]. The technique has been successfully used for measurements of physical ageing in thermoplastics [6, 21], and has the potential for evaluation of diffusive bonds between titanium alloys [22]. It has also been shown that using the noncollinear wave-mixing technique for isothermal-epoxy-cure monitoring enables users to estimate four transition points in the curing process instead of the three points provided by linear ultrasonics [21, 27]. However, the limited range of experimental work involving noncollinear wave-mixing techniques means that broader experimental verification of the techniques—at different measurement conditions and for different applications—is necessary. In particular, the important effect of deviations from perfect resonant conditions has not yet investigated. In theory of noncollinear wave mixing a fulfillment of resonant conditions assumes homogeneity of a specimen. In practice this assumption is not always satisfied and both the initial and scattered waves can deviate from the theoretical raypaths due to anomalies in geometry and material properties.

Investigation of such deviation effects is the main subject of this study. Here we focus initially on numerical and experimental investigations of the noncollinear wave-mixing technique for the evaluation of vertical interfaces formed by two tightly pressed together polyvinyl chloride (PVC) plates. This technique is then also applied to investigating subsurface micro-cracks in polymethyl methacrylate (PMMA).

The first part of the paper contains a brief review of the noncollinear wave-mixing technique. The review is supported by numerical modeling and analysis of the scattered secondary field in the far field zone. The analysis is performed for two different wave-mixing situations: (1) wave-resonance conditions are perfectly fulfilled, and (2) wave-resonance conditions are imperfect, i.e., deviation from resonance conditions exists within the system. The imperfect resonance conditions are introduced into the system as a small deviation in the interaction angle from the perfect resonance angle between two initial waves.

The second part contains experimental data. The experiments are performed both for detection of the kissing-bond interface between two tightly pressed together PVC plates and for detection of subsurface micro-cracks in PMMA. An immersion ultrasonic-measurement technique was used for the experiments.

II. Noncollinear wave mixing technique

The geometry involved in noncollinear wave mixing is shown in Fig. 1. The nonlinear wave interaction requires satisfaction of the resonance conditions [20]:

$$\omega_r = \omega_1 \pm \omega_2, \quad (1)$$

$$\mathbf{k}_r = \mathbf{k}_1 \pm \mathbf{k}_2. \quad (2)$$

where frequencies ω_1 , ω_2 , and ω_r represent two initial waves and the resonant wave, respectively. The parameters $\mathbf{k}_1$, $\mathbf{k}_2$, and $\mathbf{k}_r$ are their wave vectors, where $k_i = \omega_i/c_i$, c_i is the phase velocity of the corresponding wave and $i = 1, 2, r$. The geometry shown in Fig. 1 involves the acoustic axes of two incident waves ($\mathbf{k}_1$ and $\mathbf{k}_2$ vectors) and the generated resonant (nonlinear) wave beam ($\mathbf{k}_r$ vector) in the interaction volume V , which is formed by two intersecting initial wave beams. The analyzed geometry contains four parts:

1. The wave-propagation path of the first initial wave $\mathbf{k}_1$;
2. The wave-propagation path of the second initial wave $\mathbf{k}_2$;
3. Volume V where the nonlinear wave interaction occurs;
4. The wave-propagation path of the nonlinear wave $\mathbf{k}_r$.

Fig. 1. Problem geometry of the noncollinear wave mixing, where $\mathbf{k}_1$ and $\mathbf{k}_2$ are the initial wave vectors, $\mathbf{k}_r$ is the resonance wave vector, V is the interaction volume with non-zero third-order elastic constants, α is the angle between $\mathbf{k}_1$ and $\mathbf{k}_2$. $\mathbf{r}'$ is the radius vector of integration inside volume V .

The first two parts ($\mathbf{k}_1$ and $\mathbf{k}_2$) influence: (a) the fulfillment of the resonance conditions, and (b) the amplitude of the resonance wave. If the resonant conditions 1 and 2 are satisfied, the $\mathbf{k}_1$ and $\mathbf{k}_2$ vectors intersect at the resonant angle α_r which can be determined from the equation:

$$\left(\frac{\omega_1}{c_1}\right)^2 + \left(\frac{\omega_2}{c_2}\right)^2 \pm 2 \frac{\omega_1 \omega_2}{c_1 c_2} \cos \alpha_r = \left(\frac{\omega_1 \pm \omega_2}{c_r}\right)^2. \quad (3)$$

The frequencies ω_1 and ω_2 are fixed in most practical experiments, and therefore a deviation of an actual angle α from the resonant angle α_r can result from two different factors involved in the system: (1) local variation in medium properties—e.g., one or both ultrasonic wave velocities c_1 and c_2 , and (2) geometrical variations in the medium, e.g., surface waviness.

The ultrasonic wave velocity may vary locally depending on a local treatment such as thermal processing of metal (annealing) [28] or cyclic loading (fatigue) [29]. A natural and local variation in sound speed is observed in concrete and rock materials. When noncollinear wave mixing is performed using an immersion or contact ultrasonic technique, the surface geometrical conditions also influence the fulfillment of the resonance conditions. For example, the experimentally estimated surface waviness of a PVC pipe indicates that the surface waviness is in the range of $\pm 5^\circ$. A poor alignment of a test specimen or its curvature also causes a deviation in the interaction angle α from perfect resonance conditions. Angular shift $\Delta\theta$ can reach 0.2° when a 1 mm offset from a horizon of a 300 mm plate is presented in the measurement system, where θ is the initial wave incident angle from a fluid to a solid. Knowing the PVC properties and applying the Snell's law, the deviation of the actual angle α from the resonant angle α_r can be evaluated.

Despite the initial waves ideally having constant amplitudes, ultrasonic losses and wave dissipation of the initial waves $\mathbf{k}_1$ and $\mathbf{k}_2$ can vary due to local medium properties. Thus, the resonance wave amplitude also varies due to changes in the interacting wave amplitudes. In a general case, the changes may be caused by second- or third-order elastic constants. The third-order elastic constants may have an influence because the initial waves used for noncollinear wave mixing are generated at a relatively high power. Hence, part of the initial wave energy is transformed into higher-order harmonics, which means that more of the initial wave energy is transformed to the harmonics, due to the increase in material nonlinearity. Therefore, the initial waves with reduced energies participate in the nonlinear wave interaction within the volume V .

Volume V , where the initial waves intersect, represents the informative material volume that provides linear and nonlinear material responses, i.e., second- and third-order material elastic constants are involved in the response [20]. Therefore, this (third) part influences the linear and nonlinear behavior of the noncollinear wave-mixing technique, which provides a target signal of the nonlinear measurement.

The volume factor for the scattered secondary field in the far field has the form of the integral [20]:

$$V_r^\pm = \int_V \sin \left(\Delta_r^\pm - \left(\mathbf{k}_1 \pm \mathbf{k}_2 - \frac{\omega_1 \pm \omega_2}{v_r} \hat{\mathbf{r}} \right) \cdot \mathbf{r}' \right) dV, \quad (4)$$

where $\mathbf{r}'$ is the radius vector of integration inside volume V (geometry shown in Fig. 1). Symbol r can be either “L” or “S”, denoting the type of scattered wave; it is “L” (longitudinal) in our analysis. $\mathbf{r} = r\hat{\mathbf{r}}$ $|\hat{\mathbf{r}}| = 1$ is the radius vector from the center point of the interaction region and observation point. $\Delta_r^\pm$ is the phase factor. The nonlinear wave amplitude is proportional to the volume factor. The integrand in Eq. 4 generally oscillates unless the resonant conditions are satisfied when it becomes constant, providing a constructive interference for all the corresponding points within volume V . Integrals with the form of Eq. 4 are rather common in optical applications and are known for containing angular side lobes. However, for the purposes of nonlinear wave mixing, they have not yet been evaluated, except for the case of a small sphere [20]. Evaluation of the integrals in Eq. 4 is important for estimating the expected amplitude of the nonlinear component, as well as the spatial resolution of scanned images.

The fourth part, the wave propagation path of the nonlinear wave $\mathbf{k}_r$, behaves in a linear way. This means that after the nonlinear wave is excited, changes in linear material properties exclusively influence just the amplitude of the scattered wave. Therefore, any local change presented in the wave propagation path will affect the nonlinear signal. This signal will also be affected by the different factors (variations in medium properties, geometrical variations in medium) in the first two parts ($\mathbf{k}_1$ and $\mathbf{k}_2$) of the analyzed geometry (see Fig. 1).

III. A numerical analysis of the nonlinear wave beam pattern under imperfect wave-mixing conditions

We conducted a numerical analysis to investigate the dependence of nonlinear-wave-beam behavior on deviations from resonant conditions in a PVC scanning experiment [21]. Within this analysis, the wave interaction angle α varies within volume V when other parameters (ω_1 , ω_2 , and V) remain constant. The interaction $SV(\omega_1)+L(\omega_2)\rightarrow L(\omega_1+\omega_2)$ is used where SV and L are the shear vertical and longitudinal waves, respectively. Fig. 2 shows the modeled geometry when the frequency ratio $\omega_2/\omega_1 = 1.5$ and source probes have 10 mm diameters. Inclination angles $\theta_{1,2}$ of the sources are 53° and 32° , respectively. In this numerical analysis, wave propagation is considered only in the PVC plate.

Fig. 2. (Color online) Scheme of the $SV(\omega_1)+L(\omega_2)\rightarrow L(\omega_1+\omega_2)$ wave-mixing configuration in PVC. Sources S_1 , S_2 excite the initial waves ($\mathbf{k}_1$ and $\mathbf{k}_2$), while the nonlinear wave ($\mathbf{k}_r$), is recorded in receiver R. The excited beams have the inclination angles $\theta_{1,2}$. Symbols SR denote the specularly reflected waves. Numbers show the distances in mm.

To estimate the volume factor V_r^+ , it is necessary to know an interaction volume of the initial waves. Using a 3D ray-tracing technique, we evaluated the interaction volume V for the configuration shown in Fig. 2. (For the sake of simplicity, beam spreading was not considered. In our case this simplification was valid, because beam fields of the sources S_1 and S_2 were in the near field zone in water. The beam spreading was not also significant in PVC due to the low ultrasonic wave velocities within the material, relatively high frequencies of the initial waves and relatively short far field beam range before and during intersection of the beams. Moreover, a dispersion of ultrasonic wave velocities is little within the analyzed frequencies [6].) Accounting for the beam spread increases the interaction volume and slightly changes the shape of volume V . This volume increase results in a sharper peak for the resonant wave $\mathbf{k}_r$. The estimated wave interaction volume V is shown in Fig. 3.

Fig. 3. (Color online) Interaction volume V estimated using 3D ray tracing. The beam spread is not considered in the estimation. Acoustic axes of the initial waves are lying in the x_0z plane. Color bar indicates z coordinate values within the interaction volume V .

When a complex interaction volume is involved in the analysis, the integral presented in Eq. 4 must be integrated numerically. The Gaussian quadrature method with the Legendre rule was employed for the numerical integration. The developed integration routines were verified using an analytical solution for the spherical interaction volume [20].

Initially, the radiation patterns of the volume factors (beam patterns) are calculated when the resonance conditions are perfectly satisfied. In these calculations, it is assumed that the PVC medium is infinite. Figure 4 presents the central projections of the calculated beam at two orthogonal planes, xz and yz . The beam patterns have different widths, which is important to take into account when the wave-mixing technique is used where high spatial resolution is required. The calculated beam pattern also shows the presence of side lobes that have different configurations at the two orthogonal planes.

Fig. 4. (Color online) Radiation patterns of the volume factor in the far field at perfect resonance conditions: (a) xz plane and (b) yz plane. $\Delta\alpha$ is the deviation angle from the ideal resonance angle α_r .

To investigate the influence of deviation in the wave interaction angle α on nonlinear wave interaction, we fixed the shear wave vector $\mathbf{k}_1$, and rotated the longitudinal wave vector $\mathbf{k}_2$ in the $x0z$ plane. This rotation was performed in two directions from the perfect interaction angle α_r , enabling investigation in the far field, when the interaction angle α is larger or less than α_r .

The used deviations of α from α_r are listed in Table 1. The selected deviations are different for the different directions of rotation, because numerical results have shown that the scattered secondary field in the far field is more sensitive to a decrease in the interaction angle in the analyzed case. Thus, the ultrasonic beam field decays more rapidly, due to the decrease in the interaction angle α , than the field when the angle α increases. Numerical calculations of the nonlinear beam patterns are performed in 3D space. Two planes, $x0z$ and $y0z$, are used for visualization of the predicted data.

Table 1. Used values of the incident waves deviation angles. Signs “+” (“-”) indicate increase (decrease) of the wave interaction angle α relative to the resonant angle α_r .

#	1	2	3	4
$\Delta\alpha^+$, deg.	+2	+4	+6	+8
$\Delta\alpha^-$, deg.	-2	-4	-5	-6

The computed scattered secondary field in the far field is presented in Figs. 5-8. These numerical results show three features of the nonlinear beam field when the resonance conditions are imperfect: (1) a rotation in the xz plane, (2) a decrease in the beam field amplitude, and (3) beam splitting. The beam splitting is not seen directly from the figures, but it can be explained as follows: When there is a local material inhomogeneity of smaller volume than the wave-interaction volume, two nonlinear wave-interaction conditions are possible: perfect or imperfect. In this case, at least two coupled secondary fields will be generated. The numerical results also show that the side lobes are much less affected by the interaction angle deviations than the main lobe.

Fig. 5. (Color online) Radiation patterns of the volume factor for xz projections when the interaction angle increased by the step of: (a) $\Delta\alpha = +2^\circ$, (b) $\Delta\alpha = +4^\circ$, (c) $\Delta\alpha = +6^\circ$ and (d) $\Delta\alpha = +8^\circ$.

Fig. 6. (Color online) Same as on Fig.5 but for yz projections.

Fig. 7. (Color online) Radiation patterns of the volume factor for xz projections when the interaction angle decreased by the step of: (a) $\Delta\alpha = -2^\circ$, (b) $\Delta\alpha = -4^\circ$, (c) $\Delta\alpha = -5^\circ$ and (d) $\Delta\alpha = -6^\circ$.

Fig. 8. (Color online) Same as in Fig. 7 but for yz projections.

IV. Experimental study of the noncollinear wave-mixing technique: A kissing-bond interface

In this section, we experimentally demonstrate the effect of a kissing-bond interface on the nonlinear wave component of the wave-mixing technique, specifically showing the beam splitting effect. A detail logistics of experimental parameters choice for a scheme form Fig. 2 is considered in [21].

Two PVC plates of 25 mm thickness were used for simulation of a vertical interface with kissing-bond properties [30]. The geometry of the specimen and coordinate system is presented in Fig. 9. The specimen was prepared as follows. Two holes were drilled in a solid PVC plate of commercial grade from the current stock. After drilling, the plate was cut in the middle along the x axis. To obtain smooth surfaces for the two PVC plates, and to maintain the best acoustic contact between the plates, both cut surfaces were processed simultaneously, using a milling machine. Finally, the specimen was assembled using two bolts, with the bolts tightened as much as possible using hand keys. In a similar way, an aluminum test specimen was used for simulation of a simplified surface-crack model [31].

The acoustic channel configuration shown in Fig. 2 was used in the measurements. A receiver of 3 mm diameter and spherically focused (31.75 mm focal distance) was used for reception of ultrasonic signals. Two sources of 10 mm diameter were excited by burst signals of 40 and 60 cycles at 2 MHz and 3 MHz, respectively. In this study, long signals were used to obtain a full saturation of the nonlinear wave signal, i.e., a signal with a uniform amplitude distribution. The long signals approximate the excitation of the quasi-monochromatic waves and compensate for any propagation time delay due to the possible geometrical misalignment between a test specimen and acoustic channel.

Fig. 9. PVC specimen geometry and coordinate system. Numbers show the distances in mm.

For initial characterization of the test specimen, we carried out a pulse-echo scanning of the PVC specimen with the kissing bond at 5 MHz center frequency. Figure 10 presents the experimental results in the form of a C-scan image; a peak-to-peak amplitude was used for the data representation. The interface and its geometry is readily seen in the scan (Fig. 10). Ultrasonic signal reflections from the top specimen surface indicate a small misalignment of two connected PVC plates. In this case, up to a 25% amplitude decrease is observed in the experimental data (Fig. 10a). The C-scan image formed from the back-wall reflected signals (Fig. 10b) clearly indicates fixing bolts and two different states of the interface between the PVC plates: (a) a typical kissing-bond interface, and (b) a “semi open” interface, where a significant decrease in ultrasonic signals is observed. This part of the interface is assumed to be “semi open” because a visual inspection of the interface from both sides of the specimen does not indicate an increase in the gap between the two plates. Also, no significant geometry changes are observed in this C-scan image. This partial interface opening between the two PVC plates was formed by tightening of the plates by the two bolts. Such a construction of the test specimen enables investigation of the interface with different properties: the semi-open interface with a functional transition to the kissing-bond interface.

Fig. 10. (Color online) Linear ultrasonic C-scan images of the interface between two PVC plates: (a) visualization of the top surface geometry, and (b) visualization of the back-wall ultrasonic-signal-amplitude distribution in the investigated geometry. A_{pp} is the peak-to-peak amplitude of ultrasonic signal. a.u. denotes the arbitrary units. The measurements are performed in a pulse-echo configuration at center frequency of 5 MHz. The amplitudes are scaled to the maximum amplitude at each scan.

Using nonlinear ultrasonics, we scanned the PVC specimen at two different configurations:

- a) The straight-angle configuration, i.e., the interface plane and the plane of the initial wave transducers S_1 and S_2 (see Fig. 3) are in one plane $x0z$ or in the planes parallel to the $x0z$ plane.
- b) The right-angle configuration, i.e., the interface plane is in the $y0z$ plane and the initial wave transducers S_1 and S_2 are in the $x0z$ plane (see Fig. 3), i.e. the interface and transducers S_1 and S_2 are in the orthogonal planes.

Nonlinear ultrasonic scanning results for both configurations are presented in Fig. 11. The figure shows a better detection of the interface than with linear ultrasonics (see Fig. 10b), i.e., there is a more significant decrease in the nonlinear wave energy observed in the semi-open interface zone. The nonlinear ultrasonics shows a higher sensitivity to the kissing bond interface than the linear. For example, the nonlinear wave energy decreases approximately 50% due to the kissing bond interface (175 mm – 227 mm within y axis, Fig. 11a). The corresponding linear wave peak-to-peak amplitude decreases 25% (Fig. 10b). However, an estimation of the interface geometry is strongly dependent on the orientation of the interface and the initial wave transducer planes.

Fig. 11. (Color online) Nonlinear ultrasonic C-scan images of the interface between two PVC plates: (a) the straight-angle configuration, and (b) the right-angle configuration. E is the energy of the nonlinear ultrasonic signal. The nonlinear signal energies are scaled to the maximum energy at each scan.

To verify the semi open interface hypothesis, we inserted a Teflon tape of 0.1 mm thickness between the PVC plates. This tape significantly changes wave-propagation conditions through the interface, owing to the significant difference between the PVC and Teflon material properties. The changed specimen configuration was rescanned using nonlinear ultrasonics; Figure 12 shows the scanning results. It is seen that in the presence of the

Teflon tape, the ultrasonic response from the interface is similar to the case of the semi-open interface (see Fig. 11).

Fig. 12. (Color online) Nonlinear ultrasonic C-scan images of the interface with the Teflon tape between two PVC plates: (a) the straight-angle configuration, and (b) the right-angle configuration. The nonlinear signal energies are scaled to the maximum energy at each scan.

The nonlinear wave experiments represented in Figs. 11 and 12 show that a more significant increase in nonlinear wave signal energy does not occur in the interface zone than in the undamaged zone. The decrease in the nonlinear wave signal energy is predominant in the experimental results measured from the interface zone, and a more significant increase of the energy is not observed. It is reported [13, 32] that the harmonic generation technique shows an increase in harmonics in the imperfect interfaces.

We conducted scanning of the nonlinear-wave-field distribution for a solid PVC plate and two joined plates when the semi-open interface zone (without the Teflon insert) was between the sources. Both configurations (straight- and right-angle) were investigated experimentally. During the scans, the source probes were fixed, but the receiver moved. The experimental results are presented in Figs. 13 and 14.

Figure 13 shows the measured nonlinear wave signal distribution when the wave mixing is performed in the solid homogenous PVC plate. It is seen that a single beam is presented in the experimental results, which agree well with the numerical prediction (see Fig. 4), despite the fact that the distribution is not ideally symmetric.

When the wave-interaction conditions are affected by the imperfect interface, the nonlinear wave field becomes more complex (see Fig. 14). Beam splitting is observed in both straight- and right-angle measurement configurations. In our experiments, two side lobes are observed; more lobes could be present within the entire

field, but because of the beam rotation and overly low energy, these beams go undetected. The experimental results confirm our numerical predictions when the wave-resonance conditions are not fulfilled (see Figs. 5-8 for the numerical predictions).

In the straight-angle configuration, the measured peak energies of the side lobes I and II are 34% and 10% (Fig. 14a) respectively, of the main lobe peak energy, which is 100%. Using the right-angle configuration, the measured peak energies are 31% and 12% of side lobes I and II, respectively. This shows that in our case a small difference exists between the corresponding side lobes of the two configurations (the straight and right angles). Measured peak energies of the main lobes are 16% and 31% for the straight- and right-angle configurations, respectively, of the main lobe peak energy in the solid PVC plate. This clearly indicates the significant decrease of the nonlinear wave energy due to the imperfect wave interaction conditions.

Fig. 13. (Color online) Measured distribution of the nonlinear wave energy when wave mixing is performed in the solid PVC plate. Only the single acoustic beam is observed in the experimental results. The nonlinear signal energies are scaled to the maximum energy at the scan. The contour level is 0.03.

Fig. 14. (Color online) Measured distribution of the nonlinear wave energy when the semi-open interface is located between the joint PVC plates: (a) straight-angle configuration, and (b) right-angle configuration. The white “+” sign marks the center of the field where the interaction occurs in the solid PVC plate. The nonlinear signal energies are scaled to the maximum energy at each scan. The contour level is 0.03.

V. Investigation of subsurface micro-cracks in PMMA using the nonlinear wave-mixing technique

Two 12 mm thick PMMA specimens containing subsurface micro-cracks were scanned using linear and nonlinear ultrasonics. The specimens were prepared in the following way: A PMMA plate with dimensions 80 mm $\times$ 200 mm was wrapped in adhesive tape. A stripe of the tape $\sim$ 20 mm width was removed in the middle of the specimen from one side. Then, a three-point bending and acetone treatment of the uncovered surface was used for the growth of micro-cracks in the specimens. In this way, up to 3-4 mm deep subsurface micro-cracks were produced on a single side of the specimen.

Figure 15 shows the arrangement of the acoustic channel used in the subsurface micro-cracked PMMA scanning experiments, where $\theta_1 = 36^\circ$ and $\theta_2 = 25^\circ$. Most of experiments were performed employing the configuration shown in Fig. 15a, i.e., focusing the acoustic axes of the initial waves closely to the surface of the specimen. Some verification experiments were performed using the acoustic channel configuration shown in Fig. 15b, i.e., the acoustic axes of the initial waves were focused closely to the middle of the specimen. Using both arrangements, the parameter of the acoustic channel was not changed, only the specimen was moved up 5 mm with respect to the ultrasonic transducers. This also affected the focal point of the receiving transducer, but in our case it was important to detect relative changes in the nonlinear-wave-signal instant visualization of the

subsurface micro-crack geometry. Thus, we believe that the experimental results are comparable between these two different experimental configurations.

Fig. 15. (Color online) $SV(\omega_1)+L(\omega_2)\rightarrow L(\omega_1+\omega_2)$ wave-mixing configuration for the nonlinear wave mixing in PMMA: (a) wave focusing near the surface, and (b) wave focusing near the middle of the specimen. Numbers show the distances in mm.

Figure 16 shows pulse-echo scanning results from the subsurface cracked PMMA specimen: surface profile image (Fig. 16a), and distribution of the detected micro-cracks using the back-wall reflected ultrasonic signals (Fig. 16b). The micro-cracks are readily seen in the specimen owing to wave dissipation. However, the change in the peak-to-peak amplitude of the ultrasonic signals is not significant—approximately 10%.

Fig. 16. (Color online) Pulse-echo C-scan images from the PMMA specimen with the subsurface micro-cracks: (a) surface profile, and (b) the subsurface micro-cracks shown. The amplitudes are scaled to the maximum amplitude at each scan.

Figure 17 shows C-scan images formed from the nonlinear ultrasonic data when the initial waves are generated from the undamaged and damaged sides of the PMMA specimen, respectively. The presented results correspond to the right-angle experimental configuration described above. The wave-mixing results from the undamaged side shows an almost linear response for the nonlinear measurements (Fig. 17a), i.e., only a decrease in the nonlinear wave signal is observed in the presence of the micro-cracks. This could be explained by the fact that the focal point of the initial waves very near the specimen surface and the micro-cracks from the opposite side of the specimen would not significantly affect the interaction volume of the initial waves.

When the wave mixing was performed from the micro-cracked side of the specimen, the nonlinear wave scanning response was different from the previous case. In this case, a slight increase in the is observed in the nonlinear wave energy (see Fig. 17b and marked areas by ellipse). In the presented data, the maximum increase is 8.7% in comparison to the response from the undamaged PMMA specimen zone. The corresponding maximum decrease is 14% in the analyzed data. However, the decrease in the nonlinear wave energy is predominant in the experimental data.

Fig. 17. (Color online) Nonlinear ultrasonic C-scan images of the PMMA specimen with subsurface micro-cracks: the scanning results when the initial waves are generated from the undamaged (a) and damaged (b) sides of the specimen. Zones in which an increase in the nonlinear signal energy is observed are marked by ellipsis. The right-angle configuration is used. The nonlinear signal energies are scaled to the maximum energy at each scan.

We repeated the C-scanning experiments using nonlinear ultrasonics and the straight-angle configuration for the same PMMA specimen. The corresponding results are shown in Fig. 18. Experimental results show a lower sensitivity in the measurements of the micro-cracks, and an almost linear response is observed in the data.

Fig. 18. (Color online) Nonlinear ultrasonic C-scan images of the PMMA specimen with the subsurface micro-cracks: the scanning result when the initial waves are generated from the undamaged (a) and damaged (b) sides of the specimen. The straight-angle configuration is used. The nonlinear signal energies are scaled to the maximum energy at each scan.

To verify nonlinear ultrasonic experimental results, nonlinear ultrasonic measurements were performed in which the focal point of the initial waves was below the specimen's surface, i.e., close to the middle of the specimen (see Fig. 15b).

Figure 19 shows C-scan images of the PMMA specimen at two different measurement configurations: straight and right angles. The measurements were performed from the micro-cracked side of the specimen. Results from the opposite side are not repeated here, because more significant changes from the previous results (see Figs. 17a and 18a) were not observed in the data.

Figure 19 shows that the change in the focal point does not change the measurement results significantly (see previous results in Figs. 17b and 18b).

Fig. 19. (Color online) Nonlinear ultrasonic C-scan images when the focal point is below the PMMA surface: (a) right- and (b) straight-angle configurations, respectively. The nonlinear signal energies are scaled to the maximum energy at each scan.

Figure 20 shows nonlinear ultrasonic C-scan images of the additional PMMA specimen with subsurface micro-cracks from both sides of the specimen. The ultrasonic scanning was performed using the configuration presented in Fig. 15a from a single side of the specimen. It is seen (in Fig. 20a) that a very sharp increase in nonlinear wave energy is measured from a certain location of the specimen. The measured energy increase is 100% compared to the response from the undamaged PMMA specimen zone. After this measurement, the specimen was immersed in water and degassed for 20 min using a vacuum pump. Then the specimen was rescanned again, with the corresponding result shown in Fig. 20b. It is seen that the increase in the nonlinear wave signal disappeared from the zone in which it was previously situated. However, a certain increase in energy is measured from the different location of the specimen. In this case, the increase in energy is 10% compared to the response from the undamaged PMMA specimen zone.

The straight-angle scanning results are not presented here, because C-scan images of the nonlinear ultrasonic measurements show only linear behavior.

Fig. 20. (Color online) Nonlinear ultrasonic C-scan images of the PMMA specimen with the subsurface micro-cracks: the scanning result before degassing (a) and after degassing (b) of the specimen. Zones in which the increase in nonlinear signal energy is observed are marked by ellipsis. The right-angle configuration is used. The nonlinear signal energies are scaled to the maximum energy at each scan.

Discussion

A simple numerical model for the scattered secondary field in the far field enables us to analyze and predict the scattered field under different measurement conditions, such as perfect and imperfect resonance conditions. In this study, imperfect resonance conditions were analyzed when one of the initial wave vectors was rotated in the xOz plane. However, more complex rotation is expected in real situations when one or both initial waves will be rotated in 3D space. This will result in 3D rotation of the scattered secondary field in the far field.

Moreover, the influence of the wave-interaction volume on the scattered secondary field can be investigated by employing a numerical model for the scattered secondary field in the far field. This analysis is suitable for optimization of ultrasonic transducer shapes used in generating the initial waves.

Initial experimental investigations of the kissing-bond interface between two PVC plates showed that a decrease in nonlinear wave energy is expected in the measurements when a fixed acoustic channel is used in the investigations. However, more advanced experimental investigations should be performed to understand the response of the nonlinear ultrasonic measurement when the noncollinear wave-mixing technique is used for inspection of the inhomogeneous media. Investigators must explore different frequencies and find the most sensitive frequency range for imperfect interfaces. Additionally, more sophisticated investigations should be performed when the initial wave-mixing conditions are imperfect, i.e., the initial wave-mixing is performed under imperfect resonance conditions.

Initial experimental investigations of subsurface micro-cracks in PMMA demonstrated that a significant energy increase in the nonlinear wave signal can be expected. In our experiments, the energy increase was fixed up to 100%. However, the detected local energy increases of the nonlinear wave signal were not dominant in the nonlinear wave response from the subsurface micro-cracks. Therefore, it is not clear how or what properties of the subsurface micro-cracks increase nonlinear signal energy.

CONCLUSIONS

A numerical analysis of a nonlinear-ultrasonic-beam pattern in the far field has shown that the beam rotates, decreases in beam volume, and splits when the wave-resonance conditions are imperfect, i.e., there is a small deviation from the resonance conditions in the system. Experimental results demonstrate the features similar to those of the numerical analysis when inhomogeneities generate extra scattered wave beams. A decrease in the signal amplitude of the nonlinear wave was also observed experimentally.

Nonlinear ultrasonic experimental results have proved that a significant decrease in nonlinear wave energy is observed in the presence of a kissing-bond interface or micro-cracks. A significant increase in nonlinear wave energy has not been observed in the experiments, but in some cases the energy increases up to 100%. This energy increase was observed when the planes of the initial wave sources and micro-cracks were orthogonal.

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