Accurate analytical modeling of flat plate solar collectors: Extended correlation for convective heat loss across the air gap between absorber and cover plate

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Abstract

The well-established correlation of Hollands et al. (1976), which is applicable to isothermal boundaries and Rayleigh numbers up to 10^5 , underestimates the convective heat loss across the air gap of flat plate solar collectors with tube-and-sheet type absorbers both in normal operation and at stagnation. Two reasons for this discrepancy were identified. 1) The Rayleigh number of the air gap above absorbers with highly selective coatings can be three times as high as the application limit. 2) The absorber is not isothermal during normal operation. Based on a literature study and theoretical considerations the application limit of the correlation was extended to $Ra = 3 \cdot 10^5$. By means of an analytically derived correction parameter, the correlation was adapted to non-isothermal boundary conditions.

For the cost- and efficiency optimization of flat-plate collectors an accurate analytical model was developed, based on the model of Duffie and Beckman (1991), enhanced by the new correlation for convective heat loss between absorber and cover plate and further correlations presented by Eismann (2015). The model was validated against data of standardized collector tests. It is able to predict both the thermal efficiency and the stagnation temperature within the uncertainty limit of the standardized test method EN 12975-2 (CEN 2006).

Keywords

Solar collector, convective heat loss, analytical model, thermal efficiency, stagnation temperature, cost reduction

1 Introduction

The efficiency of solar collectors shown in Figure 1 is experimentally determined by standardized test methods. The empirical model, according to the standard EN 12975-2 (CEN 2006), describes the efficiency η of a solar collector as a function of the dimensional parameter, $x = (T_{fav} - T_a)/G$, which is a function of the solar irradiation, G , and the difference between average fluid temperature, T_{fav} , and ambient temperature, T_a . The conversion factor, η_0 , is the efficiency for the case where the average fluid temperature is at ambient temperature. The conversion factor and the two heat loss coefficients, a_1 , and a_2 , are the main results of efficiency tests.

Figure 1 Cross-section of a typical flat plate collector with bottom and side wall insulation.

$$\eta = \eta_0 - a_1 x - a_2 x^2 G \quad (1)$$

The influence of the uncertainties of measured quantities on the efficiency were discussed by Mathioulakis et al. (1999), Müller-Schöll and Frei (2000) and within the European QAIST (Quality Assurance in Solar Thermal Heating and Cooling Technology) project by Fischer et al. (2011), Weissmüller et al. (2012) and by Kovacs (2012). For an accredited test laboratory the standard uncertainty of the efficiency is about 3 % (Kovacs 2012; p.67, Table 31). From the engineering point of view the uncertainty of the analytical model should be smaller than the uncertainty of test results. It would then be possible to thoroughly optimize solar collectors in an early stage of the developing process where changes in the design are still inexpensive.

As will be shown in the following chapters, the convective heat transfer across the air gap between absorber and cover plate presents the most significant uncertainty of analytical models. All available correlations are based on experiments or numerical simulations with isothermal boundaries, whereas tube-and-fin type

absorbers are not isothermal. The widely used correlation of Hollands et al. (1976) is valid only for isothermal boundary conditions and up to Rayleigh number of 10^5 . Furthermore, the Rayleigh number of the gap above highly selective absorbers can exceed the application limit by a factor of three. For these reasons the convective heat transfer across the air gap of flat plate collectors is underestimated in both normal operation and stagnation conditions, which leads to an overestimation of efficiency by up to 20 % and of stagnation temperature by about 10 K. A more general correlation, which forms the basis of the correlation derived in this article, has been presented by Eismann (2014).

1.1 Specification of goals and methods

This article provides a new, detailed collector model which is at least as accurate as the standardized test method.

Instead of dedicated experiments empirical data of 22 flat plate collectors which were tested according to the European standard, EN 12975-2 (CEN 2006) were used for validation.

These collectors were modelled, based on test reports and detailed specifications provided by the manufacturers. All flat plate collectors are similarly built. The housings are made of extruded aluminum profiles and sheet metal bottom plates. All absorbers are made of a single metal sheet with selective coating and a meander type absorber tube. Straight segments of the tube run horizontally beneath the absorber sheet. All collectors have a cover plate made of glass.

2 Convective heat transfer across the air gap of flat plate collectors

2.1 Flow patterns and heat transfer in gaps with isothermal boundaries

De Graaf and van der Held (1953) derived Nusselt numbers from experiments on air gaps with inclination angles between 0 and 90 degrees. The upper plate was water-cooled. The temperature of the lower plate was set by electrical heating. Along the edges of the hot and cold plates the gap was bridged by glass panes which allowed for the observation of flow patterns. An optical method based on the principle of Schmidt and Saunders (1938) was utilized which bases on the temperature dependency of the refractive index of air. In order to interpret their results the method is briefly outlined: A beam of light entering a medium with a gradient of refractive index is deflected by an angle, γ , proportional to the length, l , of the light path within the medium and proportional to the gradient of the refractive index perpendicular to the incident beam. The gradient of the refractive index of gases is proportional to their density gradient, thus, $\gamma \sim \partial\rho/\partial y \cdot l / \rho$ (Holman 2001, p. 320). Instead of a single beam, De Graaf and Van Der Held (1953) used a light sheet which was sent through the gap in slope direction. The image of the light sheet was projected on the shutter in front of a photographic plate which was exposed for about two seconds in each experiment.

Their results for inclination angles between 10 and 20 degrees are summarized as follows. Below a first critical Rayleigh number of about $Ra_{c,l} = 2000$ the image of the light sheet remained a straight line, indicating a stratified temperature profile where heat is transferred by conduction only. Due to buoyancy a laminar flow

developed. Hot air rose along the hot plate while cold air moved downwards beneath the cold cover plate. This so called “base flow” occupied the whole gap and was present at all inclination angles and superimposing all flow regimes. Above the first critical Rayleigh number the stratified temperature profile became unstable and a regular pattern of stable convection rolls developed, with their axes in the upslope direction. This was deduced from the appearance of a fairly stable, wavy line. The cycle period of the waves was interpreted as distance between the axes of the rolls, which was found to be about three times the gap size. The heat transfer was increased significantly above that of pure conduction. By increasing the temperature of the hot plate the amplitude of the waves became higher while the cycle period remained the same. At a second critical Rayleigh number of about $Ra_{c,II} \approx 30\,000$ the image of the light sheet became irregular and time dependent. This was interpreted as the transition to turbulent flow. At inclination angles between 30 to 60 degrees, however, De Graaf and Van Der Held (1953) reported a direct transition from a stratified base flow to turbulent flow.

The transitions between these flow patterns were subsequently investigated by many authors. Hart (1971a), who used water in his experiments, observed the first transition from stratified base flow to steady rolls in upslope direction at all inclination angles up to 80 degrees. For Rayleigh numbers above the first critical Rayleigh number, $Ra_{c,I}$, but still below the transition to turbulent flow, he found an intermediate transition from steady rolls to wavy, meandering rolls. This intermediate transition was investigated in a following paper (Hart 1971b) by linear stability analysis. The critical Rayleigh number for the intermediate transition, $Ra_{c,int}$, varies strongly with the inclination angle and reaches a minimum between 30° and 60° . Above 30° inclination the region for stable stationary rolls between the first and intermediate transition, $Ra_{c,int} - Ra_{c,I}$, is rather small and not easy to identify as will be demonstrated below.

The apparent discrepancy between the results of De Graaf and Van Der Held (1953) and Hart (1971a) can be explained by the effect of the Prandtl number, $Pr = \rho v c_p / \lambda$, on the stability of laminar, buoyancy driven shear flows. The first transition, from stratified base flow to steady cellular flow, is independent of the Prandtl number. This was shown experimentally by Krishnamurti (1970a), (1970b) and numerically by Bühler et al. (1979). For large aspect ratios above $l_g/s_g \approx 50$ the first critical Rayleigh number varies only with the inclination angle, $Ra_{c,I} = 1708/\cos\phi$. The critical Rayleigh number for the intermediate transition, i.e. from steady cellular flow to time dependent cellular flow, however, increases with the Prandtl number. This was shown experimentally by Krishnamurti (1970a), (1970b). Using linear stability analysis, Clever and Busse (1977) derived stability curves for water at room temperature which are in qualitative agreement with the experimental results of Hart (1971b; Fig.4). Above an inclination angle of 30 degrees the difference between the critical Rayleigh numbers of the first and the intermediate transition is about $\Delta Ra \approx 500$ for water ($Pr = 6.7$) and only $\Delta Ra \approx 10$ for air ($Pr = 0.71$) (Clever and Busse 1977; Fig.10). In the experiments by De Graaf and Van Der Held (1953) on a 22.9 mm wide gap the difference of $\Delta Ra = 10$ corresponds to a change of the temperature difference of less than 0.01 K, which is probably far below the resolution of their setup. Based on these considerations, their reported direct transition to turbulence is reinterpreted as the intermediate transition from straight rolls to wavy, time dependent rolls. The closeness of the two critical Rayleigh numbers, $Ra_{c,I}$ and $Ra_{c,int}$, for inclination angles above 30 degrees was experimentally confirmed by Ruth et al. (1980).

The intermediate transition from straight rolls to wavy, time dependent rolls is not associated with a distinct change of slope of the Nusselt number since the flow pattern is essentially the same, albeit time dependent. There is, however, yet another transition which significantly influences the growth of the Nusselt number. Hollands et al. (1975) derived from their conduction layer model the relevant second critical Rayleigh number of $Ra_{c,II} = 5830$ for horizontal gaps. Above this Rayleigh number the flow develops a boundary layer structure with a turbulent core. This was the crucial step towards their general correlation (Hollands et al. 1976) for arbitrary inclination angles between 0° and 60° and isothermal boundaries. Since its publication almost forty years ago this correlation has been widely adopted by scientists and engineers. Their correlation expresses the Nusselt number as the sum of three terms.

$$Nu = 1 + Nu_{conv I} + Nu_{conv II} \quad (2)$$

The unity term for heat conduction is always present whereas the convective terms are activated only above their respective critical Rayleigh numbers. The convective terms are functions of the modified Rayleigh number Ra_ϕ which is defined as the product of the Rayleigh number and the cosine of the inclination angle between the horizontal plane and the collector plane.

$$\begin{aligned} Nu_{conv I} &= 1.44 \left[1 - \frac{1708}{Ra_\phi} \right] \left[1 - (\sin(1.8\phi))^{1.6} \frac{1708}{Ra_\phi} \right] ; \quad Ra_\phi \geq 1708 \\ Nu_{conv II} &= \left[\left(\frac{Ra_\phi}{5830} \right)^{1/3} - 1 \right] ; \quad Ra_\phi \geq 5830 \end{aligned} \quad (3)$$

$$Ra_\phi = Ra \cdot \cos \phi = \frac{g \beta s_g^3 \rho_g c_g (T_p - T_c)}{\nu \lambda} \cos \phi \quad (4)$$

The correlation was experimentally validated for inclination angles between 0° and 60° , an aspect ratio of $l_g/s_g = 48$ and Rayleigh numbers up to 10^5 , which is sufficient for collectors with non-selective absorbers. With highly selective coatings, however, the radiative heat transfer is suppressed. This results in higher temperature differences and Rayleigh numbers up to $3 \cdot 10^5$. A correlation which is valid for Rayleigh numbers up to $Ra = 10^6$ and 60° inclination was presented by ElSherbiny et al. (1982).

$$\begin{aligned} Nu_1 &= \left\{ 1 + \left[0.0936 \cdot Ra^{0.314} / (1 + G) \right]^7 \right\}^{1/7} \\ G &= 0.5 / \left[1 + (Ra/3160)^{20.6} \right]^{0.1} \\ Nu_2 &= \left[0.104 + 0.175 / (l_g / s_g) \right] Ra^{0.283} \\ Nu &= \max \{ Nu_1, Nu_2 \} \end{aligned} \quad (5)$$

This correlation will be used in chapter 2.3 to extend the application limit of the correlation by Hollands et al. (1976).

2.2 Heat transfer in gaps with non-isothermal boundaries

Based on the hypothesis that the discrepancies between simulations and test results are caused by disagreement of boundary conditions, a number of experimental and numerical investigations have been undertaken.

Schinkel and Hoogendoorn (1983) kept the cool upper plate isothermal while the lower plate was heated with constant heat flux. They measured an 11 % higher heat loss compared to isothermal conditions. Furthermore, they showed by numerical simulations, albeit for an air gap with an aspect ratio, l_g/s_g , of only 6, that the Nusselt number in the lower part of the air gap is significantly higher than in the upper part. Hellström, Karlsson et al. (1988) pointed out that the cover plate is also non-isothermal. For their experiments they built a model collector with an acrylic cover plate and an absorber made of eight finned tubes running horizontally. The absorber was heated by resistive heating foils attached to the backside of each strip. In one experiment the strips were kept at the same temperature by individual heating. Because of the temperature stratification within the gap, the lowermost strip suffered a 5 times higher heat loss than the uppermost, which is in qualitative agreement with the numerical results of Schinkel and Hoogendoorn (1983).

The first direct measurements of the convective heat transfer under real conditions were carried out by Bartelsen, Janssen and Rockendorf (1993). They used a specially designed flat plate collector with variable gap size. The absorber was made of a number of identical absorber strips of the MTI-type (Rockendorf 2012) which were connected in parallel by headers. Three inclination angles of 30, 45 and 60 degrees were investigated. The experiments were run at their indoor facility which provided excellent stability and repeatability. In general, the heat transfer coefficient was distinctively higher compared to the value predicted by the correlation of Hollands et al. (1976). For an inclination angle of 30° and an average temperature difference of 33 K between absorber and cover plate the effect of flow direction was investigated. The results for horizontal flow and upward flow in slope direction were almost the same for all gap sizes. From this important result it can be concluded that their results are also applicable to upward flow in meander type absorbers. For downward flow the heat loss was increased by more than 10 % compared to upward flow, which is in qualitative agreement with the results of both Hellström et al. (1988) and Schinkel and Hoogendoorn (1983).

An important clue about the effect of the temperature distribution on the heat transfer was given by Chao et al. (1981). They derived average Nusselt numbers for the convective heat transfer in inclined, rectangular enclosures by numerical simulation. For a triangular temperature distribution the heat transfer was higher than for isothermal boundaries. The difference increased with decreasing inclination angle. Although the enclosure had a very low aspect ratio of 2:1:1, these results are in qualitative agreement with the experiments of Bartelsen et al. (1993).

Föste et al. (2011) investigated a flat plate collector with double glazing both experimentally and by simulation. One of their tasks was to characterize the heat transfer between absorber and the first glass pane. Surprisingly, the heat transfer coefficient derived from CFD simulations was close to the value predicted by the correlation of Hollands et al. (1976) while the experimental value was 15-20 % higher. This discrepancy could be explained by a too restricted model region which was defined by the full length and thickness of the

collector and by only the width corresponding to the distance between two neighboring absorber tubes. Thus, the flow inside the gap was not able to develop as it would have in the real gap. Both Krishnamurti (1970a) and Hart (1971a) have shown the selecting effect of lateral boundaries on the possible modes within a certain flow regime.

Error! Reference source not found. shows the heat transfer coefficient measured by Bartelsen et al. (1993) for an inclination angle of 45 degrees and an average temperature difference of 33 K, as derived from figure 2 of their article. Instead of the gap size, the Rayleigh number was chosen as the independent variable. The error bars were added under the assumption that the uncertainty of the average temperatures, on which the

Figure 2 Heat transfer coefficients measured (Bartelsen et al. 1993) and calculated (Hollands et al. 1976) for an inclination angle of 45°.

heat transfer is based, is 0.5 K.

The heat transfer coefficient predicted by the correlation of Hollands et al. (1976) is about 20 – 30 % lower for the same conditions. Furthermore, it shows a local minimum at $Ra_{\phi} \approx 2400$, which marks the end of the conductive regime whereas the measured values of Bartelsen et al. (1993) decrease monotonically with increasing Rayleigh number. At Rayleigh numbers above $Ra = 20\,000$ the measured heat transfer coefficients decrease less than the predicted values.

2.3 Extended correlation for isothermal and non-isothermal boundaries

By replacing the exponent, $1/3$, in the third term of equation (2) by an exponent of 0.39 the correlation of Hollands et al. (1976) closely fits to the correlation of ElSherbiny et al. (1982) for Rayleigh numbers above 10^5 , with a maximum deviation of $+9\%$ around $Ra = 16\,000$ (Figure 3). The deviation from the original correlation of Hollands et al. (1976) reaches its maximum of -7.5% at the application limit $Ra \cdot \cos(60^\circ) = 50\,000$. In the same figure, the extended correlation, equation (6), is evaluated for the non-isothermal case with a correction parameter $R_c = 1$ and a coefficient $C = 0.29$ which are defined in chapters

Figure 3 Nusselt number for 60° inclination calculated by the extended correlation for isothermal boundary conditions ($R_c = 0$) and non-isothermal boundary conditions ($R_c = 1$), compared to the correlations of Hollands et al. (1976) and ElSherbiny et al. (1982).

2.3.1 and 2.3.2 respectively.

Local temperature variations of the absorber plate during normal operation cause density gradients in the air above the absorber plate. Based on the chapters 2.1 and 2.2, three hypotheses are stated.

1. The first critical Rayleigh number, $Ra_{c,I}$, which indicates the limit of the conduction regime decreases with increasing inhomogeneity of the temperature distribution.
2. The second critical Rayleigh number, $Ra_{c,II}$, above which the flow develops boundary layer structure, also decreases with increasing inhomogeneity of the temperature distribution.
3. The local temperature inhomogeneity of a stagnating absorber is irrelevant. Both critical Rayleigh numbers assume the same values as for the isothermal case. The correlation of Hollands et al. (1976), with an exponent of 0.39 instead of $1/3$ in the third term of equation (2) is applicable to stagnating flat plate collectors.

A dimensionless correction parameter, R_c , will be introduced to adapt the correlation for the thermal boundary conditions, reflecting both the state of operation and the thermo-physical properties of the collector. The correction parameter shall assume values between 0 and 1 . For $R_c = 0$ the extended correlation

reduces to the form of Hollands et al. (1976). For $R_c = 1$ both critical Rayleigh numbers are set to zero. A linear relationship between the critical Rayleigh numbers and the correction parameter, R_c , is proposed. A further correction is necessary because, as **Error! Reference source not found.** shows, the measured values above $Ra = 20\,000$ decrease distinctively slower with increasing Rayleigh number than the predicted values. Therefore, the third term, Nu_{convII} , will be multiplied by a term which is bigger than unity in normal operation and which becomes unity for isothermal boundary conditions. This term shall be a linear function of the correction parameter, R_c . The following form of the correlation satisfies the above stated requirements:

$$\begin{aligned} Nu_{cond} &= 1 \\ Nu_{convI} &= 1.44 \left[1 - \frac{1708}{Ra_\phi + 1708R_c} \right] \left[1 - (\sin(1.8\phi))^{1.6} \frac{1708}{Ra_\phi + 1708R_c} \right] \\ Nu_{convII} &= \left[\left(\frac{Ra_\phi + 5830R_c}{5830} \right)^{0.39} - 1 \right] (1 + C \cdot R_c) \end{aligned} \quad (6)$$

2.3.1 Derivation of the correction parameter R_c

The correction parameter was derived from the analytical model of Duffie and Beckmann (1991) which is briefly summarized. The core of this model is the solution of the stationary energy equation of the absorber fin, f , which is indicated in Figure 1. From the global irradiation, G , the part, $S = G\tau_c\alpha_p$, is absorbed by the absorber plate. Applying Fourier's law at the fin base results in the useful gain per unit length in flow direction,

$$\dot{q}_f = F_0 f [S - U_L (T_b - T_a)], \quad (7)$$

with the ambient temperature, T_a , the fin width, f , the temperature at the fin base, T_b , the overall heat loss coefficient, U_L , and the standard fin efficiency, F_0 .

$$F_0 = \frac{\tanh\left(f\sqrt{U_L/\lambda_p s_p}\right)}{f\sqrt{U_L/\lambda_p s_p}} \quad (8)$$

The useful gain per unit length of the whole periodic section, w , of a single plate absorber, i.e. the absorber strip, is expressed by the following equation.

$$\dot{q}_w = F' (2f + b) [S - U_L (T_f - T_a)] \quad (9)$$

The quantity, F' , is the local collector efficiency factor, which takes the bond conductivity and the heat transfer from the tube wall to the fluid into account.

$$F' = \frac{1/U_L}{w \left[\frac{1}{U_L (R_b b + R_e F_0 2f)} + \frac{1}{C_b} + \frac{1}{\alpha_{c,t-f} d_t \pi} \right]} \quad (10)$$

The two correction factors for non-ideal absorbers, R_b and R_e , are taken from Eismann and Prasser (2013). Integration over the whole length of the absorber strip results in the useful gain of the whole collector, related to the inlet temperature, T_{fa} ,

$$\dot{Q}_u = A_p F_R \left[S - U_L (T_{fa} - T_a) \right], \quad (11)$$

with the heat removal factor,

$$F_R = \frac{\dot{m} c_l}{A_p U_L} \left[1 - \exp \left(- \frac{A_p F' U_L}{\dot{m} c_l} \right) \right]. \quad (12)$$

In order to relate the useful gain to the arithmetic average of the inlet and outlet temperature, T_{fav} , the heat removal factor is replaced by the collector efficiency factor, F_{av} .

$$F_{av} = \frac{F_R}{1 - \frac{A_p U_L F_R}{2 \dot{m} c_f}} \quad (13)$$

The following definition of the correction parameter, R_c , satisfies the required characteristics stated above.

$$R_c = 2 \frac{F_R}{F_{av}} - 1 = \exp \left(- \frac{A_p F' U_L}{\dot{m} c_f} \right) \quad (14)$$

The thermo-physical properties of the collector are reflected by the product of absorber area, A_p , local collector efficiency factor, F' , and overall heat loss coefficient, U_L . The state of operation is defined by the product of mass flow rate, $\dot{m}$, and heat capacity of the fluid, c_f . The dependency of the correction parameter on the flow rate is shown in Figure 4 for collector no. 11 (see Table 4 in the appendix).

Figure 4 Correction factor, R_c , for the collector no. 11 (Table 1 in the appendix) as a function of the flow rate per square meter of absorber area.

For “high flow” operation at 40 l/hm^2 , the correction parameter is about 0.9. For “low flow” operation at 10 l/hm^2 the temperature inhomogeneity is less pronounced and the correction parameter is only about 0.7.

2.3.2 Determination of the constant C

While the correction factor, R_c , is completely defined by the analytical model, the constant, C , in the third term of equation (6) has yet to be determined. However, both the material properties and the experimental efficiency characteristics of a solar collector vary within certain tolerances. Modelling of the collectors based on specifications alone would result in an unacceptable large uncertainty. The following sensitivity study shows a way to eliminate the uncertainties of the three leading parameters: The absorptance, α_p , of the absorber, the transmission coefficient, τ_c , of the cover plate and the conversion factor, η_0 , of the empirical model in equation (1) .

For this study the collector characterized by the dataset number 5 (see Table 4 in the appendix) was used. Since all collectors are similarly built, the conclusions drawn from this analysis are also valid for all other collectors. Figure 5 shows the influence of certain changes of test conditions on the efficiency. The dimensional ratio, $x = (T_{fav} - T_a)/G$, characterizes the state of stationary operation. The influence of the irradiation, G , and the ambient temperature, T_a , is considered negligible. The influence of sky temperature, however, is not negligible and has to be accounted for.

Figure 5 Deviation of the efficiency from the nominal value caused by changes in test conditions.

Figure 6 shows the influence of certain material properties on the efficiency. Changes of the bond and plate conductivity have only little influence. The heat flux within the plate, hence the efficiency of the collector, decreases with increasing average fluid temperature. Therefore, the deviation of the efficiency from the nominal value also decreases with increasing average fluid temperature.

Figure 6 Deviation of the efficiency from the nominal value caused by changes of material properties.

An increase of the emissivity of 0.01 has approximately the same effect as a 15 % higher conductivity of the bottom insulation. Their influence on the efficiency is approximately proportional to the difference between ambient and average fluid temperatures. The uncertainties of these four properties cannot be reduced. On the other hand, the absorptance, α_p , of the absorber coating and the transmission coefficient, τ_c , of the cover plate influence only the conversion factor, η_0 , and have no effect on loss terms.

It is therefore possible to eliminate the uncertainties of those three properties by the following iterative procedure. Data sets of 22 collectors were created, based on the test results, their specific test conditions and the specifications as provided by the manufacturers. These collectors were then simulated using the flat-plate collector model of Duffie and Beckman (1991), with the extended correlation for the convective heat transfer across the air gap, equation (6), the correlation for the edge effect of non-ideal absorbers (Eismann and Prasser 2013) and further correlations presented in Eismann (2015). For all collectors, an initial value for $C = 0.5$ was assumed. In a first procedure, the absorptance of the absorber and the transmission coefficient of the glass plate was adjusted until the calculated conversion factor differed no more than 0.0005 from the value given by the test report. The absorptance was adjusted within the tolerance of ± 0.01 , as listed in the data sheet. If necessary, the transmission coefficient was adjusted, too. All other values were kept as they are. After this procedure the difference between the simulated and experimentally determined efficiencies at $x = (T_{fav} - T_a)/G = 0.05$ and $0.1 \text{ Km}^2/\text{W}$ were calculated.

$$\Delta\eta = \left(\frac{\eta - \eta_{\text{exp}}}{\eta_{\text{exp}}} \right)_{x=0.05} + \left(\frac{\eta - \eta_{\text{exp}}}{\eta_{\text{exp}}} \right)_{x=0.1} \quad (15)$$

A new value of the constant, C , was chosen until the condition $|\Delta\eta| < 0.002$ was met. Since a change of the constant, C , slightly influences the conversion factor, both steps were repeated in each iteration.

3 Results and discussion

3.1 Convective heat transfer across the air gap

3.1.1 Constant C

Figure 7 shows the constant C in the third term of the extended correlation (6), $\text{Nu}_{\text{conv},II}$, for each collector. The difference between indoor and outdoor tests is too small to draw any conclusions. The scatter is mainly due to the uncertainties of the experiment, the heat conductivity of the bottom insulation and the emissivity

Figure 7 Constant, C , for the third term, $\text{Nu}_{\text{conv},II}$, of the extended correlation for convective heat transfer across the air gap of flat-plate collectors.

of the absorber coating.

The results for the collectors no. 19 – 22 clearly deviate from the general trend. According to the information from the manufacturer, these were prototypes built with a sample batch of a new type of bottom insulation. There are three possible reasons for the deviation. A) A batch of insulation as used in collector no. 5 was sent by supplier of the insulation by mistake. This insulation has a 13 % lower conductivity which would result in values of, C , indicated by the dashed markers. B) The emissivity of all 4 absorber was not 0.043 but by fortune only 0.02. However, this is rather unlikely, because only two of the absorbers were prototypes

whereas the other two were serial products taken from stock. C) The deviation results from uncertainties in the measurement.

Discarding these four data points results in an average value of $C = 0.29$ and a mean deviation of $\delta_C = 0.13$ from the average. Replacing the data of the bottom insulation with the “supposedly correct” data would result in almost the same result $C = 0.28 \pm 0.13$ over all 22 data sets. Accepting the results without correction would lead to an average of $C = 0.23 \pm 0.17$. It is suggested to use the average value of the data sets 1 – 18 of $C = 0.29$, but the uncertainty of the whole data set of $\delta_C = \pm 0.17$. The influence of the uncertainty, δ_C , on the the efficiency depends not only on the state of operation but also on the ratio of top loss coefficient to the overall heat loss coefficient which is different for all collectors. Figure 8 shows the difference of the efficiency related to the efficiency calculated for their individual constant, C_0 , for collectors no. 5 and 11.

Since the average of the constant, C , is based on simulations of 22 individual collectors the uncertainty of the constant, C , is represented by the mean deviation, $\delta_C = n^{-1} \sum |C_k - \langle C \rangle|$, rather than the standard deviation.

Figure 8 Influence of the uncertainty on the constant, C , on the efficiency for two collectors no. 5 and no. 11.

3.1.2 Heat transfer coefficient

Figure 9 shows the heat transfer coefficient for an inclination angle of 45° for different values of the correction parameter, R_c . The thick curve is the heat transfer coefficient predicted by the extended correlation (6) and a correction parameter of $R_c = 0$. With increasing correction parameter, the heat transfer coefficient increases while its minimum in the region of $Ra_{c,l} = 1708/\cos\phi$ becomes less pronounced. For two reasons, the experimental values of Bartelsen et al. (1993) are still higher than those predicted by the extended correlation. 1) Gaps between the edges of neighboring absorber strips, which are never perfectly straight, cause additional heat loss (Fischer 2011, p.20). 2) In contrast to tube-and-sheet absorbers MTI absorber strips have a symmetrical cross section. Therefore, the gap width above the absorber tube is reduced by half the

Figure 9 Heat transfer coefficient for 45° inclination and different correction parameters, R_c . diameter of the absorber tube.

3.2 Uncertainties of analytical and empirical model

3.2.1 Efficiency

The efficiencies of the collectors 1 – 18 are calculated by the analytical model using both the original correlation of Hollands et al. (1976) and the extended correlation, equation (6). The average of the relative differences between analytical and empirical efficiencies is presented in Figure 10. The analytical model using the correlation of Hollands et al. (1976) overestimates the efficiency at $x = (T_{fav} - T_a)/G = 0.1 \text{ Km}^2/\text{W}$ by about 18 %. The analytical model using the extended correlation predicts the efficiency within a relative uncertainty of about 6 % at $x = 0.1 \text{ Km}^2/\text{W}$. Assuming a global uncertainty of the measured efficiency of $\delta\eta_{\text{exp}} = 3 \%$, the uncertainty of the experimental efficiency related to the averages of the measured efficiencies

at $(T_{fav}-T_a)/G = 0.1 \text{ Km}^2/\text{W}$ is about 9 %. Thus, the uncertainty of the extended analytical model lies well within the uncertainty of the empirical model.

Figure 10 Left axis: Average relative deviation of the simulated efficiencies from the average of the experimental values for collectors no. 1 – 18. Right axis: Average relative uncertainty of the experimental efficiency value.

The curvature of the relative differences between analytical and empirical efficiency is positive. This seems to indicate a slightly stronger growth of the heat losses than predicted by the model. However, neither the present model nor the limited number of collectors allow a further discussion of this very small effect.

3.2.2 Stagnation temperature

In the course of a full collector test the stagnation temperature is measured at a certain point on the backside of the absorber plate. The position of the sensor is defined by 1/3 of the absorber dimension in slope direction below its upper edge and by the centerline between its left and right edge. There is no literature available on the temperature distribution of stagnating absorbers except the work of Föste et al. (2011) on a collector with double glazing. Because of the heat advected by the base flow the average temperature in the upper half of the absorber will be higher than in the lower half. Furthermore, the temperature of the absorber will decrease towards the edges because of the heat losses across the side walls of the collector. Based on these qualitative considerations, the measured temperature can be roughly interpreted as the maximum absorber temperature at 45° inclination, which is in agreement with temperature distributions presented by Föste et al. (2011, diag. 4.2, p. 91): For 45° inclination, 1000 W/Km² global irradiation of and 30 °C ambient temperature the maximum absorber temperature of their absorber was about 222 °C whereas the average temperature was

about 13 K lower. For collectors with single glazing a smaller difference between maximum and average temperature has to be expected because the top loss coefficient is higher.

For 10 of the 22 collectors the stagnation temperature was measured. The stagnation temperature at individual test conditions was calculated by both the empirical model, equation (1), and the analytical model utilizing equation (6). The results are shown in Figure 11. The average stagnation temperature calculated by the analytical model lies about 5 °C below the measured maximum value, which is reasonable for collectors with single glazing. The large spread of the measured values reflects the uncertainties of the temperature measurement itself and the local change of gap size due to local thermally induced deformations. On the other hand, the empirical model underestimates the average stagnation temperatures by about 20 K. This follows from the the empirical model itself which implicitly reflects the non-isothermal boundary conditions

Figure 11 Comparison between measured (maximum) and calculated (average) stagnation temperature of 10 different collectors.

and therefore higher convective heat losses in normal operation.

4 Conclusions

For the cost- and efficiency optimization of flat-plate collectors an accurate analytical model was developed. The model is particularly useful in an early stage of the developing process where changes in the collector design are still inexpensive. The uncertainty of the efficiency and the average stagnation temperature lie well within the uncertainty of the empirical model. The model is based on the model of Duffie and Beckman (1991), enhanced by improved and new correlations for the following effects:

The correlation of Hollands et al. (1976) for the convective heat transfer across air gaps with isothermal boundaries, valid for inclination angles between 0° and 60° and Rayleigh numbers up to $Ra = 10^5$, was extended in two ways. 1) By fitting to the correlation of ElSherbiny et al. (1982) the application limit was

increased up to $Ra = 10^6$ which covers the range of Rayleigh numbers in the air gap above highly selective absorbers. 2) The extended correlation was adapted for inhomogeneous temperature distributions of tube- and sheet absorbers by means of a correction parameter, R_c , and a constant, C . The correction parameter, R_c , which reflects both the thermo-physical properties of the collector and its state of operation, was analytically derived from the collector model of Duffie and Beckman (1991). The constant; C , was determined during the validation process.

Based on a sensitivity study the uncertainties of the three leading parameters were eliminated: The transmission coefficient, τ_c , of the cover plate, the absorptance, α_p , of the absorber coating and the conversion factor, η_0 , of the empirical model. Thereby the uncertainty of the constant, C , was considerably reduced.

Further correlations by Eismann (2015) were implemented: Heat bridges caused by absorber tubes, influence of thermo-mechanical strain on gap size and thickness of bottom insulation insulation as well as temperature dependency of the absorber emissivity.

The collector model was validated against empirical data of 22 flat plate collectors which were tested according to the European standard EN 12975-2 (CEN 2006). The uncertainty of the enhanced collector model lies well within the uncertainty of empirical test results.

Irreducible uncertainties of the material properties, the measurements and the test conditions result in a corresponding uncertainty of the constant, C , of the extended correlation. For a more accurate estimation, dedicated experiments are necessary.

Table 1 Dimensions, thermo-physical properties, efficiency characteristics, and stagnation temperature of 22 collectors used in the analysis.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	1
m ²	2.284	2.284	2.330	2.330	2.330	2.310	2.500	2.500	2.500	2.329	2.329	1.838	1.838	1.838	2.534	2.53
m	6.620	6.620	6.652	6.652	6.652	6.564	7.010	7.010	7.010	6.874	6.874	5.954	5.954	5.954	7.140	7.14
m	0.103	0.103	0.1	0.1	0.1	0.097	0.1	0.1	0.1	0.07	0.07	0.115	0.115	0.115	0.115	0.11
m	2.084	1.142	2.092	1.212	1.212	1.212	1.130	1.130	1.130	1.130	1.130	1.200	1.200	1.200	1.200	2.37
m	1.142	2.084	1.212	2.006	2.006	2.006	2.500	2.500	2.500	2.500	2.500	1.777	1.777	1.777	2.370	1.20
m	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.030	0.030	0.060	0.060	0.060	0.060	0.06
m	0.0539	0.0527	0.0492	0.0489	0.0489	0.0512	0.0518	0.0518	0.0518	0.0296	0.0300	0.0595	0.0595	0.0595	0.0595	0.058
cm	3.10E-02	3.10E-02	3.14E-02	3.14E-02	3.14E-02	3.14E-02	3.54E-02	3.54E-02	3.54E-02	3.54E-02	3.54E-02	3.71E-02	3.71E-02	3.71E-02	3.71E-02	3.71E-02
cm	1.37E-04	1.37E-04	1.40E-04	1.40E-04	1.40E-04	1.40E-04	1.76E-04	1.76E-04	1.76E-04	1.76E-04	1.76E-04	1.79E-04	1.79E-04	1.79E-04	1.79E-04	1.79E-04
cm	3.65E-07	3.65E-07	3.90E-07	3.90E-07	3.90E-07	6.45E-07	6.45E-07	6.45E-07	6.45E-07	6.45E-07	6.45E-07	7.69E-07	7.69E-07	7.69E-07	7.69E-07	7.69E-07
m	0.01	0.01	0.0433	0.0433	0.0433	0.0393	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
m	0.0363	0.0363					0.0245	0.0245	0.0245	0.0195	0.0195	0.0296	0.0296	0.0296	0.0296	0.029
m	0.030	0.030					0.010	0.010	0.010	0.010	0.010	0.020	0.020	0.020	0.020	0.02
cm	3.08E-02	3.08E-02					2.99E-02	2.99E-02	2.99E-02	2.99E-02	2.99E-02	3.08E-02	3.08E-02	3.08E-02	3.08E-02	3.08E-02
cm	1.27E-04	1.27E-04					2.05E-04	2.05E-04	2.05E-04	2.05E-04	2.05E-04	1.27E-04	1.27E-04	1.27E-04	1.27E-04	1.27E-04
cm	3.36E-07	3.36E-07					5.71E-08	5.71E-08	5.71E-08	5.71E-08	5.71E-08	3.36E-07	3.36E-07	3.36E-07	3.36E-07	3.36E-07
-	-	0.04	0.04	0.95	0.95	0.04	0.95	0.95	0.95	0.95	0.95	0.04	0.04	0.04	0.04	0.0
-	-	0.25	0.25	0.04	0.04	0.04	0.25	0.25	0.25	0.25	0.25	0.04	0.04	0.04	0.04	0.0
-	-	0.04055	0.04055	0.04005	0.04005	0.0358	0.02975	0.02975	0.02975	0.027125	0.027125	0.0358	0.0358	0.0358	0.0358	0.035
m2	2.284	2.284	2.303	2.303	2.303	2.303	2.500	2.500	2.500	2.319	2.324	1.819	1.819	1.819	2.512	2.51
-	-	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.950	0.95
-	-	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95
-	-	0.043	0.043	0.043	0.043	0.043	0.043	0.043	0.043	0.088	0.088	0.043	0.043	0.043	0.043	0.04
-	-	0.0005	0.0005	0.0005	0.0005	0.0005	0.00025	0.00025	0.00021	0.00025	0.0003	0.00018	0.00018	0.00018	0.00018	0.0001
cm	210	210	210	210	210	210	385	385	385	385	210	385	385	385	385	38
cm	0.088	0.087	0.091	0.091	0.091	0.100	0.104	0.104	0.104	0.116	0.145	0.091	0.091	0.091	0.091	0.09
m	0.0030	0.0030	0.0040	0.0040	0.0040	0.0030	0.0040	0.0040	0.0030	0.0040	0.0040	0.0030	0.0030	0.0030	0.0030	0.003
m	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.001
cm	20	20	15	15	15	20	15	15	15	15	15	20	20	20	20	2
-	-	0.0070	0.0070	0.0110	0.0110	0.0072	0.0092	0.0092	0.0092	0.0082	0.0082	0.0090	0.0090	0.0090	0.0090	0.009
-	-	0.07	0.052	0.087	0.08	0.055	0.03	0.03	0.03	0.03	0.03	0.064	0.064	0.064	0.064	0.06
-	-	0.910	0.910	0.910	0.910	0.910	0.910	0.910	0.910	0.910	0.910	0.910	0.910	0.910	0.910	0.91
m ²	800	800	800	800	800	800	800	800	800	800	806	877	870	870	870	871
°C	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	25.5	25.7	25.9	23.5	23.5	23.5	23.5
°C	Goforth	Goforth	Goforth	Goforth	Goforth	Goforth	Goforth	Goforth	Goforth	22.6	23.8	23.8	20.7	20.7	20.7	20.7
n/s	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.4	3.5	3.4	3.0	3.0	3.0	3.4
h	92	92	72	166	72	72	125	37.5	100	100	100	153.34681	153.34681	160.0406	202.22306	220.6091
%	0.33	0.33	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-	-	0.805	0.827	0.797	0.811	0.802	0.814	0.798	0.770	0.811	0.779	0.795	0.797	0.802	0.780	0.79
K	3.600	4.090	3.500	3.950	3.601	3.527	3.757	3.478	3.650	4.160	4.140	3.342	3.354	3.200	3.241	3.50
K ²	0.0070	0.0055	0.0160	0.0061	0.0140	0.0120	0.0147	0.0154	0.0161	0.0145	0.0108	0.0160	0.0160	0.0150	0.0140	0.012
m ²	outdoor	outdoor	outdoor	outdoor	outdoor	outdoor	outdoor	outdoor	outdoor	indoor	indoor	indoor	indoor	indoor	indoor	indoor
m ²	1000	1031	1084	1063	1084	1063	1084	1063	988	811	871					851
°C	30	24	27	27	27	27	27	27	32.9	31	30.9					2
°C	Goforth	Goforth	Goforth	Goforth	Goforth	Goforth	Goforth	Goforth	22.6	23.8	23.8					2
°C	209	212	212	212	212	203	211	170	183.6							17

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